Karlsruher Institut für Technologie

Improved theoretical constraints on BSM Models

Florian Staub | Planck 2018, Bonn, 21st May 2018

KARLSRUHE INSTITUTE OF TECHNOLOGY, ITP & IKP

Mainly based on

FS; Phys.Lett. B776 (2018) 407-411, [1705.03677]
Krauss, FS; Eur.Phys.J. C78 (2018) no.3, 185, [1709.03501]
Braathen, Goodsell, Krauss, Opferkuch, FS; Phys.Rev. D97 (2018) no.1, 015011, [1711.08460]
Goodsell, FS; [1805.TODAY]
Krauss, FS; [1805.TODAY]

Outline

Introduction

- (Non-supersymmetric) models with extended Higgs sectors usually introduce new couplings

Simplest model: real singlet extension

$$V = \frac{1}{2}\lambda_H |H|^4 + \frac{1}{2}\lambda_{HS}|H|^2 S^2 + \frac{1}{2}\lambda_S S^4 + m_H^2 |H|^2 + \frac{1}{2} m_S^2 S^2$$

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- The couplings are **theoretically constrained by**
  - **Unitarity constraints**:
    $\{|\lambda_H|, |\lambda_{HS}|, \frac{1}{2}|6\lambda_S + 3\lambda_H \pm \sqrt{4\lambda_{HS}^2 + 9(-2\lambda_S + \lambda_H)^2}|\} < 8\pi$
  - **Vacuum stability constraints**: $\lambda_H > 0$, $\lambda_S > 0$, $\lambda_{HS} > -4\sqrt{\lambda_H \lambda_S}$
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These constraints are often derived/applied using important **assumptions/approximations**

Tree-level perturbative unitarity

Tree-level perturbative unitarity

- The tree-level **perturbative unitarity** conditions impose constraints on $2 \to 2$ **scattering processes**
- The scattering matrix $\mathscr{M}$ is decomposed in partial waves as

$$\mathscr{M} = 16\pi \sum_J (2J+1) a_J P_J(\cos\theta) \simeq 16\pi a_0$$

J: is the angular momentum; $P_J(\cos\theta)$: Legendre polynomials

- To first order and at tree-level, the **maximal eigenvalue of the matrix** is constrained to

$$|(a_0^{\max})| \leq \frac{1}{2}$$

Unitarity constraints in BSM literature

- In order to find $a_0^{\max}$ one needs to calculate **all possible scalar $2 \to 2$ processes**:

The scattering matrix in BSM models can become big!

(THDM: 36×36, Georgi-Machacek: 91×91)

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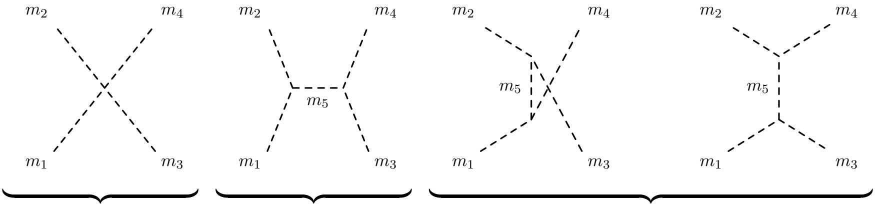

$$\frac{\lambda\sqrt{1-\frac{4m^2}{s}}}{8\pi} \to \frac{\lambda}{8\pi}$$

$$\frac{\kappa^2\sqrt{1-\frac{4m^2}{s}}}{8\pi(M^2-s)} \to \mathbf{0}$$

$$\frac{\kappa^2\log\left(\frac{M^2}{-4m^2+M^2+s}\right)}{8\pi\sqrt{s\left(s-4m^2\right)}} \to \mathbf{0}$$

Large s approximation

Common approach

The unitarity constraints for BSM models are usually derived under **assumption that the scattering energy is very large**:

- → only point interactions are non-vanishing
- → effects from **cubic couplings neglected**
- → effects from **EWSB neglected**

It seems that the validity of the large s approximation was never checked!

