A theorist’s view on losses

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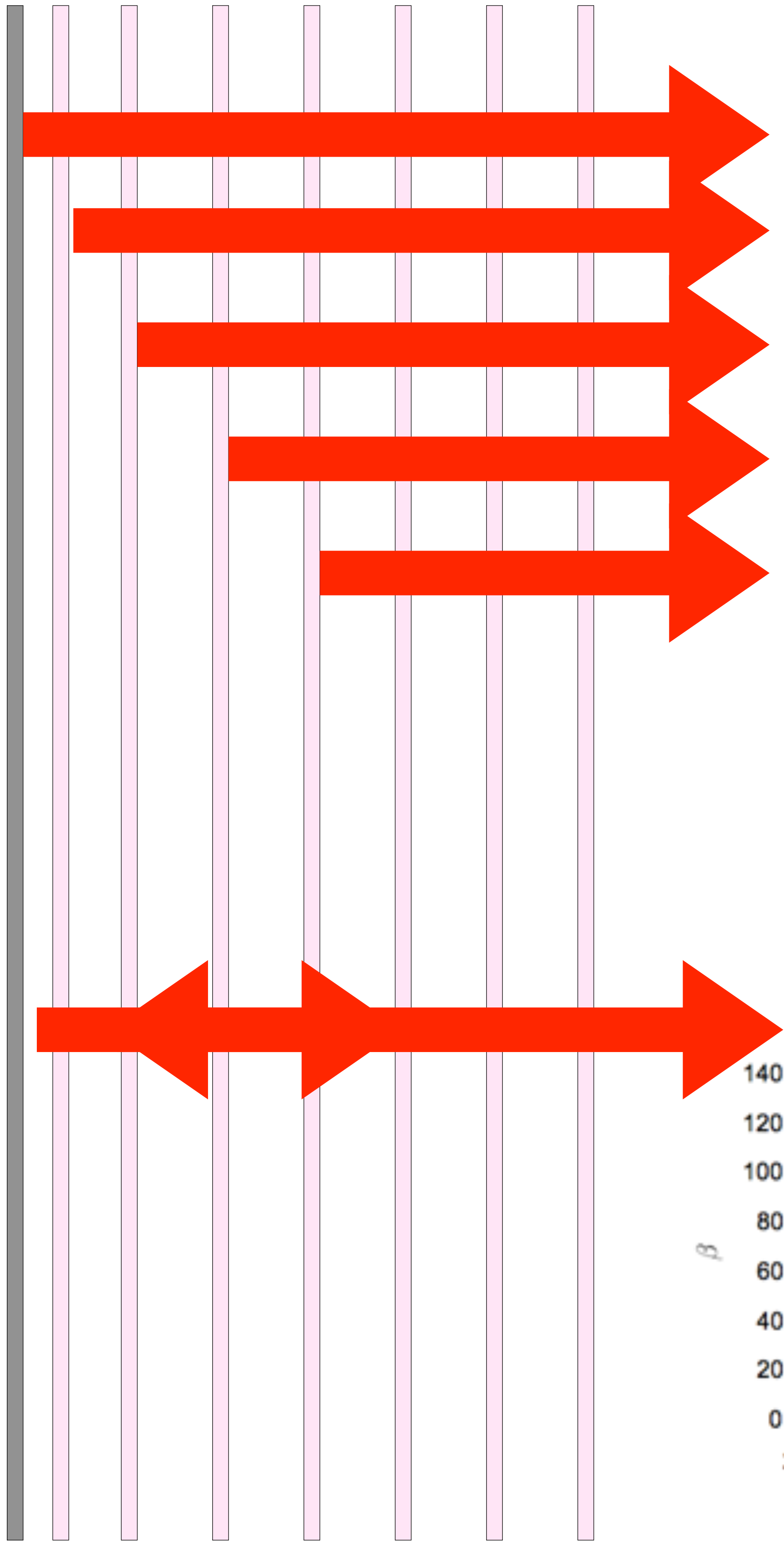
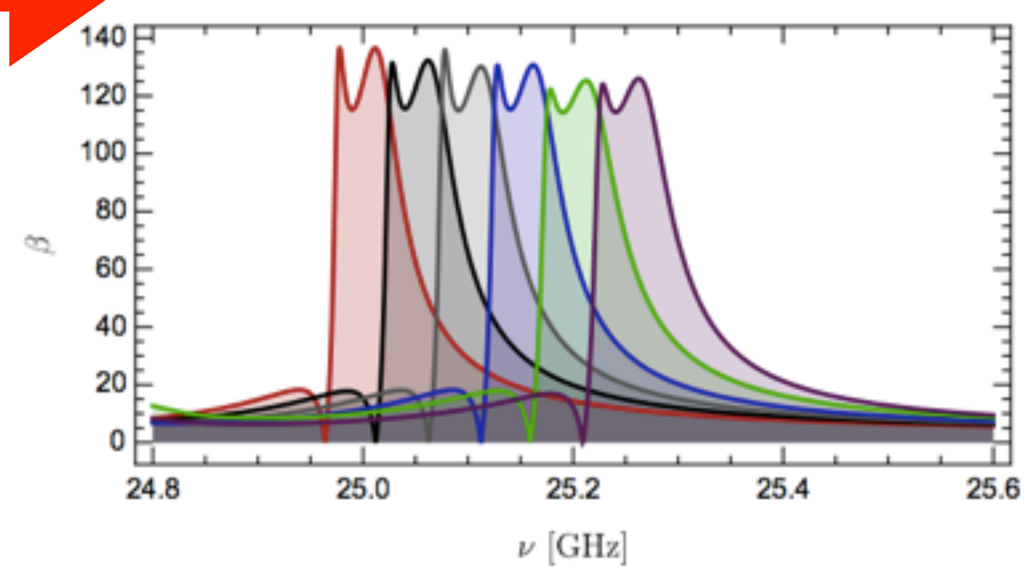
Concept of dielectric haloscope

