The Toric SO(10) F-theory Landscape and Tensor-Matter Transitions

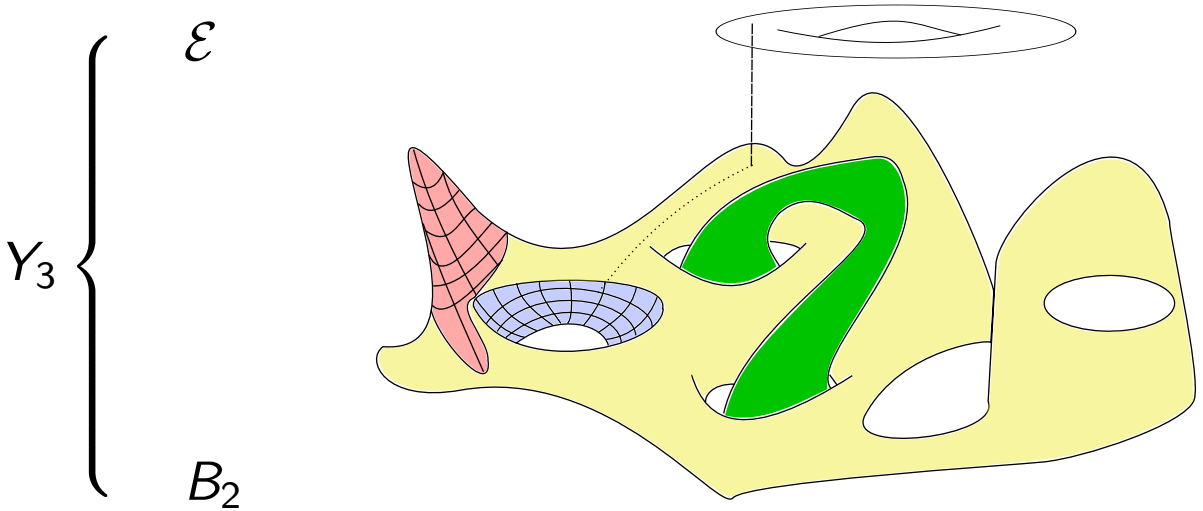
Paul-Konstantin Oehlmann

Virginia Polytechnic Institute and State University

Based on
- arXiv:1709.06609 with: W. Buchmueller, M. Dierigl, F. Ruehle
- arXiv:1804.07386 with: M. Dierigl, F. Ruehle

Rheinische Friedrich Wilhelms Universtitaet, Bonn
Planck 2018, May 23rd 2017

What is F-theory



Geometrization of strongly coupled IIB Strings

Interpret IIB axio-dilaton as complex structure τ of a **torus** $\mathcal{E}$ as

$$\tau = C_0 + i\frac{1}{g_{IIB}}$$

fibered over the **physical base** B_2 such that the **total space** is **Calabi-Yau** Y_3

Why F-theory

F-theory geometrization most powerful in 6d

- **Flexibility** to construct 6d SUGRA theories with exotic representations
- **Map out** field of **consistent** 6d string **vacua** VS. swampland [Taylor'06; Vafa'06]
- Classification of 6d **superconformal field theories** [del Zotto, Heckman, Rudelius, Vafa...'14]

Why F-theory

F-theory geometrization most powerful in 6d

- **Flexibility** to construct 6d SUGRA theories with exotic representations
- **Map out** field of **consistent** 6d string **vacua** VS. swampland [Taylor'06; Vafa'06]
- Classification of 6d **superconformal field theories** [del Zotto, Heckman, Rudelius, Vafa...'14]

Good Starting Point for Phenomenology

Start from 6d SUGRA before going to 4d [Raby'06; Buchmueller, Dierigl, Ruehle, Schweizer'17]

- **Engineer** SU(5), SO(10), E_6 with possible U(1) **flavor group**
- Important **representations** realizable **5**; **10**; **16**; **27**

Why F-theory

F-theory geometrization most powerful in 6d

- **Flexibility** to construct 6d SUGRA theories with exotic representations
- **Map out** field of **consistent** 6d string **vacua** VS. swampland [Taylor'06; Vafa'06]
- Classification of 6d **superconformal field theories** [del Zotto, Heckman, Rudelius, Vafa...'14]

Good Starting Point for Phenomenology

Start from 6d SUGRA before going to 4d [Raby'06; Buchmueller, Dierigl, Ruehle, Schweizer'17]

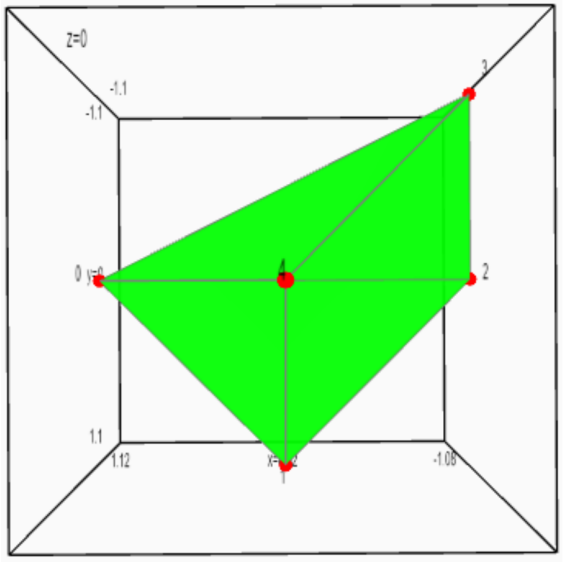
- **Engineer** SU(5), SO(10), E_6 with possible U(1) **flavor group**
- Important **representations** realizable **5**; **10**; **16**; **27**

