CONFORMAL SYMMETRY IN STANDARD MODEL AND GRAVITY

Tomislav Prokopec, ITP, Utrecht University

Stefano Lucat and T. Prokopec, arXiv:1705.00889 [gr-qc]; 1709.00330 [gr-qc];1606.02677 [hep-th]
T. Prokopec, Leonardo da Rocha, Michael Schmidt, Bogumila Swiezewska, in preparation

Desy, 27-11-2017

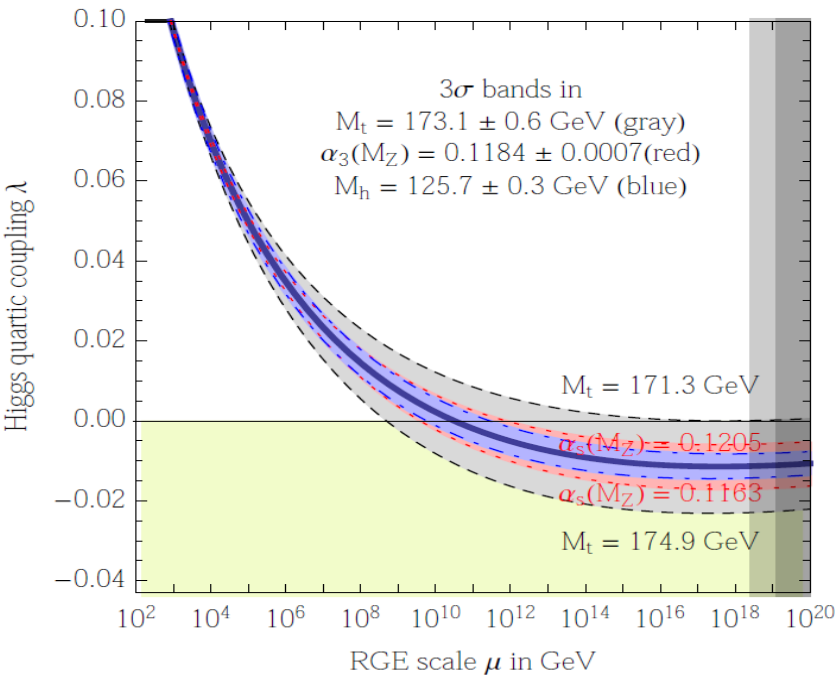
CONTENTS

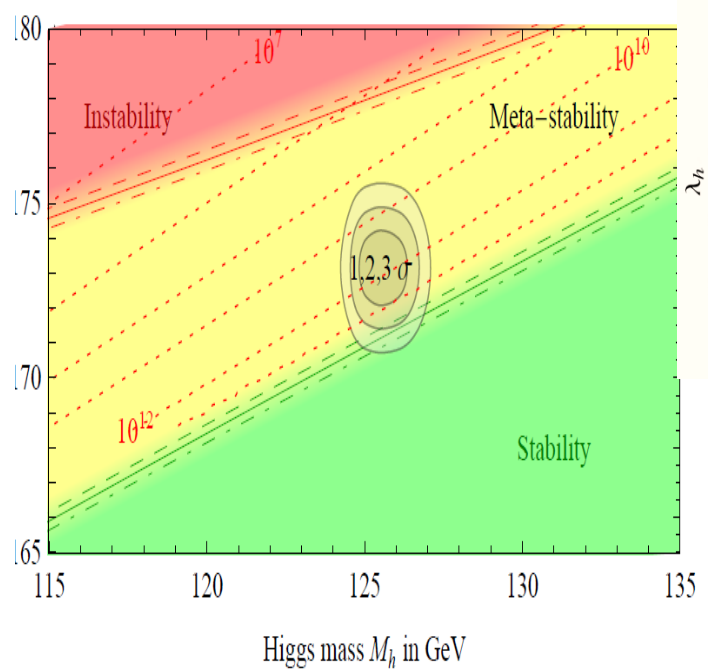
PHYSICAL MOTIVATION

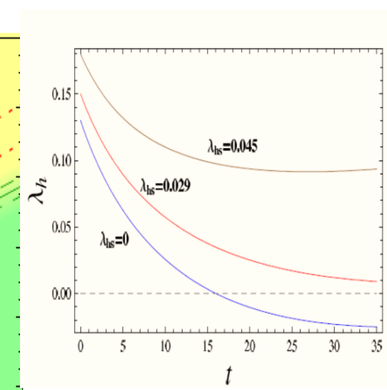
PHYSICAL MOTIVATION

- AT LARGE ENERGIES THE STANDARD MODEL IS <u>ALMOST</u> CONFORMALLY INVARIANT.
- HIGGS MASS AND KINETIC TERMS BREAK THE SYMMETRY
- OBSERVED HIGGS MASS: $m_H = 125.3\mathrm{GeV}$ is close to the stability bound
- STABILITY BOUND: $m_H \approx 130\mathrm{GeV}$: CAN BE ATTAINED BY ADDING SCALAR