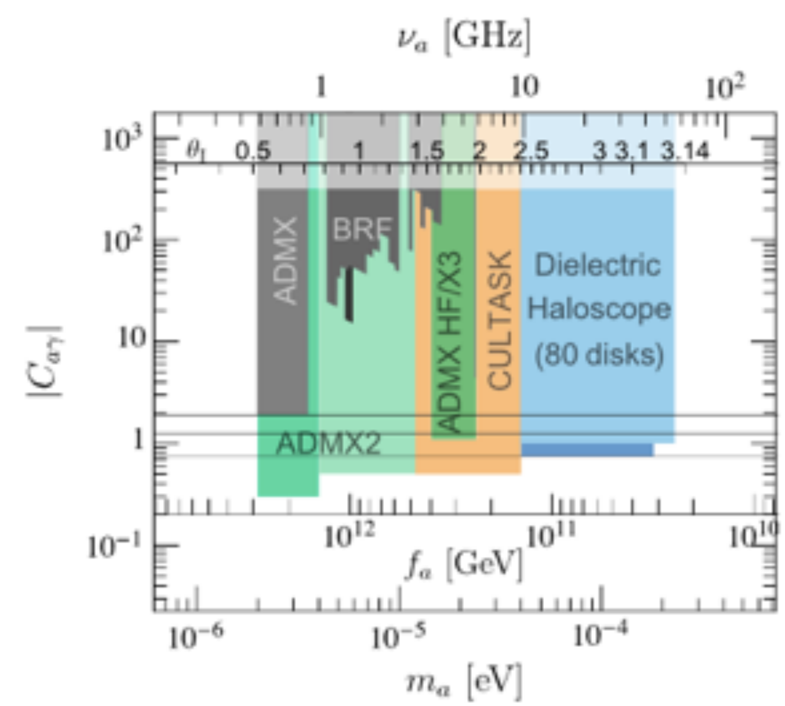


- monochromatic axion-induced EM radiation $\omega \simeq m_a$
- large Area $A \gg 1/\omega^2$
- constructive interference
- cavity effects

$$P \sim \left[2.2 \times 10^{-27} \frac{\mathrm{W}}{\mathrm{m}^2} \left(\frac{B}{10\mathrm{T}}\right)^2 C_{a\gamma}^2\right] \times A \times \beta^2$$

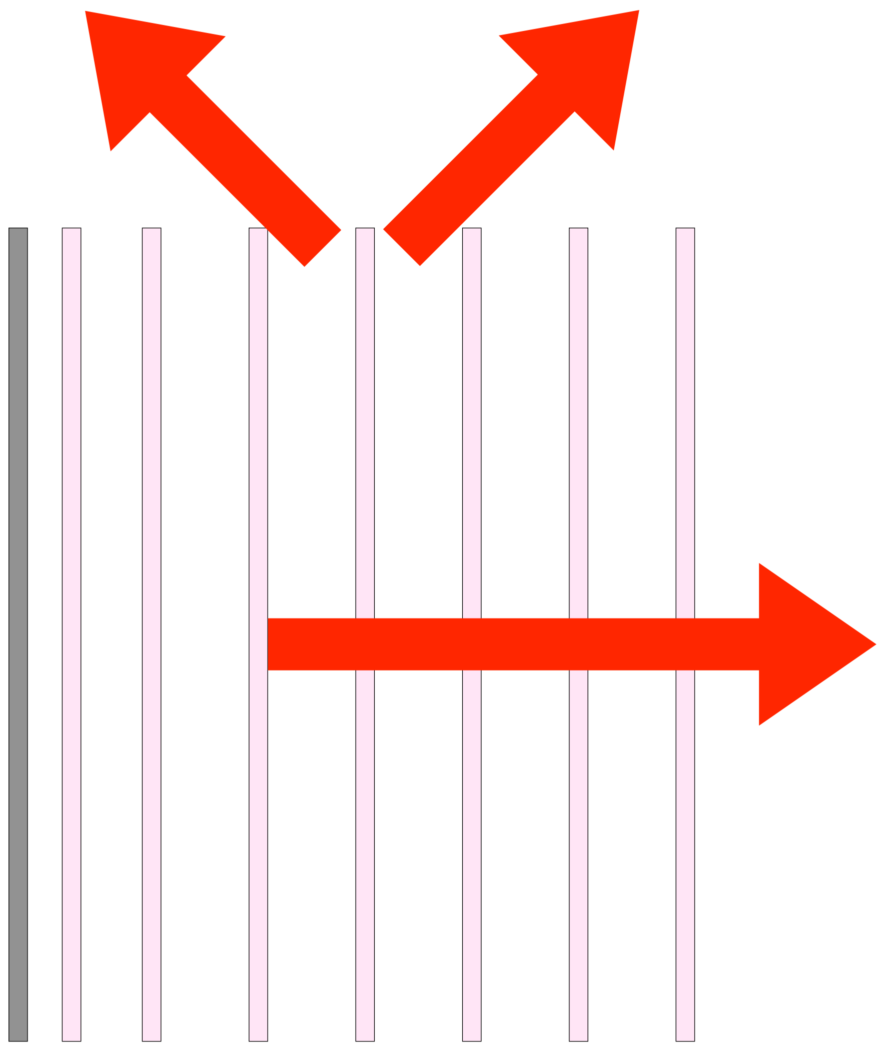
- $m_a \sim 25 - 200\,\mu\mathrm{eV}$
- $A \sim 1\mathrm{m}^2$
- $\beta^2 \sim 10^4$ **with** $O(80)$ **disks, # channels** $\delta\omega_\beta \propto N/\beta^2$





