

Froggatt-Nielsen mechanism in a model with 331-gauge group

Niko Koivunen

University of Helsinki



K. Huitu and N. Koivunen, arXiv:1706.09463 [hep-ph]

Planck, Bonn, 23.5.2018

The Flavour Problem

Three fermion families. Why?

⇒ 331 -models have the answer!

The origin of fermion mass hierarchy?

$$\left\{ \begin{array}{l} y_e = 2.94 \times 10^{-6} \\ y_\mu = 6.07 \times 10^{-4} \\ y_\tau = 1.02 \times 10^{-2} \end{array} \right. \quad \left\{ \begin{array}{l} y_u = 1,32 \times 10^{-5} \\ y_c = 7.33 \times 10^{-3} \\ y_t = 1 \end{array} \right. \quad \left\{ \begin{array}{l} y_d = 2.76 \times 10^{-5} \\ y_s = 5.46 \times 10^{-4} \\ y_b = 2.4 \times 10^{-2} \end{array} \right.$$

⇒ Froggatt-Nielsen mechanism is the answer!

Froggatt-Nielsen mechanism

- Introduce new gauge singlet scalar, ϕ , called the **flavon**, and global $U(1)_{FN}$ symmetry.
- SM fermions, the Higgs and Flavon are charged under the $U(1)_{FN}$. Charges are such that SM Yukawa couplings are forbidden.
- Effective operator is allowed:

$$c_{ij} \left(\frac{\phi}{\Lambda} \right)^{n_{ij}} \bar{f}_{L,i} H f_{R,j} + h.c. \rightarrow \underbrace{c_{ij} \left(\frac{v\phi}{\sqrt{2}\Lambda} \right)^{n_{ij}}}_{y_{ij}^f} \bar{f}_{L,i} H f_{R,j} + \dots$$

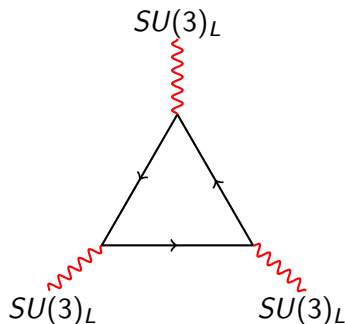
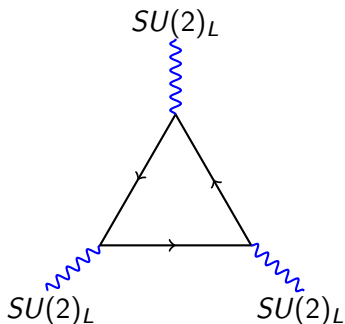
- Yukawa hierarchy determined by the charge assignment

$$n_{ij} = -\frac{1}{q(\phi)} [q(\bar{f}_{L,i}) + q(f_{R,i}) + q(H)]$$

The 331 – gauge extension

$$\text{Standard model} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\text{331 – model} = SU(3)_C \times SU(3)_L \times U(1)_X$$



Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets. Possible only with 3 generations!

331-model

- Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets. Possible only with 3 generations!
- One quark family must be in different representation than the others (due to anomaly cancellation)
⇒ flavour changing neutral current at tree-level!
- Freedom in electric charge definition: $Q = T_3 + \beta T_8 + X$. Two types of models: $\beta = \pm\sqrt{3}$ and $\beta = \pm\frac{1}{\sqrt{3}}$.

The Model

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X$$

Fermion representations:

$$\text{Triplets: } L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ \textcolor{blue}{N}_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ \textcolor{green}{U} \end{pmatrix}_L \sim (3, 3, \frac{1}{3})$$

$$\text{Antitriplets: } Q_{L,2} = \begin{pmatrix} s \\ c \\ \textcolor{red}{D}_1 \end{pmatrix}_L \sim (3, 3^*, 0), \quad Q_{L,3} = \begin{pmatrix} b \\ t \\ \textcolor{red}{D}_2 \end{pmatrix}_L \sim (3, 3^*, 0)$$

$\textcolor{blue}{N}_{L,i}$ = new neutrino-like fields, $\textcolor{green}{U}$ = new up-type quark, $\textcolor{red}{D}_1, \textcolor{red}{D}_2$ = new down-type quarks.

- 6 triplets and 6 antitriplets $\Rightarrow$ Gauge anomalies cancel!
- **Generic feature of 331-models:** Quark generations in different representations $\Rightarrow$ Flavour Changing Neutral Currents!

