Axions in a highly protected gauge symmetry model

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based on arXiv:1804.01112
in collaboration with E. Dudas and **S. Pokorski**

Centre de Physique Théorique - École Polytechnique

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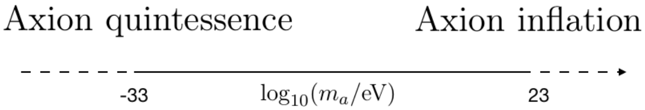
Outline

Motivation and model

Mass range in axion models:



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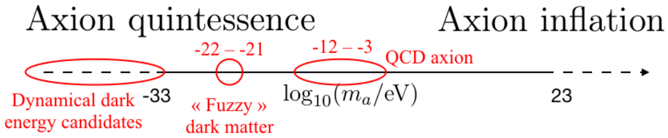

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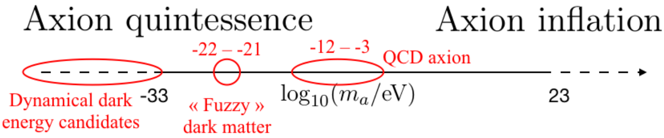

Small masses: **need for a controlled explicit breaking**

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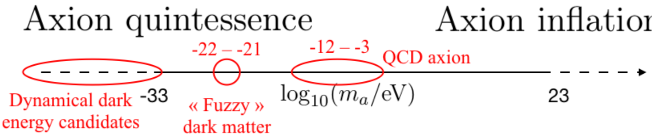

Small masses: **need for a controlled explicit breaking**

However: **global symmetries expected to be broken by quantum gravity effects**

Hawking (1987), Giddings & Strominger (1988), Banks & Seiberg (2010)

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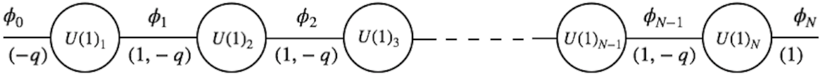
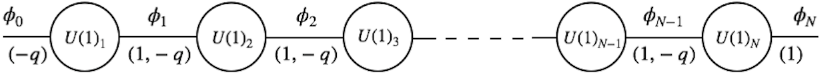


$$\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{N} F_{\mu\nu,i}F_i^{\mu\nu} - \sum_{k=0}^{N} |D_\mu \phi_k|^2 - V(|\phi_0|^2, |\phi_1|^2, \ldots)$$

Ahmed & Dillon (2017), Coy Frigerio & Ibe (2017), Choi Im & Shin (2017), within discussions about the **clockwork mechanism**: Choi & Im (2016), Kaplan & Rattazzi (2016), Giudice & McCullough (2017)



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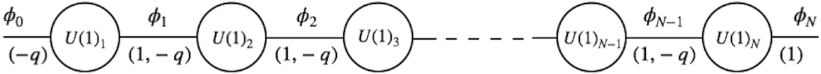

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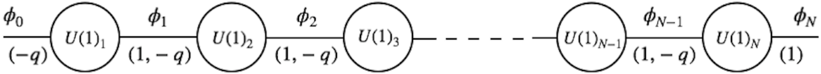

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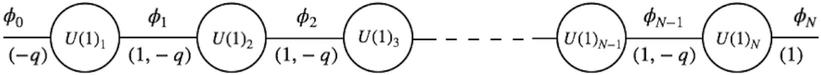

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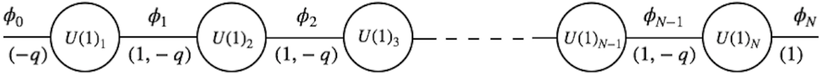
Can **protect a QCD axion** or explain **very-low DM masses**



