Primordial Magnetic Fields: Cosmological Signatures and Improved Probes

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Observed Extragalactic Magnetic Fields

Magnetic fields in galaxies: tens of μGauss in magnitude, coherent over kiloparsecs

16 *A. Brandenburg, K. Subramanian / Physics Reports 417 (2005) 1–209*

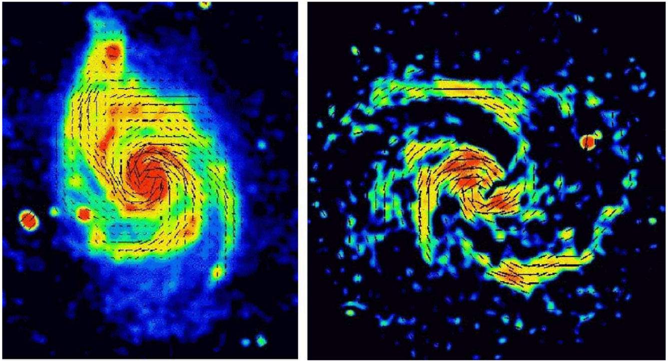
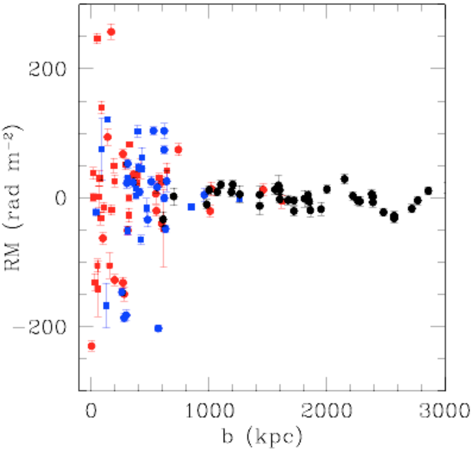


Fig. 2.9. Left: M51 in 6 cm, total intensity with magnetic field vectors. Right: NGC 6946 in 6 cm, polarized intensity with magnetic field vectors. The physical extent of the images is approximately $28 \times 34\,\mathrm{kpc}^2$ for M51 (distance 9.6 Mpc) and $22 \times 22\,\mathrm{kpc}^2$ for NGC 6946 (distance 7 Mpc). (VLA and Effelsberg. Courtesy R. Beck.)

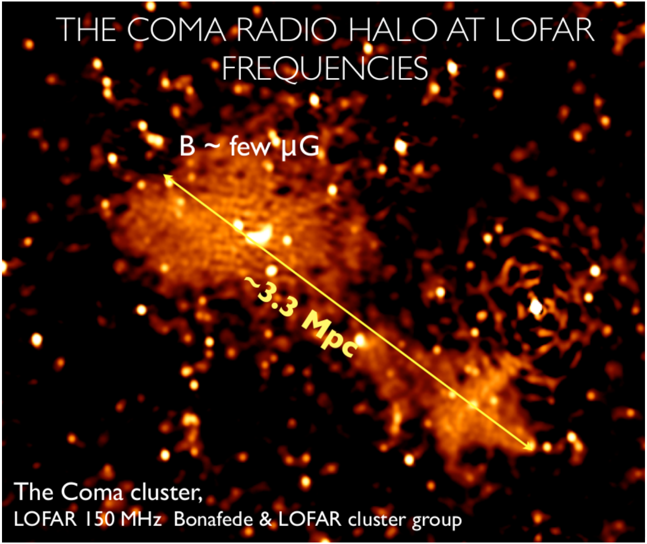
Galaxy Cluster-Scale Observations of Magnetic Fields

Faraday Rotation through rich Abell clusters to background polarized radio sources

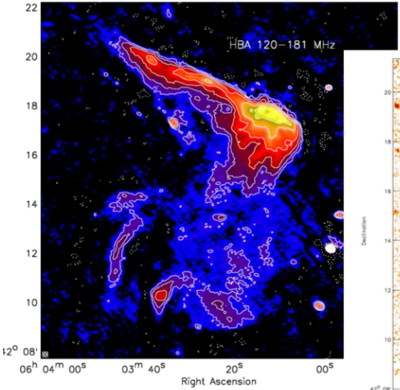


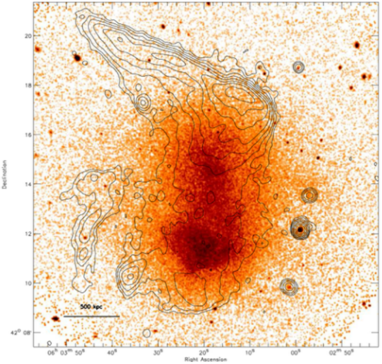
Clarke 04

Coma Cluster Observations of Magnetic Fields



The “toothbrush” cluster
van Weeren & LOFAR cluster group 2016





Bonafede+ 16, van Weeren+ 16

Primordial Cosmic Magnetic Field - Motivation

- Magnetic fields are ubiquitous out to large scales > 10 kpc

  $\vec{B}$ observed at few μGauss level in galaxies: both coherent & stochastic [e.g. Beck 16]

  Growth of $\vec{B}$ via either dynamo amplification or flux freezing + collapse

  $\longrightarrow$ A **seed** $\vec{B}$ **field** is required

  These seed $\vec{B}$ fields may be of astrophysical or **primordial origin**

- Evidence for equally strong $\vec{B}$ in high redshift ($z \sim 2$) galaxies

  [Bernet+ *Nature* 08, Kronberg+ 08, Mao 17]

  Enough time for dynamo to act?

- *Fermi*/*LAT* constraints on γ**-ray halos around TeV Blazars**

  Lower limit: $\vec{B} \geq 3 \times 10^{-16}$ G on intergalactic $\vec{B}$ [Neronov & Vovk, *Science* 10] .

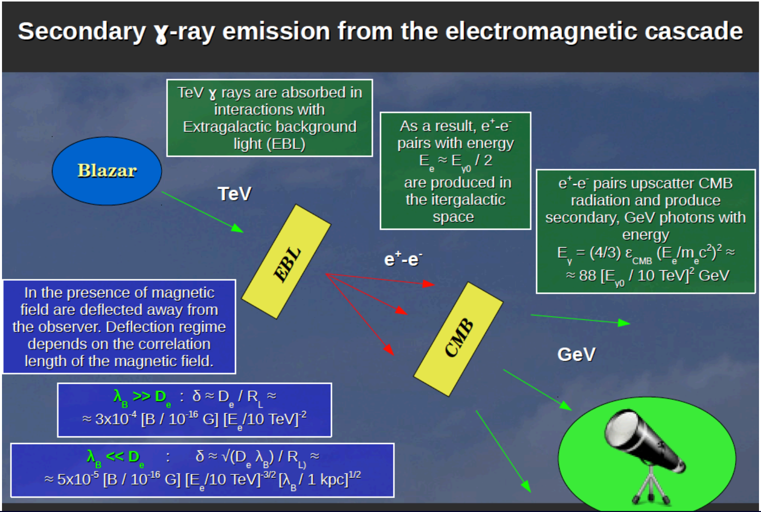
  [Tavecchio 11]

  Also time delays between primary and secondary emission: $\vec{B} \geq 10^{-17}$ G

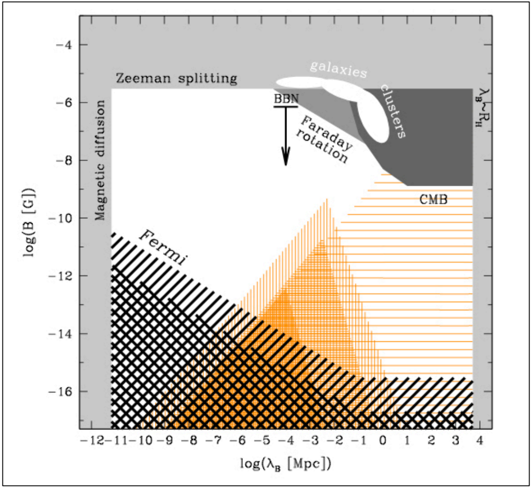
  [Dermer 11, Taylor 11]

GeV halos around TeV Blazars - Intergalactic Magnetic Field

diagram: Vovk (2011)