Our Goal

1. **Phenomenology:** Classify 6d $SO(10) \times U(1)^n$ vacua **as a starting point** for GUT model building
   Observe: vacua are **related** by
   - **perturbative transitions**: Higgsings
   - **non-perturbative transitions**: pass through superconformal points
2. **Understand and generalize those transitions** to ADEFG×U(1) groups

Outline

Toric Classification of SO(10) Theories

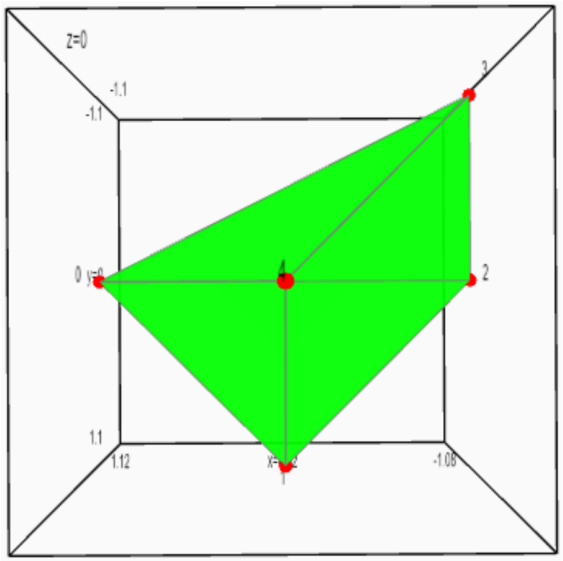


Strategy to engineer SO(10) theories

- Start with **non-generic** description of **torus fiber** with U(1) gauge group

  [Braun,Grimm,Keitel,Klevers]
- **Torus geometry** $p = 0$ in an ambient space encoded in **2d polytope**

Toric Classification of SO(10) Theories



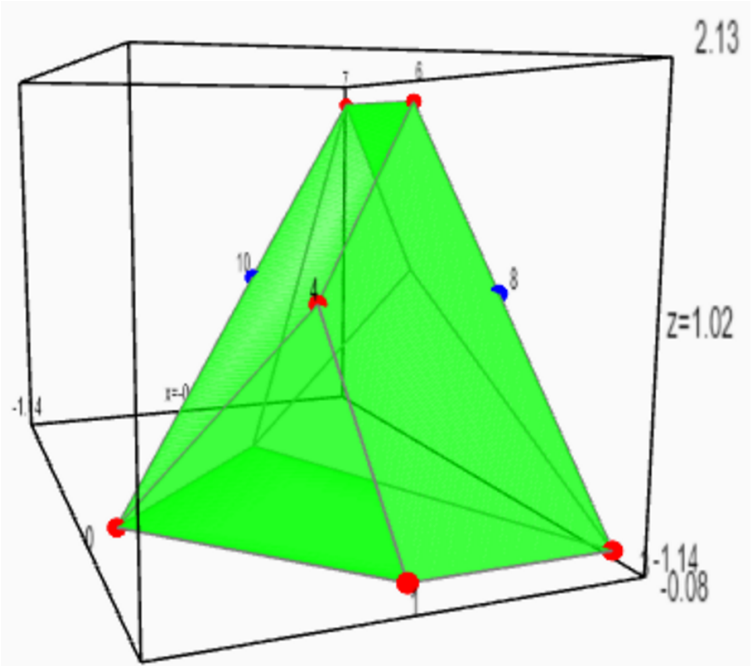
$$p = s_1 u^3 e_1^2 + s_2 u^2 v e_1^2 + s_3 u v^2 e_1^2 + s_4 v^3 e_1^2 + s_5 u^2 w e_1 + s_6 u v w e_1 + s_7 v^2 w e_1 + s_8 u w^2 + s_9 v w^2$$

Strategy to engineer SO(10) theories

- Start with **non-generic** description of **torus fiber** with U(1) gauge group

  [Braun,Grimm,Keitel,Klevers]
- **Torus geometry** $p = 0$ in an ambient space encoded in **2d polytope**