Full Calculation

[Goodsell,FS,1805.TODAY]

- **We perform a full calculation** for mass eigenstates including propagator diagrams
- We are exactly reproducing the SM results by Lee, Quigg & Thacker

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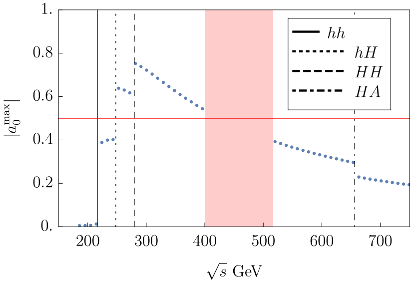
- We are exactly reproducing the SM results by Lee, Quigg & Thacker
- In order to be conservative, **we generously cut out poles**,e.g



→ One can see already an enhancement at small $\sqrt{s}$!

Singlet model: Estimate

For the singlet model, we want to get an estimate for $a_0(hS \to hS)$:

- The full expression is:

$$16\pi a_0(hS \to hS) = -\frac{\lambda_{HS}}{16\pi s\left(s-m_S^2\right)\sqrt{m_h^4-2m_h^2\left(m_S^2+s\right)+\left(m_S^2-s\right)^2}}\times$$

$$\Bigg[-\left(m_h^4-2m_h^2\left(m_S^2+s\right)+\left(m_S^2-s\right)^2\right)\left(-\lambda_{HS}v^2+m_S^2-s\right)$$

$$+\lambda_{HS}sv^2\left(s-m_S^2\right)\log\left(\frac{m_h^4-2m_h^2m_S^2+m_S^4-m_S^2 s}{s\left(2m_h^2+m_S^2-s\right)}\right)$$

$$+3m_h^2 s\left(s-m_S^2\right)\log\left(\frac{m_h^2 s}{m_h^4-m_h^2\left(2m_S^2+s\right)+\left(m_S^2-s\right)^2}\right)\Bigg]$$

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- If we assume a **light singlet**

$$v^2 \lambda_{HS}^2 \gg m_h^2 \gg m_S^2$$

we find for s **close to the threshold**

$$16\pi a_0(hS \to hS) \sim \frac{\lambda_{HS}^2 v^2}{m_h^2} < 8\pi$$

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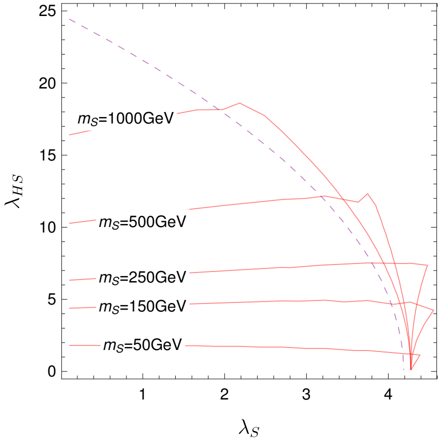
There is an enhancement of $\frac{\lambda_{HS} v^2}{m_h^2}$ for light singlet masses at small s

Singlet model: Numerical result



- Much stronger constraints on λ_{HS}
- We used $\sqrt{s} < 2500$ GeV which causes weaker constraints for λ_S

THDMs: light Higgs Window

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- Is this the case also in THDMs? → **Let's consider $M_H < 125$ GeV**

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$$\frac{a_0^{\max,s}}{a_0^{\max,s\to\infty}} \sim \frac{2}{3}\sqrt{\frac{m_A}{m_h}}\log\frac{m_A}{m_h}$$