Constraints on Cosmic Magnetic Fields



[adapted from Neronov & Vovk, *Science* 10]

Magnetogenesis

- No single compelling mechanism for the origin of strong primordial $\vec{B}$ fields

  [e.g. Durrer & Neronov 13, Subramanian 16]

- *Breaking conformal invariance of EM* [e.g. Subramanian 10, 15; Martin & Yokoyama 08]

$$\begin{aligned} S = & \int \sqrt{-g}\, d^4x b(t)[-\frac{f^2(\phi, R)}{16\pi} F_{\mu\nu}F^{\mu\nu} - g_1 RA^2 \\ + & g_2 \theta F_{\mu\nu}\tilde{F}^{\mu\nu} - D_\mu\psi(D^\mu\psi)^* \;] \end{aligned} \tag{1}$$

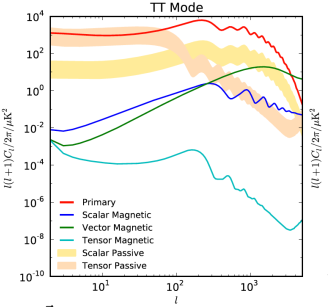
- $\boxed{\text{Inflation}}$ - but backreaction, strong coupling or running of coupling constants

- Phase transitions - QCD, EW - but produce small correlation length for $\vec{B}$, require inverse-cascade [e.g. Vachaspati 91, Sigl 97, Copi 08 Kahniashvili 14, 13]

CMB Probes and Limits on Primordial Magnetic Fields

PMF affect BBN, CMB, First Stars, Reionization, λ_{Jeans}, 21 cm, LSS, UHECR..

- CMB power spectrum: modes from $\vec{B}$

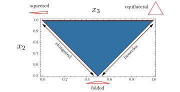


for $|\vec{B}| \sim$ 5nG, [J. R. Shaw & A. Lewis, PRD 10]

- Alfven modes (vortical vector mode, overdamped) [Jedamzik+ 98, Subramanian+ 98], metric perturbations: passive mode $\frac{\Delta T}{T}|_{passive} \propto \ln\left(\frac{T_B}{T_\nu}\right)$ [Shaw+ 10]
- *Planck* Power spectrum constraints: $B_0 \leq 4.4$ nG, ≤ 2.1 nG (scale-invariant $\vec{B}$) Planck Paper XIX 2015]

Beyond TT Power Spectrum

- CMB Polarization power spectrum, Faraday rotation, Dissipation of $\vec{B}$ energy changing recombination history, Spectral distortion, Non-Gaussianity



[Yadav & Wandelt 10]

- **Magnetic Bispectrum** Scalar Passive $\boxed{B_0 \leq 2 \text{ nG}}$ **[P. Trivedi, T. R. Seshadri & K. Subramanian, PRD 2010]**
- **Magnetic Trispectrum** Scalar Passive $\boxed{B_0 \leq 0.7, \quad 0.05 \text{ nG}}$

[P. Trivedi, K. Subramanian & T. R. Seshadri, PRL 2012, PRD 2014]

Why is **CMB Non-Gaussianity** from Cosmic **Magnetic** Fields Important?

NG from Inflationary models:

Small fluctuations in the field
(linear order dominates)

$\downarrow$

Gaussian statistics for field fluctuations

$\downarrow$

Gaussian statistics for CMB
temperature anisotropy
at lowest order

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}\left[\Phi_G^2(\mathbf{x}) - \langle\Phi_G^2(\mathbf{x})\rangle\right] + g_{NL}\Phi_G^3(\mathbf{x})$$

Inflationary CMB non-Gaussianity **only** arises only from **higher order** effects

NG from Magnetic Fields:

Magnetic energy densities & stresses
inherently quadratic in $\vec{B}$ field:

$$\Omega_B, \Pi_B \propto |\vec{B}|^2$$

$\downarrow$

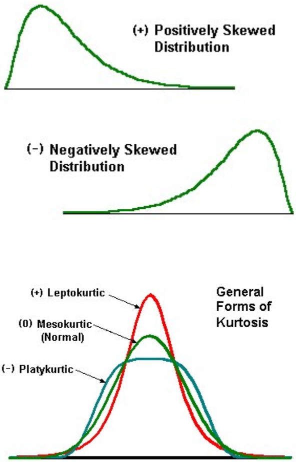
Even for a purely Gaussian $\vec{B}$ field,
Ω_B, Π_B entirely non-Gaussian

$\downarrow$

Non-Gaussianity fluctuations in CMB
anisotropy induced by $\vec{B}$ field

Magnetic fields source CMB non-Gaussianity **even** at **lowest order**

Non-Gaussian Distributions, Primordial NG



$$\frac{\Delta T(\boldsymbol{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\boldsymbol{n})$$

$$a_{lm} = \int d\boldsymbol{n} \; \frac{\Delta T(\boldsymbol{n})}{T} \; Y^{*}_{lm}(\boldsymbol{n})$$

$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$$

$$T^{m_1 m_2 m_3 m_4}_{l_1 l_2 l_3 l_4} = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} a_{l_4 m_4} \rangle$$

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL} \left[\Phi^2_G(\mathbf{x}) - \langle \Phi^2_G(\mathbf{x}) \rangle \right] + g_{NL} \Phi^3_G(\mathbf{x})$$

$$\frac{B_\Phi(k_1, k_2, k_3)}{P_\Phi(k_1) P_\Phi(k_2) + perm.} = 2 f_{NL}$$

[Komatsu & Spergel 01]

$$l_1(l_1+1) l_3(l_3+1) \, \boldsymbol{b_{l_1 l_2 l_3}} \approx \mathbf{10^{-18} \, f_{NL}}$$

$-10 < f^{loc}_{NL} < 74$ and $-8.9 < f^{loc}_{NL} < 13.9$ (95 % C.L.)

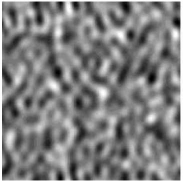
[WMAP7 Komatsu et al. 2011 & Planck 2015]

Properties of Assumed Cosmic Magnetic Field

Homogeneous cosmic $\vec{B}$-fields: v strict limits from CMB quadrupole, anisotropic homogeneous model.

Instead we consider a **stochastic $\vec{B}$-field** $\vec{B}(\vec{x}, t)$

- Magnetic Field: Stochastic. Statistically homogeneous and isotropic.



[Realization: I. Brown & R. Crittenden 05]

- Magnetic field $\longrightarrow$ velocity field
  On scales > galactic scales: velocities very small $\longrightarrow$ $\vec{B}$ -fields do not change.
  [Jedamzik, Katalinic & Olinto 98, Subramanian & Barrow 98]

- Highly conducting cosmic plasma $\rightarrow$ $\vec{B}$-fields frozen into matter

$$\Longrightarrow \quad \vec{B}(\vec{x}, t) = \frac{\vec{b}_0(\vec{x})}{a^2(t)}$$

Power Spectrum of the Magnetic Field

Assumed to be a non-helical Gaussian Random Field. Statistical properties of $\vec{B}$ specified completely by 2-point correlation function - power spectrum $M(k)$

$$\langle b_i(\vec{k}) b_j^*(\vec{q}) \rangle = (2\pi)^3 \delta(\vec{k} - \vec{q}) P_{ij}(\vec{k}) M(k)$$

$\rightarrow$ Completely determined by $M(k)$

$P_{ij}(\vec{k}) = (\delta_{ij} - k_i k_j / k^2)$ is the projection operator that ensures $\vec{\nabla} \cdot \vec{b}_0 = 0$

$\langle \vec{b}_0 \cdot \vec{b}_0 \rangle = 2 \int \frac{dk}{k} \Delta_b^2(k)$ with $\Delta_b^2 = k^3 M(k) / 2\pi^2$

Form of $M(k)$:
$M(k) = A\, k^n$ with a cutoff at Alfven wave damping scale.

