SO(3) family symmetry, the axion and the flavor puzzle

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arXiv: 1706.03116; MR**,** C.A.Vaquera-Araujo, J.W.F. Valle, F. Wilczek
arXiv: 1805.08048; MR**,** J.W.F. Valle, F. Wilczek

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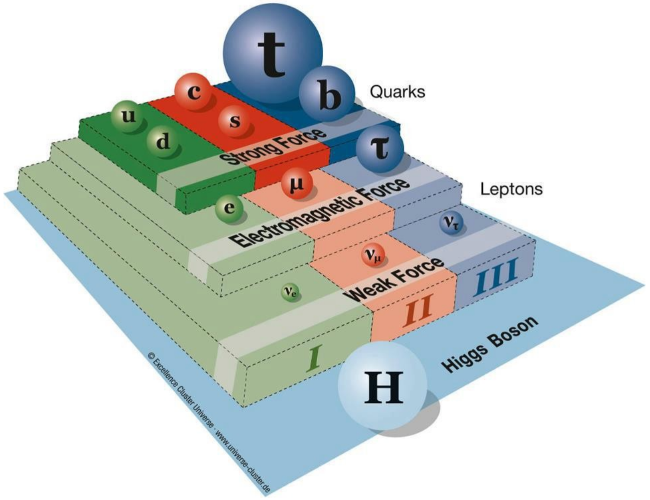
Planck 2018, Bonn

SOME OPEN QUESTIONS...

- **Family structure & flavor puzzle**

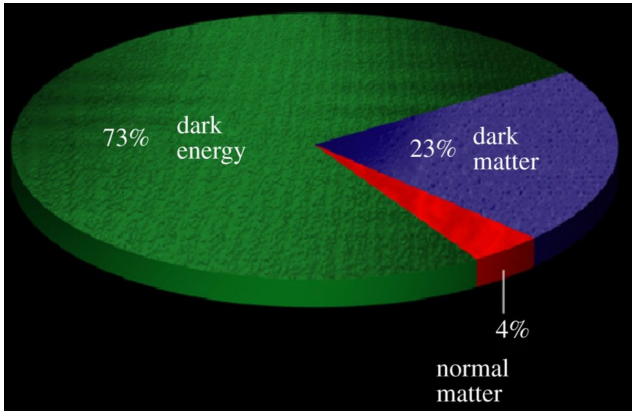


- **Neutrino mass**

- **Strong CP-problem**

$$\Delta\mathcal{L} = \frac{g^2\theta}{16\pi^2}\epsilon^{\alpha\beta\gamma\delta}\, G^a_{\alpha\beta}G^a_{\gamma\delta}$$

- **Nature of Dark Matter**



What about the family replication?

Comprehensive Unification

See: arXiv: 1706.03116

FAMILY UNIFICATION USING SPINORS

- **Spinors ARE the answer to family replication**
  (Gell-Mann, Ramond, Slansky, 1980; Wilczek & Zee, 1982)

$$SO(18) \to SO(10) \times SO(8)$$
$$\mathbf{256} \to (\mathbf{16}, \mathbf{8}) + (\overline{\mathbf{16}}, \mathbf{8}')$$

- **Hypercolor hypothesis: some SO(8) subgroup confines ALL but the observed SM families.**

$$SO(18) \to SO(10) \times SO(3)_F \times SO(5)_{HC}$$
$$\mathbf{256} \to (\mathbf{16}, \mathbf{3}, \mathbf{1}) + (\mathbf{16}, \mathbf{1}, \mathbf{5}) + (\overline{\mathbf{16}}, \mathbf{2}, \mathbf{4})$$

COMPREHENSIVE UNIFICATION

(MR, Vaquera, Valle, Wilczek; 2017)

- **SO(18) in 5-dimensional space-time.**
- **Orbifold symmetry breaking:** $SO(18) \rightarrow SO(10) \times SO(8)$

$$\mathbf{256} \rightarrow (\mathbf{16},\mathbf{8})^{++} + (\overline{\mathbf{16}},\mathbf{8}')^{-+}$$

- **Higgs mechanism:**

$$SO(10) \times SO(8) \rightarrow SO(10) \times SO(5)_{HC}$$

$$(\mathbf{16},\mathbf{8})^{++} + (\overline{\mathbf{16}},\mathbf{8}')^{-+} \rightarrow 3 \times (\mathbf{16},\mathbf{1})^{++} + (\mathbf{16},\mathbf{5})^{++} + 2 \times (\overline{\mathbf{16}},\mathbf{4})^{-+}$$