Toric Classification of SO(10) Theories

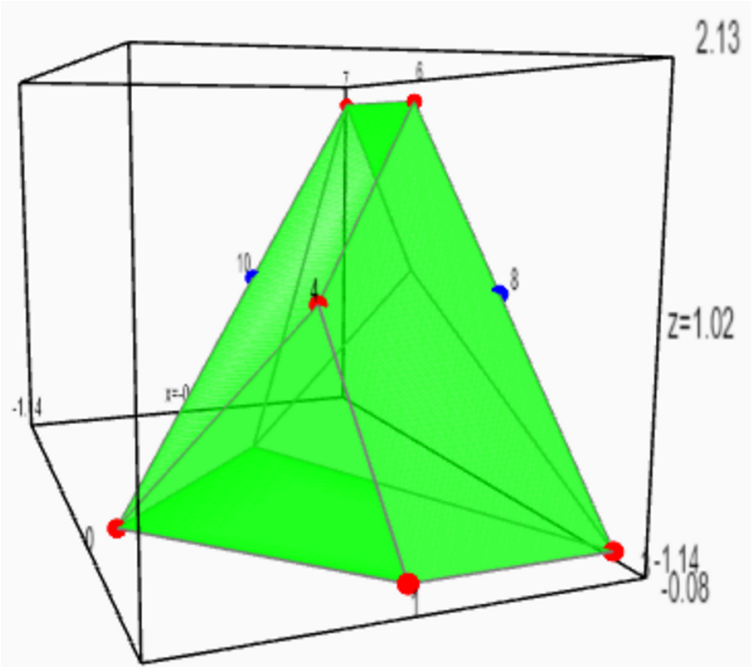


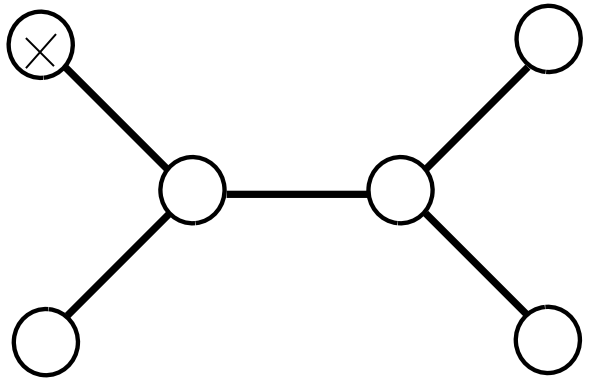
$$p = s_1 e_1^2 f_0 f_2^2 f_4 f_1 u^3 + s_2 e_1^2 f_0^2 f_2^2 f_3 f_4 f_1^2 f_5 u^2 v + s_3 e_1^2 f_0^2 f_2 f_3 f_1 u v^2 + s_4 e_1^2 f_0^3 f_2 f_3^2 f_1^2 f_5 v^3$$
$$+ s_5 e_1 f_2 f_4 u^2 w + s_6 e_1 f_0 f_2 f_3 f_4 f_1 f_5 uvw + s_7 e_1 f_0 f_3 v^2 w + s_8 f_2 f_3 f_4^2 f_1 f_5^2 u w^2 + s_9 f_3 f_4 f_5 v w^2$$

Strategy to engineer SO(10) theories

- **Enhance** the 2d polytope to a 3d **top** representing the torus fiber within a general Calabi-Yau [Candelas,Font; Bouchard,Skarke]
- height>0 rays give ADE resolution divisors D_{f_i} restricting to a base divisor $\mathcal{S}$

Toric Classification of SO(10) Theories





Strategy to engineer SO(10) theories

- height$=0$: the **generic fiber** encoding U(1) theory
- height>0 encodes **Dynkin label** of ADE resolution divisors
- Resolution divisors **intersect** as (negative) affine **Dynkin diagram**

$$\text{over } \mathcal{S}: \qquad D_{f_i} \cdot \mathbb{P}^1_j = -\hat{G}^{i,j}_{\text{SO(10)}} \tag{1}$$

Base Independent 6D SUGRA

Form of the **top** fixes general form of 6d SUGRA spectrum

- **Gauge group** fixed by fibral divisors
- **Matter representations** and charges fixed as well
- **Matter multiplicities** fixed as well, using prime ideal decomposition techniques

Base Independent 6D SUGRA

Form of the **top** fixes general form of 6d SUGRA spectrum

- **Gauge group** fixed by fibral divisors
- **Matter representations** and charges fixed as well
- **Matter multiplicities** fixed as well, using prime ideal decomposition techniques

Impose Calabi-Yau threefold embedding of the top

- Check: Full **Anomaly cancellation is automatic** ✓
- **Freedom** to fix 6D SUGRA spectrum by intersections of
  - K_b^{-1} anticannonical class of the base
  - $\mathcal{S}$ the divisor class of the SO(10)
  - $\mathcal{D}_A, \mathcal{D}_B$ two topological twists of fiber and base

Base Independent 6D SUGRA

Form of the **top** fixes general form of 6d SUGRA spectrum

- **Gauge group** fixed by fibral divisors
- **Matter representations** and charges fixed as well
- **Matter multiplicities** fixed as well, using prime ideal decomposition techniques

