An EFT approach to lepton anomalies

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Coy and Frigerio *in preparation*
Coy, Frigerio and Sumensari *in preparation*

Outline

1. Introduction
2. EFT of the Type-I seesaw
3. EFT approach to B anomalies

Leptonic physics

- Extensive past, present, and future experimental programmes testing copious lepton sector observables

Neutrinos:

- Precision era, with Δm_{ij}^2 measured at $\mathcal{O}(1\%)$ and $\sin^2(\theta_{ij})$ measured at $\mathcal{O}(5-10\%)$, though δ_{CP} less well known
- Mass mechanism unknown, many models proposed

Anomalous magnetic dipole moments:

- Persistent $(g-2)_\mu$ anomaly
- Improved hadronic vacuum polarisation calculation, $\gtrsim \mathbf{3.5\sigma}$ **discrepancy**, could reach 5σ in a few years at Fermilab

Semi-leptonic B decays

- Recent evidence of **LFUV** at LHCb in $b \to s$ channel
- Define the ratios

$$R_{K^{(*)}}[q^2_{min}, q^2_{max}] = \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\Gamma(B^{+(0)} \to K^{+(*)}\mu^+\mu^-)/dq^2}{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\Gamma(B^{+(0)} \to K^{+(*)}e^+e^-)/dq^2} \quad (1)$$

- SM predicts $R_{K^{(*)}} \approx 1$, LHCb finds:

$$R_K[4m_\mu^2, 1.1 \text{ GeV}^2] = 0.660^{+0.110}_{-0.070} \pm 0.024; \quad (2a)$$

$$R_K[1 \text{ GeV}^2, 6 \text{ GeV}^2] = 0.745^{+0.09}_{-0.074} \pm 0.036; \quad (2b)$$

$$R_{K^*}[1.1 \text{ GeV}^2, 6 \text{ GeV}^2] = 0.685^{+0.113}_{-0.069} \pm 0.047 \quad (2c)$$

- Combined 4σ deviation
- Also combined 4σ signal of LFUV in $b \to c$ channel

Effective Field Theory

- Many anomalies and constraints, also many models
- EFT enables a **model-independent** analysis of the data: relate observables to Wilson coefficients and study the parameter space
- Can demonstrate relative compatibility or tension between different data
- Also useful framework to study a specific model

Type-I seesaw (in collaboration with Michele Frigerio)

- Type-I seesaw among the simplest neutrino mass models
- Add n_s right-handed fermions singlet
- Need $n_s \geq 2$ for at least two non-zero neutrino masses
- Lagrangian for Type-I seesaw is

$$\mathcal{L} = \mathcal{L}_{SM} + \overline{\nu_{Ri}} i \partial\!\!\!/ \nu_{Ri} - Y_{\nu,ai} \overline{l_{La}} \tilde{H} \nu_{Ri} - \frac{1}{2} \overline{\nu_{Ri}} M_{ij} \nu^c_{Rj} + h.c., \quad (3)$$

  for $i, j = 1, \ldots, n_s$; can in general make M_{ij} diagonal
- Obtain $m_\nu \lesssim 0.1$ eV for $Y_{\nu,ai} \sim 1$ and $M_{ii} \gtrsim 10^{15}$ GeV
- Specific cases such as **inverse seesaw** and **linear seesaw** produce correct size of m_ν for much smaller M_{ii}

EFT approach

- Aim: find $\mathbf{d = 5, 6}$ **Wilson coefficients** (WCs) generated by Type-I seesaw at leading order, then calculate observables
- Weinberg operator generated at tree-level, $d = 6$ operators may be generated
  a. At tree-level, $C \sim \sum_i Y_\nu^2 / M_i^2$
  b. Via 1-loop operator mixing, $C \sim \sum_i \frac{g^2}{16\pi^2} \frac{Y_\nu^{2,4}}{M_i^2} \log\left(\frac{M_i}{\mu}\right)$
  c. Via finite 1-loop diagram(s), $C \sim \sum_i \frac{g^2}{16\pi^2} \frac{Y_\nu^{2,4}}{M_i^2}$
  d. At 2+ loops (neglected)
- Usual procedure: matching at tree-level (at each different $\mu = M_i \gg m_W$, also at $\mu = m_W$), running at one-loop
- **Dipole operators** (more generally, type c) a special case: no leading log, need to **match at one-loop**

Matching at M_i

- Integrating out sterile neutrinos with mass M_i gives[1]

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \frac{Y^*_{\nu,ai} Y^*_{\nu,bi}}{2M_i}(Q_{W,ab} + h.c.) + \frac{Y_{\nu,ai} Y^*_{\nu,bi}}{4M_i^2}\left(Q^{(1)}_{Hl,ab} - Q^{(3)}_{Hl,ab}\right), \tag{4}$$

where we define the operators

$$Q_{W,ab} = (\overline{l_{La}}\tilde{H}^*)(\tilde{H}^\dagger l_{Lb}) \tag{5a}$$

$$Q^{(1)}_{Hl,ab} = (\bar{l}_{La}\gamma_\mu l_{Lb})(H^\dagger i\overleftrightarrow{D}^\mu H) \tag{5b}$$

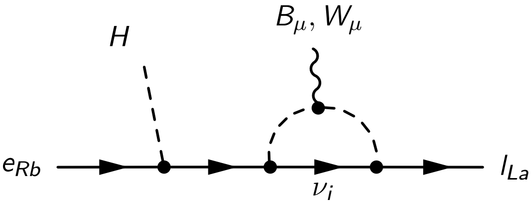
$$Q^{(3)}_{Hl,ab} = (\bar{l}_{La}\gamma_\mu \sigma^A l_{Lb})(H^\dagger i\overleftrightarrow{D}^\mu \sigma^A H) \tag{5c}$$

- This is tree-level matching
- Size of WCs dominated by contribution from lightest RH neutrino(s) integrated out

[1] Broncano, Gavela, and Jenkins, 0210271

Matching at M_i

- Need **one-loop matching** for dipole operators



- Generates dipole operators with ($\mu \gg m_W$) WCs:

$$\mathcal{L}_{EFT} \supset \frac{1}{M_i^2} \frac{Y_{\nu,ai} Y^\dagger_{\nu,bi} y_b}{192\pi^2} (g_2 Q_{eW,ab} - g_1 Q_{eB,ab}) + h.c., \quad (6)$$

where EW dipole operators are defined as

$$Q_{eB} = (\overline{l_{La}} \sigma_{\mu\nu} H e_{Rb}) B^{\mu\nu} \quad (7a)$$

$$Q_{eW} = (\overline{l_{La}} \sigma_{\mu\nu} \sigma^A H e_{Rb}) W^{A\mu\nu}. \quad (7b)$$

EFT procedure

- Can calculate[2] WCs of operators generated at 1-loop via mixing of $Q_{Hl}^{(1,3)}$ and Q_W^2
- For processes relevant below EW scale, can match SMEFT onto low-energy EFT and compute QED, QCD running[3]
- One-loop matching needed for dipole operator also at $\mu = m_W$
- **No relevant QED running** below m_W at $\mathcal{O}(e^2/16\pi^2)$, though QCD running exists (important for B anomalies)

[2] RGEs found in papers by Jenkins, Manohar, Trott, and Alonso: 1308.2627, 1310.4838, 1312.2014

[3] Jenkins, Manohar, and Stoffer: 1711.05270

Comparison with models

- Can compare EFT with calculations made in different Type-I seesaw models
- Decay width $h \to e_k \bar{e}_m$ for $k \neq m$ and $m_k \gg m_m$ calculated in model with exact $U(1)_L$ symmetry, Dirac masses for sterile neutrinos is

$$\Gamma(h \to e_k \bar{e}_m) \approx \frac{\lambda^2 m_k^2 v^2 m_h}{(4\pi)^5} \left[Y_\nu M^{-2} \log\left(\frac{M}{m_W}\right) Y_\nu^\dagger \right]_{km}^2 , \quad (8)$$

  which agrees at leading order with a previous calculation in this model[4]
- **Computationally simple** to find this leading order result

[4] Arganda et al.: 1612.09290

Comparison with models

- Including contribution from one-loop matching at $\mu = m_W$, WC of dipole operator is

$$C_{e\gamma,ab} = -\frac{ev}{\sqrt{2}}\frac{(Y_\nu M^{-2} Y_\nu^\dagger)_{ab} y_b}{96\pi^2} - \frac{ev}{\sqrt{2}}\frac{g^2 U_{ai}^* m_i^2 U_{bi} y_b}{256\pi^2 m_W^4} \quad (9)$$

- Contribution to $(g-2)_f$ is given by

$$\Delta a_f = \frac{4 m_f}{e}\mathrm{Re}[C_{e\gamma,ff}] \quad (10)$$

- Immediately see $C_{e\gamma,ff} < 0$, therefore Type-I seesaw **worsens** $\mathbf{(g-2)_\mu}$ **anomaly**, as shown previously in explicit models
- Moreover, calculate $\Gamma(\mu \to e\gamma)$ from Eqn. (9), agreement at this order with literature[5]

[5] Cheng and Li, Phys.Rev.Lett. 45 (1980) 1908

Spurion analysis

- Consider extended lepton flavour symmetry, $G_L = SU(3)_l \times U(1)_l \times SU(3)_e \times U(1)_e \times SU(3)_\nu \times U(1)_\nu$, with

$$Y_e \sim (\mathbf{3}_1, \overline{\mathbf{3}}_{-1}, \mathbf{1}_0); \quad Y_\nu \sim (\mathbf{3}_1, \mathbf{1}_0, \overline{\mathbf{3}}_{-1}); \quad M \sim (\mathbf{1}_0, \mathbf{1}_0, \overline{\mathbf{6}}_{-2})$$

- Can see several relations between the different WCs, e.g.
  - $C_{Hl}^{(1,3)} \propto Y_\nu (M^*)^{-1} M^{-1} Y_\nu + \mathcal{O}(Y_\nu^4 M^{-2})$, so proportional at tree-level
  - $C_W \propto Y_\nu^* M^{-1} Y_\nu^\dagger$
  - $C_{eH}, C_{eB}, C_{eW} \propto \alpha_1 Y_\nu (M^*)^{-1} M^{-1} Y_\nu^\dagger Y_e + \alpha_2 Y_\nu (M^*)^{-1} Y_\nu^T Y_\nu^* M^{-1} Y_\nu^\dagger Y_e$, so all suppressed by Y_e
  - Y_ν enters in $d = 6$ WCs either as $(Y_\nu M^{-2} Y_\nu^\dagger)$ or $(Y_\nu M^{-1} Y_\nu^T Y_\nu^* M^{-1} Y_\nu^\dagger)$
- Ongoing work: general (not just Type-I seesaw) spurion analysis of neutrino masses and relations to other observables

EFT analysis of B anomalies (with Michele Frigerio and Olcyr Sumensari)

- $b \to s$ anomalies suggest new physics relating to muons or electrons: possible connection to $(g-2)_\mu$ anomaly?
- Performed basis-independent search for unique $d = 6$ operator which could **simultaneously explain** both anomalies
- Could come from similar scales: for B anomalies, may expect

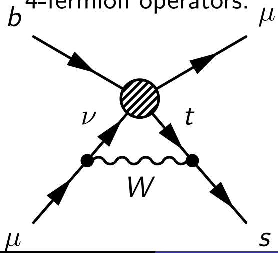
$$\mathcal{L} \sim \frac{g^2 V_{ts}^* C_i}{16\pi^2\Lambda^2} \log\left(\frac{\Lambda}{m_b}\right)\mathcal{O}_j, \tag{11}$$

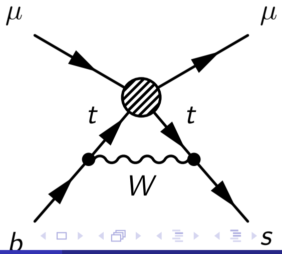
  at 1-loop, need $C_i/\Lambda^2 \sim -1/(3\text{TeV})^2$ for $\mathcal{O}_j \equiv \mathcal{O}_9, \mathcal{O}_9 - \mathcal{O}_{10}$
- For $(g-2)_\mu$, correction enters into Lagrangian as

$$\mathcal{L} \supset \frac{e^2 v C_i}{16\pi^2\sqrt{2}\Lambda^2} \log\left(\frac{\Lambda}{m_\mu}\right)\mathcal{O}_{e\gamma} \tag{12}$$

  with anomaly explained for $C_{e\gamma}/\Lambda^2 \sim 1/(10\ \text{TeV})^2$

- Considered effects from tree-level, one-loop and Barr-Zee type two-loop diagrams
- **No such operator found**
- Difficulties in a combined explanation include
  - dipole described by tensor operators, $b \to s$ transition by vector operators, little vector$\leftrightarrow$tensor operator mixing
  - $(g-2)_\mu$ is $\Delta F = 0$ but $b \to s$ is $\Delta F = 1$, so a) relative CKM suppression, and b) limited diagram topologies, e.g. for 4-fermion operators:





EFT analysis of B anomalies

- Next: **full EFT analysis** of $b \to s$ anomalies
- Consider full set of operators (including flavour structure) which can contribute either a tree-level or at one-loop to these anomalies
- Tree-level[6] and one-loop[7] analyses exist, however room to extend the existing studies
  - consider operators with **electrons** as well as muons
  - relax top dominance assumption (can be offset by CKM suppression)
  - consider additional constraints on space of WCs, e.g. from $(g-2)_\mu$

[6] Alonso, Grinstein, and Camalich, 1407.7044

[7] Celis, Fuentes-Martin, Vicente, Virto, 1704.05672

Summary

- Interesting time for leptonic physics, with indications of new physics in a variety of observables
- Demonstrated advantages of EFT approach: can be model-independent or model-specific, computationally straightforward, highlights relationships between observables
- Outlined EFT treatment of Type-I seesaw models
- Described more thorough EFT approach to B anomalies, relation to other lepton observables

Back-up Slides

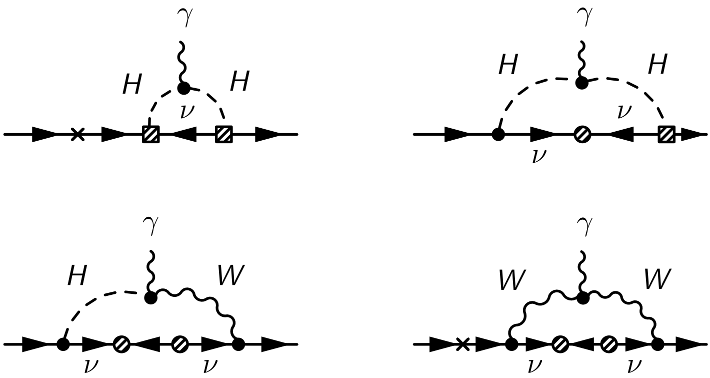
EFT procedure

- Can calculate WCs of operators generated at 1-loop via mixing of $Q_{Hl}^{(1,3)}$ and Q_W^2
- For example, RGE for C_{eH} is

$$16\pi^2 \frac{dC_{eH,ab}}{d\log\mu} = 4\lambda(C_{Hl}^{(1)}Y_e + 3C_{Hl}^{(3)}Y_e)_{ab} - 4\mathrm{Tr}[C_{Hl}^{(3)}Y_eY_e^\dagger]Y_{e,ab}$$
$$+ 2(C_{Hl}^{(1)}Y_eY_e^\dagger Y_e)_{ab} - 6g_1^2(C_{Hl}^{(1)}Y_e)_{ab} - 6g_1^2(C_{Hl}^{(3)}Y_e)_{ab}$$
$$+ \frac{4}{3}g_2^2 Y_{e,ab}\mathrm{Tr}[C_{Hl}^{(3)}] + \frac{7}{4}(C_W^\dagger C_W Y_e)_{ab} + \mathrm{Tr}[C_W^\dagger C_W](Y_e)_{ab}$$
$$\approx \frac{7y_b}{16}(Y_{\nu,ai}M_i^{-1}Y_{\nu,ic}^T Y_{\nu,cj}^* M_j^{-1}Y_{\nu,jb}^\dagger) - 2\lambda y_b(Y_{\nu,ai}M_i^{-2}Y_{\nu,ib}^\dagger), \tag{13}$$

with simplification made for $a \neq b$

One-loop matching at m_W



- Integrate out gauge bosons
- Generates contribution to dipole operator of size

$$\mathcal{L} \supset -\frac{eg^2 U^*_{ai} m_i^2 U_{bi}}{256\pi^2 m_W^4}(\overline{e_a}\sigma_{\mu\nu}(m_a P_L + m_b P_R)e_b)F^{\mu\nu} \qquad (14)$$

Leptonic physics

Charged lepton sector

- Stringent limits on CLFV **particularly for $\boldsymbol{\mu \to e}$**, with $BR(\mu \to e\gamma) \lesssim 10^{-13}$, $BR(\mu \to eee) \lesssim 10^{-12}$, while $BR(\tau \to \mu\gamma, e\gamma) \lesssim 10^{-8}$
- Hint of CLFV at LHC Run 1 in $h \to \tau\mu$ disfavoured by Run 2
- Hierarchical Yukawas $\Rightarrow$ lepton flavour universality violation
- Yukawas measured at LHC consistent with SM: $\sigma \times BR(h \to \tau^+\tau^-)_{exp}/\sigma \times BR(h \to \tau^+\tau^-)_{SM} = 1.12 \pm 0.23$; $\sigma \times BR(h \to \mu^+\mu^-)_{exp}/\sigma \times BR(h \to \mu^+\mu^-)_{SM} = 0.1 \pm 2.5$

More on lepton anomalies

- $(g-2)_e$ consistent with SM (1.3σ discrepancy), but if $\frac{\Delta a_e}{\Delta a_\mu} = (\frac{m_e}{m_\mu})^2$, should appear in next generation experiments

B anomalies

- LFUV anomalies also found in $b \to c$ channel: can define

$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)}\tau\nu)}{\Gamma(B \to D^{(*)}\ell\nu)} \tag{15}$$

where $\ell = e, \mu$

- SM predicts $R_D \approx 0.3$ and $R_{D^*} \approx 0.25$, but B factories find:

$$R_D = 0.407 \pm 0.046; \qquad R_{K^*} = 0.304 \pm 0.015; \tag{16}$$

- Combined 4σ deviation