$n = -3$ is scale-invariant

$$\Rightarrow \Delta_b^2(k) = \frac{B_0^2}{2}(n+3)\left(\frac{k}{k_g}\right)^{n+3}$$

Fixing A: In terms of variance of Magnetic Field B_0 at $k_G = 1h$ Mpc^{-1}

Bispectrum Results on Cosmic Magnetic Fields

Source of CMB Bispectrum	$b_{l_1 l_2 l_3}$	B_0 Limit (nG)
Inflationary Perturbations [e.g. Riotto 2008]	$10^{-18}\ f_{NL}$	—
Magnetic **Energy Density** Ω_B: local isosceles, equilateral [T. R. Seshadri & K. Subramanian, PRL 2009]	10^{-22}	≤ 35
Magnetic Scalar Anisotropic Stress Π_B: s-independent	6-9 $\times\ 10^{-16}$	$\boxed{\leq 3}$
Magnetic Scalar Anisotropic Stress Π_B: sq. collinear [P. Trivedi, T. R. Seshadri & K. Subramanian, PRD 2010]	-1.4 $\times\ 10^{-16}$	$\boxed{\leq 2}$

(magnetic $b_{l_1 l_2 l_3}$ for B_0 = 3nG)

- Numerical evaluation of magnetic bispectra by Nagoya Group: M. Shiraishi et al.
  Vector mode bispectrum - $B_0 \leq 16$ nG (PRD 2010) also Kahniashvili & Lavrelashvili 2012
  Tensor mode bispectrum - $B_0 \leq 2.6 - 4.4$ nG (PRD 2011) and WMAP7 constraints $B_0 \leq 3.2$ nG (2013)
- Also scalar Ω_B bispectrum Caprini et al. (2009) $B_0 \leq 10$ nG, Cai (2010) and Brown (2010)

Magnetic Trispectrum Calculation - Energy Density Ω_B

- CMB anisotropy from Ω_B : Sachs-Wolfe $\frac{\Delta T}{T}(\boldsymbol{n}) = \mathcal{R}\,\Omega_B(\boldsymbol{x}_0 - \boldsymbol{n}D^*)$, $\mathcal{R} \sim -0.04$,

  $\Delta T(\boldsymbol{n})/T = \sum_{lm} a_{lm} Y_{lm}(\boldsymbol{n})$, $\quad a_{lm} = \frac{4\pi}{i^l} \int \frac{d^3k}{(2\pi)^3}\, \mathcal{R}\, \hat{\Omega}_B(\boldsymbol{k})\, j_l(kD^*)\, Y^*_{lm}(\hat{k})$

- **Trispectrum** $T^{m_1 m_2 m_3 m_4}_{l_1 l_2 l_3 l_4} = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} a_{l_4 m_4} \rangle$

$$T^{m_1 m_2 m_3 m_4}_{l_1 l_2 l_3 l_4} = \mathcal{R}^4 \int \left[\prod_{i=1}^{4} (-i)^{l_i} \frac{d^3\boldsymbol{k}_i}{2\pi^2} j_{l_i}(k_i D^*) Y^*_{l_i m_i}(\hat{\boldsymbol{k}}_i) \right] \zeta_{1234}$$

  with ζ_{1234} defined as,

$$\zeta_{1234} = \langle \hat{\Omega}_B(\boldsymbol{k}_1) \hat{\Omega}_B(\boldsymbol{k}_2) \hat{\Omega}_B(\boldsymbol{k}_3) \hat{\Omega}_B(\boldsymbol{k}_4) \rangle$$

- This 4-pt. correlation of Ω_B is an 8-pt correlation of magnetic fields $\vec{b}(\boldsymbol{k})$
- Using Wick's Theorem: 8-pt. correlation $\rightarrow$ 105 terms involving products of 2-pt correlations.
- Out of the 105 terms, can neglect 45 vanishing terms with 1-pt delta functions and 12 unconnected terms with products of 2-pt delta functions to leave 48 terms.

Ω_B Trispectrum - Mode Coupling Integral

- A long calculation of the remaining 48 terms gives the mode-coupling integral ψ_{1234}

$$\zeta_{1234} = \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)\ \psi_{1234}$$

- In terms of the product of four $M(k)$ magnetic spectra and an angular structure involving 28 cosines α_i,β_i ... λ_i of angles between $\hat{s}$ and the $\hat{k}_i$'s and their combinations.

$$\begin{aligned}
\psi_{1234} = \frac{8}{(8\pi\rho_0)^4} \int d^3\boldsymbol{s} \qquad & M(s)M(|\boldsymbol{k}_1 + \boldsymbol{s}|) \times \\
\Big[\qquad & M(|\boldsymbol{k}_1 + \boldsymbol{k}_3 + \boldsymbol{s}|)\left(M(|\boldsymbol{k}_2 - \boldsymbol{s}|)\mathcal{F}_{\mathrm{i}} + M(|\boldsymbol{k}_4 - \boldsymbol{s}|)\mathcal{F}_{\mathrm{ii}}\right) \\
+ \quad & M(|\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{s}|)\left(M(|\boldsymbol{k}_3 - \boldsymbol{s}|)\mathcal{F}_{\mathrm{iii}} + M(|\boldsymbol{k}_4 - \boldsymbol{s}|)\mathcal{F}_{\mathrm{iv}}\right) \\
+ \quad & M(|\boldsymbol{k}_1 + \boldsymbol{k}_4 + \boldsymbol{s}|)\left(M(|\boldsymbol{k}_2 - \boldsymbol{s}|)\mathcal{F}_{\mathrm{v}} + M(|\boldsymbol{k}_3 - \boldsymbol{s}|)\mathcal{F}_{\mathrm{vi}}\right) \Big]
\end{aligned}$$

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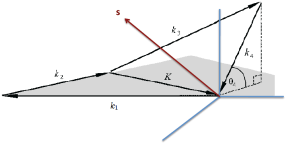

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with 72 angular terms

$$\begin{aligned}
\mathcal{F}_{\rm i} &= -1 + (\alpha_1^2+\alpha_2^2+\alpha_6^2+\beta_2^2+\beta_6^2+\gamma_6^2) - (\alpha_1\alpha_2\beta_2+\alpha_1\alpha_6\beta_6+\alpha_2\alpha_6\gamma_6+\beta_2\beta_6\gamma_6) + \alpha_1\alpha_2\beta_6\gamma_6 \\
\mathcal{F}_{\rm ii} &= -1 + (\alpha_1^2+\alpha_4^2+\alpha_6^2+\beta_4^2+\beta_6^2+\epsilon_6^2) - (\alpha_1\alpha_4\beta_4+\alpha_1\alpha_6\beta_6+\alpha_4\alpha_6\epsilon_6+\beta_4\beta_6\epsilon_6) + \alpha_1\alpha_4\beta_6\epsilon_6 \\
\mathcal{F}_{\rm iii} &= -1 + (\alpha_1^2+\alpha_3^2+\alpha_5^2+\beta_3^2+\beta_5^2+\delta_5^2) - (\alpha_1\alpha_3\beta_3+\alpha_1\alpha_5\beta_5+\alpha_3\alpha_5\delta_5+\beta_3\beta_5\delta_5) + \alpha_1\alpha_3\beta_5\delta_5 \\
\mathcal{F}_{\rm iv} &= -1 + (\alpha_1^2+\alpha_4^2+\alpha_5^2+\beta_4^2+\beta_5^2+\epsilon_5^2) - (\alpha_1\alpha_4\beta_4+\alpha_1\alpha_5\beta_5+\alpha_4\alpha_5\epsilon_5+\beta_4\beta_5\epsilon_5) + \alpha_1\alpha_4\beta_5\epsilon_5 \\
\mathcal{F}_{\rm v} &= -1 + (\alpha_1^2+\alpha_2^2+\alpha_7^2+\beta_2^2+\beta_7^2+\gamma_7^2) - (\alpha_1\alpha_2\beta_2+\alpha_1\alpha_7\beta_7+\alpha_2\alpha_7\gamma_7+\beta_2\beta_7\gamma_7) + \alpha_1\alpha_2\beta_7\gamma_7 \\
\mathcal{F}_{\rm vi} &= -1 + (\alpha_1^2+\alpha_3^2+\alpha_7^2+\beta_3^2+\beta_7^2+\delta_7^2) - (\alpha_1\alpha_3\beta_3+\alpha_1\alpha_7\beta_7+\alpha_3\alpha_7\delta_7+\beta_3\beta_7\delta_7) + \alpha_1\alpha_3\beta_7\delta_7
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with 72 angular terms