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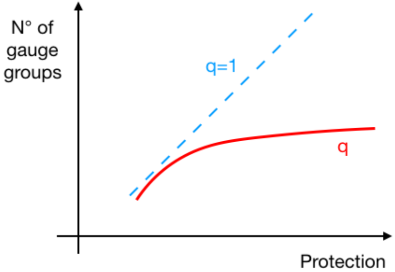
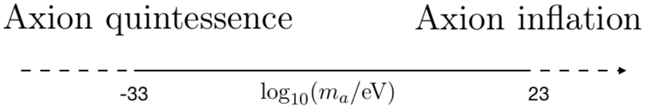


Couplings and scales in the EFT

Phenomenology of axion models: characterized by **axion mass**



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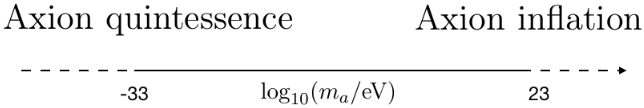

and **couplings to SM fields**

$$\mathcal{L} \supset \frac{g_{a\gamma\gamma}}{f_a} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{i g_{a,\mathrm{EDM}}}{f_a} a \overline{N}\gamma_{\mu\nu}\gamma^5 N F^{\mu\nu} + \frac{g_{aNN}}{f_a}\partial_\mu a \overline{N}\gamma^\mu\gamma^5 N + \frac{g_{aee}}{f_a}\partial_\mu a \bar{e}\gamma^\mu\gamma^5 e$$

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KSVZ model of invisible QCD axion: anomalous set of fermions

$$\mathcal{L} \supset \phi \overline{Q_L} Q_R + h.c. \xrightarrow{Q \text{ triangle loop}} \frac{a}{f} F \tilde{F}$$

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General feature:

$$\mathcal{L} \supset -\sum_i (\mathcal{O}_i \overline{\psi_{i,L}} \psi_{i,R}) \xrightarrow{\text{triangle loops}} \frac{i}{32\pi^2} \log\left(\prod_i \mathcal{O}_i\right) F\tilde{F}$$

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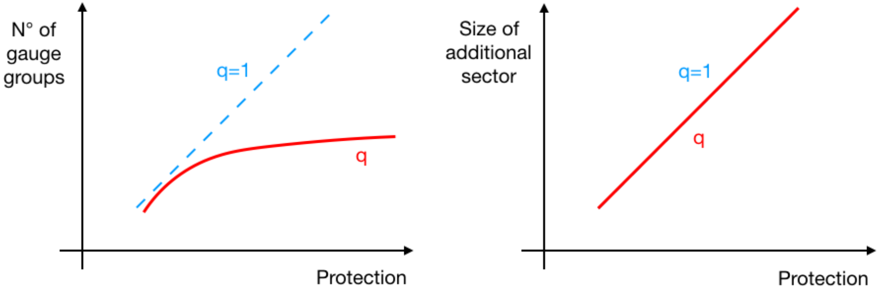

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Example: coupling to the first SM generation

$$\mathcal{L} \supset -\frac{1}{M_P}\left(\overline{u_R} H\phi_i Y_u Q_L + \overline{d_R}(H\phi_i)^* Y_d Q_L + \overline{e_R}(H\phi_i)^* Y_e L_L\right) + h.c.$$

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Site-dependent coupling to the spins derived in minimal setup

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Scales: $f \qquad \frac{f_a}{g_{a\gamma\gamma}} = \frac{f}{\sqrt{1+q^2+\ldots+q^{2N}}} \qquad \frac{f_a}{g_{aee}} = \frac{\sqrt{1+q^2+\ldots+q^{2N}}\,f}{q^i}$

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Symmetry-preserving operators:

- can be present in minimal setups
- are site-localized and display clockwork properties of the theory

Application: ALP DM detection

Detection with NMR (with $\frac{\partial_\mu a}{f_a}\overline{N}\gamma^\mu\gamma^5 N$ obtained in minimal setup):