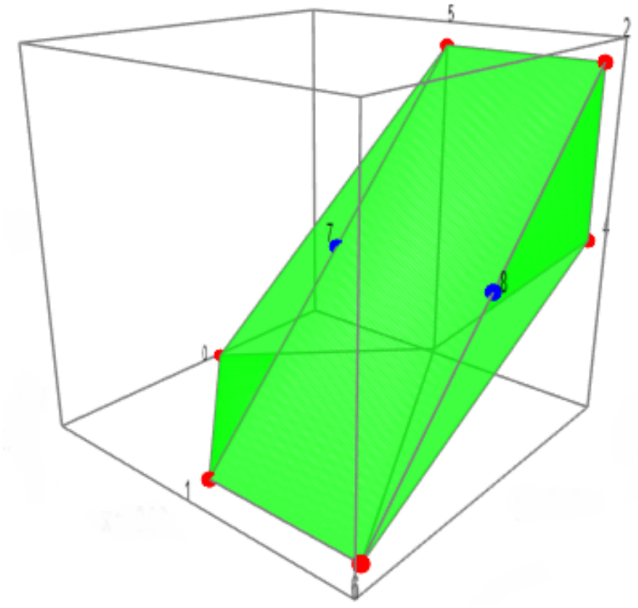
Impose Calabi-Yau threefold embedding of the top

- Check: Full **Anomaly cancellation is automatic** ✓
- **Freedom** to fix 6D SUGRA spectrum by intersections of
  - K_b^{-1} anticannonical class of the base
  - $\mathcal{S}$ the divisor class of the SO(10)
  - $\mathcal{D}_A, \mathcal{D}_B$ two topological twists of fiber and base

Rep	Multiplicity
$\mathbf{16}_{3/4}$	$(2K_b^{-1} - \mathcal{D}_A)\mathcal{S}$
$\mathbf{10}_{3/2}$	$\mathcal{D}_B\mathcal{S}$
$\mathbf{10}_{-1/2}$	$(3K_b^{-1} - \mathcal{D}_B - 2\mathcal{S})\mathcal{S}$
$\mathbf{16}_{-1/4}$	$(\mathcal{D}_A - \mathcal{S})\mathcal{S}$
$\mathbf{45}_0$	$1 + \frac{1}{2}\mathcal{S}(\mathcal{S} - K_b^{-1})$

Rep	Multiplicity
$\mathbf{1}_3$	$(K_b^{-1} - \mathcal{D}_A + \mathcal{D}_B)\mathcal{D}_B$
$\mathbf{1}_2$	$6(K_b^{-1})^2 + K_b^{-1}(-5\mathcal{D}_A + 4\mathcal{D}_B - 2\mathcal{S})$ $+\mathcal{D}_A^2 + \mathcal{D}_A(2\mathcal{D}_B + \mathcal{S}) - \mathcal{D}_B(2\mathcal{D}_B + 5\mathcal{S})$
$\mathbf{1}_1$	$12(K_b^{-1})^2 + K_b^{-1}(8\mathcal{D}_A - \mathcal{D}_B - 25\mathcal{S})$ $-\mathcal{D}_B^2 - 4\mathcal{D}_A^2 + 6\mathcal{S}^2 + \mathcal{D}_A(\mathcal{D}_B + 4\mathcal{S})$
$\mathbf{1}_0$	$18 + 11(K_b^{-1})^2 + 3\mathcal{D}_A^2 + 2\mathcal{D}_B^2 + 10\mathcal{S}^2 - 2\mathcal{D}_A\mathcal{D}_B$ $-K_b^{-1}(3\mathcal{D}_A + 4\mathcal{D}_B + 15\mathcal{S}) + 5(\mathcal{D}_A - \mathcal{D}_B)\mathcal{S}$

Transitions Between Theories

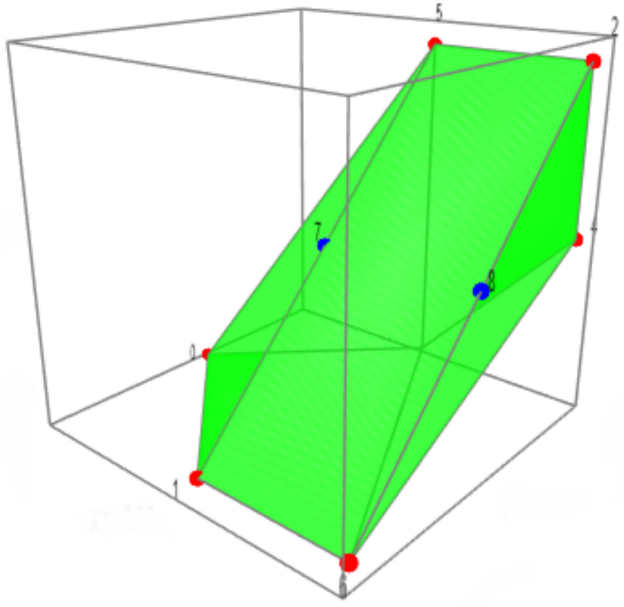


An organization principle

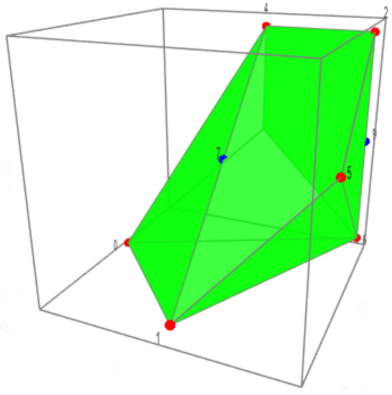
Often theories are **geometrically related by transitions** (blow-ups/downs):

