Solving the Hierarchy Problem Discretely

Solving the Hierarchy problem with Higgs partners

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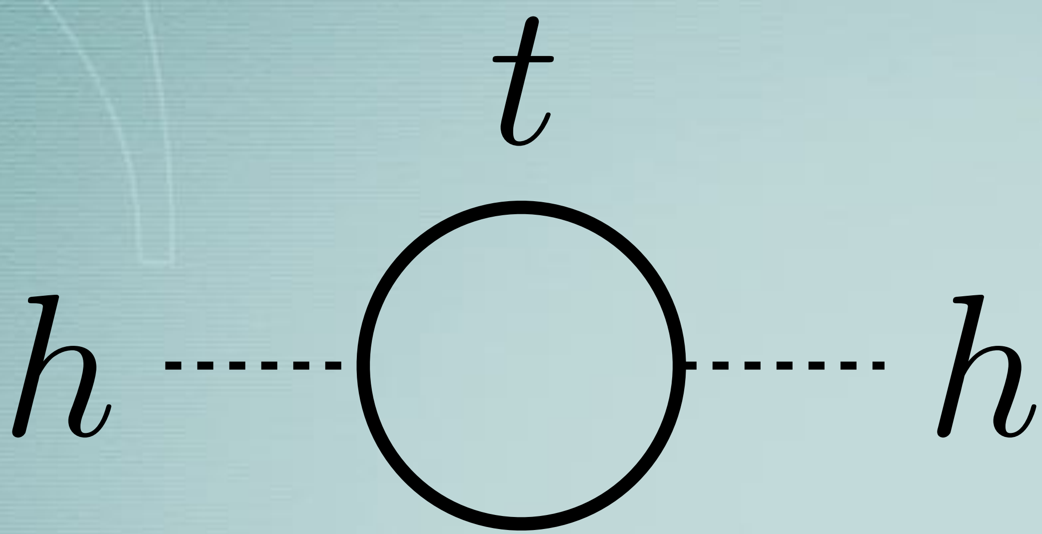
AH : 1802.10093

Naturalness

- Particle Physics has a love affair with Naturalness
- Naturalness : IR physics should not depend sensitively on small variations of UV parameters

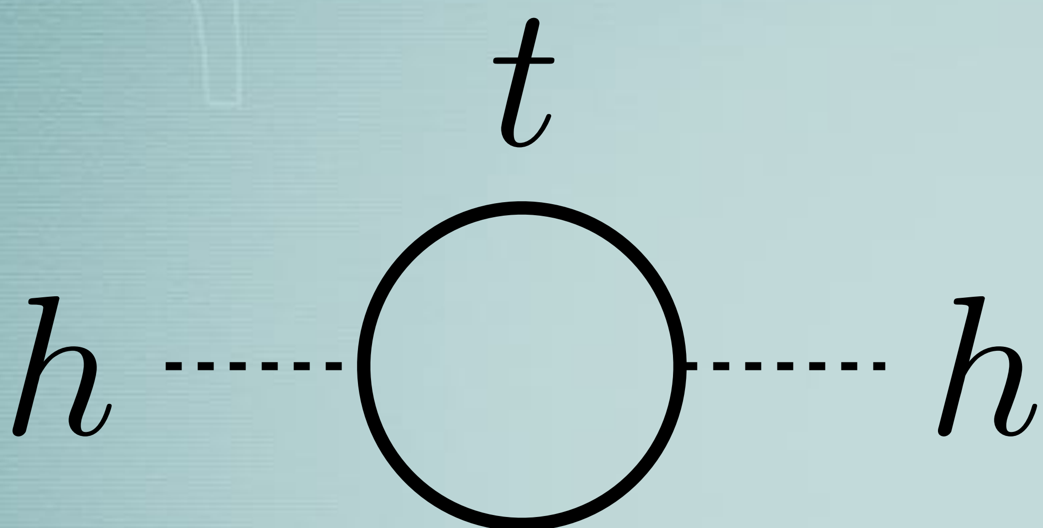
Naturalness

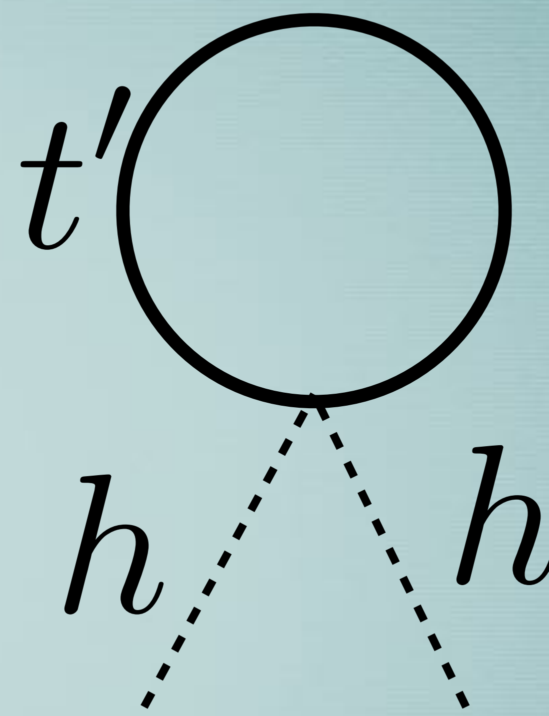


$$\delta m_h^2 \sim -\frac{3y_t^2\Lambda^2}{8\pi^2}$$

- Expectation : Naturalness requires additional light degrees of freedom to cancel quadratic divergence

Naturalness





- Traditional solution - Introduce top partners to cancel the quadratic divergence
    - SUSY, Composite Higgs, Twin Higgs, Little Higgs, …

Naturalness

- Non Traditional Solutions
  - Multiverse, Relaxion, NNaturalness, …

1. Brute force statistics to get a vacua/copy with the correct Higgs mass
2. Story to explain how we got there

Naturalness

- Traditionalish solution - Use Higgs partners + moduli to cancel the quadratic divergence
- Like Traditional solutions
    - Some particles have partners
- Like Non-Traditional solutions
    - Partners are not responsible for cancelling the quadratic divergence

Naturalness

- Imagine that the Higgs mass is controlled by a scalar (moduli) vev
  - 2 Higgs doublet model, MSSM, Twin Higgs, Multiverse, Relaxion, NNaturalness, …
- What if the minimum of the moduli is such that any large mass the Higgs might have is naturally cancelled by the moduli vev?

Naturalness

$$\frac{d^2}{d(m_h^2)^2} V(m_h^2)|_{m_h^2=0} < 0$$

- Very difficult to realize a situation where this occurs
- If moduli only controls the Higgs mass, then Higgs mass of zero is always a maximum

Naturalness

- Way around this involves Higgs partners + a discrete symmetry!
    - Unfortunately, not enough time to describe details about how discrete symmetries can be used to solve the general hierarchy problem
- Thus obtain a solution that centers around Higgs partners and NOT top partners!