$$\zeta_{1234} = \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)\frac{-8\,(24\pi)\,A^4\,k_1^{2n+3}k_2^n k_3^n}{(8\pi\rho_0)^4}\left[\frac{(2^n)(4n+3) - (n+3)}{(4n+3)(n+3)}\right]$$

with algebraic steps similar to the inflationary trispectrum, we perform the integrals over the angular parts of $(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4, \boldsymbol{K})$. This gives,

$$
\begin{aligned}
T^{m_1 m_2 m_3 m_4}_{l_1 l_2 l_3 l_4} &= \left[(-768)\,\frac{\mathcal{R}^4}{\pi^7}\right]\left(\frac{A}{(8\pi\rho_0)}\right)^4 \\
&\times \left\{\frac{(2^n)(4n+3)-(n+3)}{(4n+3)(n+3)}\right\} \\
&\times \int dr_1 r_1^2 \int dr_2 r_2^2 \int dk_1 k_1^2 k_1^{2n+3} j_{l_1}(k_1 D^*) j_{l_1}(k_1 r_1) \\
&\times \int dk_2 k_2^2 k_2^n j_{l_2}(k_2 D^*) j_{l_2}(k_2 r_1) \int dk_3 k_3^2 k_3^n j_{l_3}(k_3 D^*) j_{l_3}(k_3 r_2) \\
&\times \int dk_4 k_4^2 j_{l_4}(k_4 D^*) j_{l_4}(k_4 r_2) \times \sum_{LM} (-1)^{L-M} \\
&\times \int dK K^2 j_L(K r_1) j_L(-K r_2) \\
&\times \int d\Omega_{\hat{r}_1} Y_{l_1 m_1}(\hat{r}_1) Y_{l_2 m_2}(\hat{r}_1) Y_{LM}(\hat{r}_1) \\
&\times \int d\Omega_{\hat{r}_2} Y_{l_3 m_3}(\hat{r}_2) Y_{l_4 m_4}(\hat{r}_2) Y_{L\,-M}(\hat{r}_2)
\end{aligned}
$$

Trispectrum Constraints on Primordial Magnetic Fields

We compare our magnetic trispectrum

$$\boxed{\mathrm{T}^{l_1 l_2}_{l_3 l_4}(L)} \simeq \boxed{-5.8 \times 10^{-29}} \left(\frac{n+3}{0.2}\right)^3 \left(\frac{B_{-9}}{3\ \mathrm{nG}}\right)^8 \frac{h_{l_1 L l_2} h_{l_3 L l_4}}{l_1(l_1+1) l_2(l_2+1) l_3(l_3+1)}$$

to the standard CMB trispectrum [Okamoto & Hu 02, Kogo & Komatsu 06] sourced by inflationary perturbations.

$$T^{l_1 l_2}_{l_3 l_4}(L) \approx 9\, C^{SW}_{l_2} C^{SW}_{l_4} \left[4 f_{NL}^2 C^{SW}_L + g_{NL}\left(C^{SW}_{l_1} + C^{SW}_{l_3}\right)\right] h_{l_1 L l_2} h_{l_3 L l_4}$$

Taking the first term and using WMAP7 limits

$$T^{l_1 l_2}_{l_3 l_4}(L) \approx \boxed{5.4 \times 10^{-27} \tau_{NL} \ \text{or}\ 7.78 \times 10^{-27} f_{NL}^2} \frac{h_{l_1 L l_2} h_{l_3 L l_4}}{l_2(l_2+1) l_4(l_4+1) L(L+1)}$$

The magnetic field limits are obtained by taking the one-eighth power of the ratio of primordial to magnetic trispectra, which gives

$$\boxed{\mathrm{B}_0 \leq 19\ \mathrm{nG}} \ \text{using}\ \tau_{NL} \tag{2}$$

[P. Trivedi, T. R. Seshadri & K. Subramanian, PRL 2012]

Passive Mode Trispectrum - sourced by Scalar Anisotropic Stress

- More complex: 16 times the number of operators
- ~ 1500 angular terms instead of only 72.

s-independent angular terms evaluate to:

$$\begin{aligned}\mathcal{F}^{\text{s-indep}}_{\Pi_B\Pi_B\Pi_B\Pi_B} = \quad & -13 + 9(\theta_{12}^2 + \theta_{13}^2 + \theta_{14}^2 + \theta_{23}^2 + \theta_{24}^2 + \theta_{34}^2) \\ & -27(\theta_{12}\theta_{13}\theta_{23} + \theta_{12}\theta_{14}\theta_{24} + \theta_{13}\theta_{14}\theta_{34} + \theta_{23}\theta_{24}\theta_{34}) \\ & +27(\theta_{12}\theta_{13}\theta_{24}\theta_{34} + \theta_{12}\theta_{14}\theta_{23}\theta_{34} + \theta_{13}\theta_{14}\theta_{23}\theta_{24})\end{aligned}$$

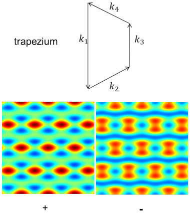
Also performed numerical calculation in flat-sky limit

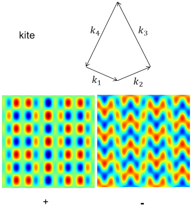
- $\mathbf{T}^{\mathbf{l}_1\mathbf{l}_2}_{\mathbf{l}_3\mathbf{l}_4}(\mathbf{L}) \approx \mathbf{10}^{-19}$, this is $\sim\ \times\mathbf{10}^{10}$ **greater** than Ω_B trispectrum
- Magnetic field limits from trapezium and kite configurations

$\boxed{\mathrm{B}_0 \leq 1.3 \text{ nG}}$ using τ_{NL} and $\boxed{\mathrm{B}_0 \leq 0.7 \text{ nG}}$ using f_{NL}^2

  - cf. bispectra (2 nG), the $\vec{B}$- limits from trispectra are upto **three times as strong**.

[P. Trivedi, T. R. Seshadri & K. Subramanian, PRL 2012 & PRD 2014]





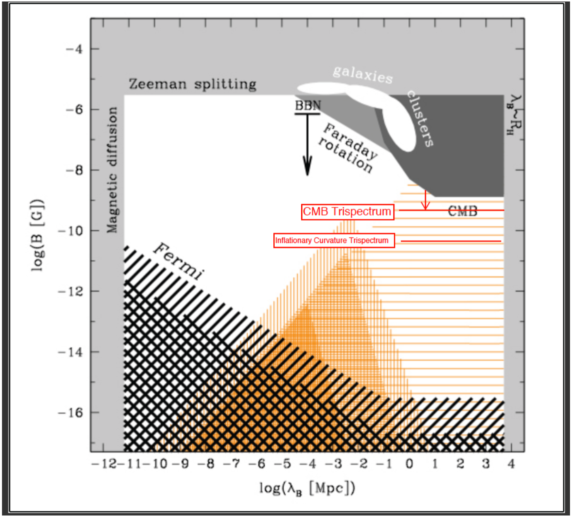
[Figures: Lewis 11]

Planck Results and Improved Magnetic Field Constraints

- *Planck* TT power spectrum 2015: $B_0 <$ 4.4, 2.1 nG similar to WMAP7 $B_0 < 3.5$nG
- Bispectrum: $-8.9 < f_{NL}^{loc} < 13.9$ constraint tighter than $-10 < f_{NL}^{loc} < 74$ WMAP7
- Trispectrum: $\tau_{NL} < 2800$ constraint tighter than $-6000 < \tau_{NL} < 33,000$ WMAP7
- Improved *Planck* τ_{NL} constraint allows sub-nanoGauss magnetic constraint $B_0 < 0.7$ nG independent of any inflation assumption about $\tau_{NL} - f_{NL}$ relation
- *Planck* Non-Gaussianity + Our magnetic trispectrum calculation
  $\rightarrow$ **Robust sub-nanoGauss magnetic field upper limit (flat-sky: 0.6 nG)**
- Recent Inflationary Magnetic Curvature Mode (Bonvin et al 2013) $\rightarrow$
  Can dominate other magnetic modes, lower trispectrum constraint to 0.05 nG