- **Higgs transitions**: $SO(10) \times [U(1)]$

Transitions Between Theories



generic fiber
blown-down

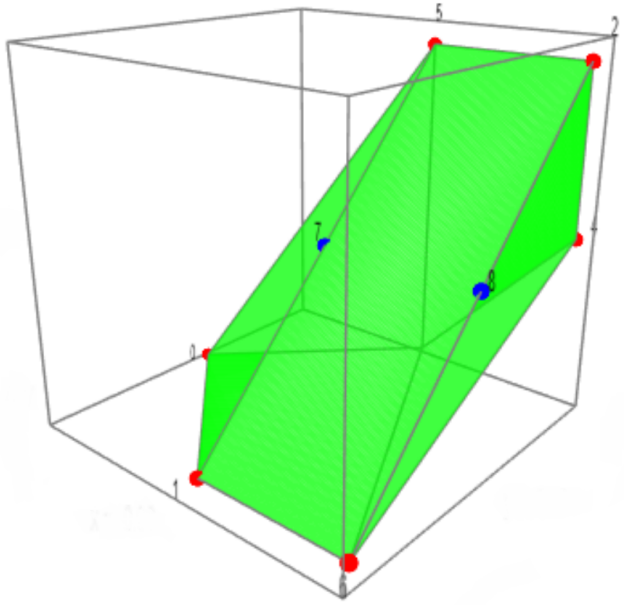


An organization principle

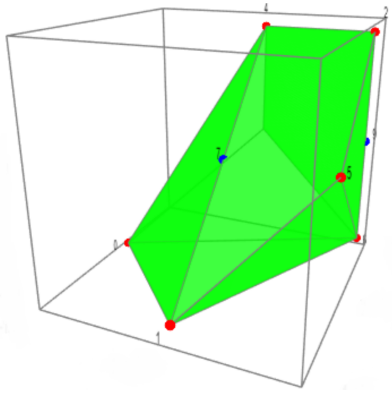
Often theories are **geometrically related by transitions** (blow-ups/downs):

- **Higgs transitions**: $SO(10) \times [U(1) \to \mathbb{Z}_3]$ by VEV $\langle (\mathbf{1})_{(3)} \rangle$

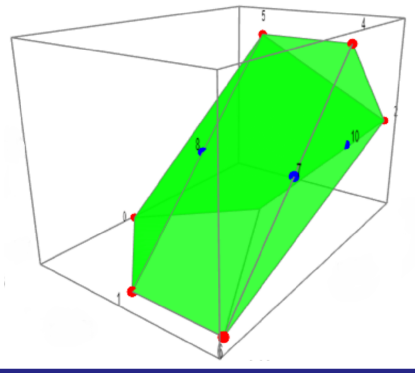
Transitions Between Theories



generic fiber
blown-down



Non-Flat
Blow-up

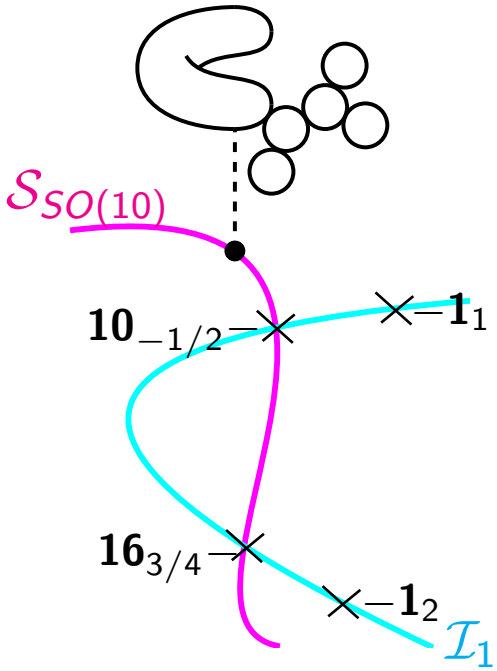


An organization principle

Often theories are **geometrically related by transitions** (blow-ups/downs):

- **Higgs transitions**: $SO(10) \times [U(1) \to \mathbb{Z}_3]$ by VEV $\langle (\mathbf{1})_{(3)} \rangle$
- **non-toric blow-up** over points in the SO(10) divisor
  $\to SO(10) \times [U(1)] +$ Super Conformal Matter [Bershadsky, Johanson'94]

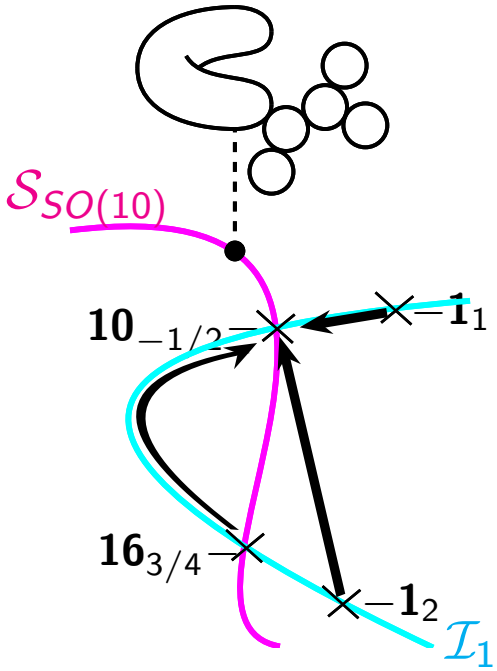
Superconformal Transitions



Tensor-Matter Transitions

1. Start with $SO(10) \times [U(1)]$ 6d **SUGRA** model

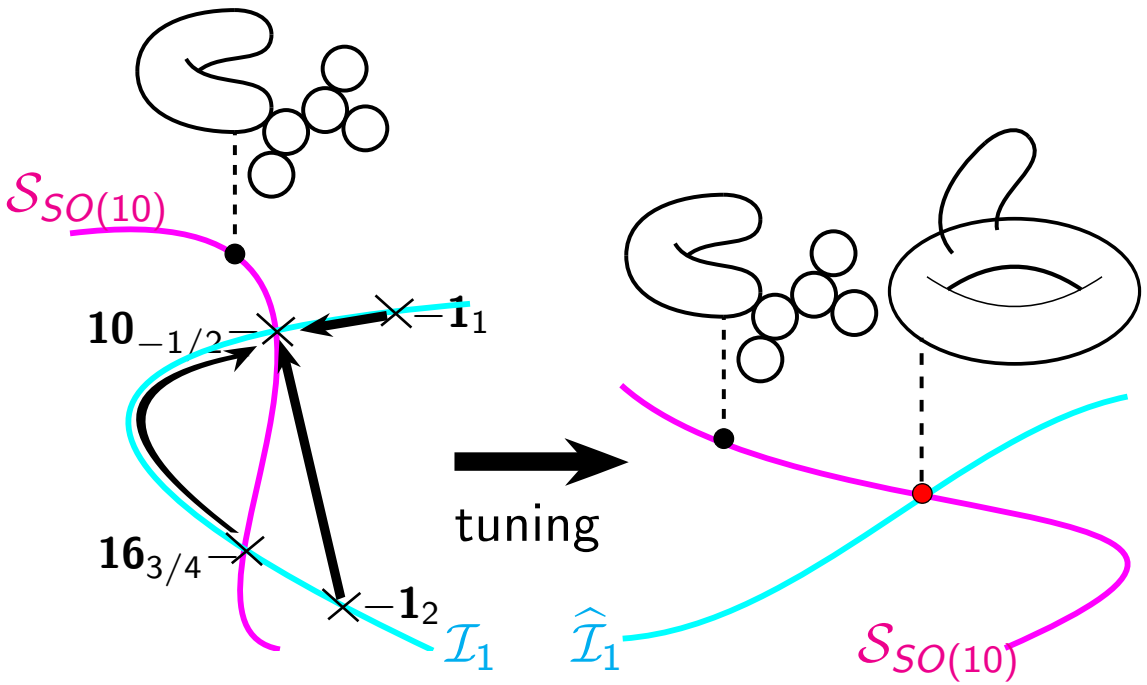
Superconformal Transitions



Tensor-Matter Transitions

1. Start with $SO(10) \times [U(1)]$ 6d **SUGRA** model
2. Tune representations **onto** each other

Superconformal Transitions

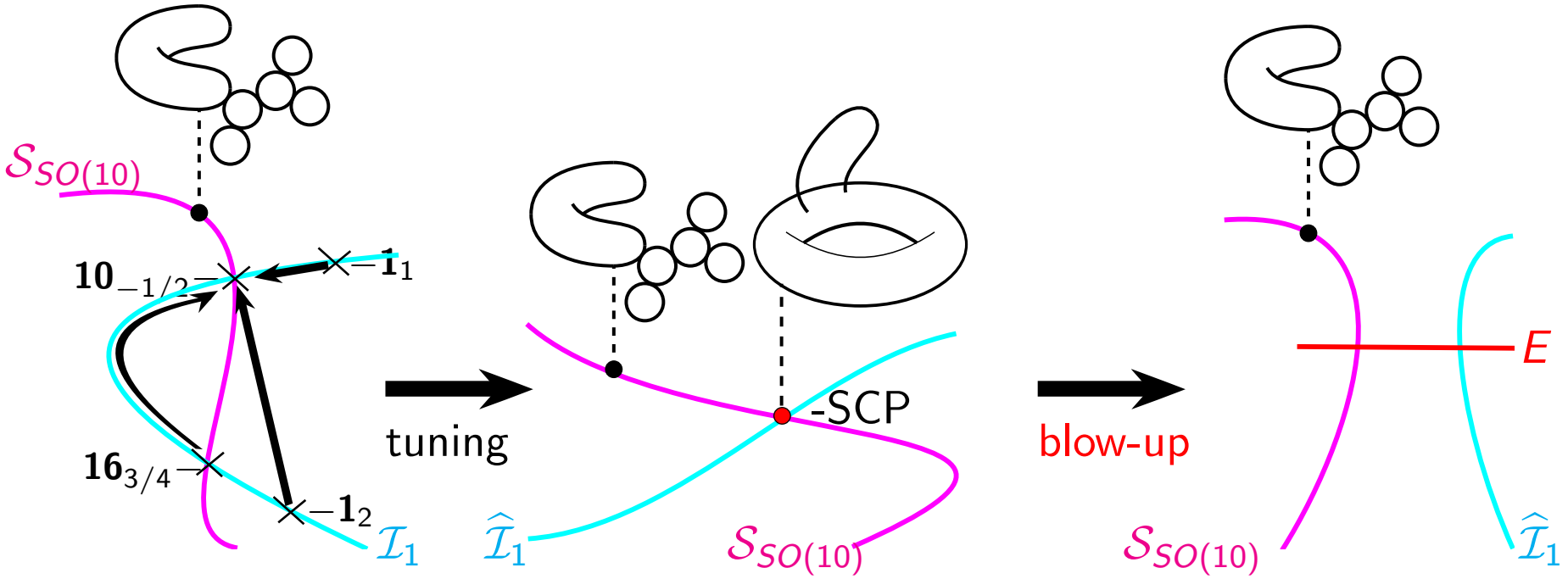


Tensor-Matter Transitions

1. Start with $SO(10) \times [U(1)]$ 6d **SUGRA** model
2. Tune representations **onto** each other
   Results in a **superconformal point(SCP)** with a **non-flat fiber** over it

$$(\mathbf{16}_{3/4} \oplus \mathbf{10}_{-1/2} \oplus \mathbf{1}_2 \oplus \mathbf{1}_1 \oplus \mathbf{1}_0) \xrightarrow{\text{merge}} \text{SCP}$$

Superconformal Transitions



Tensor-Matter Transitions

1. Start with $SO(10) \times [U(1)]$ 6d **SUGRA** model
2. Tune representations **onto** each other
   Results in a **superconformal point(SCP)** with a **non-flat fiber** over it
   $$(\mathbf{16}_{3/4} \oplus \mathbf{10}_{-1/2} \oplus \mathbf{1}_2 \oplus \mathbf{1}_1 \oplus \mathbf{1}_0) \xrightarrow{\text{merge}} \text{SCP} \xrightarrow{\text{blow-up}} 1 \cdot \text{Tensor}$$
3. **Blow-up** the superconformal point **in the base**
   $\rightarrow$ **tensor branch** of the SCP: **well defined SUGRA** again

Global Matter-Tensor Transitions

Superconformal points correspond to (4,6,12) **non-minimal** singularities/small **heterotic instantons**/ **E-string** theories/**tensionless non-critical strings**

[Seiberg,Witten'96; Ganor Hanany'96; Morrison Vafa'97]

Generically two 6d **SUGRA** theories are **related** by

Global Matter-Tensor Transitions

Superconformal points correspond to (4,6,12) **non-minimal** singularities/small **heterotic instantons**/ **E-string** theories/**tensionless non-critical strings**

[Seiberg,Witten'96; Ganor Hanany'96; Morrison Vafa'97]

Generically two 6d **SUGRA** theories are **related** by

Content		
Vectors:	**V**	$-n$ "**Higgsing**"
Hypers:	**H**	
Tensors:	**T**	

Global Matter-Tensor Transitions

Superconformal points correspond to (4,6,12) **non-minimal** singularities/small **heterotic instantons**/ **E-string** theories/**tensionless non-critical strings**

[Seiberg,Witten'96; Ganor Hanany'96; Morrison Vafa'97]

Generically two 6d **SUGRA** theories are **related** by

Content		
Vectors:	**V**	$-n$ "**Higgsing**"
Hypers:	**H**	
Tensors:	**T**	const.

Anomaly	
Grav^4:	$\mathbf{H} - \mathbf{V} + 29\mathbf{T} = 273$

Global Matter-Tensor Transitions

Superconformal points correspond to (4,6,12) **non-minimal** singularities/small **heterotic instantons**/ **E-string** theories/**tensionless non-critical strings**

[Seiberg,Witten'96; Ganor Hanany'96; Morrison Vafa'97]

Generically two 6d **SUGRA** theories are **related** by

Content			Anomaly	
Vectors:	**V**	const.	Grav^4:	$\mathbf{H} - \mathbf{V} + 29\mathbf{T} = 273$
Hypers:	**H**	$\circlearrowleft^{-29}_{+1}$ "**Matter-Tensor**"		
Tensors:	**T**			

Global Matter-Tensor Transitions

Superconformal points correspond to (4,6,12) **non-minimal** singularities/small **heterotic instantons**/ **E-string** theories/**tensionless non-critical strings**

