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ALL-ORDERS CALCULATIONS FOR PDFs DETERMINATION

DESY - Hamburg,
Monday 15th January 2018

mostly based on:
large *x*:1507.01006, 1510.00375
small *x*:1607.02153, 1708.07510, 1710.05935

OUTLINE

- Introduction
- PDFs with large-x resummation
- PDFs with small-x resummation: evidence for BFKL dynamics in inclusive HERA data
- Conclusion and Outlook

A WORD ON PDFs

- Parton distribution functions describe the non-perturbative structure of the colliding protons
- collinear factorisation implies their universality (up to power corrections)

$$\sigma(x,Q) = \sigma_0 C\left(\frac{x}{x_1 x_2}, \alpha_s(\mu)\right) \otimes f_1(x_1,\mu) \otimes f_2(x_2,\mu)$$

- coefficient functions (NLO,NNLO, N^3LO)
- parton evolution (NLO,NNLO)
- electro-weak corrections
- quark mass effects
- target-mass corrections
- …

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measure

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measure compute

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measure compute extract

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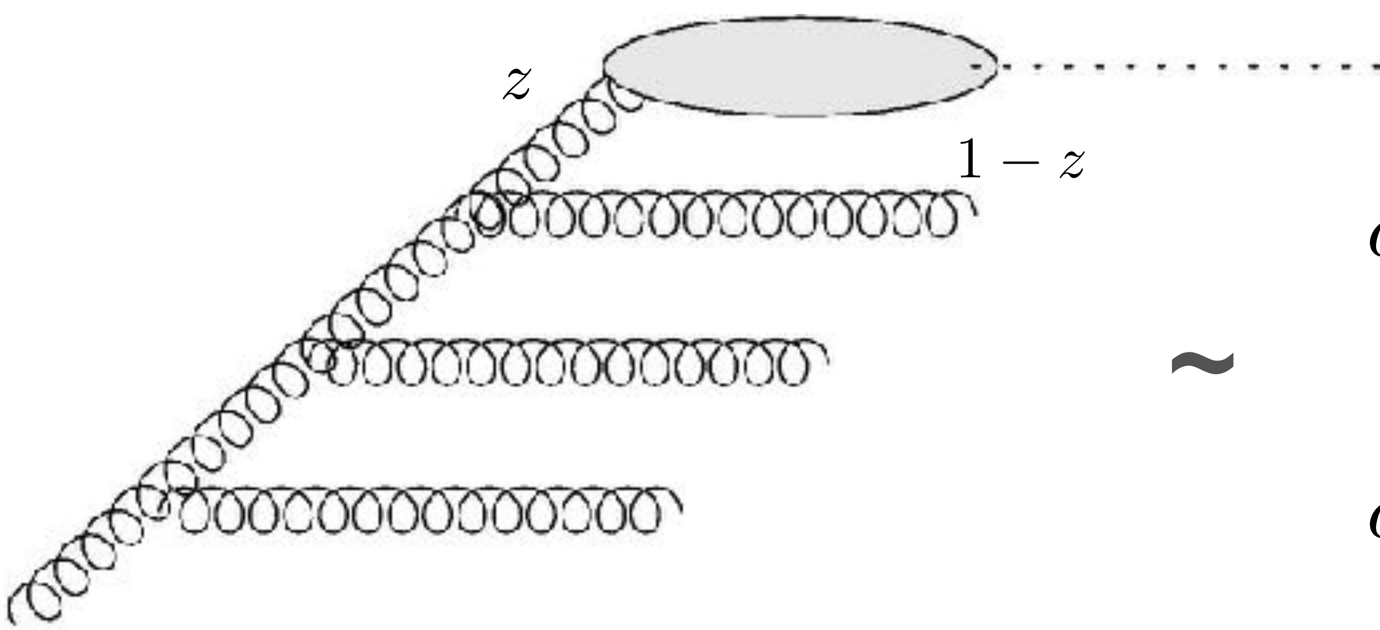
HIGHER-ORDER CORRECTIONS

- Higher-order QCD corrections correspond to emission of extra partons or virtual corrections
- these corrections are enhanced in particular regions of phase-space

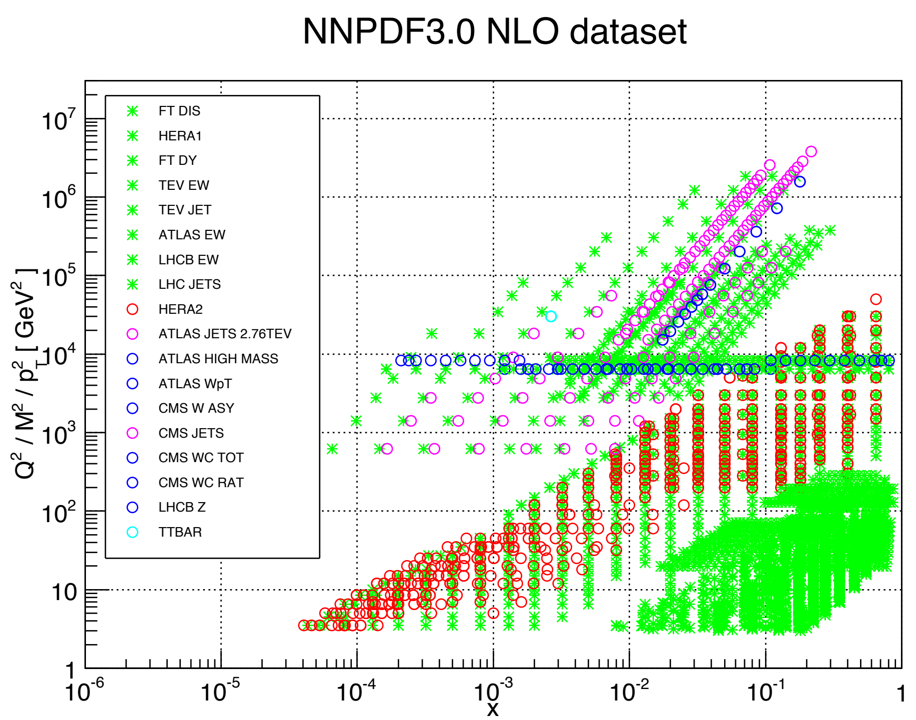


$$\sim \quad \alpha_s^k \left[\frac{\log^{2k-1}(1-z)}{1-z} \right]_+ , \; z \to 1$$

$$\alpha_s^k \frac{\log^{k-1} z}{z} , \; z \to 0$$

WE WILL MOST CONVENIENTLY WORK IN MELLIN SPACE
SOFT-GLUON RESUMMATION: Z→1 ➪ LOGS OF N
BFKL RESUMMATION: Z→0 ➪ POLES IN N (TYPICALLY AT N=0)

DATASET OF A GLOBAL FIT



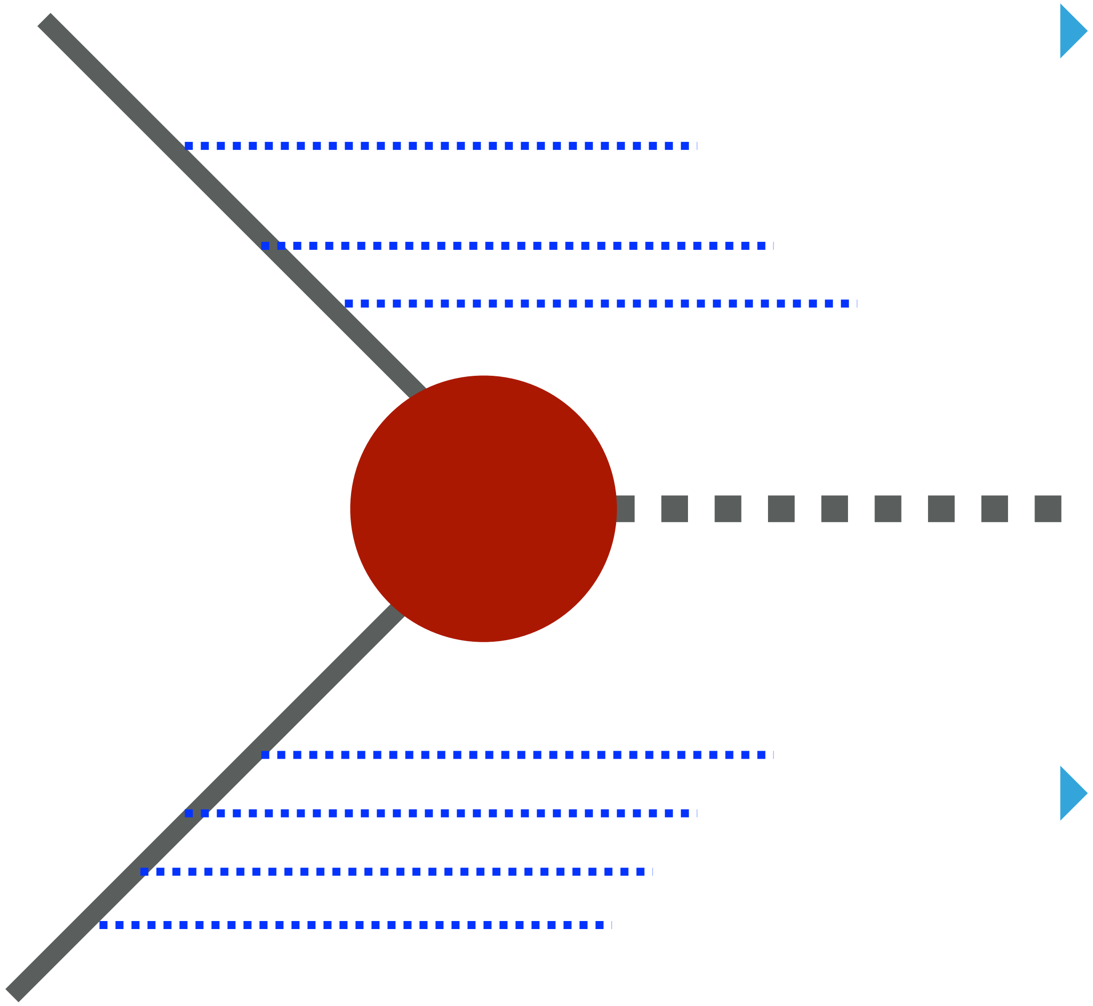
- Standard PDFs fits rely on NLO and NNLO calculations of coefficient functions and evolution
- current datasets span several order of magnitude in Q^2 and x

QUESTIONS THAT COME TO MIND

- Do we trust FO everywhere?
- Do we see evidence of all-order effects in the data?
- Is it ok to use standard PDFs with resummed calculation?

THRESHOLD (LARGE-X) RESUMMATION

PRODUCTION AT THRESHOLD



- absolute threshold: the initial-state energy is just enough to produce the final state with invariant mass Q

$$x = \frac{Q^2}{s} \to 1$$

- emissions forced to be soft, leading to log-enhanced contributions order-by-order in perturbation theory

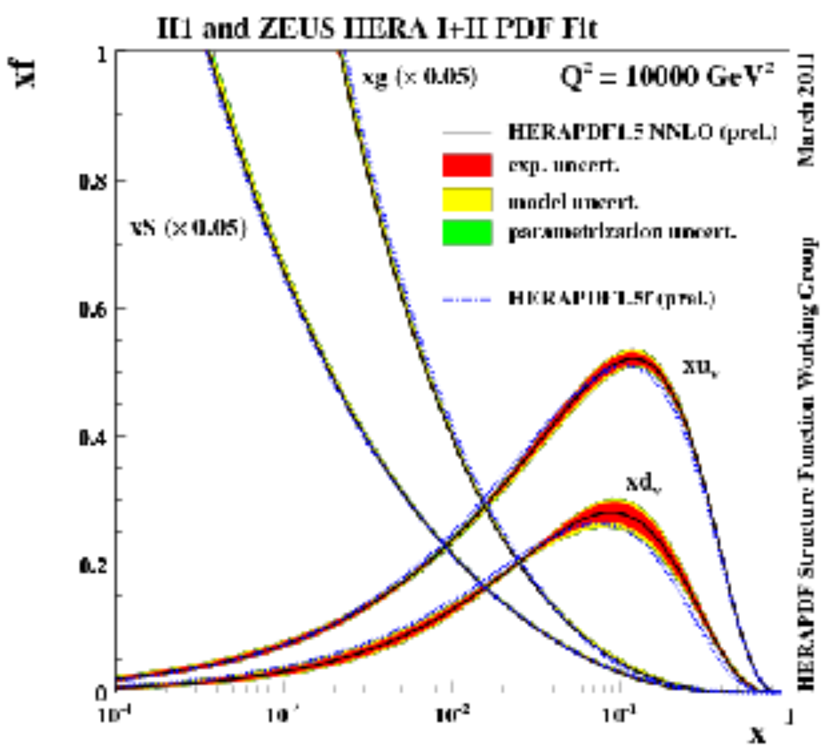
$$\text{LO}: Q^2 = \hat{s}$$

$$\text{beyond LO}: Q^2 = z\hat{s}$$

$$C(z, \alpha_s) \sim \sigma_0 \sum_{n=1} \sum_{k=-1}^{2n-1} \alpha_s^n \left[\frac{\ln^k(1-z)}{1-z}\right]_+$$

$\delta(1-z)$ →

WHY BOTHER WITH THRESHOLD AT THE LHC?

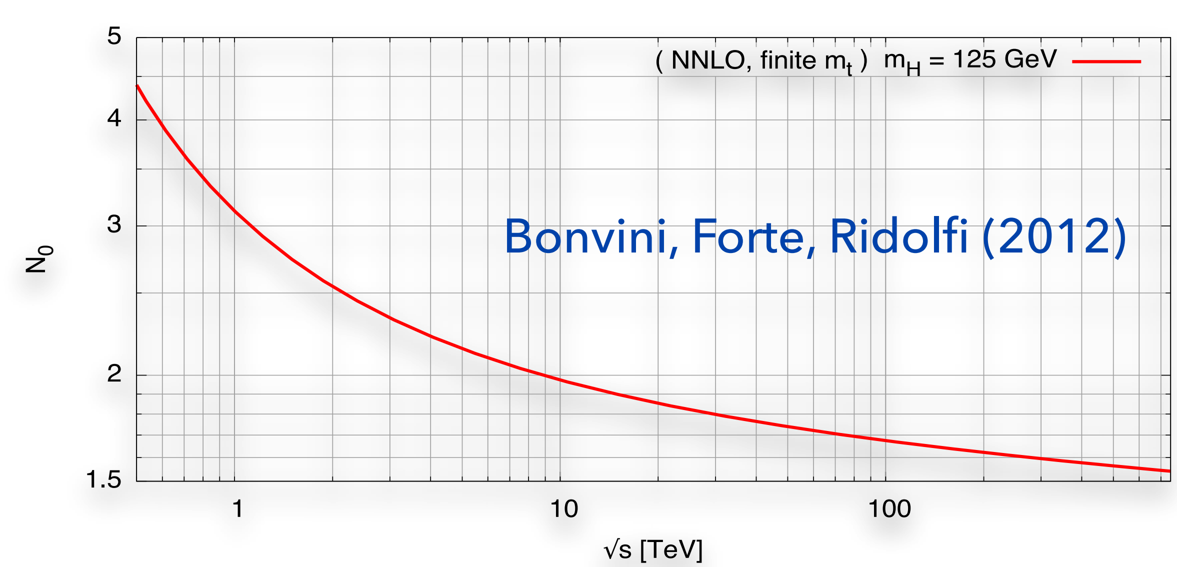


- Gluon PDF shows a steep increase at low *x*

$$\hat{s} = x_1 x_2 s$$

- region of partonic threshold is enhanced in the convolution

- more precise argument in Mellin space
- a saddle-point approximation indicates the region that gives the bulk of the contribution to the inverse Mellin integral
- this region turns out to be fairly narrow around the (real) saddle-point



THRESHOLD RESUMMATION

- **momentum space:** distributional terms for $z \to 1$
- **moment space:** terms that do not vanish at large N

$$C_{\rm res}(N,\alpha_s) = \bar{g}_0(\alpha_s,\mu_{\rm F}^2)\, \exp \bar{\mathcal{S}}(\alpha_s,N),$$

$$\bar{\mathcal{S}}(\alpha_s,N) = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left(\int_{\mu_{\rm F}^2}^{m_{\rm H}^2 \frac{(1-z)^2}{z}} \frac{d\mu^2}{\mu^2} 2A\big(\alpha_s(\mu^2)\big) + D\big(\alpha_s([1-z]^2 m_{\rm H}^2)\big) \right),$$

$$\bar{g}_0(\alpha_s,\mu_{\rm F}^2) = 1 + \sum_{k=1}^{\infty} \bar{g}_{0,k}(\mu_{\rm F}^2)\alpha_s^k,$$

Anastasiou *et al*. (2014)

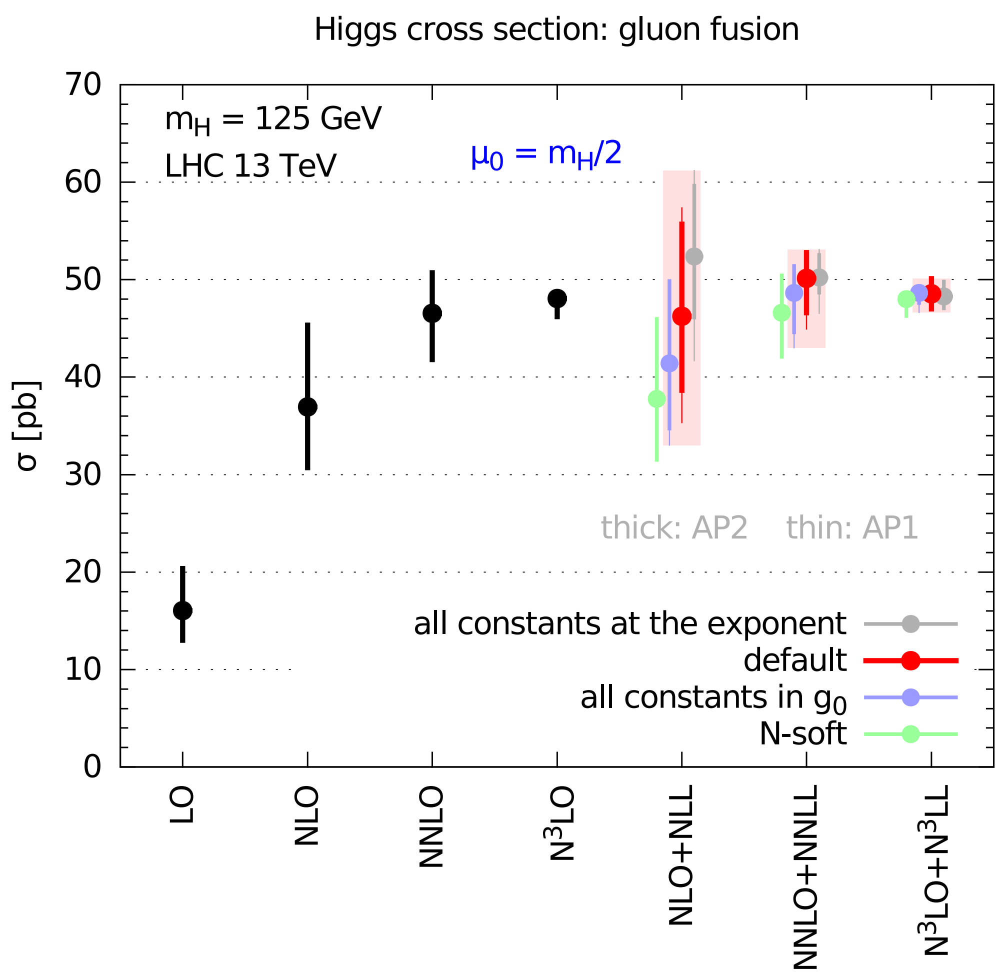
$$A(\alpha_s) = \sum_{k=1}^{\infty} A_k \alpha_s^k, \qquad D(\alpha_s) = \sum_{k=1}^{\infty} D_k \alpha_s^k,$$

Catani *et al*. (2002); Moch, Vogt (2005); Laenen, Magnea (2005) [...]

- constants can go in the exponent of in front of it
- state of the art N^3LL (but the 4-loop cusp)
- next-to-eikonal can be important (e.g. $(1-z)^2/z$)

Laenen *et al*. (2015, 2016); Larkoski, Neill, Stewart (2015)

AN EXAMPLE: HIGGS IN GLUON FUSION



- resummed (and matched) cross-section converges faster than pure FO
- resummation is perturbative, i.e. captures the effect of the first few orders, so that $N^3LO+N^3LL \sim N^3LO$
- it provides further handles to estimate uncertainty from missing higher orders (e.g. subleading logs)

order	σ [pb]	
N^3LO	$48.1^{+0.1}_{-1.8}$	scale variation
N^3LO	48.1 ± 2.0	$\overline{\text{CH}}$ at 95% DoB
N^3LO+N^3LL	48.5 ± 1.9	scale+resummation variations
all-order estimate	48.7	from accelerated fixed-order series
all-order estimate	48.9	from accelerated resummed series

N^3LO: Anastasiou *et al.* (2015);

N^3LL: Bonvini, SM (2014); Bonvini, SM, Muselli, Rottoli (2016);

see also Catani *et al.* (2014); Ahmed *et al.* (2014/2015); Schmidt and Spira (2015); … also Becher *et al.* in SCET;

PDFs AT LARGE X

Observable: $\sigma = \sigma_0 \, C(\alpha_s(\mu)) \otimes f(\mu) \left[\otimes f(\mu) \right]$

Evolution: $\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$

- coefficient functions contain large-*x* logs
- PDF evolution doesn't (in MSbar)

$$P_{gg}(x) \sim \frac{A(\alpha_s)}{(1-x)_+}$$

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Process	observable	resummation available
DIS	$d\sigma/dx/dQ^2$ (NC, CC, charm, ...)	YES
DY Z/γ	$d\sigma/dM^2/dY$	YES
DY W	differential in the lepton kinematics	NO
$t\bar{t}$	total σ	YES
jets	inclusive $d\sigma/dp_t/dY$	YES/NO

it should be easy to compute

different calculations exist at NLL(*) but no public implementation

de Florian, Vogelsang (2007, 2013); Kidonakis, Owens (2000); Liu, Moch, Ringer (2017)

- coefficient functions contain large-*x* logs
- PDF evolution doesn't (in MSbar)

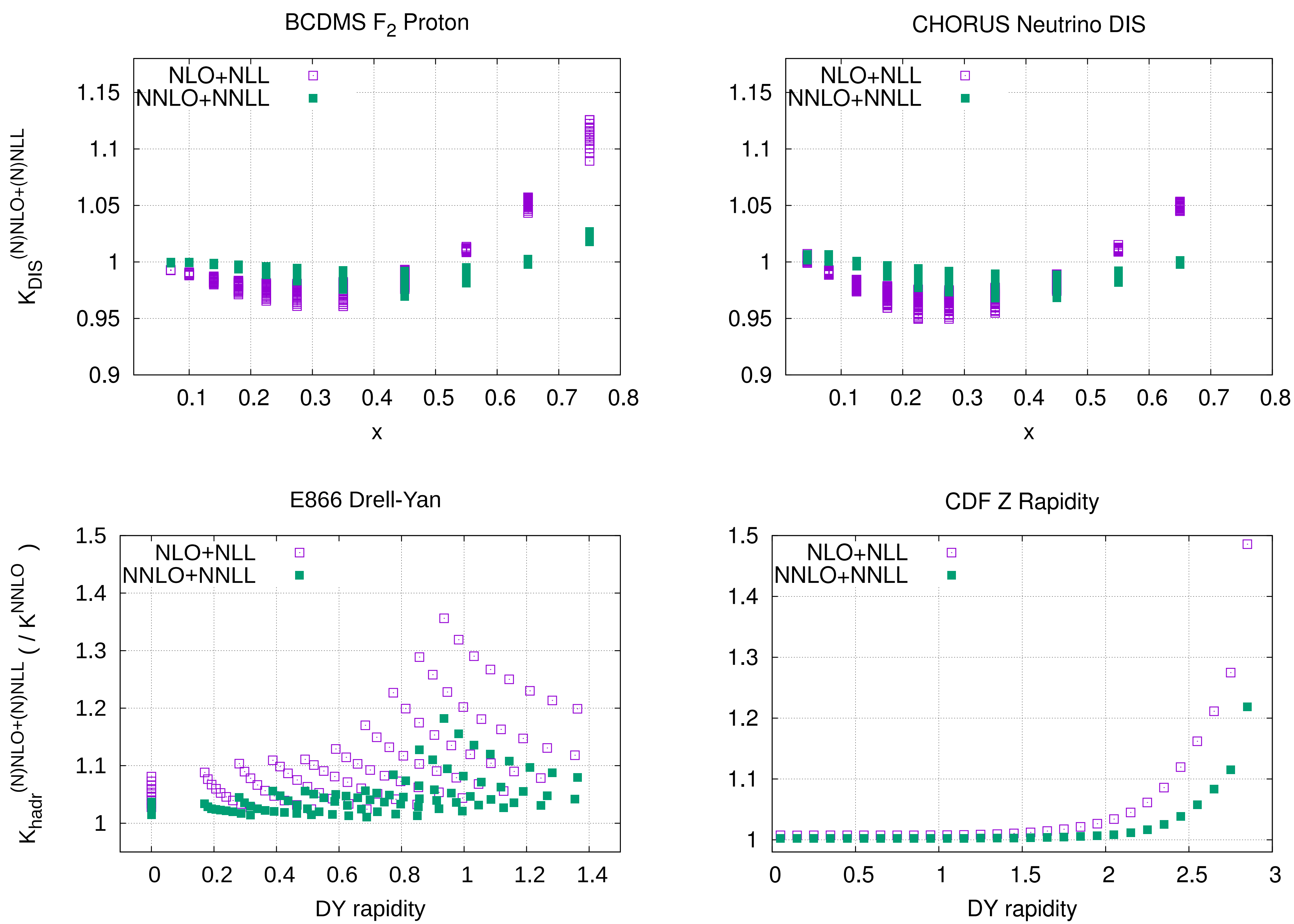
$$P_{gg}(x) \sim \frac{A(\alpha_s)}{(1-x)_+}$$

- performing a resummed fit is relatively straightforward
- data set is restricted: no jets
- (*)global vs non-global

DIS, DY available from TROLL (TROLL Resums Only Large-x Logarithms)
www.ge.infn.it/~bonvini/troll

$t\bar{t}$ available from top++ www.alexandermitov.com/software

EFFECTS ON THEORY PREDICTIONS



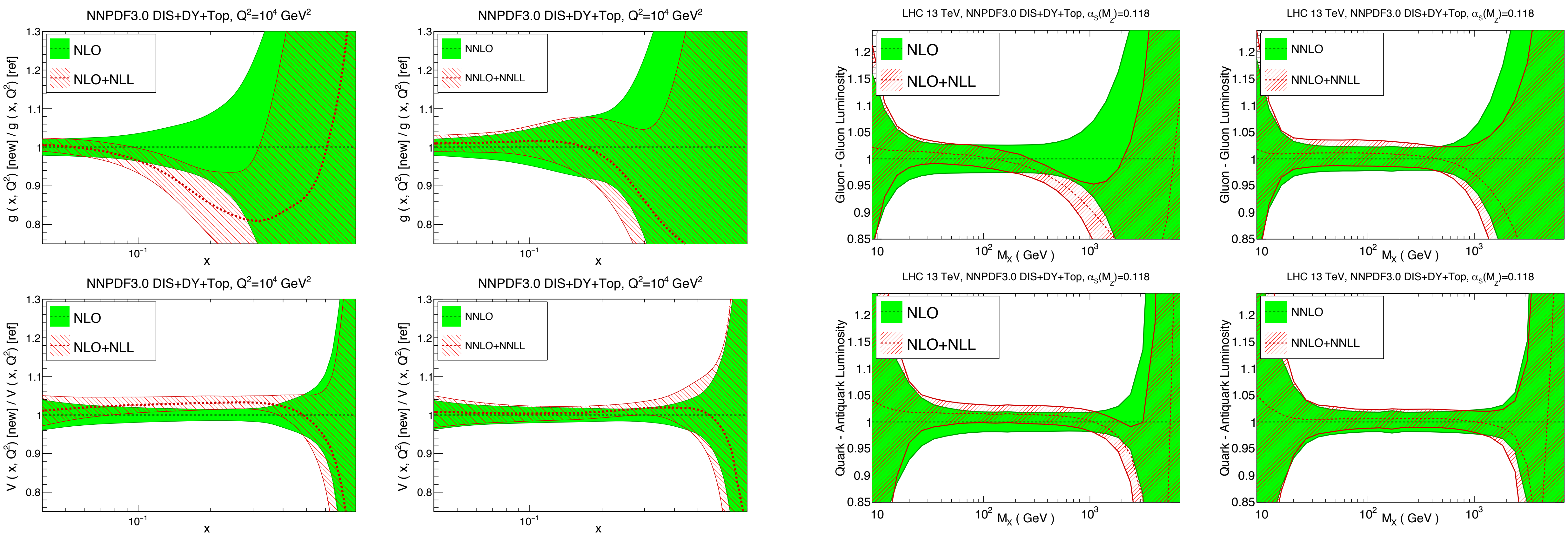
- K-factors reduced when NNLO is included: resummation is perturbative