Particle Content

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -\frac{1}{3})$$

The most general vacuum is:

$$\langle \eta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathbf{v}' \\ 0 \end{pmatrix}, \quad \langle \rho \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{v}_1 \\ 0 \\ \mathbf{v}_2 \end{pmatrix}, \quad \langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ \mathbf{u} \end{pmatrix}$$

The symmetry breaking pattern is:

$$SU(3)_L \times U(1)_X \xrightarrow{\mathbf{u}, \mathbf{v}_2} SU(2)_L \times U(1)_Y \xrightarrow{\mathbf{v}_1, \mathbf{v}'} \times U(1)_{em}, \quad (1)$$

where $\mathbf{v}_2, \mathbf{u} \gtrsim 1\text{TeV}$ and $\mathbf{v}_1, \mathbf{v}' = \mathcal{O}(100\text{GeV})$

Minimal scalar sector admits Froggatt-Nielsen mechanism!

$$\rho = \begin{pmatrix} \rho^0 + \langle \rho^0 \rangle \\ \rho^- \\ \rho'^0 + \langle \rho'^0 \rangle \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 + \langle \chi'^0 \rangle \end{pmatrix} \sim (1, 3, -\frac{1}{3}).$$

- The combination $\rho^\dagger \chi$ is a gauge singlet.
- It can carry a non-zero $U(1)$ charge.
- $\rho^\dagger \chi$ has non-zero VEV:

$$\langle \rho^\dagger \chi \rangle = \frac{v_2 u}{2} \neq 0$$

The combination $\rho^\dagger \chi$ can play the role of the flavon in the Froggatt-Nielsen mechanism!

331 Froggatt-Nielsen mechanism

Froggatt-Nielsen mechanism works the usual way:

$$(c_s^f)_{ij} \left(\frac{\rho^\dagger \chi}{\Lambda^2} \right)^{(n_f^s)_{ij}} \bar{\psi}_{L,i}^f S f_{R,j} \rightarrow \underbrace{(c_s^f)_{ij} \left(\frac{v_2 U}{2\Lambda^2} \right)^{\epsilon (n_f^s)_{ij}}}_{(y_s^f)_{ij}} \bar{\psi}_{L,i}^f (S + \langle S \rangle) f_{R,j} + \dots$$

The minimal scalar content of the 331-models with $\beta = \pm \frac{1}{\sqrt{3}}$ is enough for Froggatt-Nielsen mechanism. **No new scalars required!**

Models with $\beta = \pm \frac{1}{\sqrt{3}}$ thus explain both the number of fermion families and their hierarchy simultaneously!

Up-type quark mass matrix

$$m_u = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 y_{11}^u & v_1 y_{12}^u & v_1 y_{13}^u & v_1 y_{14}^u \\ -v' y_{21}^u & -v' y_{22}^u & -v' y_{23}^u & -v' y_{24}^u \\ -v' y_{31}^u & -v' y_{32}^u & -v' y_{33}^u & -v' y_{34}^u \\ v_2 y_{11}^u + u y_{11}^u & v_2 y_{12}^u + u y_{12}^u & v_2 y_{13}^u + u y_{13}^u & v_2 y_{14}^u + u y_{14}^u \end{pmatrix}$$

Two sources of hierarchy:

- Usual Froggatt-Nielsen hierarchy:

$$y_{ij} = c_{ij} \left(\frac{v_2 u}{2\Lambda^2} \right)^{n_{ij}} = c_{ij} \epsilon^{n_{ij}}, \quad c_{ij} = \mathcal{O}(1) \quad \text{and} \quad \epsilon < 1$$

- VEVs of two scales: $v_2, u \gtrsim 1\text{TeV}$ and $v_1, v' = \mathcal{O}(100\text{GeV})$

Higgs mediated FCNCs at tree-level

Up-type quark Yukawa couplings:

$$\mathcal{L}_{up} = \sum_{\gamma=1}^4 (y_{\rho}^u)_{1\gamma} \bar{Q}'_{L,1} \rho u'_{R,\gamma} + \sum_{\gamma=1}^4 (y_{\chi}^u)_{1\gamma} \bar{Q}'_{L,1} \chi u'_{R,\gamma} + \sum_{\alpha=2}^3 \sum_{\gamma=1}^4 (y_{\eta^*}^u)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \eta^* u'_{R,\gamma}$$

Down-type quark Yukawa couplings:

$$\begin{aligned} \mathcal{L}_{down} = & \sum_{\gamma=1}^5 (y_{\eta}^d)_{1\gamma} \bar{Q}'_{L,1} \eta d'_{R,\gamma} \\ & + \sum_{\alpha=2}^3 \sum_{\gamma=1}^5 (y_{\rho^*}^d)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \rho^* d'_{R,\gamma} + \sum_{\alpha=2}^3 \sum_{\gamma=1}^5 (y_{\chi^*}^d)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \chi^* d'_{R,\gamma}. \end{aligned}$$

Both up- and down-type quarks couple to three different scalar triplets.