A theory of moduli

- Starting point is a Z_N symmetry
- N copies of the Higgs exchanged under symmetry

$$H_i \to H_{i+1}$$

- N=3 (4) to solve the Little Hierarchy problem

Z_N Symmetry

- Moduli is a periodic scalar

$$\phi = \phi + 2\pi f$$

- Shift symmetry broken by a spurion down to a periodic scalar

$$\epsilon^2 \sin\left(\frac{\phi}{f} + \theta\right)$$

Z_N Symmetry

- Theory has a Z_N symmetry which is non-linearly realized on the scalar

$$\phi \to \phi + \frac{2\pi f}{N}$$

A theory of moduli

- Most general Lagrangian consistent with symmetries + spurions

$$V \;=\; \sum_k m^2_{H,k}(\phi) H_k H_k^\dagger + \lambda (H_k H_k^\dagger)^2$$

$$m^2_{H,k}(\phi) \;=\; -m_H^2 + \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

- Including cross quartics does not change the story

A theory of moduli

- Most general Lagrangian consistent with symmetries

$$V = \sum_k m^2_{H,k}(\phi) H_k H_k^\dagger + \lambda (H_k H_k^\dagger)^2$$

$$m^2_{H,k}(\phi) = -m^2_H + \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

Couples with a different phase to each Higgs (partner) due to non-linear realization

A theory of moduli

- Depending on the value of the moduli, Higgs mass can be positive or negative

$$\epsilon^2 \geq m_H^2$$

- Due to quadratic divergence

$$m_H^2 \sim \frac{\Lambda^2}{16\pi^2}$$

- Inclusion of cross quartics does not change anything

Tree Level Potential

Bare potential

$$V = -\Lambda_c^4 \cos\left(N\frac{\phi}{f} + \theta\right) - \sum_k \frac{m_{H,k}^4(\phi)}{4\lambda}\Theta\left(-m_{H,k}^2(\phi)\right)$$

Potential from integrating out the Higgs at Tree level

Tree Level Potential

$$V = -\sum_k \frac{m^4_{H,k}(\phi)}{4\lambda}\Theta(-m^2_{H,k}(\phi))$$

- Focus on the Higgs mass contribution to the potential

Tree Level Potential

$$V = -\sum_k \frac{m_{H,k}^4(\phi)}{4\lambda}\Theta(-m_{H,k}^2(\phi))$$

- Focus on the Higgs mass contribution to the potential when all Higgs masses are negative

$$V \propto \sum_k m_H^4 + \sum_k m_H^2 \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right) + \sum_k \epsilon^4 \cos^2\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

Tree Level Potential

$$V \propto \sum_k m_H^4 + \sum_k m_H^2 \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$
$$+ \sum_k \epsilon^4 \cos^2\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

Independent of moduli

Tree Level Potential

$$V \propto \sum_k m_H^4 + \sum_k m_H^2 \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

$$+ \sum_k \epsilon^4 \cos^2\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

Sums to 0 (equilateral triangle closes in on itself)

Tree Level Potential

$$V \propto \sum_k m_H^4 + \sum_k m_H^2 \cancelto{0}{\epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)}$$

$$+ \sum_k \epsilon^4 \cos^2\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

Sums to N/2 (generalized $\sin^2 + \cos^2 = 1$ identity)

Tree Level Potential

$$0$$

$$V \propto \sum_k m_H^4 + \sum_k m_H^2 \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

$$+ \sum_k \epsilon^4 \cos^2\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

$$N/2$$

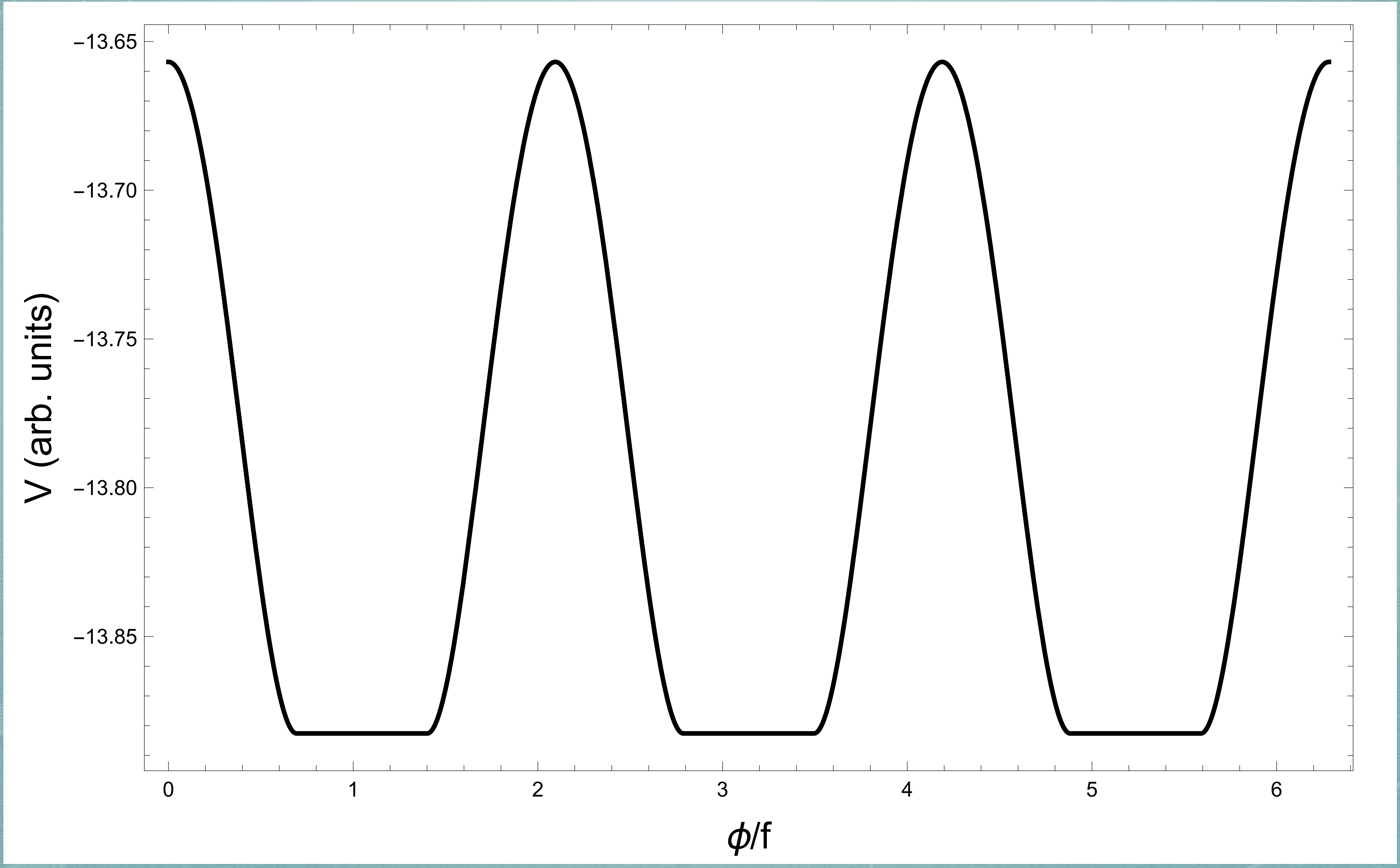
If all Higgs masses negative, then potential independent of moduli!