[P. Trivedi, T. R. Seshadri & K. Subramanian, PRL 2012 & PRD 2014]

Updated Constraints on Cosmic Magnetic Fields



[Adapted from Neronov & Vovk, *Science* 10]

Magnetic Tensor Mode Bispectrum

- Magnetic Fields also source Tensor Anisotropic Stress
- Gravitational waves (magnetic tensor modes) induce perturbations to the geodesic for photon propagation $\rightarrow$ additional temperature anisotropy in the CMB sky
- Magnetic tensor mode can dominate over the magnetic scalar modes by more than an order of magnitude in the power spectrum [Shaw & Lewis 2010]

$$\Pi^T(\boldsymbol{k})^{(\pm)} = \left[\hat{e}_{ij}(\boldsymbol{k})^{(\pm)}\right] \Pi_{ij}(\boldsymbol{k})$$

Here, $\hat{e}_{ij}(\boldsymbol{k})^{(\pm)}$ is the Fourier transform of the spin-2 polarization basis tensor $\hat{e}_{ab}^{(\pm)}$ which is defined in real space as (e.g. Hu & White 1997)

$$\hat{e}_{ij}(\boldsymbol{x})^{(\pm)} \equiv \frac{1}{2}(\hat{e}_\theta \mp i\hat{e}_\phi)_i \otimes (\hat{e}_\theta \mp i\hat{e}_\phi)_j$$

- Study tensor perturbations in perturbed FLRW metric

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \{\delta_{ij} + 2h_{ij}\}dx^i dx^j\right)$$

with $h_i^i = 0$ and $h_i^j k^i = 0$ for tensor perturbations.

Evolution of Magnetic Tensor Perturbations

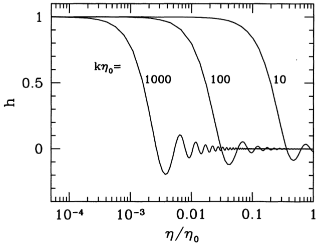
- Tensor Perturbations evolve via tensor Einstein Equation

$$\ddot{h}_{ij}(\eta,\boldsymbol{k}) + 2\frac{\dot{a}}{a}\dot{h}_{ij}(\eta,\boldsymbol{k}) + k^2 h_{ij}(\eta,\boldsymbol{k}) = \frac{8\pi\, G\, \Pi^T_{ij}(\boldsymbol{k})}{a^2}$$

with solution

$$\dot{h}_{ij}(\eta,\boldsymbol{k}) \simeq 4\pi\, G\, \eta_0^2 z_{eq}\ \ln\left(\frac{z_B}{z_\nu}\right) k\, \Pi^T(\boldsymbol{k}) \frac{j_2(k\eta)}{k\eta}$$

$$\simeq K\, \Pi^T(\boldsymbol{k})\, \frac{j_2(k\eta)}{\eta}$$



- Only large scale tensor perturbations important (not decayed) at current epoch

CMB Anisotropy from Magnetic Tensor Mode

The CMB Anisotropy is an integrated effect

$$\frac{\Delta T}{T}(\eta_0, \boldsymbol{x}, \boldsymbol{n}) = \int_{\eta_*}^{\eta_0} \dot{h}_{ij}\left(\boldsymbol{x}(\eta), \eta\right) n^i n^j d\eta$$

[Starobinskii (1985), Durrer (2000)]

we rewrite after contracting the spin-2 polarization basis tensor $\hat{e}_{ij}$ with $n^i n^j$

$$\frac{\Delta T}{T}(\boldsymbol{n}) = \frac{1}{2}\int_{\eta_*}^{\eta_0} d\eta \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\{\dot{h}^{(+)}(\boldsymbol{k},\eta) + \dot{h}^{(-)}(\boldsymbol{k},\eta)\} \sin^2\theta_k \, e^{-i(\boldsymbol{m}\cdot\boldsymbol{k}_\perp)D^*} \, e^{-ik_z D^*}. \tag{3}$$

and going to the flat sky limit yields

$$\begin{aligned} a_\ell^{(\pm)} &= \frac{1}{2}\int_{\eta_*}^{\eta_0} \frac{d\eta}{(D^*)^2} \int_{-\infty}^{+\infty} \frac{dk_z}{2\pi} \dot{h}^{(\pm)}(\boldsymbol{k},\eta) \frac{\ell^2 \; e^{-ik_z D^*}}{\ell^2 + k_z^2 (D^*)^2}. \\ &= \frac{K}{2}\int_{\eta_*}^{\eta_0} \frac{d\eta}{(D^*)^2} \int_{-\infty}^{+\infty} \frac{dk_z}{2\pi} \left[\Pi^{T\,(\pm)}(\boldsymbol{k}) \frac{j_2(k\eta)}{\eta}\right] \frac{\ell^2 \; e^{-ik_z D^*}}{\ell^2 + k_z^2 (D^*)^2}. \end{aligned} \tag{4}$$

Magnetic Tensor Mode CMB Bispectrum

The flat-sky bispectrum

$$\langle a_{\ell_1} a_{\ell_2} a_{\ell_3} \rangle = (2\pi)^2 \;\; \delta^{(2)}(\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2 + \boldsymbol{\ell}_3) \;\; B(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2, \boldsymbol{\ell}_3)$$

We write the three-point correlation function

$$\left\langle a^{(\pm)}_{\ell_1} a^{(\pm)}_{\ell_2} a^{(\pm)}_{\ell_3} \right\rangle = \left(\frac{K}{2}\right)^3 \left[\prod_{i=1}^{3} \int_{\eta_*}^{\eta_0} \frac{d\eta_i}{D_i^2} \int_{-\infty}^{+\infty} \frac{dk_{i_z}}{2\pi} \, \frac{j_2(k_i\eta_i)}{\eta_i} \, \frac{\ell_i^2 \; e^{-ik_{i_z}D_i}}{\ell_i^2 + k_{i_z}^2 D_i^2} \right] \times \left\langle \Pi^{T\,(\pm)}(\boldsymbol{k}_1) \; \Pi^{T\,(\pm)}(\boldsymbol{k}_2) \; \Pi^{T\,(\pm)}(\boldsymbol{k}_3) \right\rangle$$

Three point correlation of magnetic tensor stress

$$\left\langle \Pi^{T\,(\pm)}(\boldsymbol{k}_1) \, \Pi^{T\,(\pm)}(\boldsymbol{k}_2) \, \Pi^{T\,(\pm)}(\boldsymbol{k}_3) \right\rangle = \delta\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right) \psi_{123}$$

with mode-copling integral

$$\psi_{123} = \frac{1}{(4\pi p_\gamma)^3} \int d^3s \; M(|\boldsymbol{k}_1 + \boldsymbol{s}|) \; M(s) \; M(|\boldsymbol{s} - \boldsymbol{k}_3|) \; \mathcal{F}_{\Pi^T\Pi^T\Pi^T} .$$

Tensor Mode CMB Bispectrum - Constraint

Simple analytic estimate

$$\ell_1^2 \ell_3^2 \, b_{\ell_1 \ell_2 \ell_3} \sim \left(10^{-15}\right) \left(\frac{m_T}{10}\right) \left(\frac{n+3}{0.2}\right)^2 \left(\frac{B_{-9}}{3}\right)^6$$

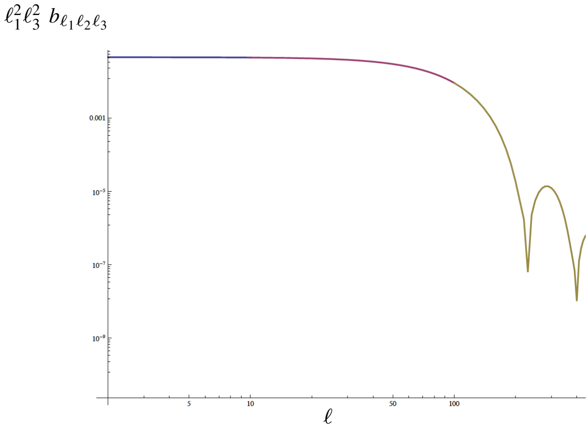
Further semi-analytic calculation of tensor bispectrum confirms an improvement of the B_0 constraint by a factor of two cf. scalar bispectrum .

Upper Limit Constraints:

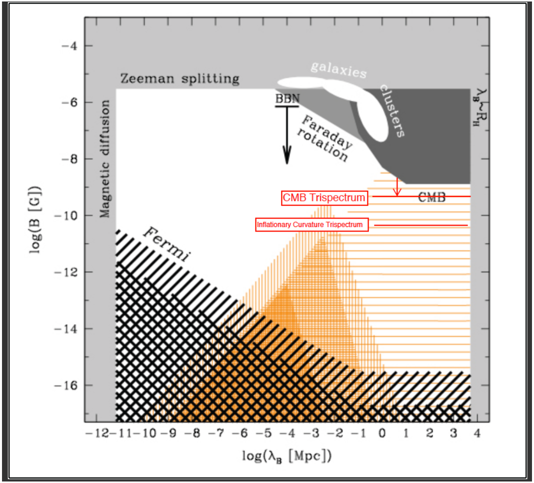
$B_0 \lesssim 2$ nG (scalar bispectrum) $\quad$ $B_0 \lesssim 1$ nG (tensor bispectrum)

Preliminary estimate of tensor mode trispectrum (4-pt correlation) $\quad B_0 \lesssim 0.4$ nG

Magnetic Bispectrum shape from semi-analytic treatment

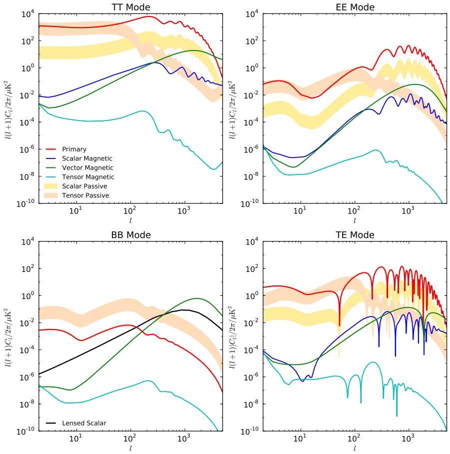


Updated Constraints on Cosmic Magnetic Fields



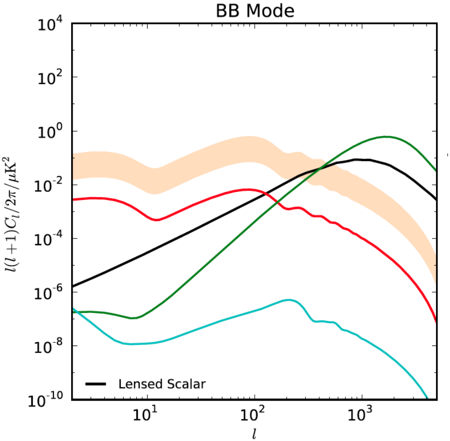
[adapted from Neronov & Vovk, *Science* 10]

Magnetic Fields also produce B-Mode Polarization



[Shaw & Lewis 10]

Magnetic Fields also produce B-Mode Polarization



[Shaw & Lewis 10]

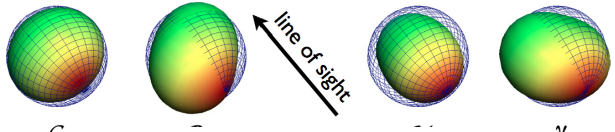
Magnetic Fields also produce B-Mode Polarization

- **$\vec{B}$ also produce (vector and) tensor mode perturbations which source B-Mode polarization!**
- B-mode polarization power spectrum: inflationary shape but amplitude $\propto B^4$
- **Trispectrum constraint on $B_0 < 1$ nG did not allow $\vec{B}$ to explain entire BICEP2 B-mode 'signal'**
- PMF could contribute to B-mode signal if detected: e.g. 2 nG made early claimed BICEP2 high r consistent with Planck r [Bonvin et al. PRL (2014)]
- Both Non-Gaussianity and Polarization constraints important for primordial $\vec{B}$

V Scalar Vector Tensor Components (2D) of Ruler Distortions

Relative Perturbation to Physical Scale of Ruler

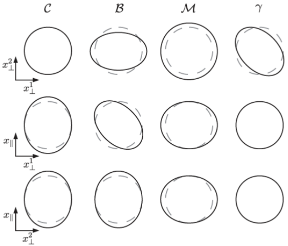
$$\frac{\tilde{r}-r_0}{\tilde{r}} = \underbrace{\mathcal{C}\frac{(\delta\tilde{x}_\parallel)^2}{\tilde{r}_c^2}}_{\text{longitudinal scalar}} + \underbrace{\mathcal{B}_i\frac{\delta\tilde{x}_\parallel\delta\tilde{x}^i_\perp}{\tilde{r}_c^2}}_{\text{Vector}} + \underbrace{\mathcal{A}_{ij}\frac{\delta\tilde{x}^i_\perp\delta\tilde{x}^j_\perp}{\tilde{r}_c^2}}_{\text{Magnification (trace) + shear (spin-2)}}$$



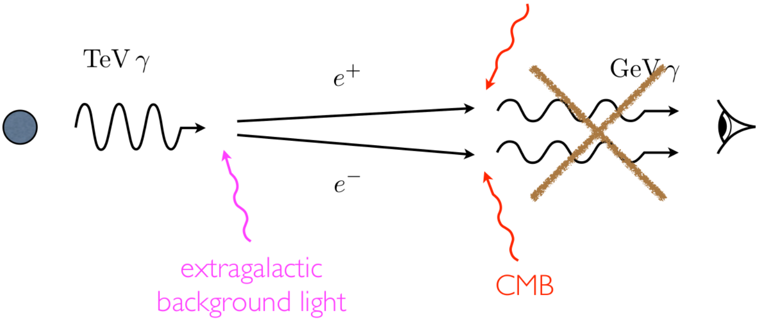
Planar Projections of Distortions: top row is plane of sky

[with Jeong & Kamionkowski]

based on Schmidt & Jeong 12

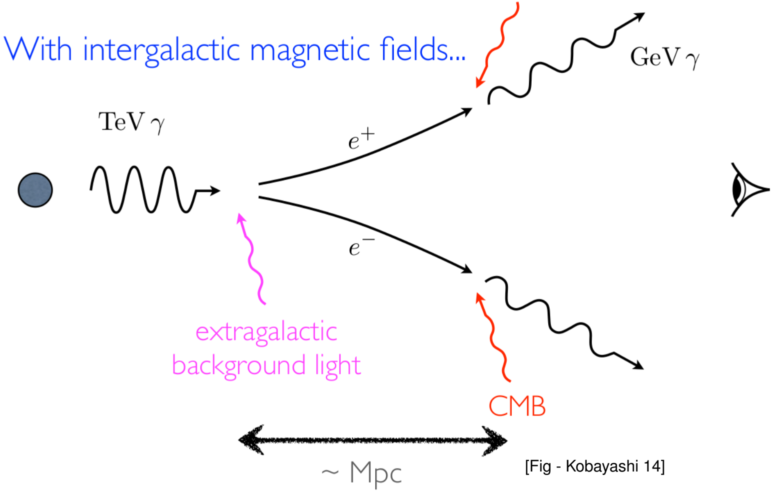


Lower Limits on Intergalactic Magnetic Fields from Blazars



[Fig - Kobayashi 14]

Lower Limits on Intergalactic Magnetic Fields from Blazars



[Fig - Kobayashi 14]

Blazar Lower Limits on B - Debate on Role of Plasma Instabilities

- Two Stream Instability - Electrostatic
- Langmuir oscillations: Pair Beam energy is drained by instability at a faster rate than inverse-Compton emission off CMB photons$\rightarrow$ Heating of the IGM
- Cold beam: Kinetic regime, Oblique mode dominates
- Broderick, Chang, Pfrommer (2012) and subsequent papers (2014,2016,2017)
- Heating of the IGM by Blazar pair beams produces several other cosmological effects

Debate whether instabilities are damped

- Non-linear Landau damping can be important
- Plasma instabilities can be stabilized $\rightarrow$ inverse-Compton should dominate e.g. Miniati & Elyiv (2013)
- Plasma instability is important e.g. Schlickeiser (2012)
- Development of plasma instabilities can be quite sensitive to beam parameters: energy distribution, angular distribution (Durrer & Neronov 2013)

Consider the Weibel Instability

Weibel Instability - Electromagnetic

- Arises due to anisotropic particle momentum distribution - gets converted into magnetic energy (Weibel 1959)
- Counter-streaming jets can excite the EM Weibel Instability leading to generation of magnetic fields (e.g. Medvedev & Loeb 1999)
- Current filamentation and transverse small-scale magnetic fields

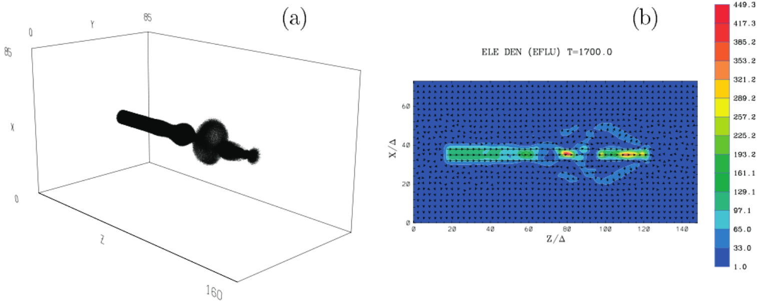
PiC Simulations and Laser-Driven Lab Experiments

- Particle-in-Cell (PiC) simulations confirm that transverse magnetic fields can be generated by the Weibel Instability
- Nishikawa et al. (2005, 2014) find that

$$\frac{B^2}{8\pi} \sim 0.3(\gamma - 1)\, m_{\rm e}\, n_{\rm b}\, c^2$$

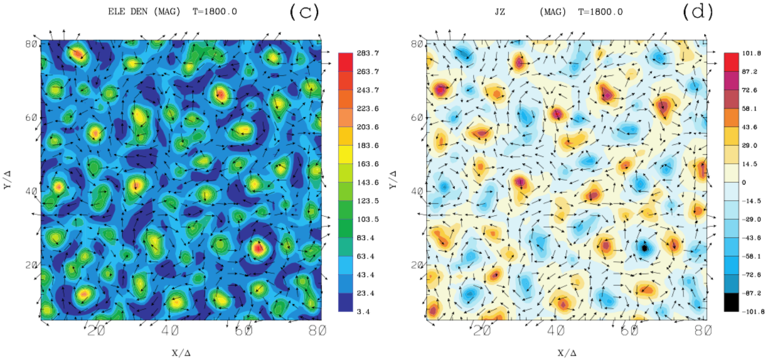
- Upto 30 % of pair beam kinetic energy goes into magnetic energy density
- Huntington et al (2015) observe magnetic field generation via Weibel Instability in interpenetrating opposing initially unmagnetised plasma flows.

PiC Simulations of Weibel Instability



[Nishikawa et al. 2005]

PiC Simulations of Weibel Instability



[Nishikawa et al. 2005]

Weibel-Generated Magnetic Fields

What is the Weibel-generated magnetic field that would suppress oblique-mode instability?

the Larmor frequency must be greater than the cooling rate Γ,

$$\frac{eB}{\gamma mc} \gtrsim 2\pi\Gamma$$

The magnetic field must exceed a value given by

$$B \gtrsim \frac{2\pi\,\gamma mc\,\Gamma}{e}$$

This yields a magnetic field lower limit,

$$B \gtrsim 1.1 \times 10^{-12}\,\text{G} \left(\frac{\gamma}{10^6}\right) \left(\frac{\Gamma}{10^{-4}\ \text{yr}^{-1}}\right)$$

Weibel-Generated Magnetic Fields

Taking a conservative value compared to PiC simulations of 10% of pair beam energy going into the generation of magnetic fields via the Weibel Instability

$$\frac{B^2}{8\pi} \sim 0.1(\gamma - 1)\, m_\mathrm{e}\, n_\mathrm{b}\, c^2$$

We find the value of the Weibel-generated magnetic fields is

$$B_\mathrm{W} \simeq \quad 2.76 \quad \times 10^{-11}\mathrm{G} \begin{cases} 0.255, & \text{if } z = 0.5 \\ 1, & \text{if } z = 1 \\ 0.444, & \text{if } z = 2 \end{cases}$$

$$\times \quad \left(\frac{EL_\mathrm{E}}{10^{45}\ \mathrm{erg\ s^{-1}}}\right)^{\frac{1}{2}} \left(\frac{E_\gamma}{1\mathrm{TeV}}\right)^{\frac{1}{2}}$$

The Weibel-generated field is larger than the lower limit for a magnetic field needed to suppress plasma instabilities.

Consequences of Weibel-Generated Magnetic Fields

We can also associate a critical beam number density n_b^{crit} with the minimum magnetic field required to suppress oblique mode instabilities and find

$$n_b^{crit} \simeq 5.3\times 10^{-25} \left(\frac{\gamma_e}{10^6}\right) \left(\frac{\Gamma}{10^{-4}\ \text{yr}^{-1}}\right)^2 \text{cm}^{-3}$$

The beam number density value taken for generation of magnetic fields via the Weibel instability was

$$n_b \simeq 3.7 \times 10^{-22}\ \text{cm}^{-3}$$

which was derived by balance with the inverse-Compton cooling rate Γ

Comparing, we find Weibel-generated fields can still suppress beam instabilities even if beam density is almost three orders of magnitude lower than assumed.

- Weibel-generated magnetic fields can suppress oblique mode instability
- Weibel-generated magnetic fields can obtain significant values for suppression for pair beam number densities set by the inverse-Compton cooling rate.
- The growth rate for the Weibel and oblique mode are comparable for a range of beam Lorentz factors
- The Weibel instability activation energy is found to be well below saturation energy thereby stabilizing it against rapid decay

Length Scales of Electron-Positron Pair Beam

The maximum lifetime for a relativistic electron against inverse-Compton up-scattering of a CMB photon can be estimated as

$$\tau_{IC} = \frac{E}{|dE/dt|} = \frac{\gamma\, m_e c^2}{(4/3)\, \sigma_T\, c\, \gamma^2\, U_\gamma} = \frac{2.33 \times 10^{12}\ \text{yr}}{\gamma}$$

which gives a mean free path of

$$l_{IC} \simeq 0.36 \left(\frac{E_e}{1\ \text{TeV}} \right)^{-1} \text{Mpc}$$

The transverse size λ_T of the beam is determined by the opening angle $(1/\gamma)$ of the beamed radiation from the blazar multiplied by the mean free path for pair production $l_{\gamma\gamma}$ which yields a much smaller scale

$$\lambda_T \simeq \frac{l_{\gamma\gamma}}{\gamma} \simeq 80\,\kappa \left(\frac{E_\gamma}{10\ \text{TeV}} \right)^{-1} \gamma_6^{-1}\ \text{pc}$$

where $\kappa \sim 1$ is a numerical factor to account for uncertainties in the measurement and models of the extragalactic background light.

On the other hand, the coherence length λ_B for the Weibel-generated magnetic fields will be set by the background IGM plasma skin depth

$$\lambda_B = \lambda_{skin} = \frac{2\pi\, c}{\omega_{p_e}} = \sqrt{\frac{\pi\, m_e c^2}{n_{\text{IGM}}\, e^2}}$$

so that

$$\lambda_B \simeq 2.3 \times 10^{-9}\, (1+\delta)^{-1/2}\, (1+z)^{-3/2}\ \text{pc}$$