⇒ **FCNCs!**

Up-type quark couplings to Higgs

Higgs is the linear combination of real parts of the neutral fields η^0 , ρ^0 , ρ'^0 , χ^0 and χ'^0 .

$$\mathcal{L}_{Yukawa} = \bar{u}'_L \frac{\Gamma'^u_h}{\sqrt{2}} u'_R h = \bar{u}_L \underbrace{(U_L^u) \frac{\Gamma'^u_h}{\sqrt{2}} (U_R^{d\dagger})}_{\Gamma_h^u / \sqrt{2}} u_R h$$

The physical Up-type Higgs-Yukawa coupling can be written as:

$$(\Gamma_h^u)_{ij} = \sqrt{2} \frac{m_j}{v_{SM}} \left[\delta_{ij} + \alpha_1 (U_L^u)_{i1} (U_L^{u\dagger})_{1j} - (U_L^u)_{i4} (U_L^{u\dagger})_{4j} \right. \\ \left. + \alpha_2 (U_L^u)_{i1} (U_L^{u\dagger})_{4j} + \alpha_3 (U_L^u)_{i4} (U_L^{u\dagger})_{1j} \right],$$

where

$$\alpha_i = \frac{SU(2)_L\text{-breaking scale}}{SU(3)_L\text{-breaking scale}} \ll 1 \Rightarrow \text{Suppression!}$$

Additional suppression from U_L^u

$$(\Gamma_h^u)_{ij} = \sqrt{2} \frac{m_j}{v_{SM}} \left[\delta_{ij} + \alpha_1 (U_L^u)_{i1} (U_L^{u\dagger})_{1j} - (U_L^u)_{i4} (U_L^{u\dagger})_{4j} \right. \\ \left. + \alpha_2 (U_L^u)_{i1} (U_L^{u\dagger})_{4j} + \alpha_3 (U_L^u)_{i4} (U_L^{u\dagger})_{1j} \right]$$

Example: Assume **hierarchy**:

$$m_{i,j}^u \leq m_{i+1,j}^u.$$

FN charges: $q(Q_{L,1}^c) = 3$, $q(Q_{L,2}^c) = 2$, $q(Q_{L,3}^c) = 0$,
expansion parameter: $\epsilon = 0.23$.

$$U_L^u \sim \begin{pmatrix} 1 & \epsilon^1 & \epsilon^3 & \epsilon^{3+x} \\ \epsilon^1 & 1 & \epsilon^2 & \epsilon^{2+x} \\ \epsilon^3 & \epsilon^2 & 1 & \epsilon^x \\ \epsilon^{3+x} & \epsilon^{2+x} & \epsilon^x & 1 \end{pmatrix} \approx 1,$$

where $x = 2 + (\log \epsilon)^{-1} \log(v'/v_2) \geq 0$. $\Rightarrow$ **More Suppression!**

The Γ_h^u texture becomes:

$$\Gamma_h^u \sim \begin{pmatrix} y_u & y_c[\alpha\epsilon^1] & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^1] & y_c & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^x] & y_c[\alpha\epsilon^x] & y_t & 1 \\ y_u[\alpha] & y_c[\alpha] & y_t[\epsilon^x] & \epsilon^x \end{pmatrix}, \quad \text{where } x \geq 0,$$

$$\alpha = \frac{SU(2)_L\text{-breaking scale}}{SU(3)_L\text{-breaking scale}} \ll 1, \quad \epsilon = 0.23$$

- The off-diagonal elements over the diagonal are largest. They are suppressed by α and ϵ .
- The suppression is enough when $SU(3)_L$ -breaking scale $= v_2, u \gtrsim 4\text{TeV}$

Summary

- Froggatt-Nielsen mechanism naturally suppresses Higgs mediated FCNCs. No tricks required!
- The minimal 331-scalar sector is sufficient in housing Froggatt-Nielsen mechanism. **No additional scalars required!**
- 331-models with $\beta = \pm \frac{1}{\sqrt{3}}$ can explain the number of fermion generations and their mass hierarchy simultaneously! We are the first to point this out in the literature.

$$\beta = (+) \frac{1}{\sqrt{3}}$$

M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D **22**, 738 (1980).