Tree Level Potential

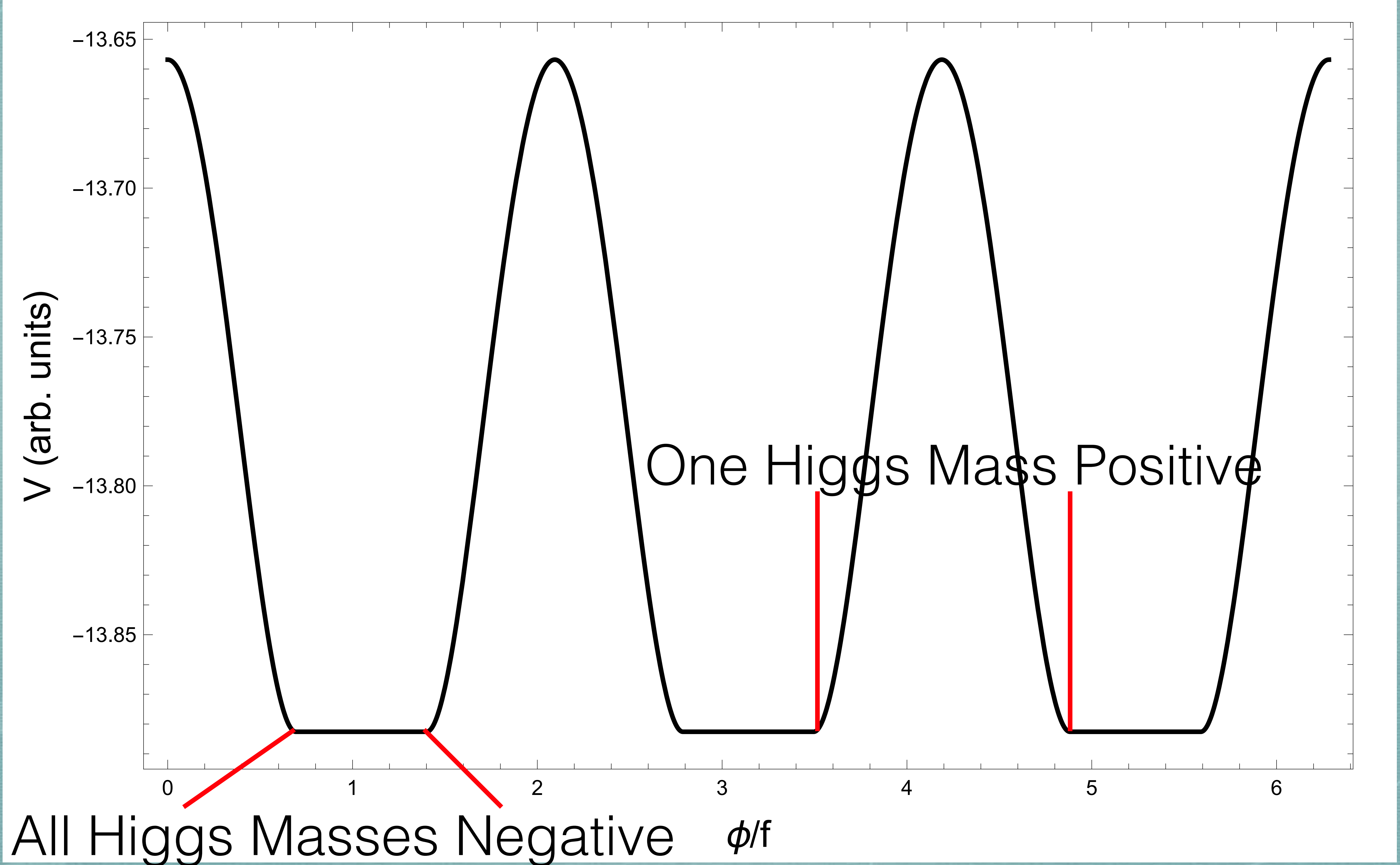
$$V \propto \sum_k m_H^4 + \sum_k m_H^2 \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right) + \sum_k \epsilon^4 \cos^2\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

If one Higgs masses positive, then sums do not become constants and potential is present