The IGM plasma frequency ω_{p_e} depends on the IGM number density of free electrons

$$n_{\text{IGM}} \simeq 2.2 \times 10^{-7}\, (1+\delta)\, (1+z)^3\, cm^{-3}$$

These lengthscales are widely separated

$$l_{\gamma\gamma} \gg l_{IC} \gg l_T \gg l_B \qquad (5)$$

with

$$10^8 \qquad 10^5 \qquad 10^2 \qquad 10^{-9} \quad \text{pc} \qquad (6)$$

Deflection of Pair Beam by Weibel-Generated Magnetic Fields

Each coherence length of the magnetic fields will produce a deflection

$$\delta \sim \frac{\lambda_B}{R_L}, \quad \text{where} \quad R_L = \frac{\gamma_e m_e c^2}{eB_\perp}$$

is the relativistic electron Larmor radius. The total deflection after N coherence lengths, where

$$N = \frac{l_{IC}}{\lambda_B} \quad \text{is then} \quad \delta \sim \frac{\lambda_B}{R_L}\sqrt{\frac{l_{IC}}{\lambda_B}} \sim \frac{\sqrt{\lambda_B l_{IC}}}{R_L}$$

assuming that the Weibel-generated magnetic fields are completely uncorrelated with each other between successive transverse screens of thickness λ_B.
The relativistic electron Larmor radius for the pair beam particles in the Weibel-generated transverse fields is

$$R_L \simeq 1.08 \text{ kpc} \left(\frac{E_e}{1 \text{ TeV}}\right)\left(\frac{B}{10^{-12} \text{ G}}\right)^{-1}$$

yielding an order of magnitude estimate for the deflection angle (in degrees)

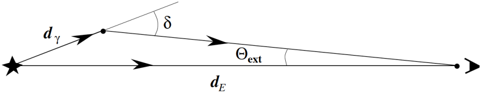
$$\delta \sim (0.04)\,(1+\delta)^{-1/4}\,(1+z)^{-3/4}\left(\frac{E_e}{1 \text{ TeV}}\right)^{-3/2}\left(\frac{B}{10^{-12} \text{ G}}\right)$$

A more careful integration over random-walk deflections (analogous to Harari et al 2002) gives the overall deflection

$$\delta \simeq \frac{1}{\sqrt{2}}\frac{B_W}{E_e}\sqrt{l_{IC}\lambda_B} \simeq (0.031)\left(\frac{E_e}{1 \text{ TeV}}\right)^{-3/2}\left(\frac{B_W}{2.8\times 10^{-11} \text{ G}}\right)\sqrt{\frac{l_{IC}}{0.36 \text{ Mpc}}}\sqrt{\frac{\lambda_B}{2.3\times 10^{-9} \text{ pc}}}\,(1+\delta)^{-1/4}\,(1+z)^{-3/4} \tag{7}$$

However this is not the observed angular extent of the GeV halo.

Angular Extent of GeV Halo after Weibel, Deflection, Inverse-Compton



[Neronov & Semikoz (2009)]

$$\Theta_{\rm ext}(E_\gamma) = \frac{l_{\gamma\gamma}}{D_\theta(z)}\delta = \frac{\delta}{\tau_\theta(z)}$$

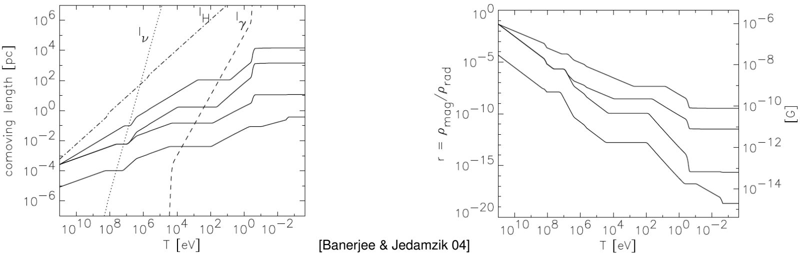
where $D_\theta = a(t_E)d_E$ is the angular diameter distance, $\tau_\theta = D_\theta / l_{\gamma\gamma}$
Numerically, the above estimate of the size of extended GeV emission around a TeV Blazar point source is

$$\Theta_{\rm ext,W} \simeq (0.003)^\circ \left[\frac{\tau_\theta}{10}\right]^{-1} \left(\frac{E_e}{1\ \rm TeV}\right)^{-3/2} \left(\frac{B}{10^{-12}\ \rm G}\right) (1+\delta)^{-1/4} (1+z)^{-3/4} \qquad (8)$$

- This angular extent is smaller than the PSF of *Fermi* or *CTA*
- **Thus we can still be sensitive to IGMF fields inspite of plasma instability effects**
- A clear observational signature of the presence of IGMF induced cascade emission around an initially point source is the decrease of the extension of the source with the increase of photon energy.

Evolution of Magnetic Fields

- PMF can evolve: magnetogenesis → recombination → galaxy formation
- Strongest Evolution: helical fields (helicity conservation) & (blue spectra)



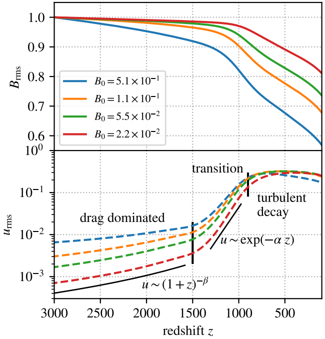
[Banerjee & Jedamzik 04]

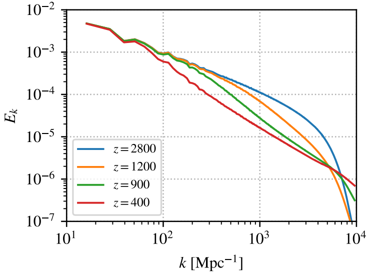
- Pre-recombination:
  - ν-drag before ν-decoupling, turbulent decay afterwards, then diffusion ∼stasis,
  - 100-1 eV: Photons free-stream at small scales → photon drag on baryons → damping
- Post-recombination: Fluid pressure and viscosity drops enormously $(n_b/n_\gamma \sim 10^{-9})$ ⟶ Turbulent Decay (dominant) and ambipolar diffusion
- Photon drag to turbulent decay transition treated as abrupt → need simulations

MHD Simulations of Magnetic Heating across Recombination

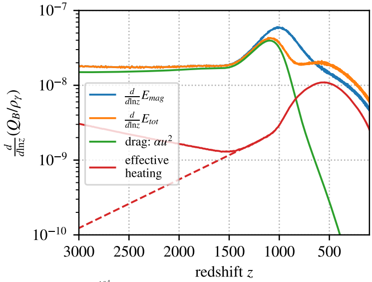
[Preliminary results: Johannes Reppin, Trivedi, Chluba, Banerjee; in prep.]

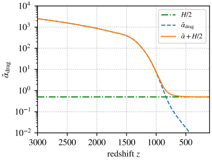
- Incompressible MHD simulations with `Pencil` code, $N = 1536^3$ resolution, cosmology with super-comoving co-ordinates, photon drag viscosity and hyperviscosity
- Magnetic energy spectrum with cutoff evolves under turbulence $\rightarrow$ power-law
- R.M.S. magnetic field decays by upto 10-40 %, mostly due to turbulent decay
- Follow baryon velocity field across epochs: drag-dominated $\rightarrow$ transition $\rightarrow$ turbulent decay





[Preliminary results: Johannes Reppin, Trivedi, Chluba, Banerjee; in prep.]





- Photon drag force falls fast as $\lambda_{\mathrm{mfp}_\gamma} \to$ horizon
- Rate of change of Magnetic energy density, rate of change of total energy density
- Total rate drag $\to$ Net Heating Rate from PMF turbulent decay
- Heating rate across epochs: drag-dominated $\to$ transition $\to$ turbulent decay
- Improving upon previous analytic and numerical work: Sethi & Subramanian 05, Schleicher+ 08, Kunze & Komatsu 14, Chluba+ 15
- Magnetic heating $\to$ changed ionization history $\to$ modified visibility function $\to$ small shifts in CMB acoustic peaks TT & EE $\to$ better constraints on PMF

Summary and Conclusions

Primordial magnetic fields could be the progenitors of large scale observed B

- **CMB non-Gaussianity more sensitive than other probes** of $\vec{B}$ at Mpc scales
- Magnetic **Trispectrum** more sensitive than magnetic **Bispectrum**
- Passive Scalar Magnetic **Trispectrum** + *Planck* $\rightarrow$ $\boxed{\mathrm{B}_0 \leq 0.7\ \mathrm{nG}}$

  (Tensor mode: 2 x improved limits; also $B_0 \leq 0.05$ nG for $\vec{B}$ curvature mode)
- Weibel instability: Blazar beams retain sensitivity to IGMF, may be testable cf. *CTA*
- MHD simulations of decaying turbulence: PMF evolving across recombination $\rightarrow$ constraints from CMB peaks
- Many orders between upper & lower limits, unclear origins: *further ideas & probes*

Thank You!