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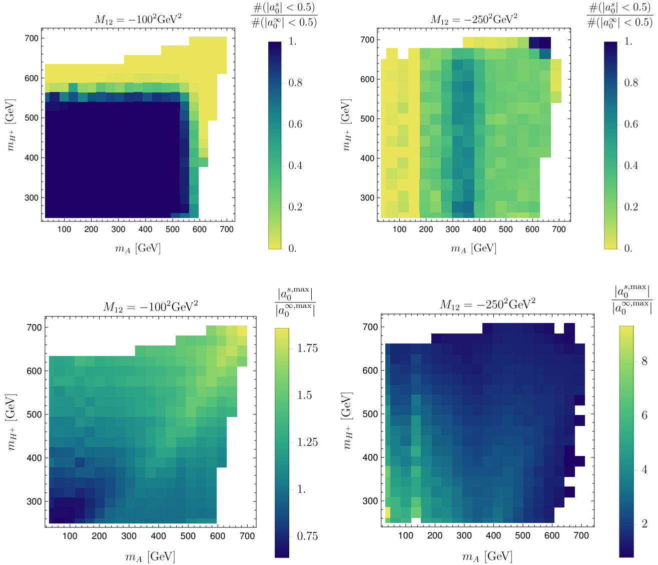
Large enhancement for large m_A or $|M_{12}|$!

Numerical Results for $m_H < m_h$

[Goodsell,FS,1805.TODAY]



- First row: ratio of points which are ruled out by the new constraints
- Second row: average enhancement

Unitarity Constraints

Overview of other results

We checked many other models/scenarios and often found that the s**-dependenent constraints can become much stronger**:

- **THDM**: [Goodsell,FS,1805.TODAY]
  - Also stronger constraints for heavy Higgs scenarios
  - (effective) cubic terms are usually more important than loop corrections
- **Triplet models**: [Krauss,FS,1805.TODAY]
  - Mass splitting between neutral and charged triplet stronger constrained
  - Mixing between triplet and doublet stronger constrained
- **Georgi Machacek model**: [Krauss,FS,1805.TODAY]
  - Strong constraints on heavy Higgs masses
  - Strong constraints on triplet contributions to ew VEV

Vacuum stability checks

Vacuum stability

'Our vacuum' might just be a local minimum in the scalar potential:

1. A **deeper minimum** might exist with different Higgs VEVs, maybe even with charge and colour breaking
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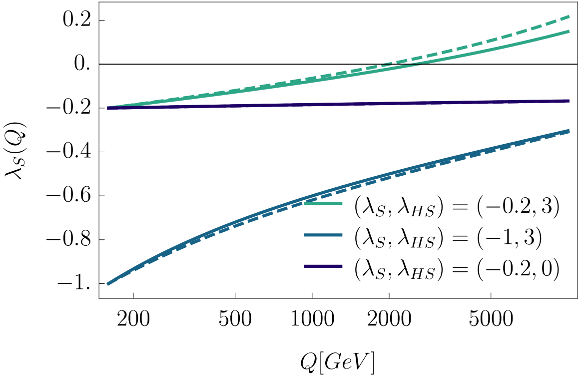
Presence of large couplings

Radiative corrections can be crucial and change the situation at the loop-level

RGE running

We can already see from the RGE running that a tree-level check might be misleading:



→ Small negative values run quickly positive in the presence of other large quartic couplings

Vacuum stability at loop-level in the THDM

We checked the **vacuum stability of the THDM** using:

1. The **RGE improved potential** to check the high-energy behaviour
2. The **full one-loop effective potential** to check for deeper minima close to the ew scale

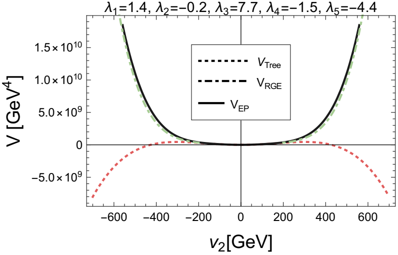
Numerical analysis

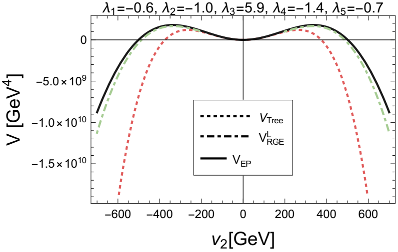
The check of the vacuum stability has been done **fully numerically** using the combination `SARAH`, `SPheno`, `Vevacious`