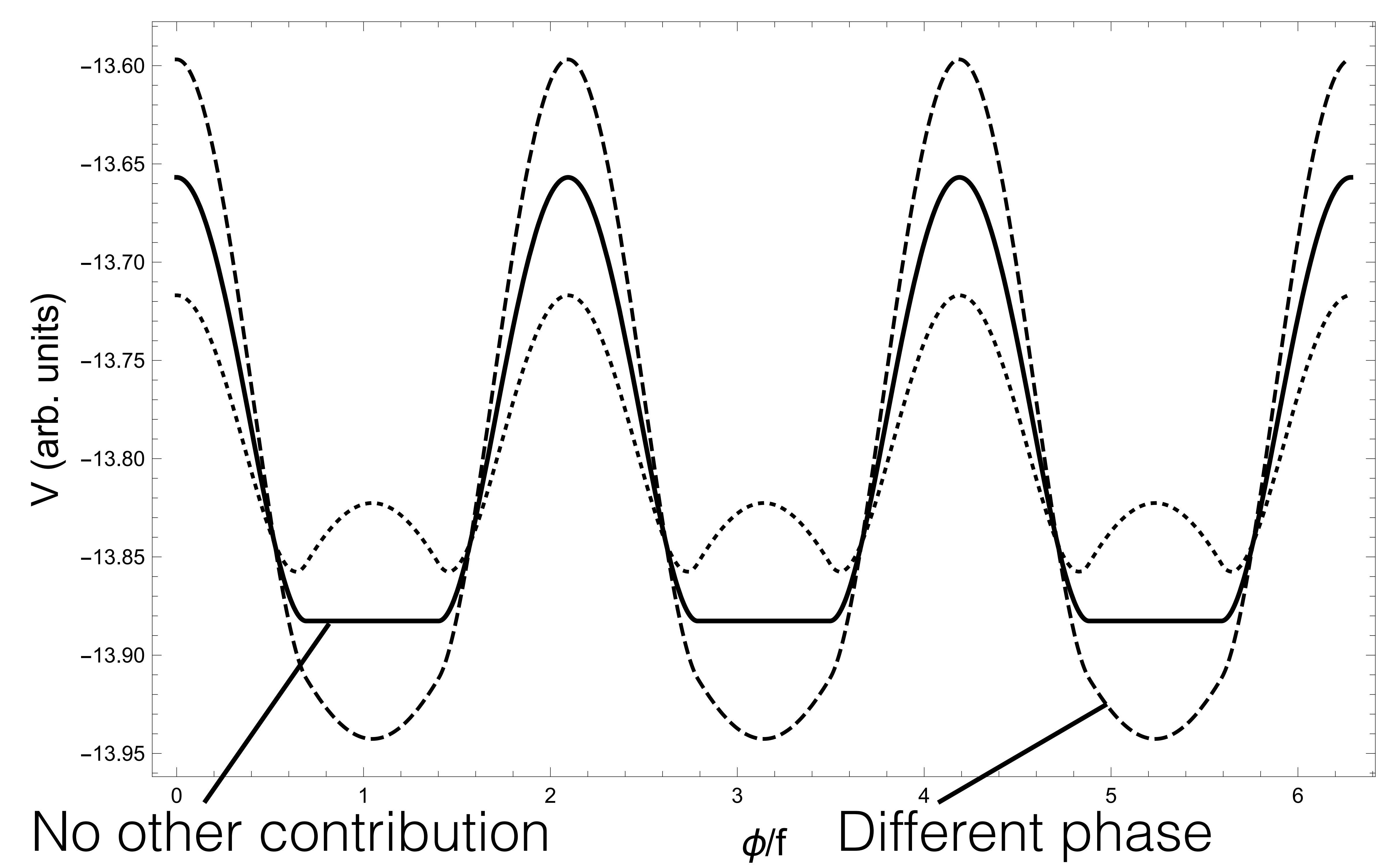
Tree Level Potential



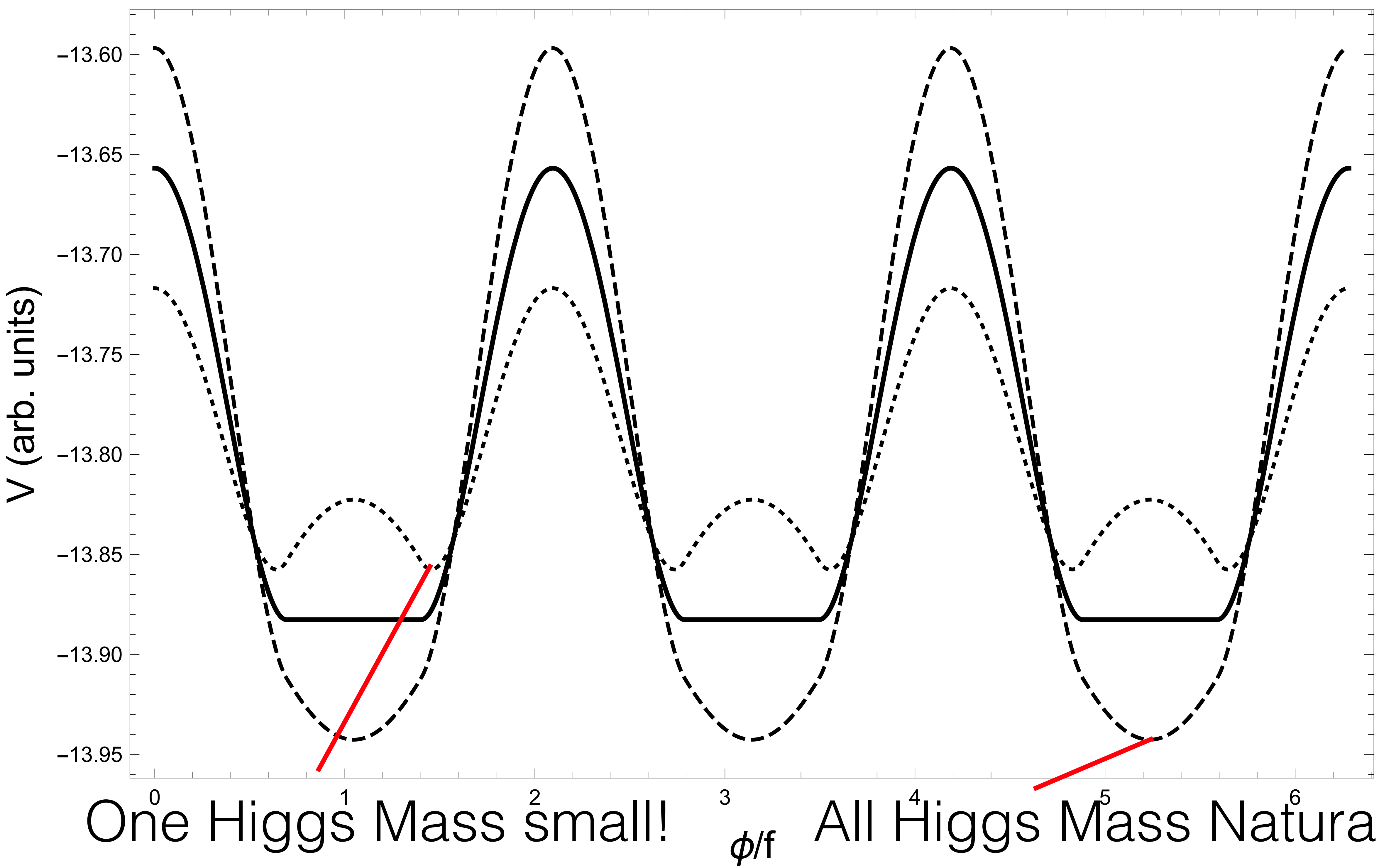
Tree Level Potential



Tree Level Potential



Tree Level Potential



One Higgs Mass small!

All Higgs Mass Natural

Tree Level Potential

- Contribution to potential from the Higgs has a moduli space
    - If all Higgs masses are negative, then it is a minimum regardless of value of the Higgs mass
- Higgs mass of zero is special!
    - It is at the edge of module space
- A small Higgs mass at the minimum is possible
    - If bare contribution pushes minimum to the edge of moduli space

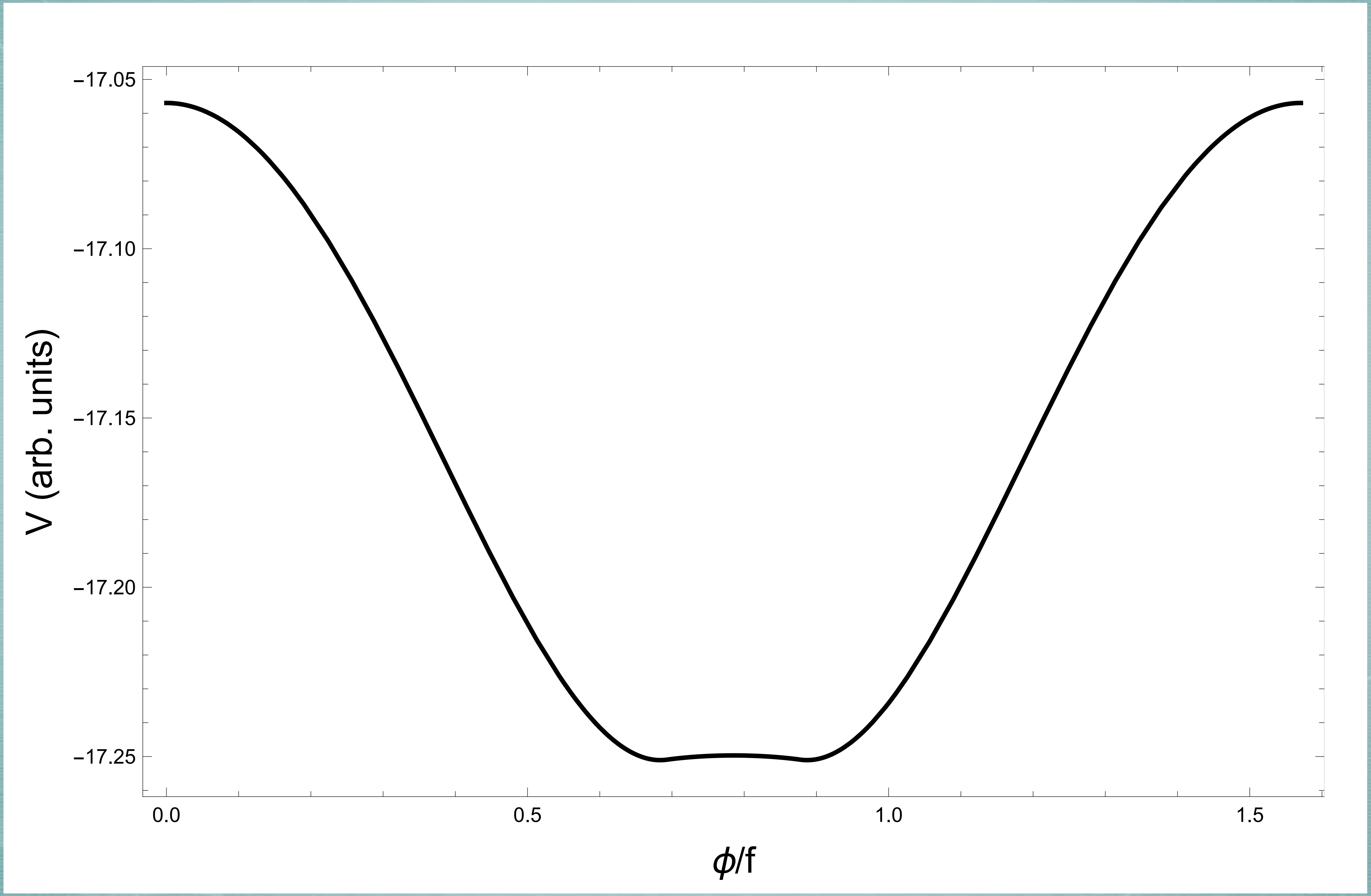
Tree Level Potential

- At tree level, choosing an exponentially suppressed bare potential leads to an exponentially light Higgs mass
- The light Higgs mass is always positive!
    - The stabilizing force is only present for positive Higgs masses

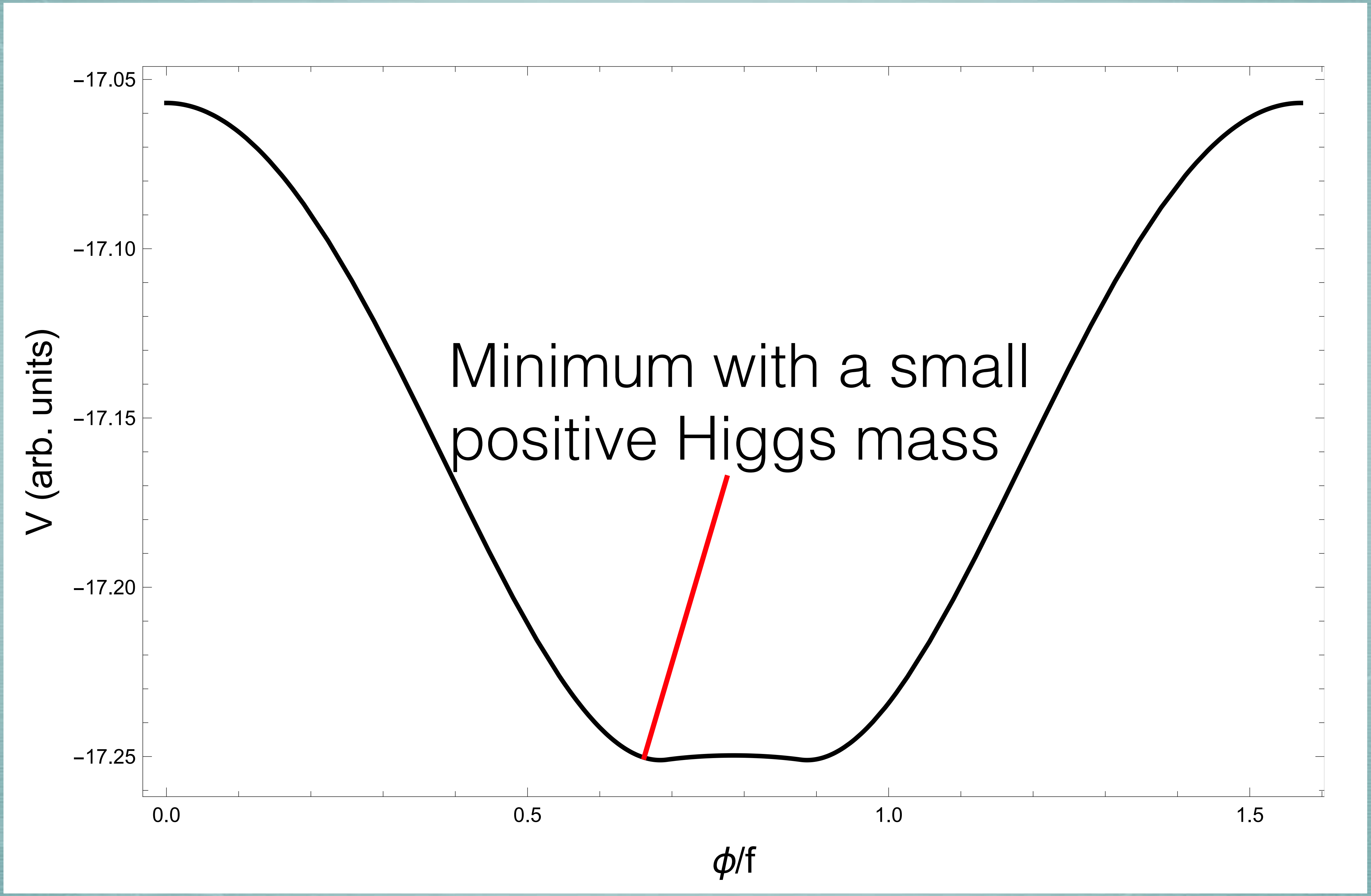
Negative Higgs Mass

- Two ways to get a negative Higgs mass
- Have a small amplitude high frequency contribution to the potential
    - Can shift the absolute minimum from small positive Higgs mass to small negative Higgs mass
    - Introduces other minimum