Fermion representations:

$$\text{Triplets: } L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ N_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ U \end{pmatrix}_L \sim (3, 3, \frac{1}{3})$$

$$\text{Antitriplets: } Q_{L,2} = \begin{pmatrix} s \\ c \\ D_1 \end{pmatrix}_L \sim (3, 3^*, 0), \quad Q_{L,3} = \begin{pmatrix} b \\ t \\ D_2 \end{pmatrix}_L \sim (3, 3^*, 0)$$

$N_{L,i}$ = new neutrino-like fields, U = new up-type quark, D_1, D_2 = new down-type quarks.

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -\frac{1}{3})$$

$$\beta = (+)\sqrt{3}$$

F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).

P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

$$\text{Leptons: } L_{L,i} = \begin{pmatrix} \nu_{L,i} \\ e_{L,i} \\ (e_{R,i})^c \end{pmatrix} \sim (1, 3, 0), \quad \mathcal{L}_m = \epsilon_{\alpha\beta\gamma} G_{ij} \bar{L}_{L,i}^\alpha (L_{L,j}^\beta)^c \eta^{\gamma*}$$

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^+ \end{pmatrix} \sim (1, 3, 0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1, 3, 1),$$

$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1, 3, -1), \quad S = \begin{pmatrix} \sigma_1^0 & h_2^- & h_1^+ \\ h_2^- & H_1^{--} & \sigma_2^0 \\ h_1^+ & \sigma_2^0 & H_2^{++} \end{pmatrix} \sim (1, 6, 0).$$

Quark Flavour Constraints

The most stringent bounds on flavour changing couplings comes from neutral meson mixing: $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$ and $D^0 - \bar{D}^0$.

Process	coupling	Current bound
$K^0 - \bar{K}^0$	$\sqrt{\text{Re}(\Gamma_{ds}^{d\ 2})}$	$([-6.3, 6.1] \times 10^{-2}) \times y_s$
$B^0 - \bar{B}^0$	$ \Gamma_{db}^d $	$< (8.8 \times 10^{-3}) \times y_b$
$B_s^0 - \bar{B}_s^0$	$ \Gamma_{sb}^d $	$< (7.8 \times 10^{-2}) \times y_b$
$D^0 - \bar{D}^0$ (Tree-level)	$ \Gamma_{uc}^u $	$< (1.4 \times 10^{-2}) \times y_c$
$D^0 - \bar{D}^0$ (Loop-level)	$ \Gamma_{ut}^u \Gamma_{ct}^u $	$< (1.5 \times 10^{-2}) \times y_t^2$

Neutrino masses

$$\mathcal{L}_{\text{Neutrino mass}} = (\overline{\nu_L} \ \overline{\nu'_L} \ \overline{(N_R)^c}) \begin{pmatrix} 0 & (m^D)^T & \frac{1}{2}m^N \\ m^D & 0 & \frac{1}{2}(m')^N \\ \frac{1}{2}(m^N)^T & \frac{1}{2}(m'^N)^T & \frac{1}{2}M \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ (\nu'_L)^c \\ N_R \end{pmatrix} + h.c.$$

$m^D, m^N \sim 100\text{GeV}$, $(m')^N, M \sim \text{GUT}$

$\Rightarrow$ sub-eV neutrinos

Linear seesaw-mechanism!

Gauge bosons

$$D_\mu = \partial_\mu - ig_3 \sum_{a=1}^8 T_a W_{a\mu} - ig_X X B_\mu,$$

$$\sum_{a=1}^8 T_a W_{a\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & W'^+_\mu & X'^0_\mu \\ W'^-_\mu & -\frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & V'^-_\mu \\ X'^{0*}_\mu & V'^+_\mu & -\frac{2}{\sqrt{6}} W_{8\mu} \end{pmatrix},$$

The neutral mass eigenstates are: **photon**, Z_μ , Z'_μ , X^0_μ and X^{0*}_μ . The charged mass eigenstates are $W^\pm_\mu$ and $V^\pm_\mu$.