Comparison for single points

- We can compare the three predictions:
  1. Tree-level potential
  2. One-loop effective potential
  3. RGE improved potential

- We usually find a **good agreement between both loop calculations**:



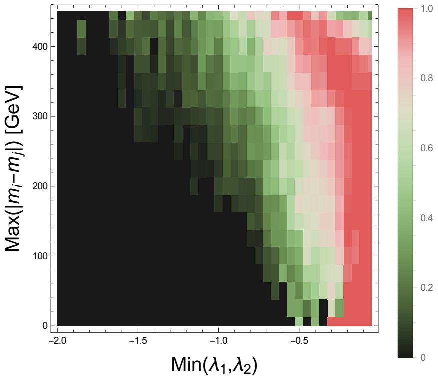


Misidentification rate

[FS,1705.03677]

- The loop effect stabilize the vacuum

→ **Tree-level checks could give wrong results**



fraction of points which:

- fail tree-level checks
- pass loop-level checks

Tree-level checks are (often) too strong for small neg. values of quartics!

Similar results

We concentrated here on UFB directions in THDMs.

Other results are:

- **THDMs**: [FS,1705.03677]
  - Meta-stable points: more than 90% of the points become stable at loop-level
- **Georgi-Machacek-Model**: [Krauss,FS,1709.03501]
  - Similar behaviour to THDMs

  → Large fraction of points become stabilized at loop-level

Finding the cut-off

Matching and Running

- To test the **high-scale behaviour** of a model, **RGE running** is needed
- The RGEs need the $\overline{\text{MS}}$ parameters as input

→ The **$\overline{\text{MS}}$ parameters** can be calculated from the **masses/mixing angles** (**'Matching'**)

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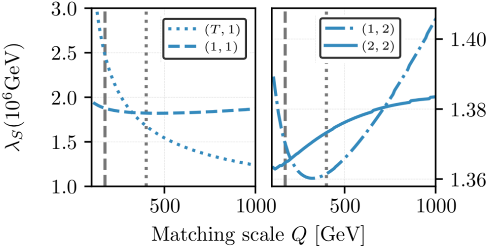
However, this gets only the leading logarithm correct!

Scale Dependence

[Braathen,Goodsell,Krauss,Opferkuch,FS,1711.08460]

- The **results shouldn't depend** (significanlty) on the choice of the **matching scale**
- **Compare** scale dependence explicitly by comparing different **loop-levels**:



- 1-loop Running & Tree-Level matching suffer from a huge uncertainty
- Best results for 2-loop running & matching

Checking the cut-off

Singlet-Model: cut-off scale

[Braathen,Goodsell,Krauss,Opferkuch,FS,1711.08460]

The **cut-off scale** depends strongly on the **loop-levels** of running/matching:

(n,m)	λ	λ_S	λ_{SH}	Λ [GeV]
$(T,1)$	0.34	1.1	-1.1	$3.2\cdot10^3$
$(T,2)$				$1.3\cdot10^4$
$(1,1)$	0.33	0.24	-0.97	$3.2\cdot10^8$
$(1,2)$				$2.5\cdot10^9$
$(2,1)$	0.32	0.18	-0.94	$2.5\cdot10^{10}$
$(2,2)$				$2.0\cdot10^{11}$