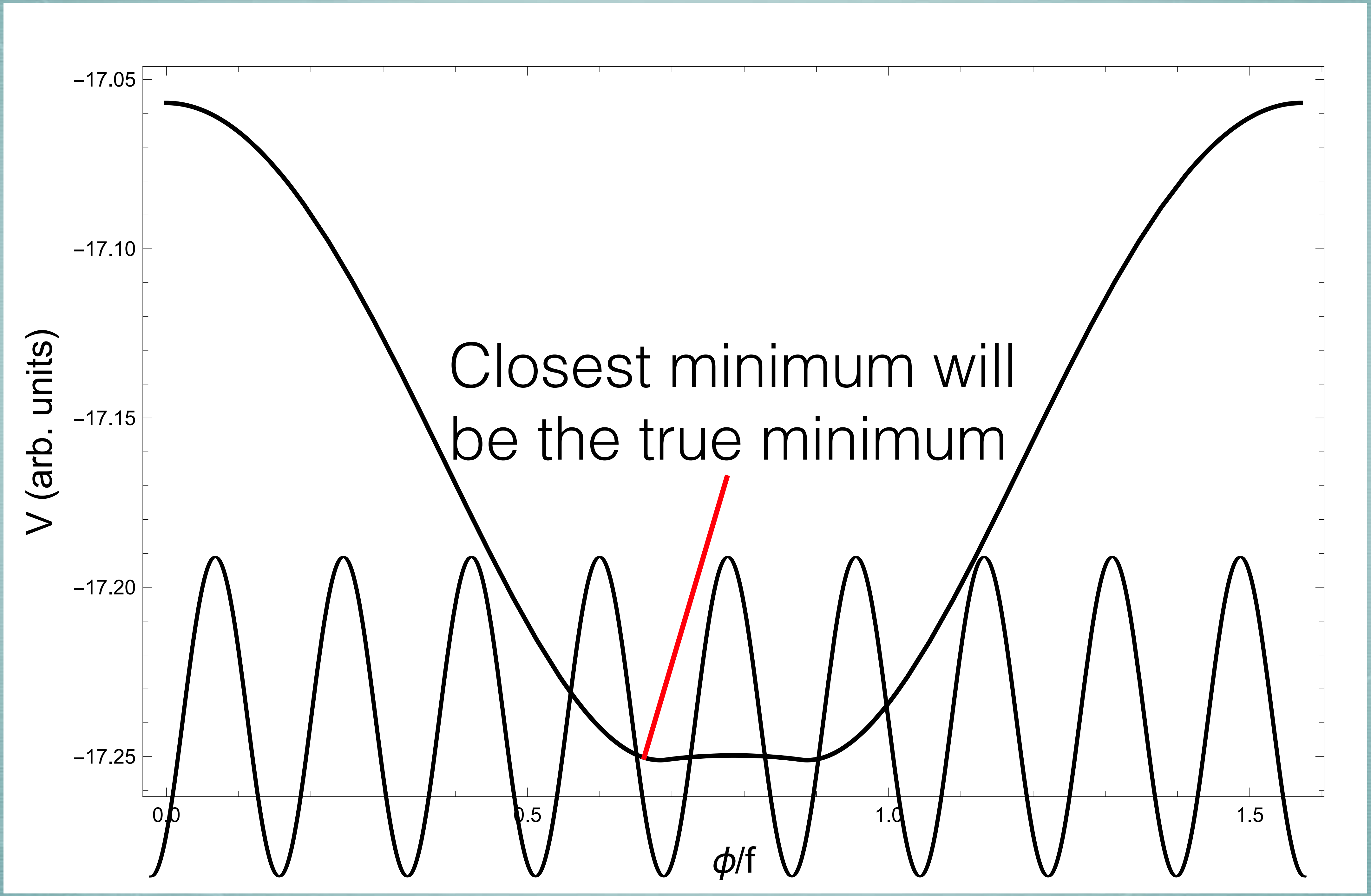
Negative Higgs Mass



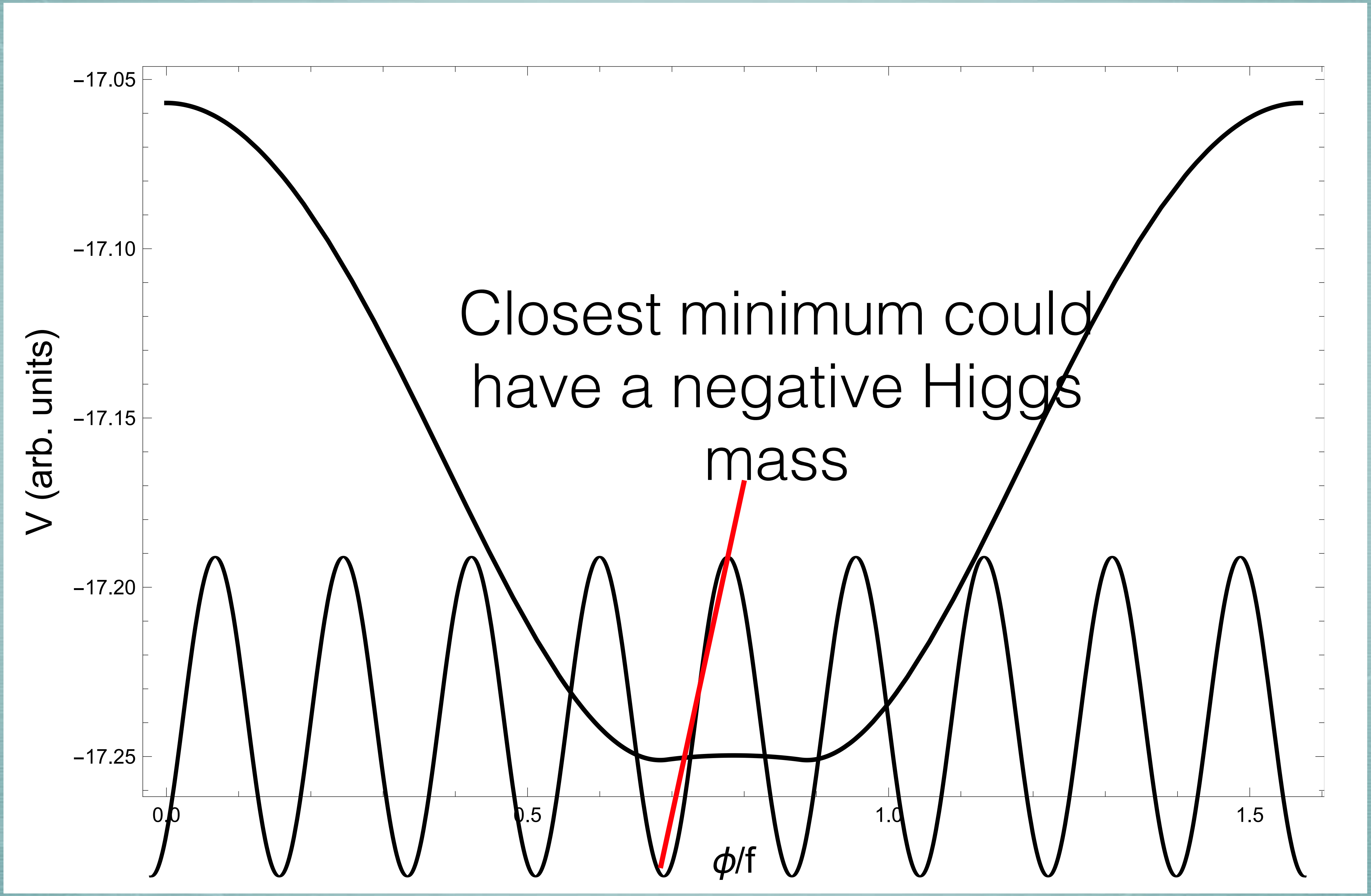
Negative Higgs Mass



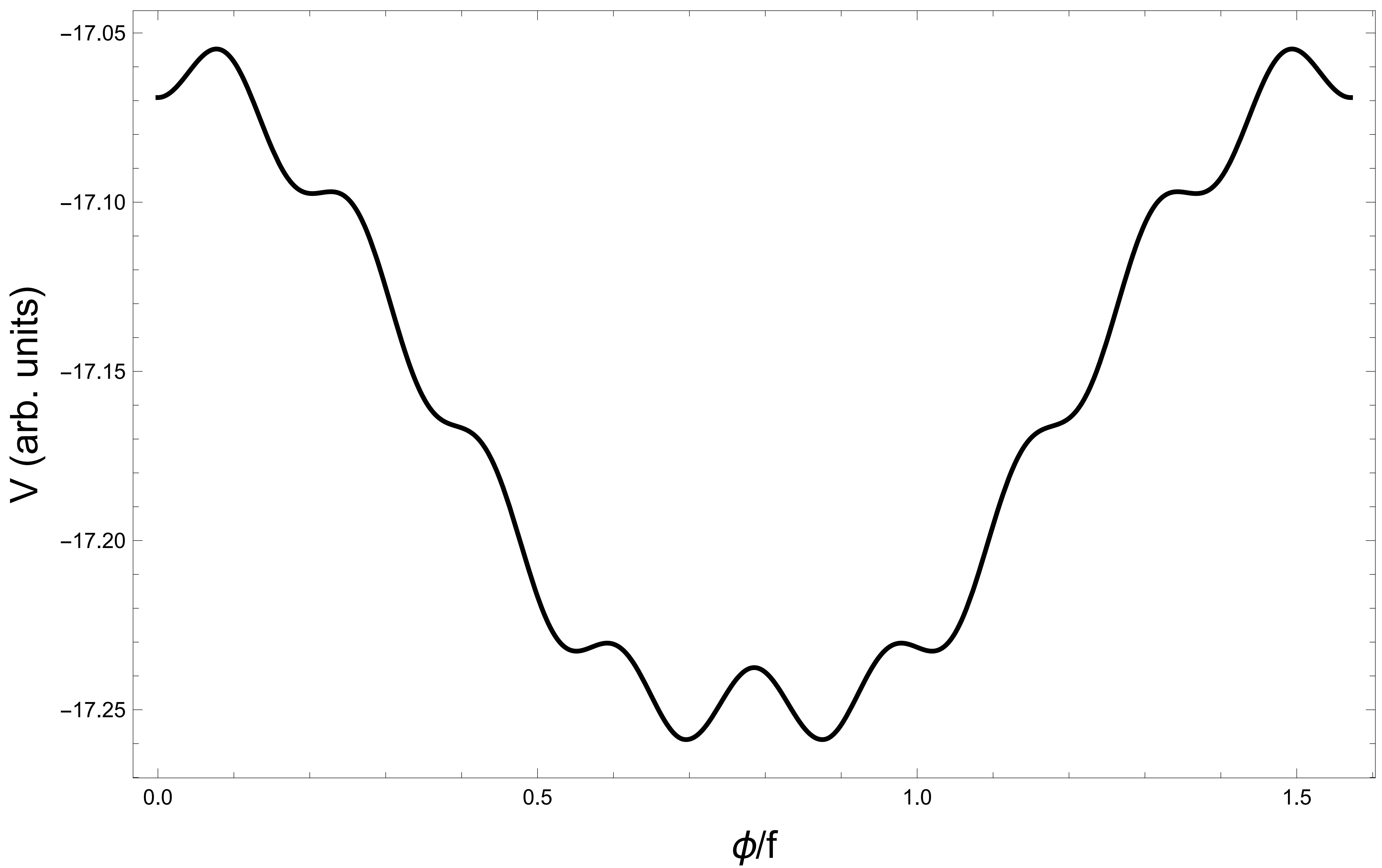
Negative Higgs Mass



Negative Higgs Mass



Negative Higgs Mass



Negative Higgs Mass

- Other option is to have 2N copies of the SM
    - New Z_2 symmetry
    - Same moduli couples to both Z_2 copies
- Have two identical light positive Higgs masses
- Softly breaking the new Z_2 allows one of the two light Higgs masses to be negative

1-loop Potential

- To solve Hierarchy problem, need additional contributions to the moduli potential to be small
- At 1-loop there is another contribution
- Thus, only solve Little Hierarchy problem

1-loop Potential

$$V_{\text{1-loop}} \sim \sum_k m_{H,k}^4(\phi) \log m_{H,k}^2(\phi)/\Lambda^2 \sim \frac{\epsilon^4}{N^2} \sin\left(\frac{N\phi}{f} + \theta\right)$$

- 1 - loop potential cannot be exponentially suppressed so that Higgs mass cannot be exponentially suppressed
- Can only solve the Little Hierarchy problem

Little Hierarchy Problem

- Completely independent of other Higgs couplings!
- Any Z_N invariant coupling works
- e.g. No need for gauge bosons partners as gauge couplings are automatically Z_N invariant

Little Hierarchy Problem

- Example : $N = 3$ or 4
  - UV cutoff of ~ 10 TeV
  - Natural Value of Higgs mass ~ TeV
  - Light Higgs mass of ~ 100 GeV

Little Hierarchy Problem

- What else needs to be duplicated?
  - Due to W/Z boson mass each Higgs charged under its own SU(2)
  - Must duplicate quarks and leptons - factor of 10 heavier than their SM respective partners
  - Must duplicate SU(3)/U(1) due to observational constraints or take a Natural SUSY approach

Pheno

- Cross quartics lead to mixing between Higgs
  - Signatures similar to that of Twin Higgs
  - Mixing tends to be smaller than in Twin Higgs
  - No EW symmetry breaking fine-tunings associated with Twin Higgs
- Only necessarily coupled particle is moduli
- Can be extremely weakly coupled as solution is completely independent of f
  - Much like the axion