Oleg Lebedev, e-Print: arXiv:1203.0156 [hep-ph]







Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia, 1205.6497 [hep-ph]

THEORETICAL MOTIVATION

THEORETICAL MOTIVATION IN SM

- HIGGS MASS TERM RESPONSIBLE FOR GAUGE HIERARCHY PROBLEM

- IF WE COULD FORBID IT BY SYMMETRY, THE GHP WOULD BE SOLVED

- THIS SYMMETRY COULD BE WEYL SYMMETRY IMPOSED CLASSICALLY

- HIGGS MASS COULD BE GENERATED DYNAMICALLY BY THE COLEMAN-WEINBERG (CW) MECHANISM

THEORETICAL MOTIVATION IN GRAVITY

- THE SYMMETRY IS BROKEN BY THE NEWTON CONSTANT AND COSMOLOGICAL TERM, G & Λ.

- G & Λ ARE RESPONSIBLE FOR GRAVITATIONAL HIERARCHY PROBLEM.

- SCALAR DILATON & CARTAN TORSION CAN RESTORE WEYL SYMMETRY IN CLASSICAL GRAVITY.

- G & Λ CAN BE GENERATED BY DILATON CONDENSATION INDUCED BY QUANTUM EFFECTS akin to THE COLEMAN-WEINBERG MECHANISM.

- IF GRAVITY IS CONFORMAL IN UV, IT MAY BE FREE OF SINGULARITIES (BOTH COSMOLOGICAL AND BLACK HOLE).

WEYL SYMMETRY IN CLASSICAL GRAVITY

CARTAN EINSTEIN THEORY

- POSITS THAT FERMIONS (& SCALARS) SOURCE SPACETIME TORSION.
- TORSION IS CLASSICALLY A CONSTRAINT FIELD
  (NOT DYNAMICAL, DOES NOT PROPAGATE)

$\Rightarrow$ CARTAN EQUATION CAN BE INTEGRATED OUT,
RESULTING IN THE KIBBLE-SCIAMA THEORY

Lucat, Prokopec, e-Print: arXiv:1512.06074 [gr-qc]

$\Rightarrow$ THIS THEORY PROVIDES ADDITIONAL SOURCE TO STRESS-ENERGY,
WHICH CAN CHANGE BIG-BANG SINGULARITY TO A BOUNCE.

- CARTAN-EINSTEIN THEORY CAN BE MADE CLASSICALLY CONFORMAL!

Lucat & Prokopec, arxiv:1606.02677 [hep-th]

CLASSICAL WEYL SYMMETRY

- WEYL TRANSFORMATION ON THE METRIC TENSOR

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{2\theta(x)} g_{\mu\nu} \qquad \mathrm{d}\tau \to \mathrm{d}\tilde{\tau} = e^{\theta(x)} \mathrm{d}\tau$$

- GENERAL CONNECTION Γ, TORSION TENSOR T, CHRISTOFFEL CON $\mathring{\Gamma}$

$$\Gamma^{\lambda}{}_{\mu\nu} = T^{\lambda}{}_{\mu\nu} + T_{\mu\nu}{}^{\lambda} + T_{\nu\mu}{}^{\lambda} + \mathring{\Gamma}^{\lambda}{}_{\mu\nu}$$

$$\delta\mathring{\Gamma}^{\mu}{}_{\alpha\beta} = \delta^{\mu}{}_{(\alpha}\partial_{\beta)}\theta \;\Rightarrow\; \delta\Gamma^{\mu}{}_{\alpha\beta} = \delta^{\mu}{}_{\alpha}\partial_{\beta}\theta \;\Rightarrow\; \delta T^{\mu}{}_{\alpha\beta} = \delta^{\mu}{}_{[\alpha}\partial_{\beta]}\theta$$

- RIEMANN TENSOR IS INVARIANT: $\delta R^{\alpha}{}_{\beta\gamma\delta} = 0$

- THIS IMPLIES THAT THE VACUUM EINSTEIN EQUATION IS WEYL INV:

$$G_{\mu\nu} = 0, \qquad \delta G_{\mu\nu} = 0$$

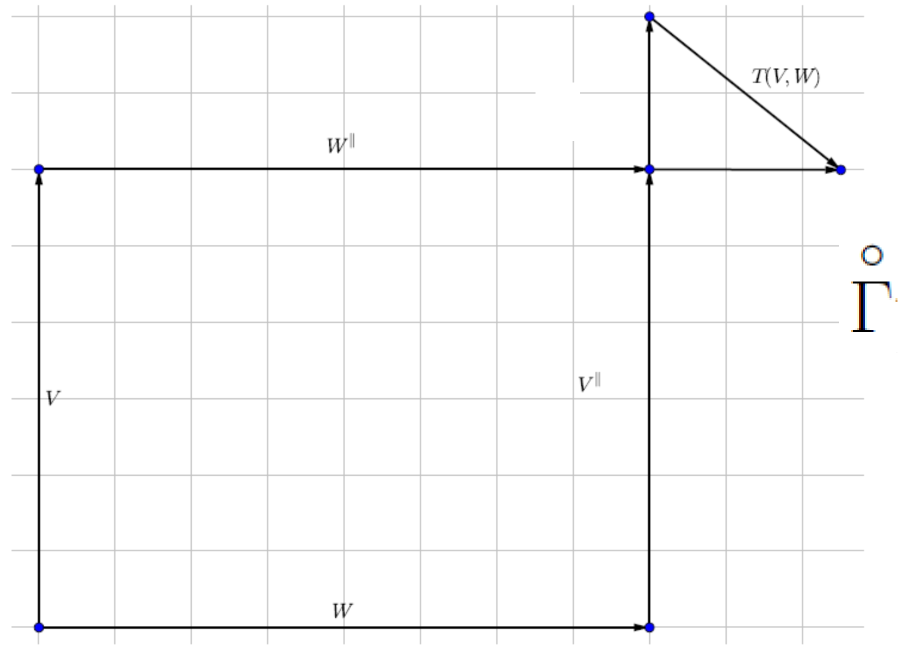
GEOMETRIC VIEW OF TORSION

- (VECTORIAL) TORSION TRACE 1-FORM:

$$\mathcal{T} \equiv \mathcal{T}_\mu \mathrm{d}x^\mu = \frac{2}{D-1} T^\lambda{}_{\lambda\mu} \mathrm{d}x^\mu$$

- TRANSFORMS AS A VECTOR FIELD:

$$\mathcal{T} \to \mathcal{T} + \mathrm{d}\theta$$



- WHEN A VECTOR IS PARALLEL-TRANSPORTED, TORSION TRACE INDUCES A LENGTH CHANGE: CRUCIAL IN WHAT FOLLOWS

PARALLEL TRANSPORT AND JACOBI EQUATION

- GEODESIC EQUATION:

$$\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau} \equiv \frac{dx^\lambda}{d\tau}\nabla_\lambda \frac{dx^\mu}{d\tau} = 0$$

$$\Gamma^\lambda{}_{\mu\nu} = T^\lambda{}_{\mu\nu} + T_{\mu\nu}{}^\lambda + T_{\nu\mu}{}^\lambda + \mathring{\Gamma}^\lambda{}_{\mu\nu}$$

$\mathring{\Gamma}$ = LEVI-CIVITA

$$T[X,Y] = -\frac{1}{2}(\nabla_X Y - \nabla_Y X - [X,Y])$$

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{[\mu\nu]} = \frac{1}{2}\left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\right)$$

$\rightarrow$ TRANSFORMS MULTIPLICATIVELY (as $1/d\tau^2$)

$$\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau} = 0 \Rightarrow e^{-2\theta(x)}\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau} = 0$$

NB: TRANSFORMATION OF $d\tau$ COMPENSATED BY TRANSFORMATION OF Γ !

- JACOBI EQUATION (JACOBI FIELDS J $\perp$ $\dot\gamma$) AND RAYCHAUDHURI EQ:

$$\nabla_{\dot\gamma}\nabla_{\dot\gamma}J + 2\nabla_{\dot\gamma}T[\dot\gamma, J] = R[\dot\gamma, J]\dot\gamma$$

$\rightarrow$ ALSO TRANSFORMS MULTIPLICATIVELY (as $1/d\tau^2$) UNDER WEYL TR

- SUGGESTS TO DEFINE A GAUGE INVARIANT PROPER TIME:

$(d\tau)_{g.i.} = \exp\left(-\int_{x_0}^{x} T_\mu dx^\mu\right) d\tau :=$ PHYSICAL TIME OF COMOVING OBSERVERS!

T. Prokopec, het Lam, e-Print: arXiv:1606.01147

WEYL SYMMETRY IN MATTER SECTOR

SCALAR MATTER

- CONFORMAL WEIGHT w_ϕ OF A CANONICAL SCALAR:

$$\phi \to e^{-\frac{D-2}{2}\theta}\phi \quad \Rightarrow w_\phi = -\frac{D-2}{2}$$

- CONFORMAL (WEYL) COVARIANT DERIVATIVE:

$$\nabla_\mu \phi = \partial_\mu \phi + \frac{D-2}{2} T_\mu \phi$$

TORSION TRACE: $\mathcal{T} \equiv \mathcal{T}_\mu \mathrm{d}x^\mu = \frac{2}{D-1} T^\lambda{}_{\lambda\mu} \mathrm{d}x^\mu$

ACTS AS A GAUGE CONNECTION!
(no i - the group is non-compact)

- CONFORMALLY INVARIANT SCALAR ACTION:

KINETIC/GRADIENT TERMS; SELF-COUPLING & COUPLING TO GRAVITY

$$\int dx^D \sqrt{-g}\,\left(-\frac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi g^{\mu\nu}\right)$$

$$\int d^D x \sqrt{-g}\left\{\frac{1}{2}\alpha^2\phi^2 R - \frac{\lambda}{4!}\phi^4\right\}$$

VECTOR & FERMIONIC MATTER

- CONFORMAL WEIGHT w_ϕ OF A CANONICAL SCALAR:

$$\psi \to e^{-\frac{D-1}{2}\theta}\psi$$

$$A_\mu \to e^{-\frac{D-4}{2}\theta}A_\mu$$

$$\Rightarrow w_\psi = -\frac{D-1}{2}, \quad w_A = -\frac{D-4}{2}$$

- CONFORMAL (WEYL) COVARIANT DERIVATIVE:

$$\mathcal{T} \equiv \mathcal{T}_\mu \mathrm{d}x^\mu = \frac{2}{D-1} T^\lambda{}_{\lambda\mu} \mathrm{d}x^\mu$$

$$\nabla_\mu \psi = \partial_\mu \psi + \frac{D-1}{2} \mathrm{T}_\mu \psi$$

- INVARIANT ACTIONS:

FERMIONS: $$\int \mathrm{d}^4x\sqrt{-g}\left[\frac{i}{2}\left(\bar{\psi}\gamma^\mu(\nabla_\mu + eA_\mu)\psi - (\nabla_\mu - eA_\mu)\bar{\psi}\gamma^\mu\psi\right) - g_y\phi\bar{\psi}\psi\right]$$

VECTORS: $$-\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) \qquad \int d^Dx\, f\, \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

NB1: IN D≠4, TORSION BREAKS GAUGE SYMMETRY!
NB2: TORSION TRACE ACTS AS A GAUGE CONNECTION (no i)!

(CLASSICALLY) CONFORMAL STANDARD MODEL & GRAVITY

- HIGGS SECTOR $$\int \mathrm{d}^D x\sqrt{-g}\left[-\frac{1}{2}(D_\mu H)^\dagger D^\mu H - \lambda_H (H^\dagger H)^2 + g_{H\Phi} H^\dagger H \Phi^2 - \lambda_\Phi \Phi^4\right]$$

COVARIANT DERIVATIVE: $$D_\mu H = \partial_\mu H + \frac{D-2}{2}\mathcal{T}_\mu H - ig\sum_a W^a_\mu \sigma^a \cdot H - ig' Y B_\mu H$$

$\Rightarrow$ CAN EXHIBIT DYNAMICAL SYMMETRY BREAKING VIA THE CW MECHANISM

T. G. Steele, Zhi-Wei Wang, arXiv:1310.1960 [hep-ph]

- DILATON ACTION:

$$S[\phi, g_{\mu\nu}] = \int dx^D \sqrt{-g}\,[-\frac{\alpha^2}{2}\ \phi^2 R - \frac{1}{2}\nabla_\mu\phi\, \nabla_\mu\phi g^{\mu\nu} - V(\phi)]$$