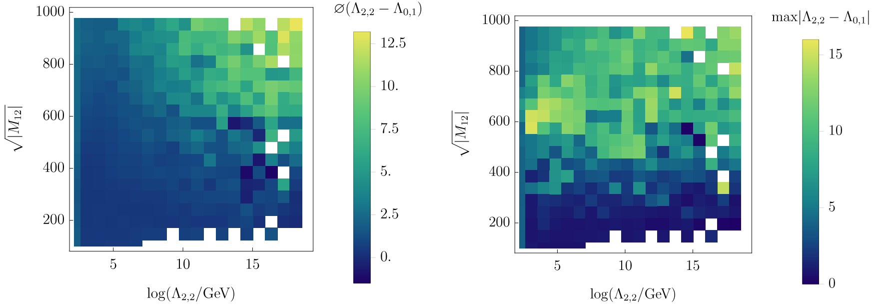
Example for Z_2 breaking case with $M_H = 700$ GeV and $\tan\alpha = 0.1$

The first order approximation (1-loop running, tree-level matching) can be **wrong by many orders of magnitude**

Checking the cut-off

THDM: changes in the cut-off

[Krauss,Opferkuch,FS,to appear]



The **cut-off can in- or decrease** at higher loop-levels:

- The **effects don't wash out** when averaging over **many points**
- **Huge differences** are visible for **specific points**

Summary

- Theoretical constraints on new couplings models are important
- **Important corrections** can appear compared to the common approaches:
  - **Unitarity constraints at small s** can be much stronger compared to the $s \to \infty$ limit
  - The **scalar potential can be stabilised by loop-effects** and UFB directions disappear
  - Predictions based on **2-loop running& matching are much more accurate** and can be **very different**

Tools

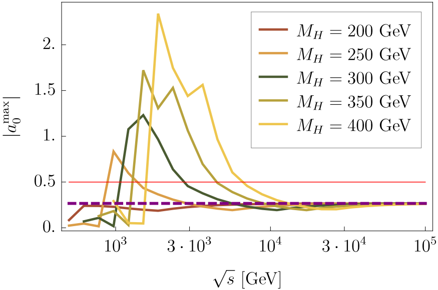
The tools to study the improved constraints for many models are public

- `SARAH`: `sarah.hepforge.org`
- `SPheno`: `spheno.hepforge.org`
- `Vevacious`: `vevacious.hepforge.org`

Backup

s Dependence

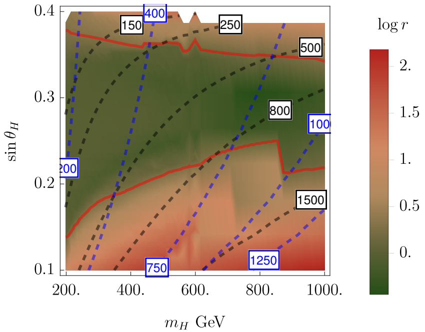
Assuming a fixed s, we find

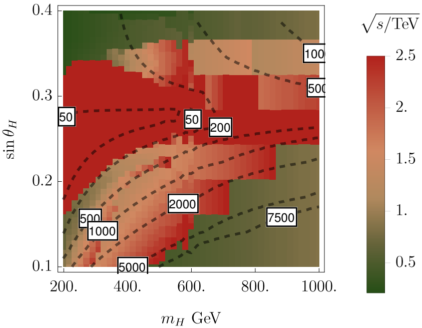


Finite s

We see that the scattering at small s can be highly enhanced compared to the large s limit!

Comparison between old and new limits





- $\log(a^s/a^{s\to\infty})$
- contours: values of m_3, m_5
- red lines: exclusion limit from new constraints

- best scattering energy
- contours: values of M_2

Vector-like extension

[Braathen,Goodsell,Krauss,Opferkuch,FS,1711.08460]

Considering

$$L = L_{\rm SM} - \left(Y_t' Q' \cdot H t' + \tilde{Y}_t' \tilde{Q}' \cdot \overline{H} \tilde{t}' + m_T \tilde{t}' t' + m_Q \tilde{Q}' Q' + \text{h.c.} \right)$$

we can check at which scale λ runs negatve: