Regularization without a renormalization scale

Paweł Olszewski

Based on:

D.M. Ghilencea, Z. Lalak, PO / Phys.Rev. D96 (2017) no.5, 055034
Standard Model with spontaneously broken quantum scale invariance

D.M. Ghilencea, Z. Lalak, PO / Eur.Phys.J. C76 (2016) no.12, 656
Two-loop scale-invariant potential and quantum effective operators

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References

S. Deser, “Scale invariance and gravitational coupling,” Annals Phys. 59 (1970)

F. Englert, C. Truffin and R. Gastmans, “Conformal Invariance in Quantum Gravity,” Nucl. Phys. B 117 (1976)

M. Shaposhnikov and D. Zenhausern, “Quantum scale invariance, cosmological constant and hierarchy problem,” Phys. Lett. B 671 (2009) 162 [arXiv:0809.3406 [hep-th]]

M. E. Shaposhnikov and F. V. Tkachov, “Quantum scale-invariant models as effective field theories,” arXiv:0905.4857 [hep-th]

C. Tamarit, “Running couplings with a vanishing scale anomaly,” JHEP 1312 (2013) 098 [arXiv:1309.0913 [hep-th]].

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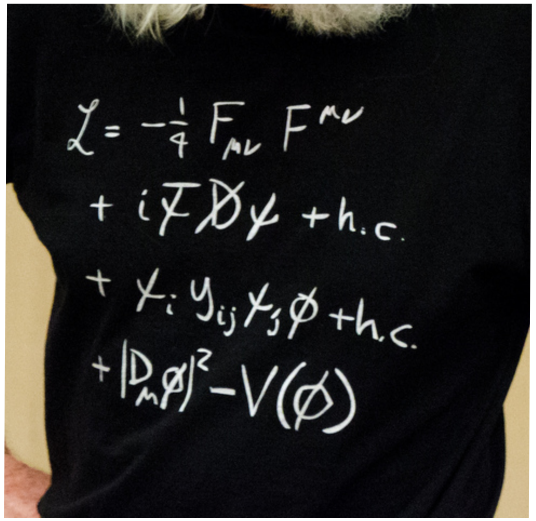
C. Tamarit, “Running couplings with a vanishing scale anomaly,” JHEP 1312 (2013) 098 [arXiv:1309.0913 [hep-th]].

Plan

1) Anomaly of the scale symmetry
2) Un-doing Coleman-Weinberg dimensional transmutation
3) Quantum scale symmetry
4) Scale symmetric Standard Model

Mass parameter in the SM



$$V(\phi) = m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

„Unnaturally” small.

Forbid dimensionful parameters altogether.

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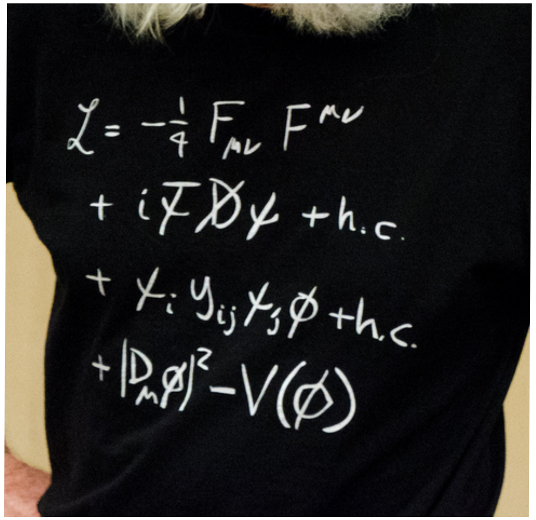

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Higgs mass via Higgs portal: new scalar field σ with a VEV

$$-\lambda_m \sigma^2 H^\dagger H \ , \quad \langle \sigma \rangle \neq 0$$

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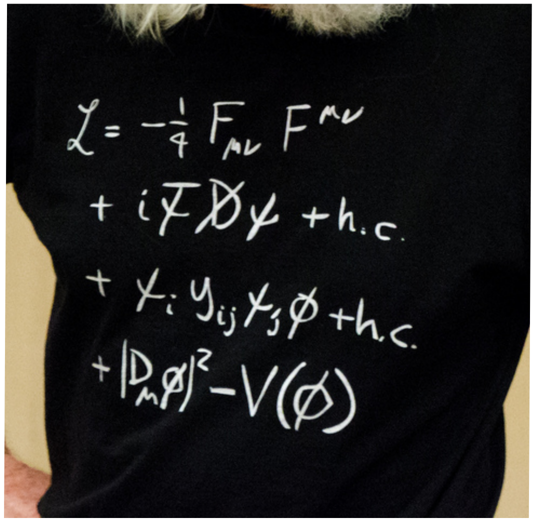

