Anomaly Cancellation in Effective Supergravity Theories from the Heterotic String: Two Simple Examples

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Supergravity from the E_8 x E_8 Heterotic String

- Goal was to study anomalies arising from the massless spectrum of the Z_3 and Z_7 toroidal orbifold compactifications of the E_8 x E_8 Heterotic String

$$\mathbb{R}^{9,1} \to \mathbb{R}^{3,1} \times X_6 \qquad E_8 \times E_8 \to E_8 \times E_6 \times G$$

$$X_6 = T^6/\mathbb{Z}_3 \qquad \mathbb{Z}_3 : G = SU(3)$$

$$X_6 = T^6/\mathbb{Z}_7 \qquad \mathbb{Z}_7 : G = U(1) \times U(1)$$

- The 4D spectrum is N=1 supergravity with vector and chiral matter supermultiplets.

- The decomposition from compactificaton

$$\{g_{MN}, B_{MN}\} \to \{g_{\mu\nu}, B_{\mu\nu}, T^{ij}, \ldots\}$$

- Gives the so-called diagonal Kahler moduli:

$$T_i = G_{i\bar{i}} + iB_{i\bar{i}}$$

- With Kahler potential

$$K = e^{g^a}\,\bar{\Phi}^a \Phi^a + g(T^i, \bar{T}^i)$$

$$g(T^i, \bar{T}^i) = \sum_{i=1}^{3} g^i(T^i, \bar{T}^i) = -\sum_{i=1}^{3} \ln(T^i + \bar{T}^i)$$

$$g^a(T^i, \bar{T}^i) = \sum_i q_i^a g^i(T^i, \bar{T}^i)$$

- Anomalies arise from the SL(2,R) modular transformation of the three diagonal Kahler moduli:

$$T^i \to \frac{a_i T_i - i b_i}{i c_i T_i + d_i} \qquad a_i b_i - c_i d_i = 1$$

- Chiral matter supermultiplets must also transform

$$\Phi^a \to \exp\left(-\sum_i q_i^a F^i\right)\Phi^a \qquad F_i = i c_i T_i + d_i$$

- These field redefinitions in turn force a Kahler transformation:

$$K \to K + \sum_i (F^i + \bar{F}^i) = K + F + \bar{F}$$

- Thus the ``modular'' anomalies are a combination of Kahler + reparametrization anomalies and we get modular/gauge/gauge, modular/gravity/gravity, and pure modular anomalies.
- Our project showed that the anomalies in these supergravity theories has the correct universal form such that they can be cancelled via the 4D Green-Schwarz mechanism

Motivation

- Modular transformations correspond to T-Duality transformations of the underlying string theory – consistency of the low energy QFT and its relation to string theory relies on the cancellation of anomalies
- On the string side, showing anomaly cancellation is easy, but one would want to ensure that the cancellation holds up purely from a QFT point of view.
- However, the QFT calculation is subtle. Previous investigations studied only subsets of the total anomaly or found a non-universal total anomaly due to difficulties with calculating the pure modular anomaly.
- Also phenomenological reasons for wanting a preserved symmetry – residual R-parity

Anomaly Cancellation Outline

- Calculate the one-loop effective action
- Introduce Pauli-Villars fields with non-invariant masses to cancel divergences*
- Anomaly arises from variation of terms involving PV mass terms in the effective action that are finite as the masses are taken to infinity
- Cancel the result via the GS mechanism
- Example:

$$\mathcal{L}_{YM} = -\sqrt{g}\frac{s}{8}\sum_a \left(F^a_{\mu\nu} - i\tilde{F}^a_{\mu\nu}\right) F_a^{\mu\nu} + \text{h.c.}$$

$$\Delta\mathcal{L}_{YM1} = -\frac{\sqrt{g}}{64\pi^2}\left(8\pi^2 b\sum_i F^i\right)\left(F^a_{\mu\nu} - i\tilde{F}_a^{\mu\nu}\right) F_a^{\mu\nu} + \text{h.c.}$$

- Cancellation by imposing:

$$\Delta s = -b\sum_i F^i$$

One-Loop Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}(g, K_R) + \sqrt{g}\frac{\ln(\Lambda^2)}{32\pi^2}(L_D + L_F) + \mathcal{L}_Q$$

- F-Term:

$$\Phi = \Phi_1 + \Phi_2 + \frac{3}{2}C_a\Phi^a_{YM} + \frac{1}{36}\Phi_X(N - 9N_G - 79) + \frac{1}{6}\Phi_w(41 + N - 3N_G) + N_G\Phi'_g$$

$$\Phi_1 = -\frac{1}{2}\left(C^M_a\Phi^a_{YM} + \Gamma^{i\alpha}_j\left(\Gamma^j_{i\alpha} + 2(T^a)^j_i W^a_\alpha\right)\right) \qquad \Gamma^i_{j\alpha} = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha Z^k\Gamma^i_{jk}$$

$$\Phi_2 = \frac{1}{3}X^\alpha\left(\Gamma_\alpha + 2(T^a)^i_i W^a_\alpha\right) \qquad X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K$$

- Quadratic:

$$\mathcal{L}_Q = -\sqrt{g}\frac{\Lambda^2}{32\pi^2}\left(\frac{1}{2}(3 + N_G - N)\mathcal{D}^\alpha X_\alpha| + (\hat{V} + M^2)(7 + 3N_G - N) + N_G\mathcal{D}^\alpha k_\alpha + \mathcal{D}^\alpha\Gamma_\alpha| + \frac{2}{x}\mathcal{D}_a\mathrm{Tr}(T^a)\right)$$

PV Fields

- A variety of fields are needed to cancel divergences and produce the correct anomaly. Many have Kahler potential & superpotential terms based on the couplings of the matter fields. As an example, a subset of chiral PV fields with non-invariant masses have

$$W(\Phi^P, \Phi'^P) = \mu_P \Phi^P \Phi'^P$$

$$K(\Phi^P, \Phi'^P) = e^{f^P} |\Phi^P| + e^{f'^P} |\Phi'^P|$$

$$f^P = \alpha^P K(Z, \bar{Z}) + \beta^P g(T, \bar{T}) + \delta^P k(S, \bar{S}) + \sum_n q_n^C g^n(T^n, \bar{T}^n)$$

Cancellation of Divergences

- To cancel divergences we require
  - Quadratic Divergences:

$$\mathrm{Tr}(\eta\Gamma_\alpha) = 0 \qquad \Gamma^i_{j\alpha} = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha Z^k \Gamma^i_{jk}$$

  - Logarithmic Divergences:

$$\mathrm{Tr}(\eta\Gamma_\alpha\Gamma_\beta) = \mathrm{Tr}(\eta\Gamma_\alpha T^a) = \mathrm{Tr}(\eta(T^a)^2) = 0$$

  - Linear Divergences:

$$\mathrm{Tr}(\eta\phi G \cdot \tilde{G}) = 0$$

  - Where $G_{\mu\nu}$ is the field strength associated with the fermion connections

$$G_{\mu\nu} = [D_\mu, D_\nu] = -\Gamma^C_{D\mu\nu} + iF^a_{\mu\nu}(T^a)^C_D + \frac{1}{2}X_{\mu\nu}\delta^C_D$$

  - And is the parameter for a chiral rotation corresponding to a T-Duality transformation:

$$\phi^C = \left(\frac{1}{2} - \alpha^C - \beta^C\right) F - \sum_i F^i q^C_i$$

Some Sum Rules

$$\sum_C \eta^C = -N - 29$$

$$\sum_C \eta^C f^C = -10K - \sum_n q_n^p g^n$$

$$\sum_C \eta^C \alpha^C \alpha^C = -4$$

$$\sum_\gamma \eta_\gamma^V = -12 - N_G$$

$$30\delta_{nm} = \sum_C \eta^C q_n^C q_m^C (1 - 2\bar{\gamma}^C)$$

Anomaly Result

- After satisfying the regularization conditions & anomaly matching conditions, we find

$$\delta\mathcal{L}_{anom} = b\int d^4\theta E(F+\bar{F})\Omega$$
$$= \frac{b\sqrt{g}\mathrm{Im}(F)}{4}\left(F_a\cdot\tilde{F}^a + R\cdot\tilde{R} + \frac{1}{2}\sum_i g^i\cdot\tilde{g}^i + \cdots\right)$$

$$8\pi^2 b = \frac{1}{24}\left(2\sum_p q_n^p - N + N_G - 21\right)$$
$$= C_a - C_a^M + 2\sum_b C_a^b q_n^b$$
$$= 30$$

Anomaly Cancellation

- In the chiral superfield description of the dilaton, we have a tree level coupling between the dilaton and the Chern-Simons anomaly superfield of the form

$$\mathcal{L} = \int d^4\theta E(S + \bar{S})\Omega = -\frac{1}{8}\int d^4\theta \frac{E}{R} S\Phi + \text{h.c.}$$

- So that the anomaly can be cancelled by allowing the dilaton to transform under a modular transformation as

$$\Delta S = -bF$$

Conclusions & Future Work

- We have shown that, for at least two examples, string-derived supergravities can be regulated in such a way that the anomalies take a universal form and can be eliminated with the GS mechanism
- Next step is to apply the procedure to more realistic string compactifications to make statements about the phenomenology of string models :
  - FIQS – Z_3 orbifold with two Wilson lines, with gauge group

$$SO(10) \times SU(3) \times SU(2) \times [U(1)]^7 \times U(1)_X$$

  - $U(1)_X$ is anomalous, but can be included in the GS mechanism
  - Hypercharge defined by a linear combination of the non-anomalous U(1)'s
  - Quark and lepton fields charged under the hypercharge and SU(3) x SU(2)
  - Hidden sector with SO(10)

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