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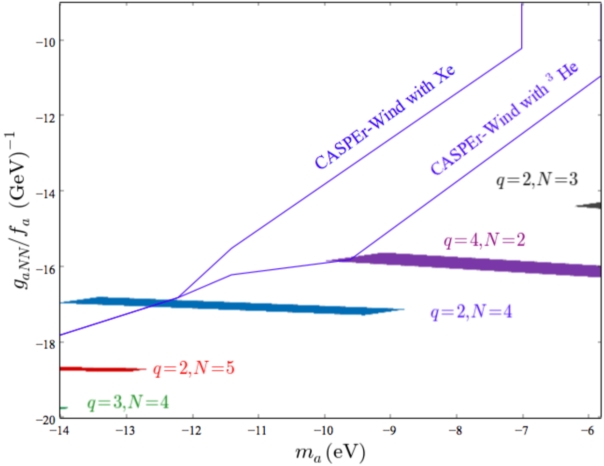
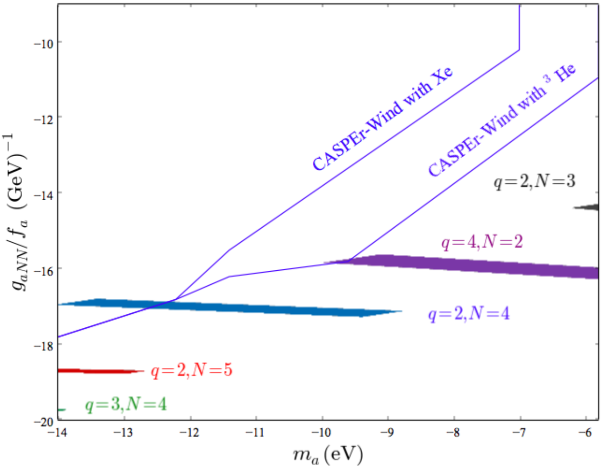
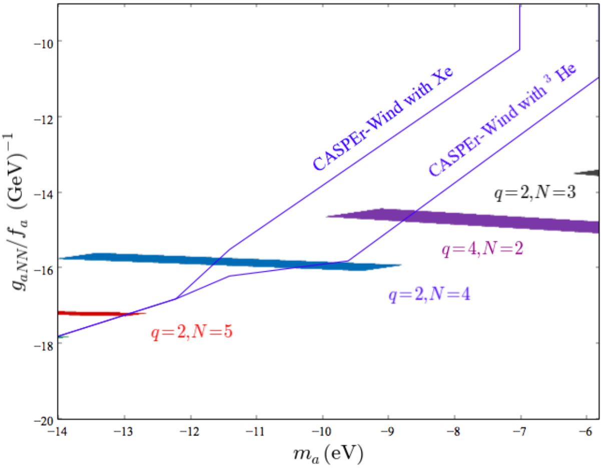


Detection with NMR (with $\frac{\partial_\mu a}{f_a}\overline{N}\gamma^\mu\gamma^5 N$ obtained in minimal setup): Coupled at site 0



Detection with NMR (with $\frac{\partial_\mu a}{f_a}\overline{N}\gamma^\mu\gamma^5 N$ obtained in minimal setup): Coupled at site N



Conclusions

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All usual axion couplings can be generated.
Symmetry-preserving couplings are generated in minimal setups and display site-dependence, whereas anomalous couplings (such as those of a QCD axion) require the use of an extended fermion sector populating every site of the quiver.

In the minimal setup, the (unavoidable) **gravity contribution is sufficient to provide the correct DM density**.
NMR-based searches, sensitive to clockwork effects, can detect such a particle.

Thank you!

Backups

KSVZ model with:

$$\mathcal{L}_{PQ} \supset \phi \overline{Q_L} Q_R + h.c. - V(|\phi|^2),\ \phi \xrightarrow{U(1)_{PQ}} e^{i\alpha}\phi \text{ and } \phi = \frac{f+r}{\sqrt{2}} e^{i\frac{a}{f}}$$

Then:

$$\text{QCD anom.} + \text{instantons} \to \mathcal{L} \supset m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u+m_d} \cos\left(\frac{a}{f} - \theta_{\rm QCD}\right)$$

Possible correction:

$$\mathcal{L}_{\not{PQ}} \supset \frac{\phi^n}{M_P^{n-4}} + h.c.$$

$\to$ **destabilizes $\boldsymbol{\theta < 10^{-10}}$ if $\boldsymbol{n < 10}$** (if $f \gtrsim 10^9$ GeV).
Indeed:

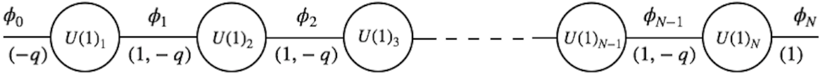
$$\frac{\phi^n}{M_P^{n-4}} \text{ term} \to \mathcal{L} \supset \left(\frac{f}{\sqrt{2}M_P}\right)^n M_P^4 \cos\left(\frac{na}{f}\right)$$

$U(1)_{PQ}$ protection: Barr and Seckel (1992)

Fields	ϕ_1	ϕ_2	$Q_L^{i=1...q}$	$\widetilde{Q}_L^{i=1...p}$	$Q_R^{i=1...p+q}$
$SU(3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$U(1)$	p	q	p	$-q$	0
$U(1)_{PQ}$	q	$-p$	q	p	0

where $\gcd(p,q)=1$ and $p+q\geq 10$

$$\mathcal{L} \supset \underbrace{\phi_1 \overline{Q_L} Y Q_R + \phi_2^* \overline{\widetilde{Q}_L} \widetilde{Y} Q_R}_{\mathcal{L}_{PQ}} + \underbrace{\frac{\phi_1^q \phi_2^{*p}}{M_P^{p+q-4}}}_{\mathcal{L}_{\not{PQ}}} + h.c.$$

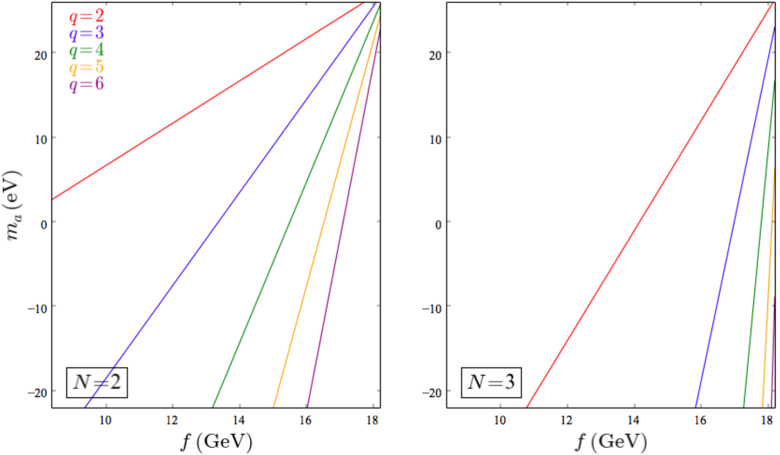


$$\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{N} F_{\mu\nu,i}F_i^{\mu\nu} - \sum_{k=0}^{N} |D_\mu \phi_k|^2 - V(|\phi_0|^2, |\phi_1|^2, \ldots)$$

Gravitational breaking:

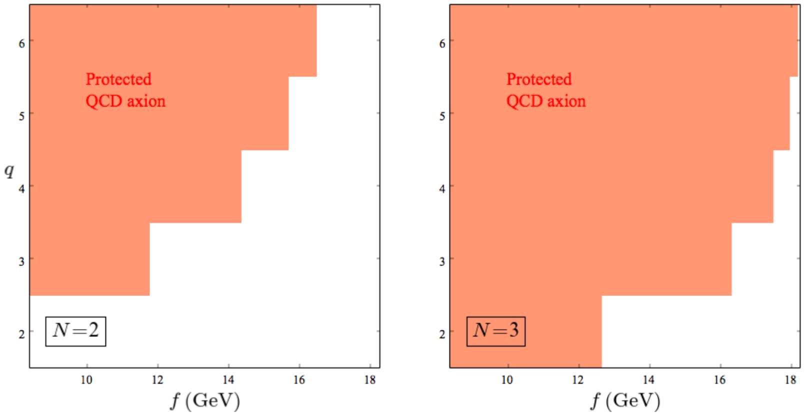
$$\mathcal{L} \supset \frac{\phi_0 \phi_1^q \ldots \phi_N^{q^N}}{M_P^{1+\ldots-4}} \rightarrow \boxed{m_a^{(\mathrm{grav})} = \left(\frac{f}{\sqrt{2}M_P}\right)^{\frac{q+\ldots+q^N-1}{2}} \sqrt{1+q^2+\ldots+q^{2N}}\, M_P}$$

Mass suppression with **few additional gauge groups**:



Application to QCD axion:

$\theta_{\rm QCD} < 10^{-10}$ if $m_a^{\rm (QCD)} > 10^5 m_a^{\rm (grav)}$



For an ALP dark matter candidate:

Masses can be **perturbative or non-perturbative**

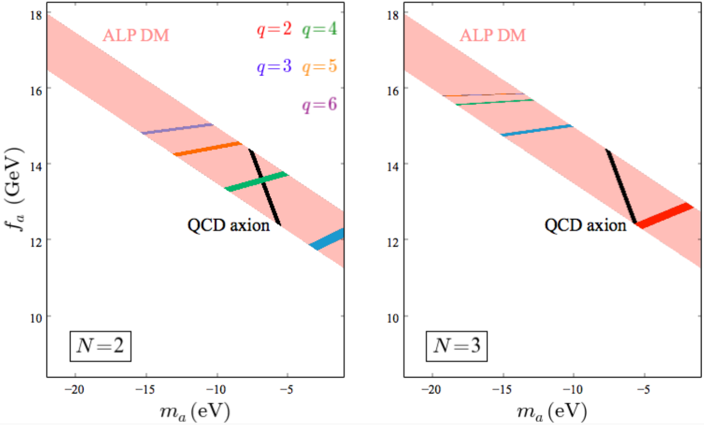
Focus here on **gravitational origin** and on **misalignment mechanism** (with pre-inflationnary breaking):

$$V = -\frac{\phi_0 \phi_1^q \ldots \phi_N^{q^N}}{M_P^{1+q+\ldots+q^N-4}} \supset -\left(\frac{f}{M_P}\right)^{1+q+\ldots q^N} M_P^4 \cos\left(\frac{a}{f_a}\right)$$

and

$$< a_{\text{init}} > = \text{random}$$

$\Omega_a h^2 = 0.12$ when:



Stability of the DM ALP's?

No anomaly: **no ALP-photon conversion** via usual $\mathcal{L} \supset \frac{a}{f_a} F\tilde{F}$

Instead: **derivative interactions** + tiny mass → **long lifetime**

Example: coupling to a heavy anomaly-free set of electrically charged fermions:

$$\mathcal{L} \supset y_1 \phi_i \overline{\psi_{R,1}} \psi_{L,1} + y_2 \phi_i \overline{\psi_{L,2}} \psi_{R,2} + h.c. \ .$$

$$\xrightarrow{\text{fermions integr.}} \mathcal{L}_{eff} \supset \frac{e^2}{48\pi^2 q^i f} \left(\frac{1}{m_1^2} - \frac{1}{m_2^2} \right) \left(\Box a F\tilde{F} - \frac{1}{2} \partial_\mu a F_{\nu\eta} \partial^\eta \tilde{F}^{\mu\nu} \right)$$

Lifetimes for the FDM: $\sim 10^{300}$s