- ACTION FOR FERMIONS:

$$\int \mathrm{d}^4 x\sqrt{-g}\left[\frac{i}{2}\left(\bar\psi\gamma^\mu(\nabla_\mu + eA_\mu)\psi - (\nabla_\mu - eA_\mu)\bar\psi\gamma^\mu\psi\right) - g_y\phi\bar\psi\psi\right]$$

- GRAVITATIONAL ACTION (LAST TERM IS BOUNDARY [GB] TERM IN D=4):

$$\int \mathrm{d}^D x\sqrt{-g}\left(\xi_1 R^2 + \xi_2 R_{\mu\nu}R^{\mu\nu} + \xi_3 R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\right)$$

NB: SM+GRAVITY CAN BE MADE WEYL INVARIANT ONLY IN D=4.

WEYL SYMMETRY IN QUANTUM THEORY AND ITS DYNAMICAL BREAKING

DECOMPOSITION OF TORSION

● TORSION TENSOR CONTAINS DD(D-1)/2=24 COMPONENTS:

● IT CAN BE BROKEN INTO SKEW SYMMETRIC PART (DUAL TO VECTOR), TORSION TRACE (VECTOR) and REMAINDER (16 components: s(12)a(13)).

USING YOUNG DIAGRAMS (THE LAST DIAGRAM IS PROBLEMATIC):

$$\text{TORSION} = \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} + \begin{array}{|c|c|c|}\hline 1 & 2 & 3\\\hline\end{array} + \begin{array}{|c|c|}\hline 1 & 2\\\hline 3 & \multicolumn{1}{c}{}\\\cline{1-1}\end{array}$$

SKEW (1) TRACE (2) MIXED REP (3)

● FORTUNATELY, MATTER & GRAVITY (AT 1 LOOP) SOURCES ONLY SKEW SYM. AND TRACE TORSION

$\Rightarrow$ ONLY THE SKEW SYMMETRIC (1) AND TRACE (2) BECOME DYNAMICAL DUE TO QUANTUM EFFECTS!!

QUANTIZATION

- CANONICAL QUANTIZATION PRESERVES WEYL SYMMETRY. INDEED:

$$\phi \to e^{-\frac{D-2}{2}\theta}\phi \qquad \pi_\phi = \sqrt{-g}n_\nu g^{\mu\nu}\bar{\nabla}_\mu\phi\,, \implies \pi_\phi \to \tilde{\pi}_\phi = \Omega^{(D-2)/2}\pi_\phi$$

IMPLIES WEYL INVARIANCE OF THE CANONICAL COMMUTATOR:

$$[\phi,\, \pi_\phi] = [\tilde{\phi},\, \tilde{\pi}_\phi] = \mathbb{1}$$

ALTERNATIVELY, IF CLASSICAL ACTION IS WEYL INVARIANT, THE PATH INTEGRAL MEASURE IS ALSO INVARIANT, HENCE GENERATING FUNCTIONAL AND EFFECTIVE ACTION ARE ALSO INVARIANT:

$$\exp\{iW[J_\phi]\} = Z[J_\phi] = \int D\phi D\pi_\phi \exp\{i\int[\pi_\phi\,\dot{\phi} - H(\pi_\phi,\phi) + J_\phi\phi]\}$$

$$\Gamma_{EFF}[\bar{\phi}, g_{\mu\nu}] = J_\phi\bar{\phi} - W[J_\phi], \quad \delta W[J_\phi]/\delta J_\phi = \bar{\phi}$$

- NB: OUR INABILITY TO PROVIDE A REGULAR/RIGOROUS DEFINITION OF THE PATH INTEGRAL MAY BREAK WEYL SYMMETRY.
- INDEED, NO REGULARIZATION SCHEME IS KNOWN (INCLUDING DIM REG) THAT DOES <u>NOT</u> BREAK WEYL SYMMETRY. PROBLEM?
- NB: ANALOGOUS CONSTRUCTION HOLDS FOR FERMIONIC & GAUGE FIELDS

WARD IDENTITIES

- PROBLEM CAN BE RESOLVED BY DEMANDING CONSTANCY OF SCALE μ IN COUNTERTERMS:

$$\nabla_\alpha \mu = 0 \quad (*)$$

- IN CONFORMAL WARD IDENTITY SCALE μ CAN BE CHOSEN AS A CONVENIENT PHYSICAL SCALE (`Weyl symmetry gauge choice/fixing')

- WHEN FULLFILLED, THE WARD IDENTITIES GUARANTEE (NON-PERTURBATIVE) REALISATION OF WEYL SYMMETRY.

- FUNDAMENTAL WARD IDENTITY:

$$\boxed{\left\langle T^{\mu}{}_{\mu} \right\rangle + \left\langle \nabla_\mu \Pi^\mu \right\rangle = 0, \quad \Pi^\mu = \frac{\delta S_m}{\sqrt{-g}\,\delta T_\mu}}$$

Stefano Lucat and T. P, 1709.00330 [gr-qc]

CONFORMAL ANOMALY

$$\left\langle T^{\mu}{}_{\mu} \right\rangle \neq 0$$

Capper, Duff (1972)

NB: IT IS REASONABLE TO DEMAND THAT – JUST LIKE GAUGE SYMMETRY – WEYL SYMMETRY CANNOT BE BROKEN.

ONE-LOOP EFFECTIVE ACTION

- CLASSICAL ACTION: $S = \int d^D x \sqrt{-g} \left\{ \frac{1}{2} \alpha^2 \phi^2 R - \frac{1}{2} (\bar{\nabla} \phi)^2 - \frac{\lambda}{4!} \phi^4 \right\}$

- INTEGRATING OUT FLUCTUATIONS IN ϕ $(in\ presence\ of\ a\ bg\ R\ and\ \phi)$ results in divergences of the form: $\propto \frac{\mu^{D-4}}{D-4} \delta^D(x - x')$

- TO RENORMALIZE AWAY THESE ONE ADDS **c.t.**'s THAT REMOVE THE DIVERGENT TERMS. THESE **c.t.**'s <u>BREAK</u> WEYL SYMMETRY!

- RENORMALIZED QUANTUM (1-LOOP) ACTION [F (G) FIELD STRENGTH ASSOCIATED WITH TRACE (SKEW) TORSION]:

$$\Gamma_{1\,LOOP} = -\frac{1}{32\pi^2} \int d^4 x \sqrt{-g} \left\{ \begin{matrix} \left[\left(\alpha^2 - \frac{1}{6} \right) \bar{R} + \alpha^2 \phi^2 \right]^2 \\ -\frac{1}{36} \left[\frac{1}{15} \bar{R}_{\alpha)(\beta\gamma)(\delta} \bar{R}^{\alpha\beta\gamma\delta} - \frac{1}{15} \bar{R}_{(\alpha\beta)} \bar{R}^{\alpha\beta} \right] \\ + F_{\alpha\beta} F^{\alpha\beta} + G_{\alpha\beta} G^{\alpha\beta} \end{matrix} \right\} \left\{ \log(\left[\left(\alpha^2 - \frac{1}{6} \right) \bar{R} + \alpha^2 \phi^2 \right] / \mu^2) + C \right\}$$

- WEYL SYMMETRY BREAKING IS "WEAK": IT APPEARS AS μ IN THE LOG(μ).

- TO GET CLOSER TO THE TRUE EFFECTIVE ACTION, RG IMPROVEMENT IS DESIRABLE, WHICH REQUIRES SOLVING CS EQUATION: $\boxed{\mu \frac{d}{d\mu} \Gamma_{RG} = 0}$

- SHOULD REPEAT BY USING GENERAL METHODS [Barvinsky, Vilkovisky]

1 LOOP EFFECTIVE ACTION 2

$$\Gamma_{1\,LOOP} = -\frac{1}{32\pi^2}\int d^4x\sqrt{-g}\left\{\begin{array}{c}\left[\left(\alpha^2-\frac{1}{6}\right)\bar{R}+\alpha^2\rho+\alpha^2\,\phi^2\right]^2\\ -\frac{1}{36}\left[\frac{1}{15}\;\bar{R}_{\alpha)(\beta\gamma)(\delta}\bar{R}^{\alpha\beta\gamma\delta}-\frac{1}{15}\bar{R}_{(\alpha\beta)}\bar{R}^{\alpha\beta}\right]\\ +F_{\alpha\beta}F^{\alpha\beta}+G_{\alpha\beta}G^{\alpha\beta}\end{array}\right\}\left\{\log(\left[\left(\alpha^2-\frac{1}{6}\right)\bar{R}+\alpha^2\rho+\alpha^2\,\phi^2\right]/\mu^2)+C\right\}$$

- WHEN RG IMPROVED, THE OFFENDING TERMS log(μ) ARE ABSORBED BY μ –DEPENDENT COUPLINGS. FORMALLY, $\Gamma_{1LOOP,RG}$ BECOMES μ –INDEPENDENT

- Riemann-SQUARED TERMS CAN BE REMOVED (REWRITTEN AS A GENERALISED GAUSS-BONNET BOUNDARY TERM. THESE ARE INTERESTING AND THEIR RELEVANCE YET TO BE STUDIED.

- SCALAR MATTER MAKE TORSION TRACE DYNAMICAL,
- FERMIONIC MATTER MAKES SKEW SYMMETRIC TORSION DYNAMICAL
- WE DO NOT YET KNOW ABOUT THE GRAVITONS

♦ CLASSICAL ACTION: 1 TENSOR + 1 m=0 SCALAR + 1 FERMION DoF

♦ QUANTUM ACTION: 1 TENSOR + 2 VECTOR + 1 MASSLESS SCALAR DoF

WARD IDENTITIES

- CONSIDER (WEYL INVARIANT) CLASSICAL ACTION:

$$S = \frac{1}{2}\int \mathrm{d}^D x \sqrt{-g}\, g^{\mu\nu}\left(\partial_\mu \phi + \frac{D-2}{2} T_\mu \phi\right)\left(\partial_\nu \phi + \frac{D-2}{2} T_\nu \phi\right)$$

- INFINITESIMAL WEYL TRANSFORMATION ω, $\Omega = 1 + \omega$

$$\delta_\omega \phi = -\frac{D-2}{2}\omega\phi$$

- TO LINEAR ORDER IN $\omega(x)$, THE ACTION CHANGES AS:

$$\delta_\omega S = \int \mathrm{d}^D x \sqrt{-g}\left(\frac{D-2}{\sqrt{-g}}\frac{\delta S}{\delta\phi(x)}\omega(x)\phi(x) - \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}(x)}\omega(x) g^{\mu\nu} - \bar{\nabla}_\mu\left(\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T_\mu(x)}\right)\omega(x)\right)$$

- WHEN THIS IS INSERTED INTO THE GENERATING FUNCTIONAL, AND ONE REQUIRES THAT THE TERM LINEAR IN ω TO VANISH, ONE GETS THE FOLLOWING WARD IDENTITY:

$$\int \mathcal{D}\phi e^{iS}\left(T^\mu_\mu + \bar{\nabla}_\mu \Pi^\mu\right) = 0 \qquad T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad \Pi^\mu = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T_\mu(x)}$$

- ♦ HERE TENSOR Π^μ IS THE DILATATION CURRENT (source for T_μ)

WARD IDENTITIES 2

- TO RECAP, WE HAVE DERIVED THE FUNDAMENTAL WARD IDENTITY:

$$\langle T_\mu^\mu \rangle + \langle \bar{\nabla}_\mu \Pi^\mu \rangle = 0 \qquad (*)$$

- SINCE $\langle T_\mu{}^\mu \rangle$ IS GENERATED BY THE TRACE ANOMALY, PART OF IT PROPORTIONAL TO THE EULER DENSITY E_4 (GAUSS BONNET), AND Box(R), WHICH ARE TOTAL DERIVATIVES, e.g.

$$\langle T_\mu{}^\mu \rangle = c_T E_4 = c_T \bar{\nabla}_\mu E^\mu,\ c_T = \text{const.}$$

- THIS MEANS THAT THE ANOMALY (except for Weyl^2) CAN BE REABSORBED INTO THE TORSION TRACE SOURCE :

$$\widetilde{\Pi}^\mu \equiv \Pi^\mu + c_T E^\mu \ \Rightarrow\ \nabla_\mu \widetilde{\Pi}^\mu = 0$$

- THAT DOES NOT MEAN THAT CONFORMAL ANOMALY HAS NO PHYSICAL CONSEQUENCE – ANALOGOUS TO CHIRAL ANOMALY - TOPOLOGICAL.

- THE FUNDAMENTAL WARD IDENTITY (*) CAN BE USED TO GENERATE HIGHER ORDER IDENTITIES (w_O= CONFORMAL WEIGHT OF O):

$$\langle T[(T_\mu{}^\mu + c_T \bar{\nabla}_\mu G^\mu) O(\{x_{(i)}\})] \rangle = w_O \sum_j \frac{\delta^D(x - x_{(j)})}{\sqrt{-g}} \langle T[O(\{x_{(i)}\})] \rangle$$

COLEMAN WEINBERG MECHANISM

S. Coleman, E. Weinberg (1973)

- MASSLESS SELF-INTERACTING SCALAR GETS A 1-LOOP CORRECTION:

$$L_\phi = -\frac{Z(\phi^2/\mu^2)}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\phi^4 - \frac{\lambda^2}{256\pi^2}\phi^4\left[\ln\left(\frac{\phi^2}{\mu^2}\right) - \frac{25}{6}\right]$$

► QUANTUM EFFECTS BREAK WEYL SYMMETRY:

SCALE μ INTRODUCED BY COUNTER-TERMS NEEDED TO RENORMALIZE

► QUANTUM EFFECTS CAN GENERATE A LOCAL MINIMUM AT $\phi \neq 0$.
THAT IS NOT PERTURBATIVELY RELIABLE, QUANTUM O(ħ) AND CLASSICAL CONTRIBUTION ARE COMPARABLE AT THE MINIMUM.

►CW USED CALLAN-SYMANZIK EQUATION TO RG IMPROVE RESULT.
THE SECOND MINIMUM THEN DISAPPEARS.

►HOPE: IN MULTIFIELD SCALAR MODELS, RELIABLE MINIMA OCCUR SUCH THAT ONE CAN USE CW MECHANISM INSTEAD THE BEH MECHANISM TO EXPLAIN MASS GENERATION. CAN BE MADE CONSISTENT WITH OBSERVATIONS IF ONE ADDS 2 SCALARS OR 1 SCALAR + 1 HIDDEN GAUGE SECTOR

T.P.,da Rocha, Schmidt, Swiezewska (2017), in progress

QUANTUM THEORY CONFRONTS OBSERVATIONS

CONFRONTING OBSERVATIONS

$$\Gamma_{1\,LOOP} \supset -\int d^4x\sqrt{-g}\,\{[\alpha(\mu)\bar{R}^2 + \beta(\mu)\phi^2 R] + \gamma(\mu)F_{\alpha\beta}\,F^{\alpha\beta}\}$$

EARLY COSMOLOGY

- CAN BE TESTED BY STUDYING INFLATIONARY MODELS GENERATED BY CONDENSATION OF SCALARON, DILATON OR LONG TORSION TRACE.

- PRELIMINARY RESULTS: CAN GET (quasi)de SITTER UNIVERSE AND NEARLY SCALE INVARIANT SCALAR SPECTRUM.

LATE COSMOLOGY

- CAN BE TESTED BY STUDYING e.g. DARK ENERGY AND STRUCTURE FORMATION, POSSIBLY DARK MATTER CANDIDATE.

- TORSION TRACE (AND MIXED TORSION) CAN BE DETECTED BY CONVENTIONAL GRAVITATIONAL WAVE DETECTORS

Stefano Lucat and T. Prokopec, arXiv:1705.00889 [gr-qc]

CONCLUSIONS AND OUTLOOK

CONCLUSIONS AND OUTLOOK

- CHALLENGE: USE FRG METHODS TO STUDY HOW THIS THEORY DIFFERS FROM THE USUAL GRAVITY, i.e. WHETHER IT IS ASYMPTOTICALLY SAFE/ADMITS UV COMPLETION.

- CHALLENGE 2: IS ANYTHING DIFFERENT WRT UNITARITY.
NOTE THAT DUE TO ABSENCE OF THE PLANCK SCALE,
THE GHOST PROPAGATOR SHOULD BE MASSLESS (WORSE?)

- CHALLENGE 3: CONFRONT THIS NOVEL THEORY AS MUCH AS POSSIBLE WITH OBSERVATIONS

- CHALLENGE 4: CAN WE GET RID OF (COSMOLOGICAL AND BLACK HOLE) SINGULARITIES?

HINT: RECALL: $(d\tau)_{g.i.} = \exp\left(-\int_{x_0}^{x} T_\mu dx^\mu\right) d\tau :=$ PHYSICAL TIME OF COMOVING OBSERVERS