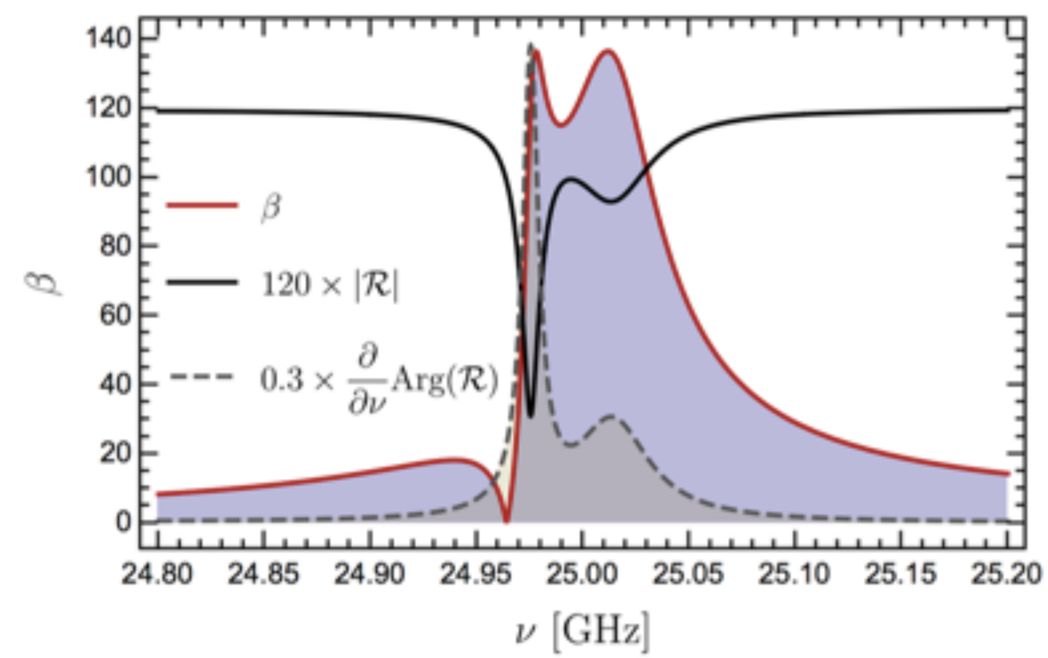
Problems (small selection)

- diffraction losses



- how do we calibrate the boost factor?

our idea is correlate it with R/phase delay

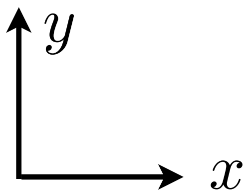


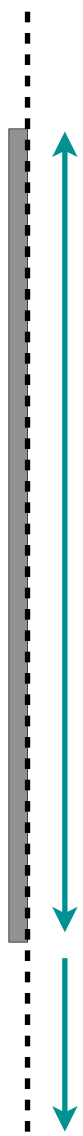
- losses are different for axion DM induced and externally induced modes!
- difficult to excite cavity exactly as axionDM!