Mirror families are decoupled: they become heavy KK modes
Extra families are confined by SO(5)$_{HC}$ (now asymptotically free)

Bottom-up approach:
$SO(3)_F$ family symmetry and axions

See: arXiv: 1805.08048

Understanding flavor using symmetries

- **Fermion masses and mixings are explained using four different mechanisms:** (Fritzsch and Xing, 99')

  - Texture zeros

  - Family symmetries

  - Radiative mechanisms

  - Seesaw mechanisms

- **From the theoretical point of view all this mechanisms rely on symmetry (and its breaking!) arguments.**

- **A huge number of possibilities arise to describe 3 families…**

  A_4, S_3, T_7, $U(1)_{FN}$, $\Delta(27)$, $SO(3)$, $SU(3)$, ...

SO(3) as THE family symmetry

- **SO(3) comes from Comprehensive Unification through the SSB:**

$$SO(18) \to SO(10) \times SO(3)_F \times SO(5)_{HC}$$

$$\mathbf{256} \to (\mathbf{16}, \mathbf{3}, \mathbf{1}) + (\mathbf{16}, \mathbf{1}, \mathbf{5}) + (\overline{\mathbf{16}}, \mathbf{2}, \mathbf{4})$$

- **As a bottom-up approach we propose a theory based in the gauge group**

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times SO(3)_F$$

The Model

	q_L	u_R	d_R	l_L	e_R	ν_R	$\mathbf{5}^u$	$\mathbf{5}^d$	$\mathbf{3}^u$	$\mathbf{3}^d$	σ	ρ
$\mathrm{SU(3)_c}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{SU(2)_L}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{U(1)_Y}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
$\mathrm{SO(3)_F}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5}$	$\mathbf{5}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5}$	$\mathbf{1}$
$\mathrm{U(1)_{PQ}}$	1	-1	-1	1	-1	-1	2	2	2	2	2	2

- **Fermions come in triplets.**
- **A duplicated Higgs, up and down-type, sector is introduced.**
- **An SM singlet, σ, breaks SO(3) and PQ at high E.**

Mass hierarchies

- **Let fermions be in SO(3) triplets, f~3.** (F. Wilczek, A. Zee, 1979)
- **Because of product rules, 3x3=1+3+5, we can use a singlet, triplet or five-plet scalar to generate fermion masses.**
- **3 and 5 are particularly interesting:**

$$M \sim \bar{f}_i(\epsilon^{ijk}\langle \mathbf{3}_j \rangle + \langle \mathbf{5}_{ik} \rangle) f_k \,, \qquad \langle \mathbf{3} \rangle = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \quad \langle \mathbf{5} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & a \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a & b \\ 0 & -b & a \end{pmatrix} \rightarrow m_1 = 0 \,, m_2 = a - b \,, m_3 = a + b$$

<u>In this context the 1st generation fermions are massless and the CKM is the identity</u>

Emergence of the CKM matrix

- **To generate 1st generation fermion mass and CKM we take perturbations around the minimum.**

$$\langle \tilde{\mathbf{5}} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a & \epsilon_1 \\ 0 & \epsilon_1 & a \end{pmatrix}, \quad \langle \tilde{\mathbf{3}} \rangle = \begin{pmatrix} b \\ 0 \\ \epsilon_2 \end{pmatrix}$$

- **Perturbations change the mass matrix:**

$$\tilde{M} = \begin{pmatrix} 0 & \epsilon_2 & 0 \\ -\epsilon_2 & -a & b+\epsilon_1 \\ 0 & -b+\epsilon_1 & a \end{pmatrix}$$

- **Generate 1st gen. Mass & quark mixing:**

$$m_1 \sim \frac{\epsilon_2^2}{m_2} \qquad \sin\theta_C \approx \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}, \qquad V_{cb} \approx \frac{\epsilon_1^u}{2a^u} - \frac{\epsilon_1^d}{2a^d},$$

Seesaw-like formula for 1st generation
Mixing angles as function of q masses

$$V_{ub} \approx \frac{\sqrt{m_d m_s}}{m_b} - \frac{\sqrt{m_u m_c}}{m_t},$$

Duplicated Higgs sector: mass relations and the axion

- **A relation between down-type quarks and charged leptons appear due to SO(3) symmetry.**

(also F. Wilczek, A. Zee, 1979)