[Seiberg,Witten'96; Ganor Hanany'96; Morrison Vafa'97]

Generically two 6d **SUGRA** theories are **related** by

Content			Anomaly	
Vectors:	$\mathbf{V}$	const.	Grav^4:	$\mathbf{H}-\mathbf{V}+29\mathbf{T}=273$
Hypers:	$\mathbf{H}$	-29 "**Matter-Tensor**"	Gauge^4:	$B_{\text{adj}}-\sum_{\mathbf{R}} H_{\mathbf{R}} B_{\mathbf{R}}=0$
Tensors:	$\mathbf{T}$	$+1$	$\text{Gauge}^2\text{Grav}^2$:	$-\frac{1}{6}\left(A_{\text{adj}}-\sum_{\mathbf{R}} \mathbf{H}_{\mathbf{R}} A_{\mathbf{R}}\right)=a\cdot b$
			.. ..	.. ..

Global Matter-Tensor Transitions

Superconformal points correspond to (4,6,12) **non-minimal** singularities/small **heterotic instantons**/ **E-string** theories/**tensionless non-critical strings**

[Seiberg,Witten'96; Ganor Hanany'96; Morrison Vafa'97]

Generically two 6d **SUGRA** theories are **related** by

Content			Anomaly	
Vectors:	**V**	const.	Grav^4:	$\mathbf{H}-\mathbf{V}+29\mathbf{T}=273$
Hypers:	**H**	$\circlearrowleft -29$ "**Matter-Tensor**"	Gauge^4:	$B_{\text{adj}}-\sum_{\mathbf{R}} H_{\mathbf{R}} B_{\mathbf{R}}=0$
Tensors:	**T**	$\circlearrowright +1$	Gauge^{2}Grav^2:	$-\frac{1}{6}\left(A_{\text{adj}}-\sum_{\mathbf{R}} \mathbf{H}_{\mathbf{R}} A_{\mathbf{R}}\right)=a\cdot b$
			.. ..	.. ..

General Analysis

- Match **Green-Schwarz** coefficients a, b of SUGRA vacua connected by Matter-Tensor transition [Sadov'96;Taylor, Kumar'10; Morrison Park'12]
- General **constraints** on matter **representation involved** in the transitions

General Results

General ADEFG$\times U(1)$ Gauge Group Classification

- **Matter representations** constraint by Casimir $A_{\mathbf{R}}/B_{\mathbf{R}}/C_{\mathbf{R}}/E_{\mathbf{R}}$ coefficients
  $\hookrightarrow$ Non-Abelian **representations uniquely** fixed, U(1) **charges almost**

General Results

General ADEFG$\times U(1)$ Gauge Group Classification

- **Matter representations** constraint by Casimir $A_{\mathbf{R}}/B_{\mathbf{R}}/C_{\mathbf{R}}/E_{\mathbf{R}}$ coefficients
  $\hookrightarrow$ Non-Abelian **representations uniquely** fixed, U(1) **charges almost**
- Other examples:
  - SU(5)$\times U(1)$ $\quad (\mathbf{10}_{q_{10}} \oplus 3 \times \mathbf{5}_{q_{5,i}} \oplus 3 \times \mathbf{1}_{q_j} \oplus \mathbf{1}_0) \rightarrow 1 \cdot \text{Tensor}$
  - SO(10) $\times U(1)$ $\quad (\mathbf{16}_{3/4} \oplus \mathbf{10}_{-1/2} \oplus \mathbf{1}_2 \oplus \mathbf{1}_1 \oplus \mathbf{1}_0) \rightarrow 1 \cdot \text{Tensor}$
  - $E_6 \times U(1)$ $\quad (\mathbf{27}_{q/3} \oplus \mathbf{1}_q \oplus \mathbf{1}_0) \rightarrow 1 \cdot \text{Tensor}$
- Representations and charges **consistent** with toric F-theory models

Summary

1. **Classified SO(10)$\times$ U(1)** models in F-theory realized as hypersurfaces in toric *tops* **independent of the base**
   - A large set of consistent 6d SUGRA vacua
2. Models often **related by transitions**:
   - **Higgs transitions**: Gauge group outside of SO(10) gets broken
   - **Tensor-Matter transitions**: Matter exchanged for superconformal point which can be blown-up without changing the gauge group
3. **General classification of tensor-matter transitions** for ADEFG $\times$ U(1) groups
   - Non-Abelian **representations uniquely** fixed, U(1) **charges almost**

Summary

1. **Classified SO(10)$\times$ U(1)** models in F-theory realized as hypersurfaces in toric *tops* **independent of the base**
   - A large set of consistent 6d SUGRA vacua
2. Models often **related by transitions**:
   - **Higgs transitions**: Gauge group outside of SO(10) gets broken
   - **Tensor-Matter transitions**: Matter exchanged for superconformal point which can be blown-up without changing the gauge group
3. **General classification of tensor-matter transitions** for ADEFG $\times$ U(1) groups
   - Non-Abelian **representations uniquely** fixed, U(1) **charges almost**

Thank you very much