Diffraction

- "Easy" exercise: 2D emission of a finite mirror

$$E(x,y) \sim e^{-i\omega t} \int \frac{d^2k}{(2\pi)^2} \widetilde{E}_k e^{ik_y y} e^{ik_x x} = e^{-i\omega t} \int \frac{dq}{2\pi} \widetilde{E}_q e^{iqy} e^{i\sqrt{\omega^2 - q^2}x}$$





boundary conditions cancel the axion induced E-field at the mirror

$$E(0,y) = -E_a$$

$$\widetilde{E}_q = \frac{\sin(qL/2)}{q}$$

but not outside (no radiation from -inf)

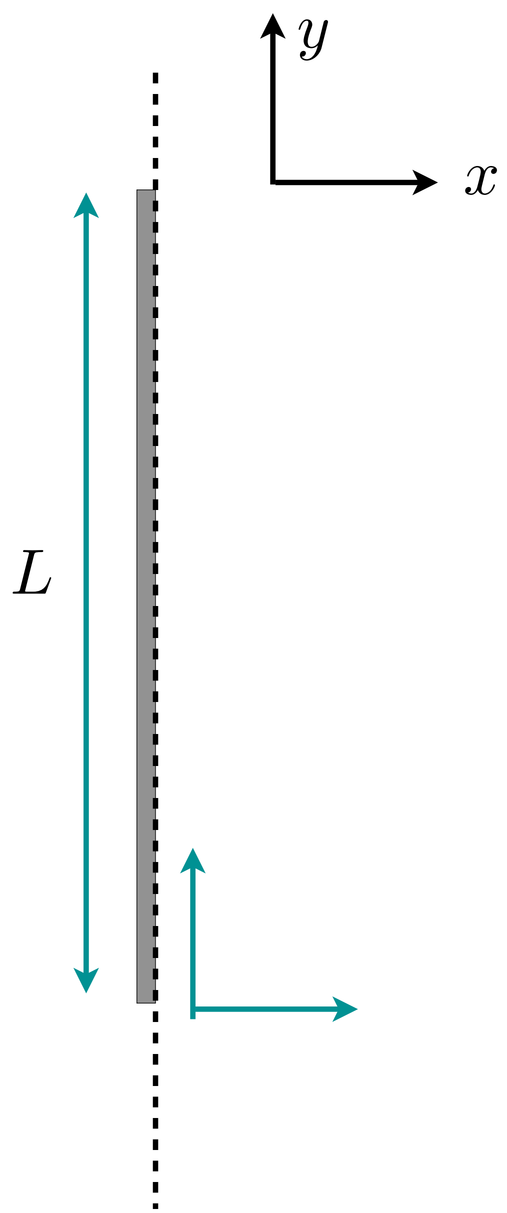
$$E(0,y) = 0$$

- neglecting polarisation, but easy to take to cylindrical 3D
- only radiation field, omega=k shell

Diffraction

- in dimensionless variables ....

$$E(x,y) \sim e^{-i\omega t} \int \frac{dqL}{2\pi} \frac{\sin(qL/2)}{qL} e^{iqL\frac{y}{L}} e^{i\sqrt{1-\frac{q^2}{\omega^2}}\omega x}$$



Modes excited are those with $qL \sim 1$

We are interested in values of $y/L \sim 1$

Here, diffraction as we increase x comes from the term $e^{i\sqrt{1-\frac{q^2}{\omega^2}}\omega x}$

typically we have $\omega x \sim \pi$ **and we hope to have** $\dfrac{qL}{\omega L} \ll 1$

$$\omega L = (100\mu\text{eV})\left(\frac{2}{\sqrt{\pi}}m\right) \sim 500$$

$$\omega L = (50\mu\text{eV})\,(0.2m) \sim 25$$

simplest approximation $e^{i\sqrt{1-\frac{q^2}{\omega^2}}\omega x} \sim e^{i\omega x} e^{-i\frac{qL^2}{2\omega L^2}\omega x}$

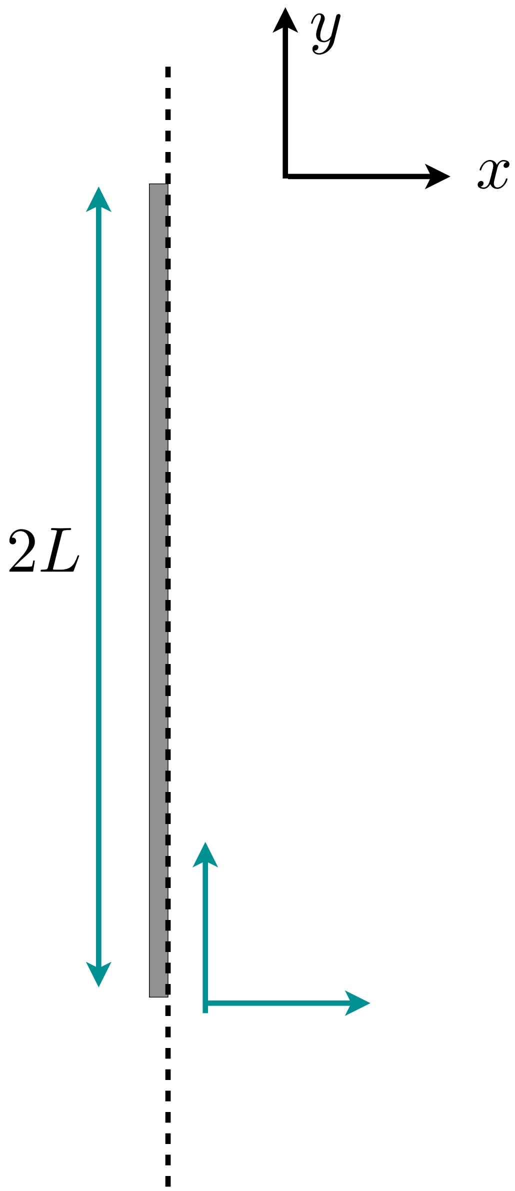
note that phase correction is $\propto \dfrac{x}{(\omega L)^2}$

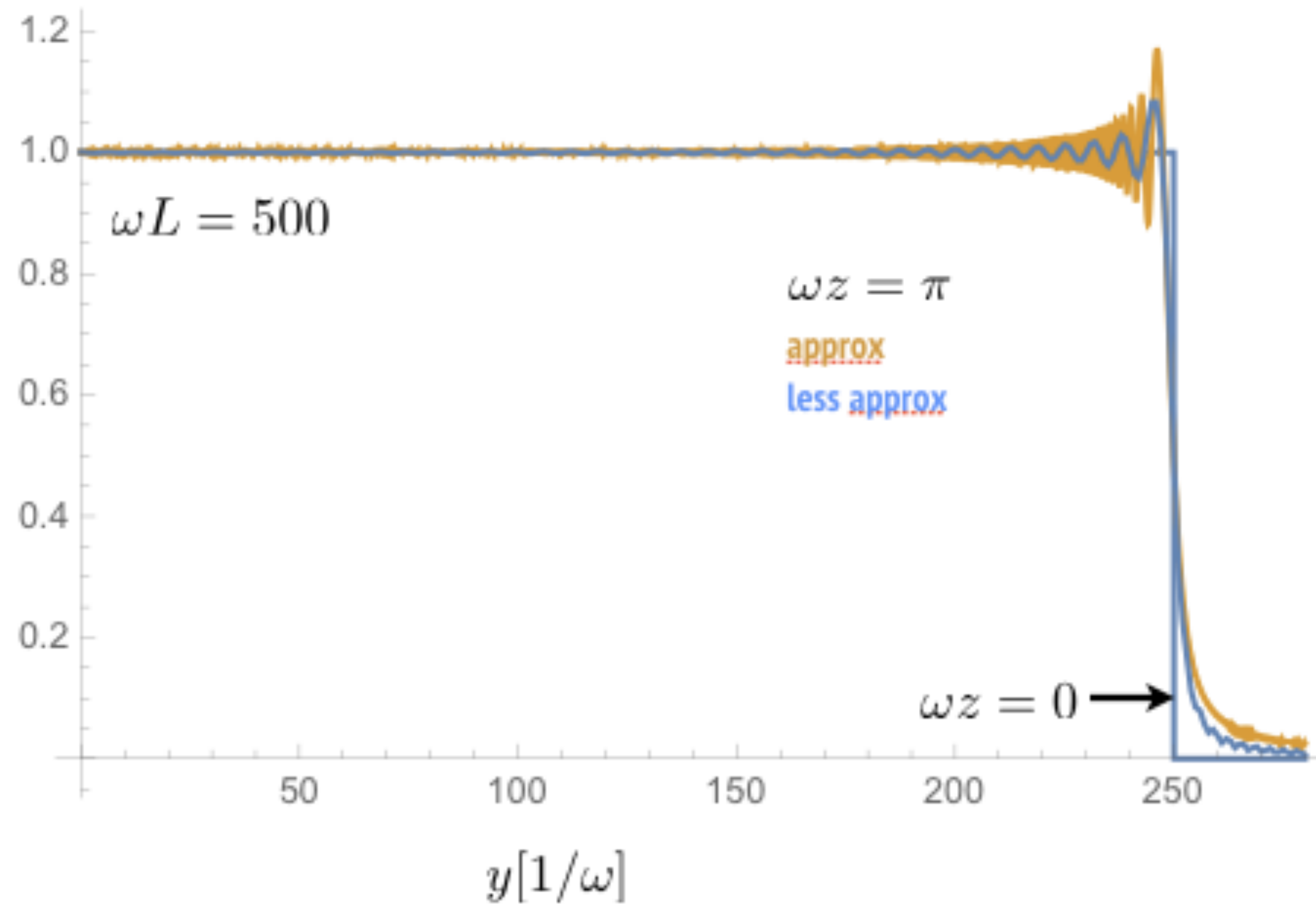
much larger than velocity effects (yet similar to understand) $\delta v \sim 1/(\omega L)$

Diffraction

- **in dimensionless variables ....**

$$E(x,y) \sim e^{-i\omega t} \int \frac{dqL}{2\pi} \frac{\sin(qL/2)}{qL} e^{iqL\frac{y}{L}} e^{i\sqrt{1-\frac{q^2}{\omega^2}}\omega x}$$

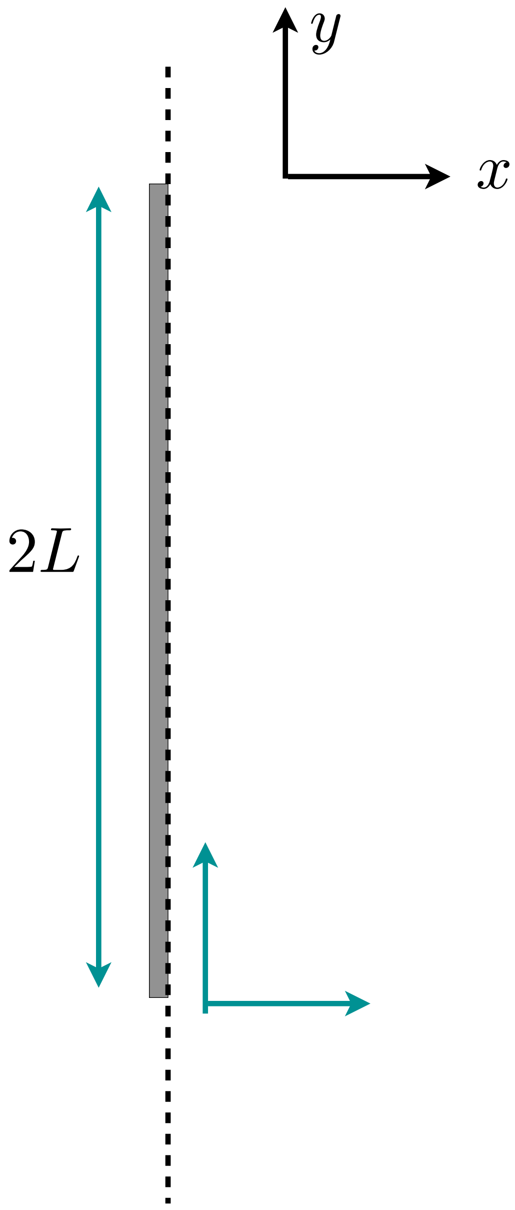


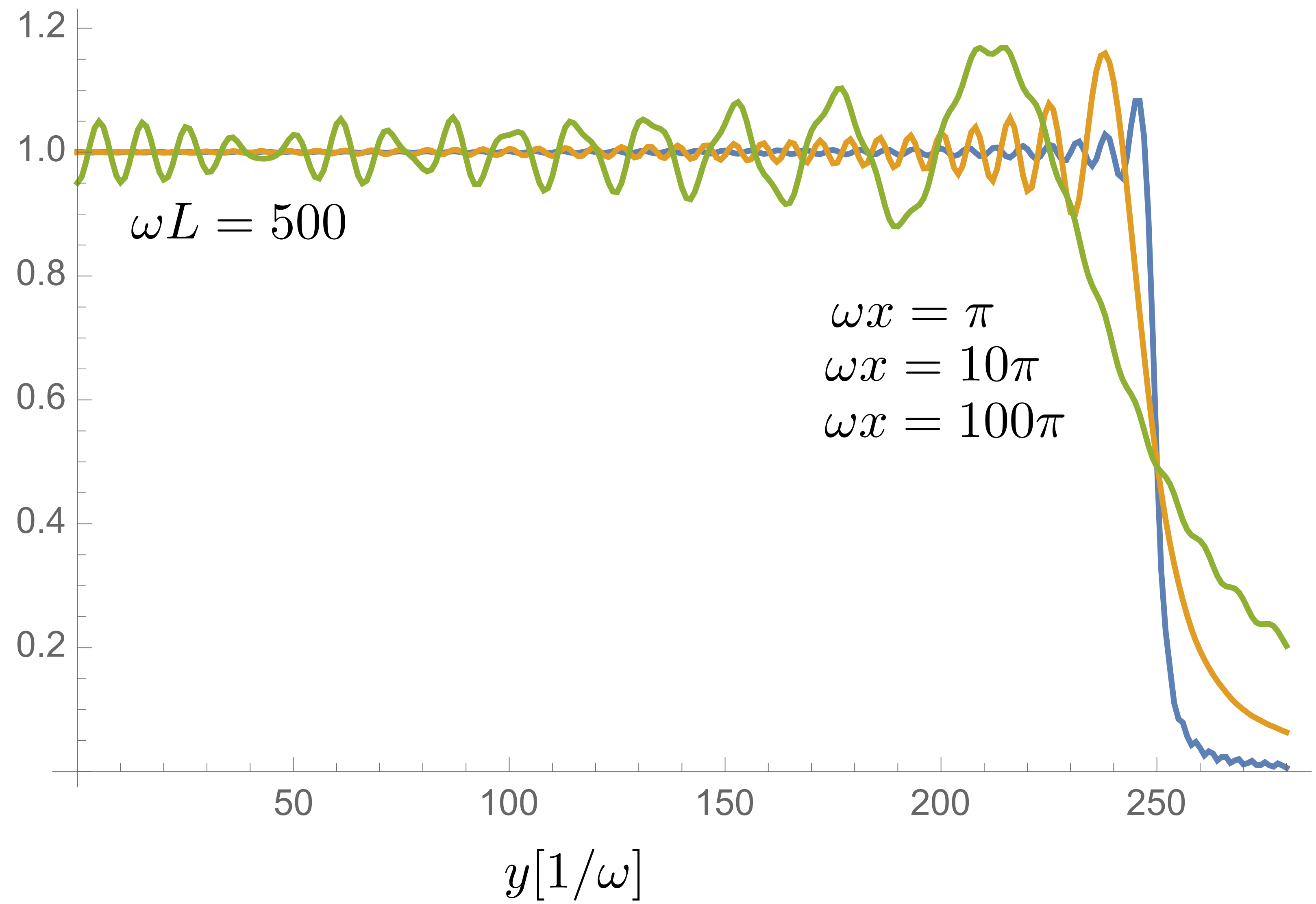


Diffraction

- in dimensionless variables ....

$$E(x,y) \sim e^{-i\omega t} \int \frac{dqL}{2\pi} \frac{\sin(qL/2)}{qL} e^{iqL\frac{y}{L}} e^{i\sqrt{1-\frac{q^2}{\omega^2}}\omega x}$$





Border effects always large, but for wL = 500 it is a small effect
Even at 100 halfwavelengths, the field is 10% coherent,

Diffraction

- Not yet quite final, need to include short distance effects, reflections, etc
- disk of diameter D,

$$\widetilde{E}_q \sim \sin(qD/2)/q$$

- emission characterised by a transverse momentum distribution and correlation

$$\delta v \sim 1/\omega D$$

- but are these modes affected by propagation through further finite-size disks?

matching at each boundary (y-dependent)
some ideas
difficult to solve self consistently

the basic idea is that every disk implies one more convolution so naively ....

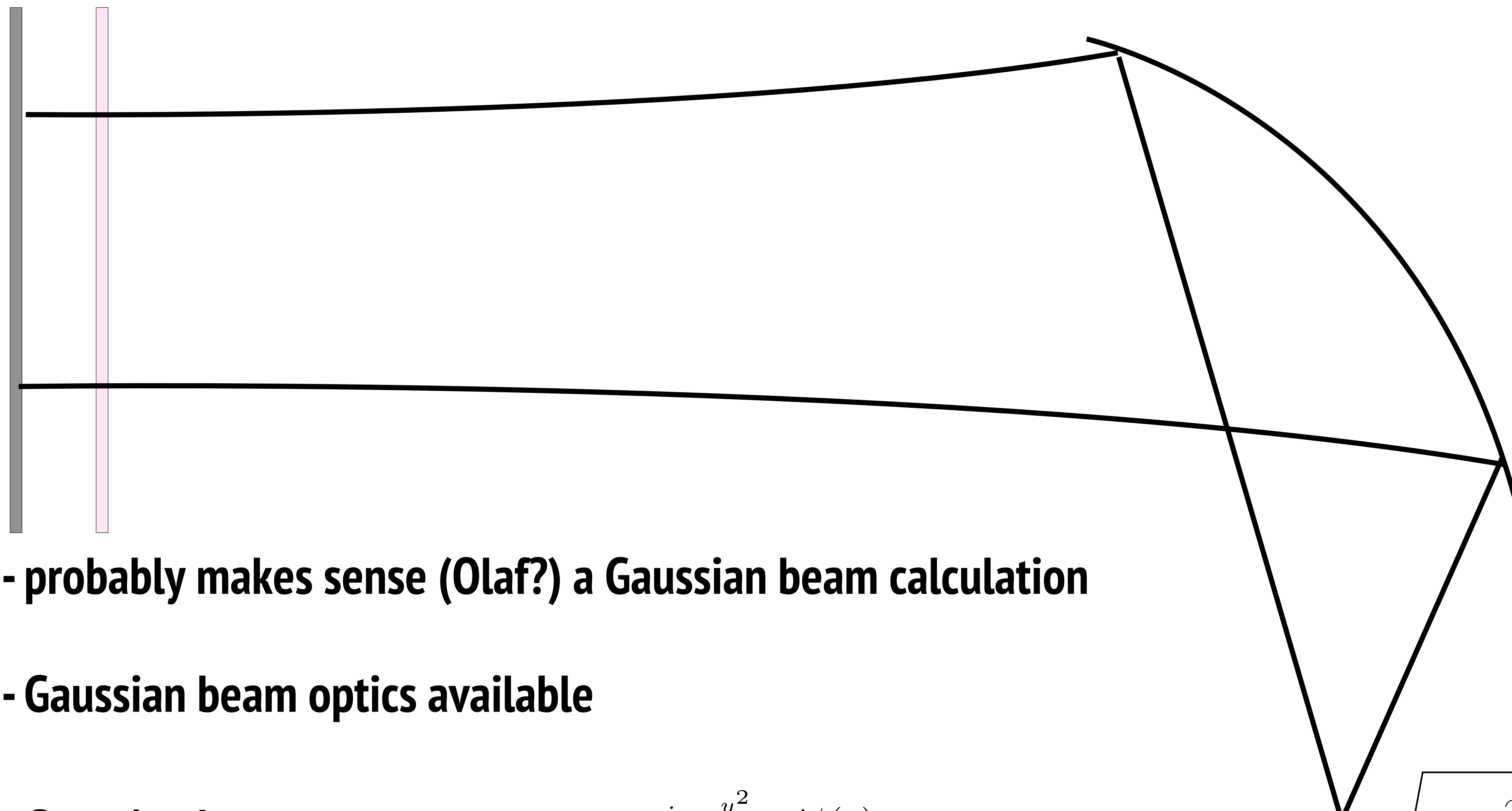
$$\widetilde{E}_{out} \sim \sin(qD/2N)/q$$

and most likely

$$\widetilde{E}_{out} \sim \sin(qD/2\sqrt{N})/q$$

what about the calibration ?

- It is quite a different calculation



- probably makes sense (Olaf?) a Gaussian beam calculation

- Gaussian beam optics available

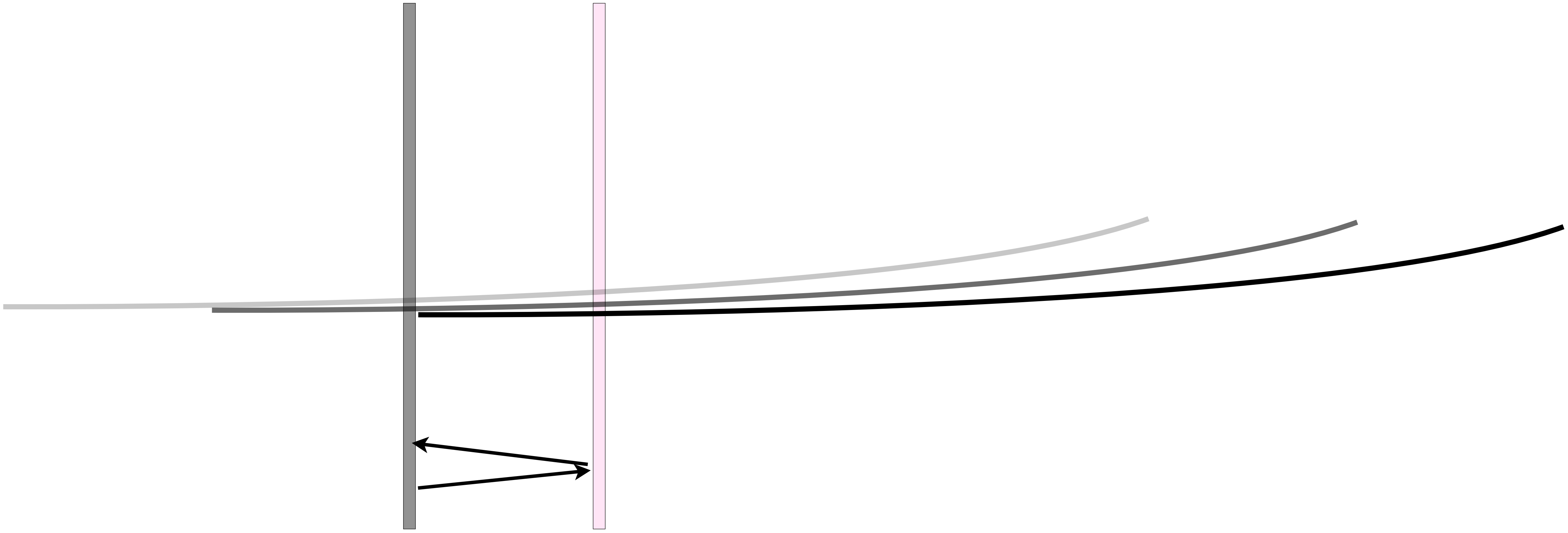
- Gaussian beams $E \sim H(y/w(x))e^{-i\omega\frac{y^2}{2q(x)}}e^{i\psi(x)}$ $\quad q = x + ix_R \quad w = w_0\sqrt{1+\frac{x^2}{x_R^2}}$

- ABCD transfer formalism "works" $\quad q_2 = \frac{Aq_1+B}{Cq_1+D} \quad x_R = \omega w_0^2/2$

Gaussian beams

- It is quite a different calculation

in the case for Gauss beams, reflexion is equivalent to a shift of the beam waist



- plane parallel resonators are unstable (this applies to aDM boosting)

Gaussian beams

plane wave multireflection solution

$$E_{\rm ref} \simeq t\, E_\gamma \sum_{b=0}^{\infty} (r^2 e^{i2\omega L})^b \to t\, E_\gamma \frac{1}{1 - r^2 e^{i2\omega L}}$$

in the case for Gauss beams, reflexion is equivalent to a shift of the beam waist

$$E_{\rm ref} \sim \sum_b (r^2 e^{i\omega L})^b \left(\frac{w_0}{w(x+2bL)}\right)^{1/2} H\left(\frac{y}{w(x+2bL)}\right) e^{-i\omega \frac{y^2}{2q(x+2bL)}} e^{i\psi(x+2bL)}$$

unfortunately this does not factorise

- **easy to do numerics (?)**
- **possible interpretation of Olaf's results:** $\omega D \sim 25, x_R \sim O(\omega DD/2) \sim 1m$
    - **Guoy phase -> shift of peaks**
    - **beam clipping ...**
    - **pure diffraction**

Conclusions

- Difraction is small if wL is large, effects ~ (wL)^2

- Some ideas how to solve the booster equations by matching multimodes across boundaries

- Gaussian beam analysis can help understanding calibration and Olaf's 20 cm results

- preliminary estimates are compatible with O(few %) losses / disk

not good reason to be worried,

plenty of reasons/ideas to sit down and compute !