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}$$

- **The relation with up-type quarks is avoided thanks to the duplicated Higgs sector.**

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \neq \frac{m_t}{\sqrt{m_u m_c}}$$

- **In the absence of vector-like quarks, the $U(1)_{PQ}$ can only be anomalous if there is a duplicated Higgs sector:**

$$[SU(3)_C]^2 \times U(1)_{PQ} \neq 0 \leftrightarrow \exists\ H_u\ \&\ H_d$$

Non-trivial flavor-axion connection

PQ breaking: DM & seesaw

- **The QCD axion is a good Dark Matter candidate with**

$$\Omega h^2 \approx 0.18\, \theta_i^2 \left(\frac{f_a}{10^{12} \text{GeV}} \right),$$

- **Misalignment angle, θ_i, of order O(0.1-1) makes the PQ scale, f_a to coincide with the seesaw scale: 10^{12}-10^{15} GeV.**

- **Recall that right handed neutrino mass, PQ & SO(3) breaking are related:**

$$\sigma \sim (\mathbf{1}, \mathbf{1}, 0, \mathbf{5})\,, \quad \mathcal{L}_M = y_M \bar{\nu}_R^c \sigma \nu_R\,,$$

- **A connexion between axion-neutrino mass emerges**

(Also present in SMASH; Ballesteros et al.)

$$m_a \sim (\Lambda_{QCD} m_\pi / v^2) m_\nu$$

CONCLUSIONS

- **SO(18) Comprehensive Unification is an attractive framework for gauge and family unification.**

- **$SO(3)_F$ turns out to be a very compelling and predictive symmetry to describe flavor.**

- **Interesting flavor-axion-neutrino connection appears in the $SO(3)_F$ theory.**

CONCLUSIONS

- **SO(18) Comprehensive Unification is an attractive framework for gauge and family unification.**

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THANK YOU!!

Back-up slides

SO(10) unification

- **SO(10) group unifies ALL SM interactions**
  (Georgi & Glashow, 1974; Minkowski, Fritzsch, 1974)

$$SO(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$

- **SO(10) spinors unify EACH family**

$$\begin{aligned}\mathbf{16} \to &(\mathbf{3}, \mathbf{2}, 1/6) + (\mathbf{1}, \mathbf{2}, -1/2) + (\bar{\mathbf{3}}, \mathbf{1}, 1/3) \\ &+ (\bar{\mathbf{3}}, \mathbf{1}, -2/3) + (\mathbf{1}, \mathbf{1}, 1) + (\mathbf{1}, \mathbf{1}, 0)\end{aligned}$$

- **Right handed neutrino is predicted: SEESAW MECHANISM!**
  (Yanagida, 1979; Glashow, 1980; Schechter & Valle, 1980)

$$m_\nu \sim \frac{v^2}{M_R}$$

SPINORS ARE THE ANSWER

$$SO(2N) \to SO(2n) \times SO(2m) \to SO(2n)$$

$$\mathbf{2}^{N-1} \to (\mathbf{2}^{n-1}, \mathbf{2}^{m-1}) + (\bar{\mathbf{2}}^{n-1}, \bar{\mathbf{2}}^{m-1}) \to 2^{m-1} \times \mathbf{2}^{n-1} + 2^{m-1} \times \bar{\mathbf{2}}^{n-1}$$

- **While tensor representations DO NOT decompose repetitively, spinors DO DECOMPOSE repetitively.**

- **We can use this to reproduce the family structure!**

EARLY ATTEMPTS WERE NOT SUCCESSFUL...

- The presence of anti-families does not allow the hypercolor hypothesis. (F. Wilczek, A. Zee; 1982)

- It is impossible to remove anti-families using Higgs mechanism: each family come with its mirror family (anti-family). (J. Bagger, S. Dimopoulos; 1982)

- **Conclusion: it seems not possible to unify the observed SM families using a SO(18) spinor.**

CHIRALITY, THE ORIGIN OF THE PROBLEM

- The SO(18) spinor, **256**, is L-H. After SSB (**16,8**) is L-H BUT ($\overline{\mathbf{16}}$**,8'**) is RIGHT-HANDED (Lorentz sense).

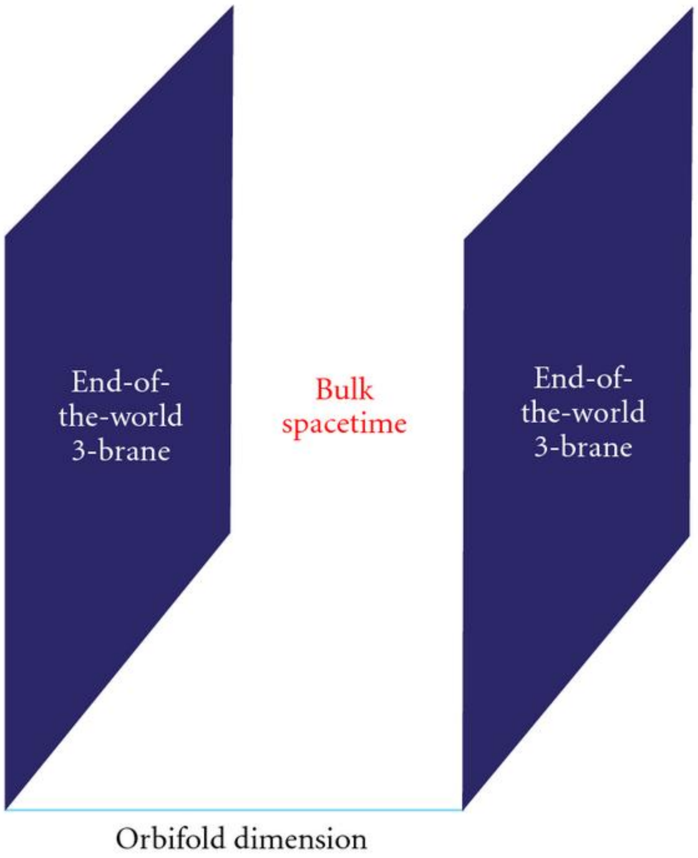
- **Chirality** is a propert of spinors that can be defined only in **EVEN** dimensions.

ORBIFOLD SYMMETRY BREAKING

- SO(18) in 5-dimensional space-time.
- Compactify the 5th dimension on an orbifold $S^1/(Z_2 \times Z_2')$. (Y. Kawamura, 1999)
- Non-trivial boundary conditions (BC) for fields.

$$\Phi(x,y) \sim P^{\Phi}\Phi(x,-y)$$

- Only fields with Neumann BC have zero modes (massless 4D fields).
- **Useful to address chirality!**



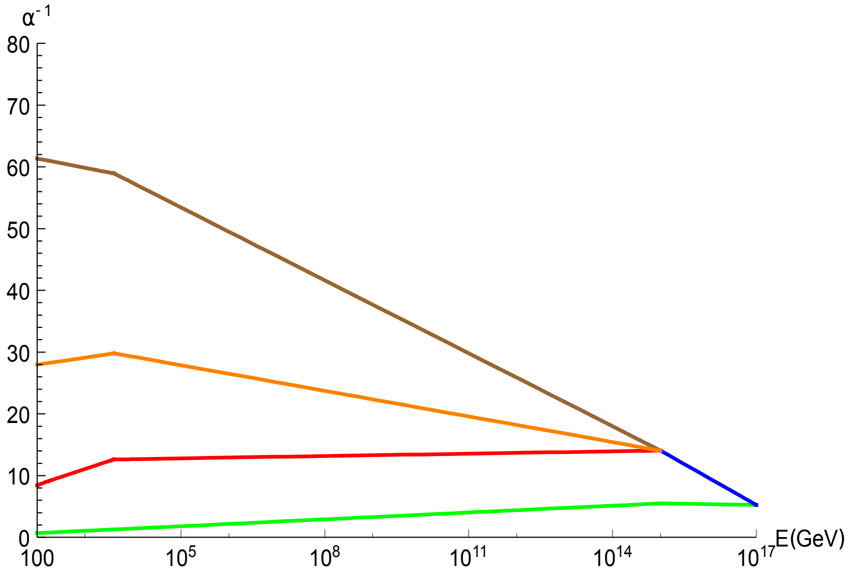
RGE in RS space

$$b_i^{RS} = \frac{1}{3}[-C_2(G)(11I^{1,0}(\Lambda) - \frac{1}{2}I^{1,i}(\Lambda))+$$
$$+ 2I^{1/2,0}(\Lambda)T_f(R) + I^{2,0}(\Lambda)T_s(R)]\,.$$

$$I^{1,0} = 1.024\,,$$
$$I^{1,i} = 0.147\,,$$
$$I^{1/2,0} = 1.009\,,$$
$$I^{2,0} = 1.005\,.$$

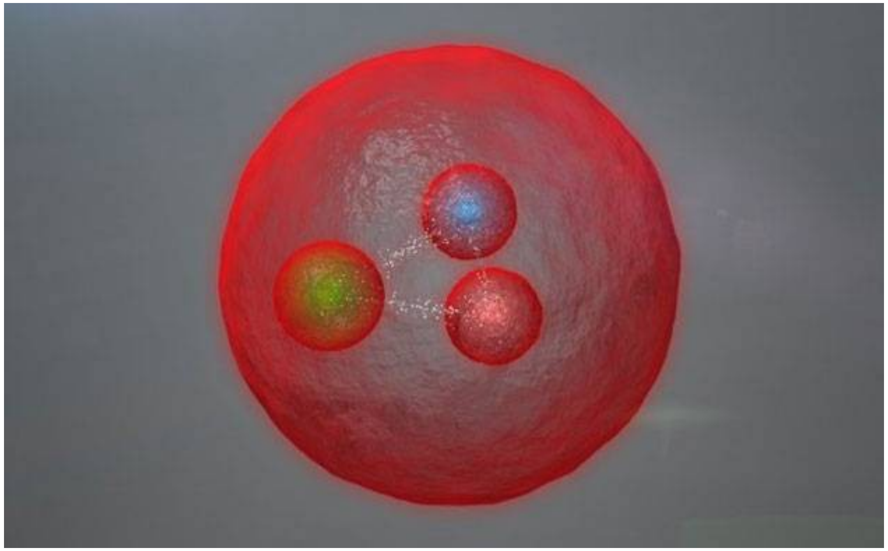
HYPERCOLOR CONFINEMENT

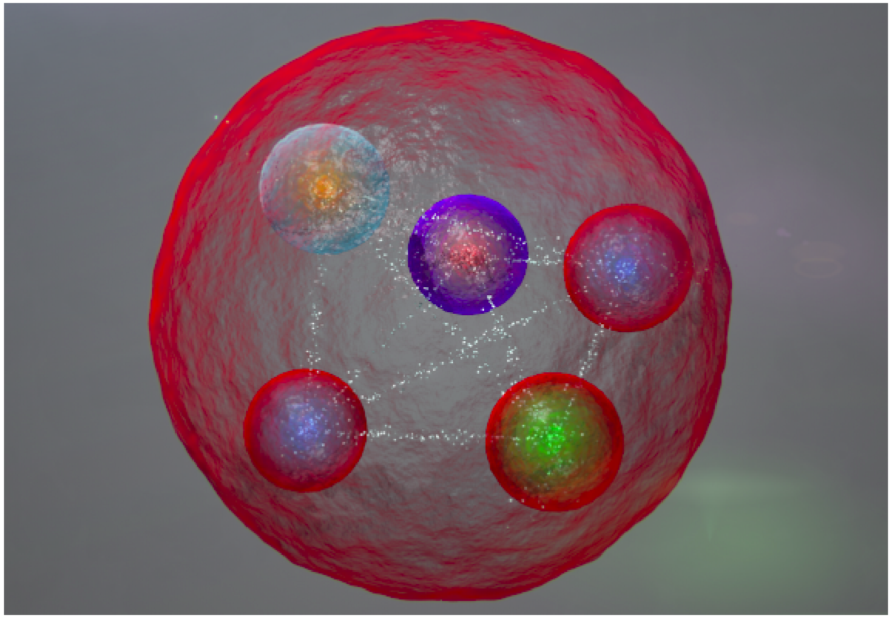
- Mirror families are decoupled: unique framework to implement the **Hypercolor hypothesis**
- Break: $$SO(10) \times SO(8) \to SO(10) \times SO(5)$$
  $$(\mathbf{16}, \mathbf{8}) \to 3 \times (\mathbf{16}, \mathbf{1}) + (\mathbf{16}, \mathbf{5})$$
- Unlike original proposals, with only (**16,8**) the SO(5) hypercolor interaction (green) is asymptotically free!



- The (**16,5**) are confined and we **only observe 3 families at low energy!**

Baryon vs Hyperbaryon





- **Baryons** are composed by **3 quarks** due to **QCD** confinement.
- Proton mass m_p= 0,938 GeV.

- **Hyperbaryons** are composed by **5 hyperquarks** due to **hypercolor** confinement.
- Lightest hyperbaryon around 10 TeV (aprox. 10000 m_p).
- **Highly stable** individually; but small contribution to energy density of universe:

$$\Omega_\chi h^2 \approx 10^{-5}\Big(M/TeV\Big)^2$$

Quantum gravity and flavor symmetries

- **The absence of interaction terms forbidden by symmetry does not distinguish between global and local symmetries.**

- **In order to avoid problems with quantum gravity effects such as wormhole tunneling or black-hole evaporation, global symmetries should be gauged.** (L.M. Krauss, F. Wilczek, 1989)

- **The apparent HUGE number of possibilities to understand flavor using symmetries is reduced to a small number of continuous symmetries.**

- **Concerning family symmetries, only SU(3) and SO(3) appear as good candidates to describe 3 chiral families.** (F. Wilczek, A. Zee, 1979)

CP violation in the quark sector

- CP violation observables are proportional to the Jarlskog invariant, *J.*
- CP violation arises from perturbations around minimum

$$\mathcal{J} = \frac{|V_{us}||V_{ud}||V_{ub}||V_{cb}||V_{tb}|}{(1-|V_{ub}|^2)} \sin\delta_{CKM}$$

$$\theta_C \approx \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \qquad |V_{ub}| \approx \frac{\sqrt{m_d m_s}}{m_b} - \frac{\sqrt{m_u m_c}}{m_t}$$

$$|V_{cb}| = \frac{\epsilon_1^u}{2k^u} - \frac{\epsilon_1^d}{2k^d}$$

The Model

	q_L	u_R	d_R	l_L	e_R	ν_R	$\mathbf{5}^u$	$\mathbf{5}^d$	$\mathbf{3}^u$	$\mathbf{3}^d$	σ	ρ
$\mathrm{SU(3)_c}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{SU(2)_L}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{U(1)_Y}$	$\frac{1}{6}$	$\frac{2}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$	-1	0	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	0	0
$\mathrm{SO(3)_F}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5}$	$\mathbf{5}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5}$	$\mathbf{1}$
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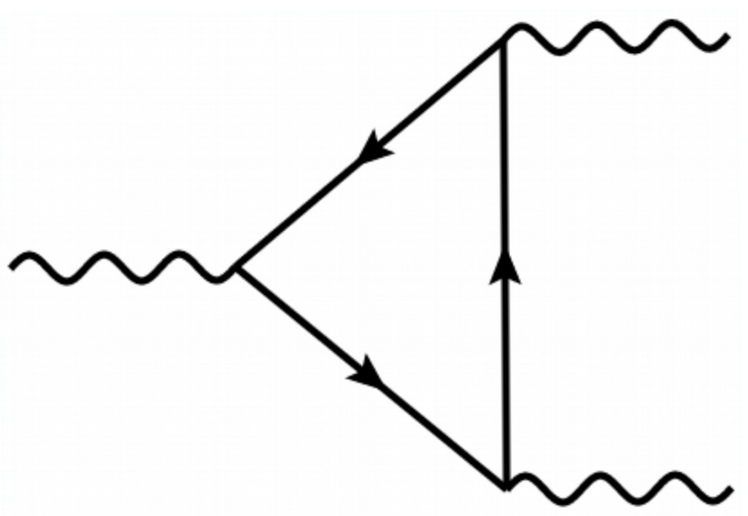
Flavor symmetries and unification

- **Constraints to flavor symmetries arise if we require compatibility with unification.**
- **Family SU(3) symmetry is quantumly inconsistent with minimal content of GUTs.**
- **Example: SO(10)xSU(3) with fermions in the (16,3) representation suffers the triangle $[SU(3)]^3$ anomaly.**
- **This leave us with only**

  one possibility...

  SO(3) symmetry



Flavor protection

- **Pseudo-Goldstone bosons or axions coupled to flavor lead to strong constraints, mainly coming from** $k^+ \to \pi^+ a$ **.**

- **This constraints the PQ breaking scale to be** $f_a \geq 4 \times 10^{11}\text{GeV}$

(Celis, Fuentes-Martin, Serodio, 2014; Calibbi, Goertz, Redigolo, Ziegler, Zupan 2016).

- **Our model ensures universal PQ charges for all families thanks to the SO(3) symmetry.**

- **SO(3) gauge familons contribute to** $K^0 - \bar{K}^0$ **and** $k^+ \to \pi^+ l^- l^+$ **however, this processes are suppressed by** f_a^2 **. This constraints the PQ scale to be** $f_a \geq 4 \times 10^{7}\text{GeV}$ **safely within the limits.**