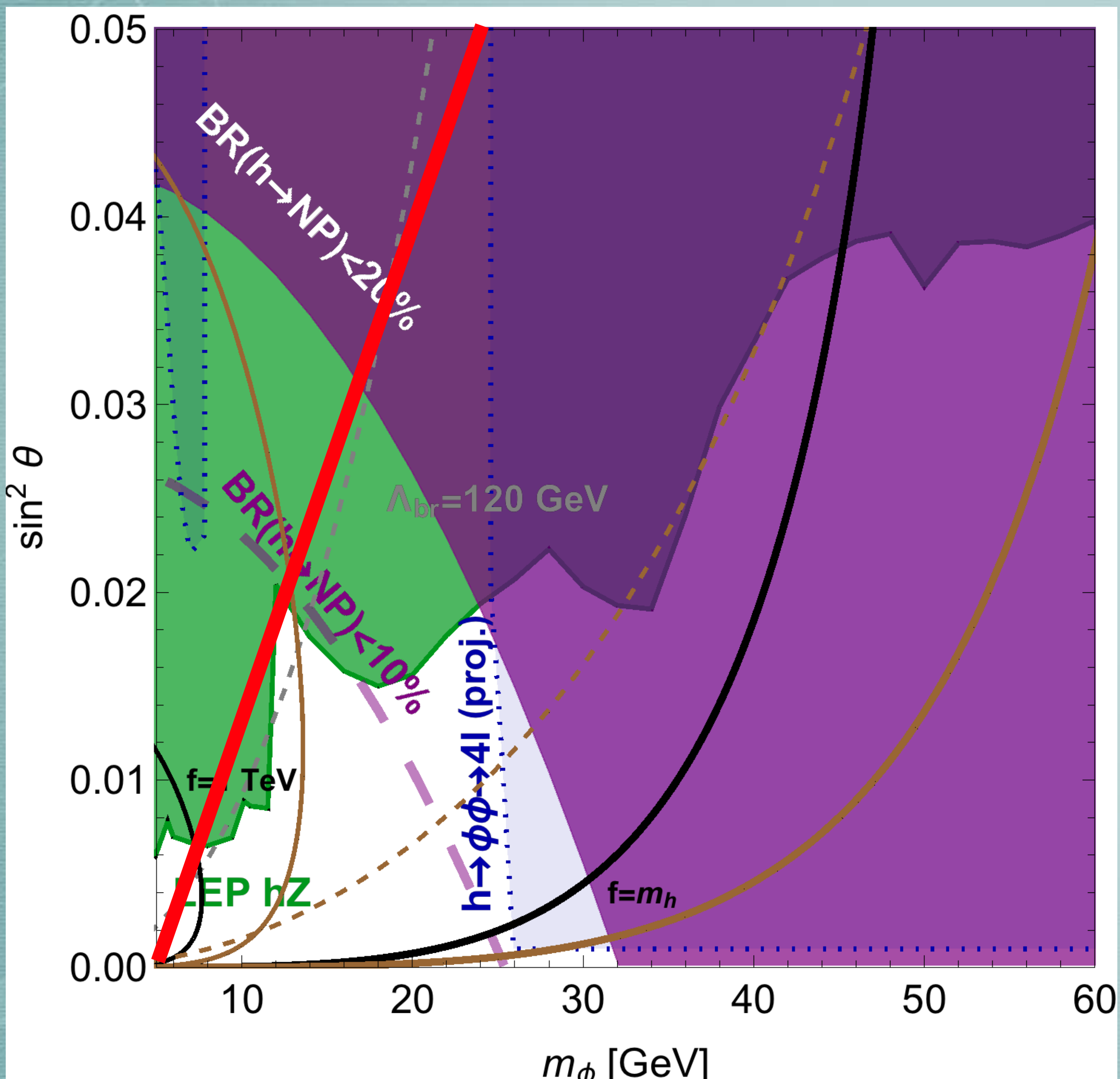
Moduli

- Parametrically, mass and mixing with Higgs

$$m_\phi \sim \mathrm{TeV}^2/f$$

$$\theta \sim \frac{\epsilon^2}{fv} \sim \frac{10\mathrm{TeV}}{f}$$

Moduli



Beam Dump

Rare meson decay

$$f \gtrsim 10^7\,\mathrm{GeV}$$

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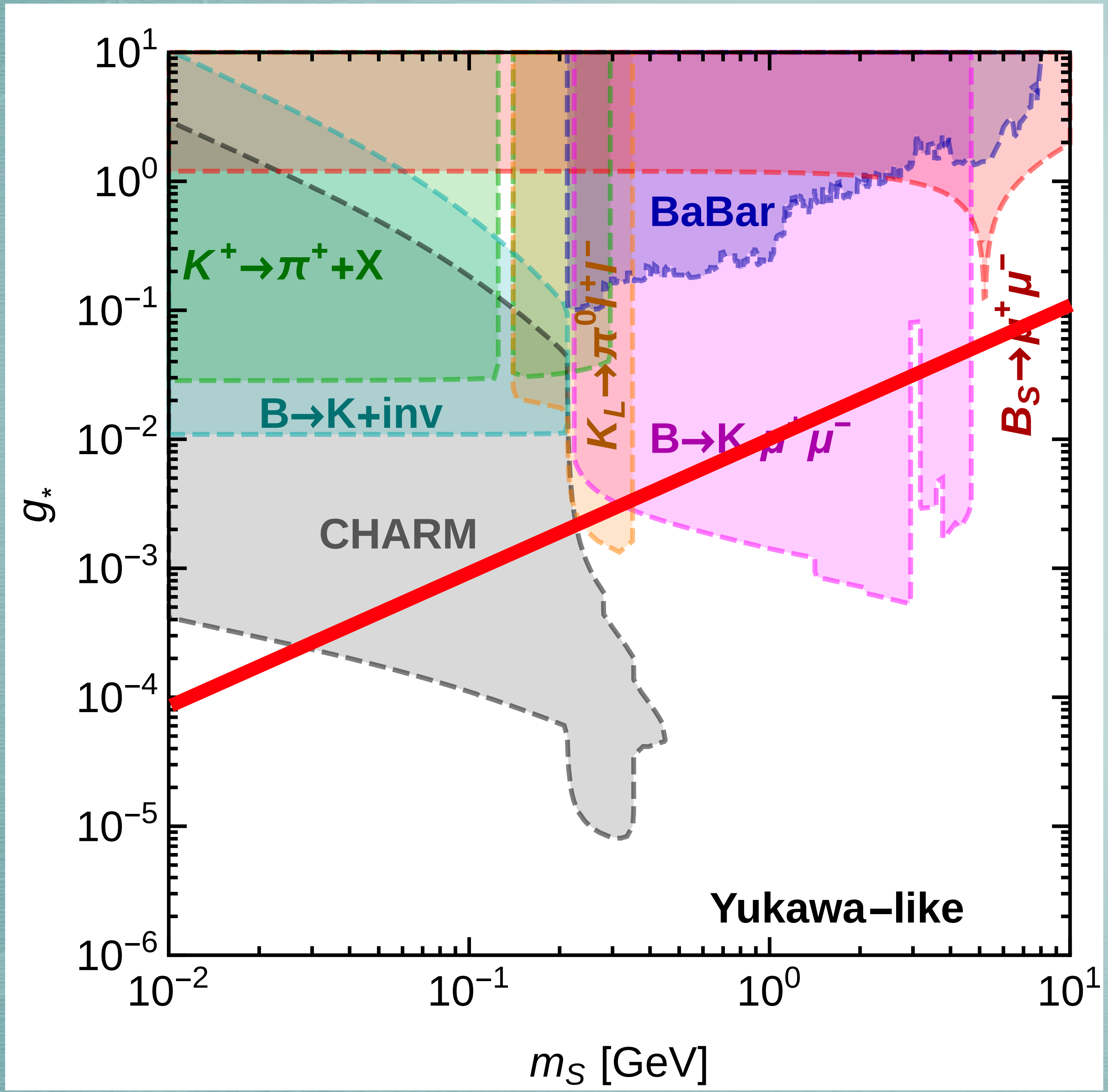
Moduli