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Global symmetry wrt. dilatations:

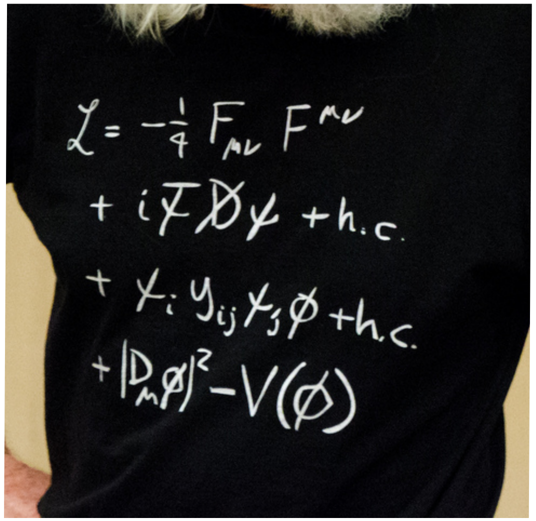
$$[D, P_\mu] = i P_\mu$$

$$\begin{aligned} U(a) &= e^{iaD} \\ \Phi(x) \to U(a)\Phi(x)U^{-1}(a) &= e^{ad_\Phi}\Phi(e^a x) \\ \delta\Phi(x) = i[D, \Phi] &= (d_\Phi + x^\nu \partial_\nu)\,\Phi(x) \end{aligned}$$

Higgs mass via Higgs portal: new scalar field ${\color{blue}\sigma}$ with a VEV

$$-\lambda_m {\color{blue}\sigma}^2 H^\dagger H \ , \qquad \langle {\color{blue}\sigma} \rangle \neq 0$$

Mass parameter in the SM



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Higgs mass via Higgs portal: new scalar field σ with a VEV

$$-\lambda_m \sigma^2 H^\dagger H \ , \quad \langle \sigma \rangle \neq 0$$

The symmetry is anomalous!

Anomaly of the scale symmetry

A symmetric action $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\phi^4$$

Noether current $$D^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)}(x^\nu \partial_\nu \phi_j + d_\phi \phi_j) - x^\mu \mathcal{L}$$

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$$T^\mu{}_\mu = \partial_\mu D^\mu = d_\phi\, \phi_j \frac{\partial V}{\partial \phi_j} - d\, V \;=\; \frac{d}{d\,\rho}\left[V(\rho^{d_\phi}\phi_j) - \rho^d V(\phi_j)\right]\Big|_{\rho=1}$$

trace of the
E-M tensor

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Homogeneity of the potential?

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$d = 4$

Quantum corrections require a dimensionfull regulator $$V = \frac{\lambda\phi^4}{4!} + \frac{1}{4(4\pi)^2}\left(\frac{\lambda\varphi^2}{2}\right)^2\left(\log\frac{\lambda\varphi^2}{2\,\overline{\mu}^2} - \frac{3}{2}\right)$$

$$T^\mu{}_\mu \;=\; \left(\phi\frac{\delta}{\delta\phi} - 4\right)V$$

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$$T^\mu{}_\mu \;=\; \left(\phi\frac{\delta}{\delta\phi} - 4\right)V \;=\; \frac{3\lambda^2}{(4\pi)^2}\,\frac{\phi^4}{4!} \;=\; \beta_\lambda{}^{(\text{1-loop})}\,\frac{\phi^4}{4!} \;\neq\; 0$$

Anomaly of the scale symmetry

$d = 4$

$$T^{\mu}_{\mu} \sim \boxed{\sum_g \beta_g \frac{\partial}{\partial g} \mathcal{L}_{\text{matter}}} + \boxed{\frac{-a}{(4\pi)^2} \tilde{R}_{\alpha\beta\gamma\delta} \tilde{R}^{\alpha\beta\gamma\delta} + \frac{c}{(4\pi)^2} C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta}}$$

(matter-related) scale anomaly

$g_{\mu\nu} = g_{\mu\nu}(x)$, gravitational anomaly

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Insertion of composite operators

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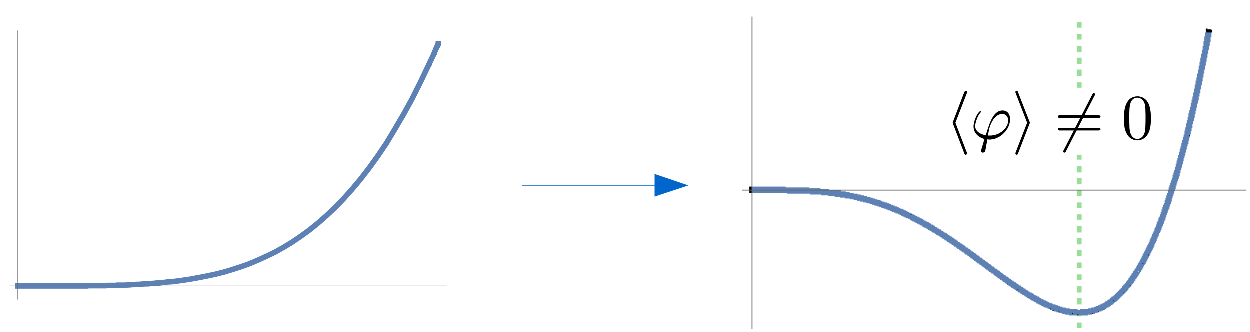
Insertion of composite operators

Dimensional transmutation by Coleman and Weinberg:

$$\varphi^4 \longrightarrow \varphi^4 + \alpha\,\varphi^4 \log \varphi^2$$



Un-doing the CW dim. transmutation with nonrenorm. interactions

$d = 4$

$$V^{(1)} \sim \phi^4 \log\frac{\phi^2}{\mu^2} + \overbrace{\left(-2\phi^4\frac{\sigma'}{\Lambda} + \phi^4\frac{\sigma'^2}{\Lambda^2} + \ldots\right)}^{\overset{!}{=} \phi^4 \log\left(\frac{\Lambda}{\Lambda+\sigma'}\right)^2}$$

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$$t \in \mathbb{R}$$

$$\mu = e^t \mu_0 = e^t \Lambda$$

$$\sigma \equiv \Lambda + \sigma'$$

$$\langle \sigma \rangle \equiv \Lambda$$

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$\sigma \equiv \Lambda + \sigma'$

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$i[D, \Lambda] = \Lambda$

$i[D, \mu] = \mu$

ren. scale charged under dilatations !

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$i[D, \mu] = \mu$

ren. scale charged under dilatations !

For consistency add also:

$$V^{(0)} += \sum_{n=1}^{\infty} \lambda_n \frac{\phi^{4+2n}}{(\Lambda + \sigma')^{2n}}$$

Un-doing the CW dim. transmutation with nonrenorm. interactions

$d = 4$

$$V_{\text{eff}}(\phi, \sigma) = V^{(0)} + V^{(1)} + \ldots$$

- regularized $\mu \sim \langle \sigma \rangle$
- nonrenormalizable $\Lambda \sim \langle \sigma \rangle$
- homogenous:

$$\left(\phi \frac{\delta}{\delta \phi} + \sigma \frac{\delta}{\delta \sigma} - 4 \right) \begin{array}{l} V^{(0)} \\ V^{(1)} \\ \vdots \end{array} = 0$$

Un-doing the CW dim. transmutation with nonrenorm. interactions

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$$\boxed{\left(\phi \frac{\delta}{\delta \phi} + \sigma \frac{\delta}{\delta \sigma} - 4\right) \begin{matrix} V^{(0)} \\ V^{(1)} \\ \vdots \end{matrix} = 0}$$

scale symmetry

Un-doing the CW dim. transmutation with nonrenorm. interactions

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$$\boxed{\begin{aligned} \left(\phi \frac{\delta}{\delta\phi} + \sigma \frac{\delta}{\delta\sigma} - 4\right) V^{(0)} &= 0 \\ V^{(1)} & \\ \vdots & \end{aligned}}$$

scale symmetry

Does adding
nonrenormalizable interactions
work this way at
all loop orders
*and **beyond the effective potential**?*

Yes! *Here's why...*

Un-doing the CW dim. transmutation with nonrenorm. interactions

Dim-Reg $d\!=\!4\!-\!2\varepsilon$

Consider *evanescent* interactions, $\lim_{\varepsilon\to 0} = 0$:

e.g. $V_{\rm new}^{(0)} = \varepsilon \left(\phi^2\sigma'^2 + \frac{\phi^2\sigma'^3}{\Lambda} + \frac{\phi^4\sigma'}{\Lambda} + \frac{\phi^4\sigma'^2}{\Lambda^2} + \ldots \right) + \varepsilon^2 \ldots$

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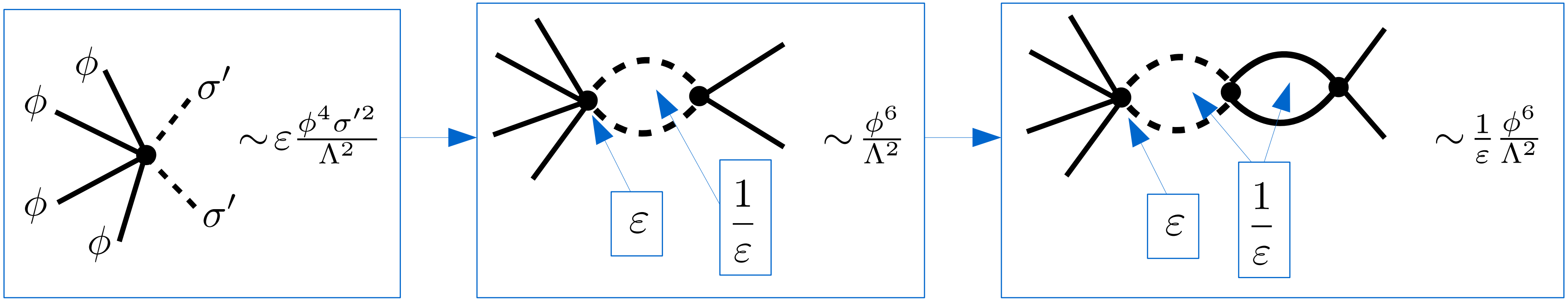
produces **new finite corrections** and **poles** in ε



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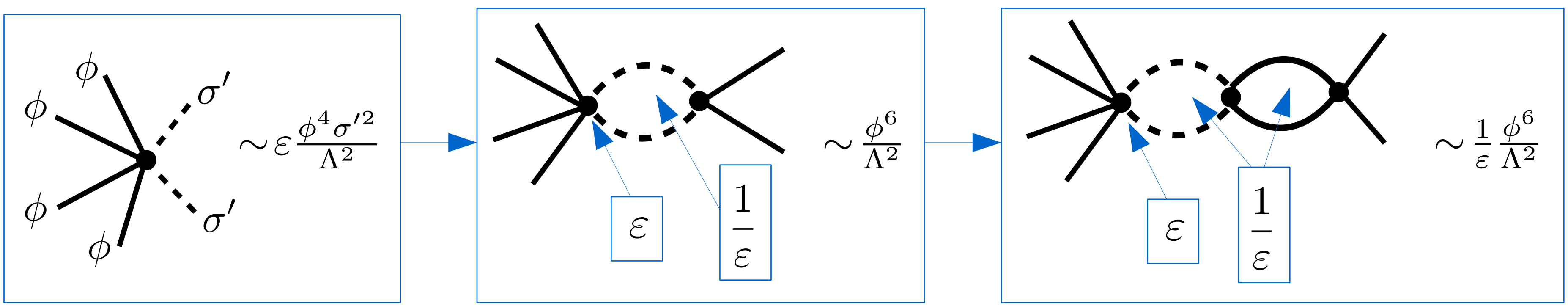

but **not new overlapping divergencies** $\longrightarrow$ new terms can be subtracktet away

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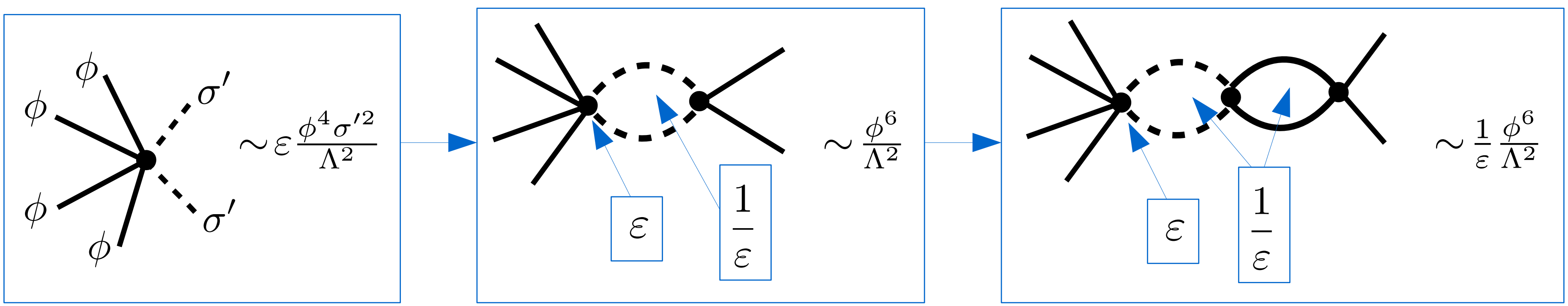

but **not new overlapping divergencies** → new terms can be subtracktet away

Conjecture 1.

evanescent interactions $\underset{\sim\,\varepsilon}{}$ **=** change of the renormalization scheme

$\sim \varepsilon$

(MS with evanescent interactions **=** new corrections and running wrt MS without evanescent int.)

Un-doing the CW dim. transmutation with nonrenorm. interactions

Dim-Reg $d = 4 - 2\varepsilon$

Recall calculation of the β functions, $\mu = e^{t}\mu_0\,, \quad \lambda = \lambda(t)\,,$

schematically: $\lambda\,\Phi^4 \longrightarrow \mu^{2\varepsilon}\underbrace{Z}_{1+\delta(\lambda)}\lambda\,\Phi^4\,, \quad [\Phi] = 1 - \varepsilon$

$$0 = \frac{\mathrm{d}\,(e^{t})^{2\varepsilon} Z(\lambda)\,\lambda}{\mathrm{d}\,t} \Rightarrow \frac{\mathrm{d}\,\lambda}{\mathrm{d}\,t} = \beta_\lambda$$

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and consider also the nonrenorm. part: $\displaystyle\sum_{n=0}^{\infty} (e^t)^{N_1\varepsilon} \mu_0^{N_2\varepsilon}\, \lambda_n \frac{\Phi^n}{\Lambda^{n-4}}$

where $[\mu_0] = [\Lambda] = 1$

hence normally: $N_1(n) = N_2(n) = n-2$

Un-doing the CW dim. transmutation with nonrenorm. interactions

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Conjecture 2.

One can choose $N_1(n) = a\,n + b \neq n-2$

as long as $4a + b \stackrel{!}{=} 2$

without modifying the β_{λ_n} functions.

Un-doing the CW dim. transmutation with nonrenorm. interactions

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Conjecture 2.

One can choose $N_1(n) = a\,n + b \neq n-2$

as long as $4a + b \overset{!}{=} 2$

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$\longrightarrow$

Henceforth take

$N_1(n) = const = 2$

Un-doing the CW dim. transmutation with nonrenorm. interactions

Dim-Reg $d=4-2\varepsilon$

Conclusion:

$$\mu = \mu(\sigma) = e^t f(\sigma)\,,$$

$$\text{where} \left\{ \begin{array}{l} [f(\sigma)] = 1 \\ f \text{ analytic near } f(\langle\sigma\rangle) \equiv \mu_0 \equiv \Lambda \\ t \in \mathbb{R} \end{array} \right\} \Rightarrow \boxed{\mu(\sigma) = e^t\, \sigma^{\frac{1}{1-\varepsilon}}}$$

is perfectly allowed.

Un-doing the CW dim. transmutation with nonrenorm. interactions

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is perfectly allowed. The *broken phase* succesfully mimicks an ordinary model. E.g.:

$$\boxed{\mu^{2\varepsilon} \frac{\phi^{4+2n}}{\sigma^{2n}} = (e^t)^{2\varepsilon}\, (\mu_0^\varepsilon)^{(2+2n)}\, \frac{\phi^{4+2n}}{\Lambda^{2n}} \left(1 + \mu_0^\varepsilon \frac{\sigma'}{\Lambda}\right)^{-2n+\frac{2\varepsilon}{1-\varepsilon}}}$$

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$$\left.\begin{cases} [f(\sigma)] = 1 \\ f \text{ analytic near } f(\langle\sigma\rangle) \equiv \mu_0 \equiv \Lambda \\ t \in \mathbb{R} \end{cases}\right\} \Rightarrow \boxed{\mu(\sigma) = e^t\, \sigma^{\frac{1}{1-\varepsilon}}}$$

is perfectly allowed. The *broken phase* succesfully mimicks an ordinary model. E.g.:

$$\boxed{\mu^{2\varepsilon}\frac{\phi^{4+2n}}{\sigma^{2n}} = (e^t)^{2\varepsilon}\,(\mu_0^\varepsilon)^{(2+2n)}\,\underbrace{\frac{\phi^{4+2n}}{\Lambda^{2n}}\left(1+\mu_0^\varepsilon\frac{\sigma'}{\Lambda}\right)^{-2n+\frac{2\varepsilon}{1-\varepsilon}}}_{\text{nonrenormalizable series with some evanescent coefficients (conjecture 1.)}}}$$

nonrenormalizable series with
some ***evanescent coefficients*** *(conjecture 1.)*

Un-doing the CW dim. transmutation with nonrenorm. interactions

Dim-Reg $d=4-2\varepsilon$

Conclusion:

$$\mu = \mu(\sigma) = e^t\, f(\sigma)\,,$$

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n-independent
(conjecture 2.)

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similarly for gauge theories: $-\frac{1}{4g^2} \mu(\sigma)^{-2\varepsilon} F_{\mu\nu} F^{\mu\nu}$

Quantum scale symmetric effective lagrangian

No scale anomaly in

$$\mathcal{L}^{(0)}(\phi,\sigma) = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\sigma)^2 - \overbrace{\mu^{2\varepsilon}(\sigma)}^{\text{„dynamical" regulator}} \left[\underbrace{V(\phi,\sigma)}_{\text{renormalizable, classicaly scale-invariant}} + \sum_{n=0} \lambda_n \frac{\phi^{4+2n}}{\sigma^{2n}} \right]$$

$\mathbb{Z}^2 \times \mathbb{Z}^2$
$\phi \to -\phi$
$\sigma \to -\sigma$

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Insertion of
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→ unbroken scale transformations to all orders

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go to *broken phase*

$$\mathcal{L}^{(0)}(\phi_0+\phi',\sigma_0+\phi')$$

Insertion of

$$[\partial_\mu D^\mu]$$

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compute loop corrections (in momentum expansion) & RGE functions *β, γ*

$$\mathcal{L}_{\text{eff}}(\phi,\sigma) = -V_{\text{eff}}(\phi,\sigma) + \ldots$$

- Homogenous function (no mass-parameters, only vev's)
- $\mathbb{Z}^2 \times \mathbb{Z}^2$ sym.
- Satisfies Callan-Symanzik eq.

Quantum scale symmetric effective lagrangian

RG-improvement:

$$\mu = e^{t}\mu_0 \ , \qquad \lambda(t)\,\phi^4 + \frac{\lambda^2(t)\phi^4}{64\pi^2}\,\log\left(\frac{\phi}{e^{t}\sigma}\right)^2 + \ldots \longleftarrow$$

Choose

$$t = t(\phi,\sigma) \sim \log\frac{\phi}{\sigma}$$

to avoid large logs.

Quantum scale symmetric effective lagrangian

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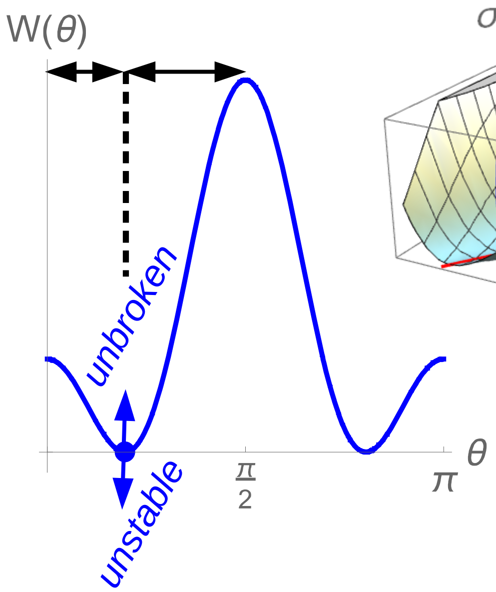

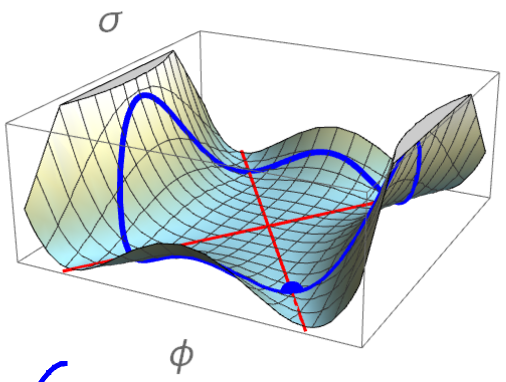

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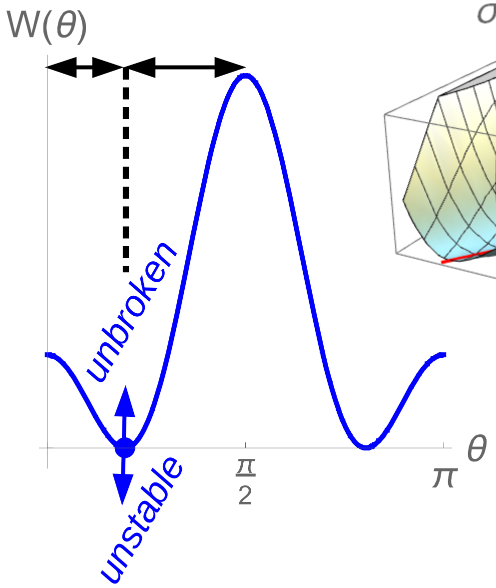
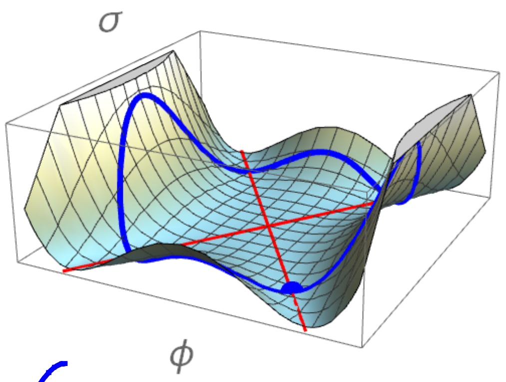
$$\exists_{\theta=\theta_0}\, W(\theta_0) = W'(\theta_0) = 0$$

renormalization condition,
similar to choosing C.C.





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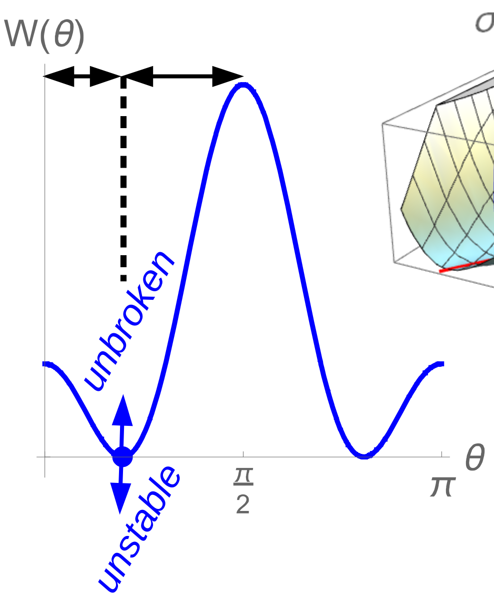

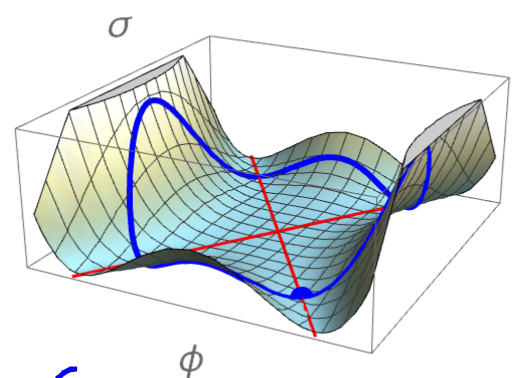

- **Hierarchy** of scales via **aligning** the flat direction $\perp\ \phi$ $\rightarrow$ $\theta_0 \approx \dfrac{\phi_0}{\sigma_0} \ll 1$

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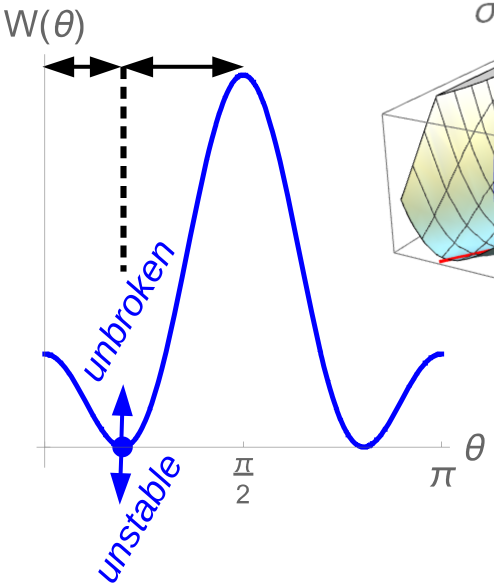

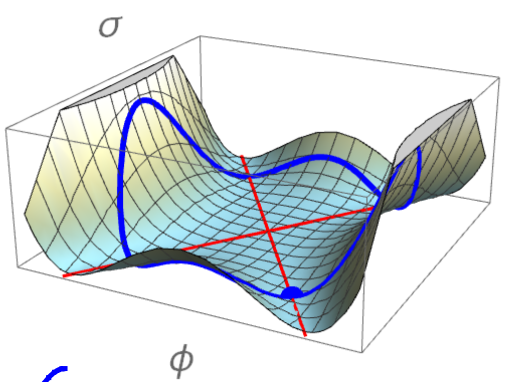

- **Hierarchy** of scales via **aligning** the flat direction $\perp\ \phi$ ► $\theta_0 \approx \dfrac{\phi_0}{\sigma_0} \ll 1$
- New perspective on **naturalness**: is this alignement stable wrt. embedding in a UV completion?

Quantum scale symmetric SM + σ

$$H = \begin{pmatrix} 0 \\ \frac{\phi}{\sqrt{2}} \end{pmatrix}$$ (electroweak vacuum $\longrightarrow$ electroweak flat direction)

$$\boxed{\mathcal{L}_{SM}\Big|_{\substack{m^2=0 \\ \mu=\mu(\sigma)}} + \frac{1}{2}(\partial\sigma)^2 - \lambda_m |H|^2\sigma^2 - \frac{\lambda_\sigma}{4}\sigma^4 + \sum_{n=0}\lambda_n \frac{|H|^{4+2n}}{\sigma^{2n}}}$$

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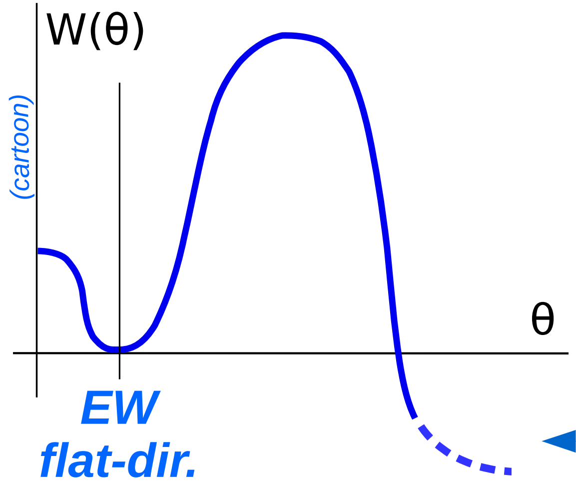

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- unstable
- unbounded below

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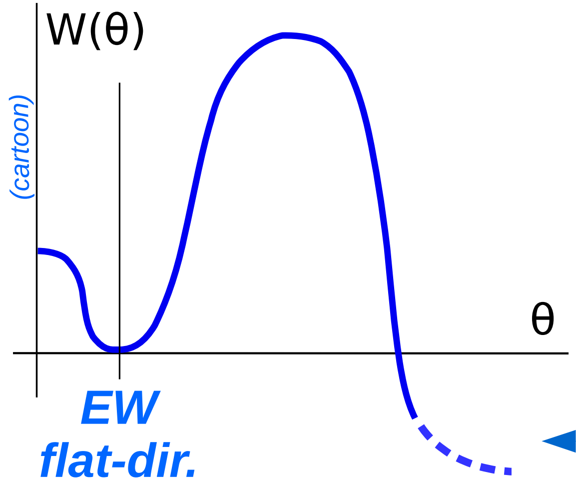

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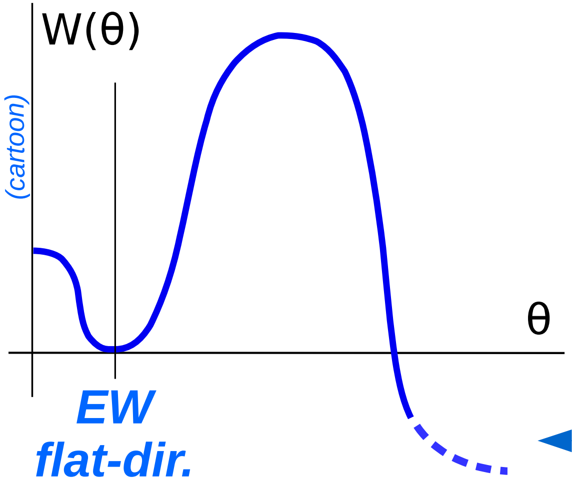

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Tunneling via **2-dim instanton** (Coleman's bounce), in the presence of nonrerm. terms.

(Even stronger) motivation to stabilise the $\boldsymbol{V_{eff}}$ completely: $\lambda_{\text{eff}} \overset{!}{>} 0$

Summary

1) You may use **a field as the scale *μ*** in Dim-Reg to preserve scale symmetry at the quantum level.

2) The price to pay: infinitely many nonpolynomial ϕ/σ operators and corresponding couplings: **nonrenormalizability**.

3) Minimal subtraction scheme involves **evanescent interactions**.

4) Presence of a **flat direction** $\leftarrow$ tuning.

5) **Naturalness: aligning** the flat direction perpendicular to Higgs

6) **Instability = unboundedness below**

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Thank You!