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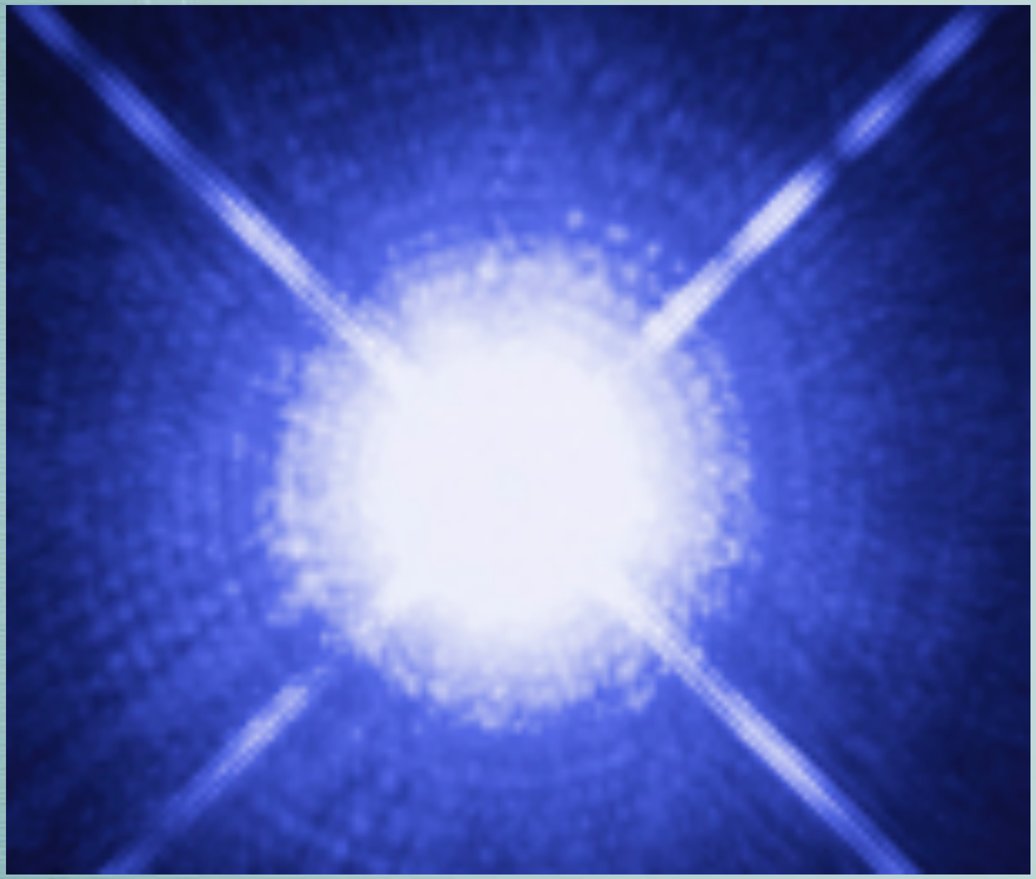
Moduli

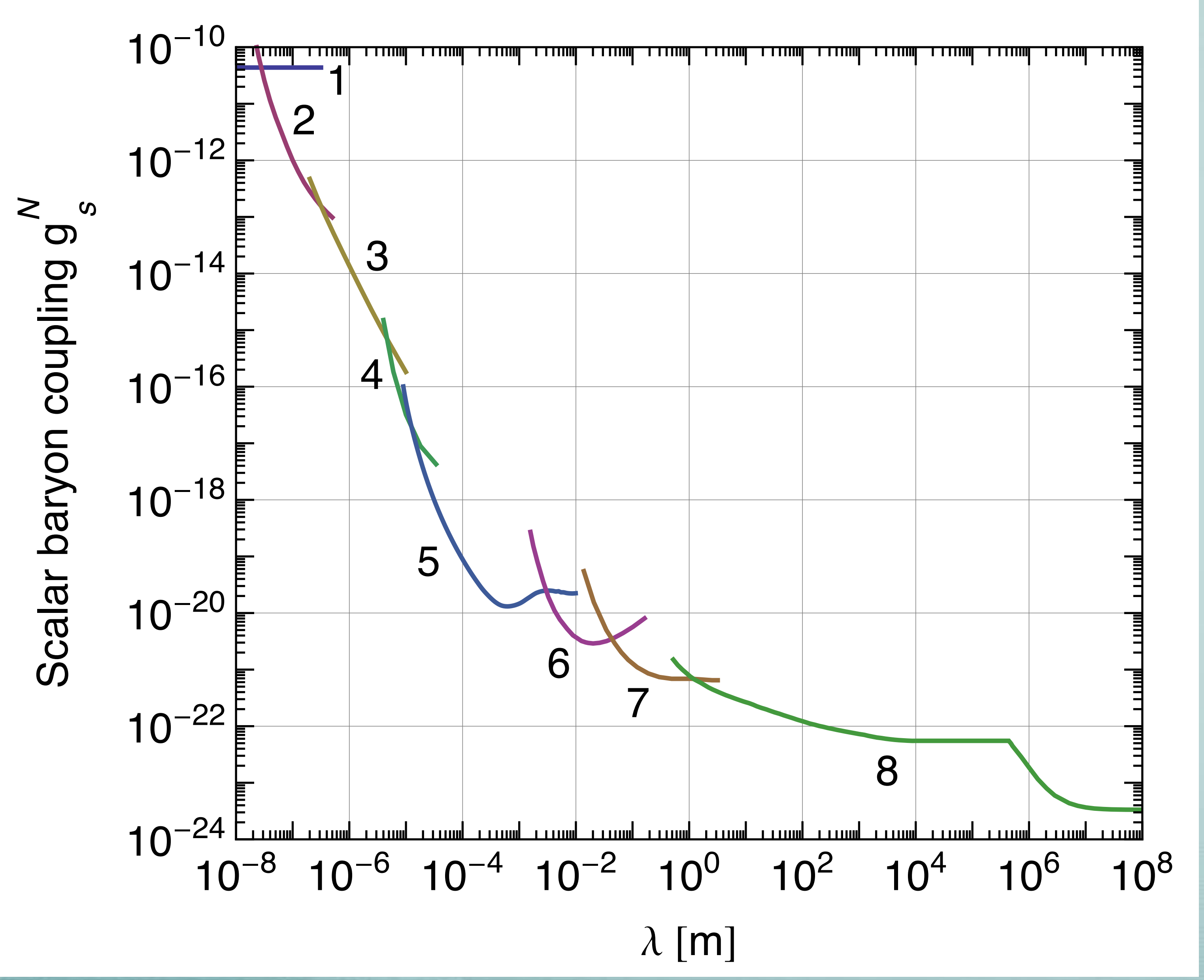


Stellar Cooling

$$10^{13}\ \mathrm{GeV} \gtrsim f \gtrsim 10^{10}\ \mathrm{GeV}$$

Constrains three orders of magnitude

Moduli



Fifth Force

$$f \gtrsim 10^{17}\ \mathrm{GeV}$$

1205.1776

Moduli

$$10^{10}\,\mathrm{GeV} \gtrsim f \gtrsim 10^{7}\,\mathrm{GeV} \qquad 10^{17}\,\mathrm{GeV} \gtrsim f \gtrsim 10^{13}\,\mathrm{GeV}$$

$$0.1\,\mathrm{GeV} \gtrsim m_\phi \gtrsim 100\,\mathrm{keV} \qquad 100\,\mathrm{eV} \gtrsim m_\phi \gtrsim 10\,\mathrm{meV}$$

- Two rough regions of space for the moduli
  - Small hole around 5-10 GeV
- Moduli can be dark matter and has roughly all of the same features as the axion
  - Slightly heavier, scalar vs pseudo scalar, overclosing the universe, etc.

Conclusion

- Naturalness has been THE paradigm for particle physics for a long time
  - Still new solutions being discovered!
  - Gives hope that there might exist other new solutions
- Can solve Little Hierarchy problem using Higgs partners rather than top partners!
  - Moduli can be probed in new and exciting ways
  - Different ways to test these theories than what is currently being explored