PDFs FIT WITH THRESHOLD RESUMMATION

Experiment	NNPDF3.0 DIS+DY+top			
	NLO	NNLO	NLO+NLL	NNLO+NNLL
NMC	1.39	1.34	1.36	1.30
SLAC	1.17	0.91	1.02	0.92
BCDMS	1.20	1.25	1.23	1.28
CHORUS	1.13	1.11	1.10	1.09
NuTeV	0.52	0.52	0.54	0.44
HERA-I	1.05	1.06	1.06	1.06
ZEUS HERA-II	1.42	1.46	1.45	1.48
H1 HERA-II	1.70	1.79	1.70	1.78
HERA charm	1.26	1.28	1.30	1.28
DY E866	1.08	1.39	1.68	1.68
DY E605	0.92	1.14	1.12	1.21
CDF Z rap	1.21	1.38	1.10	1.33
D0 Z rap	0.57	0.62	0.67	0.66
ATLAS Z 2010	0.98	1.21	1.02	1.28
ATLAS high-mass DY	1.85	1.27	1.59	1.21
CMS 2D DY 2011	1.22	1.39	1.22	1.41
LHCb Z rapidity	0.83	1.30	0.51	1.25
ATLAS CMS top prod	1.23	0.55	0.61	0.40
Total	1.233	1.264	1.246	1.269

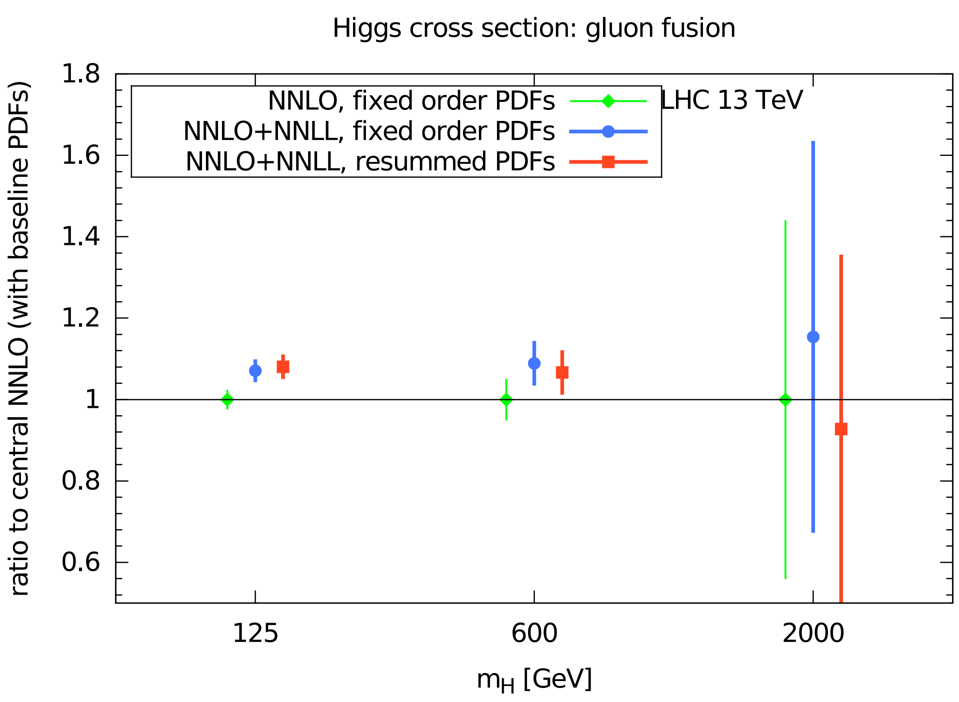
- as expected: visible effects at NLO+NLL are very much reduced at NNLO+NNLL
- χ^2 slightly worse because of DY fixed-target experiments
- this remains a puzzle

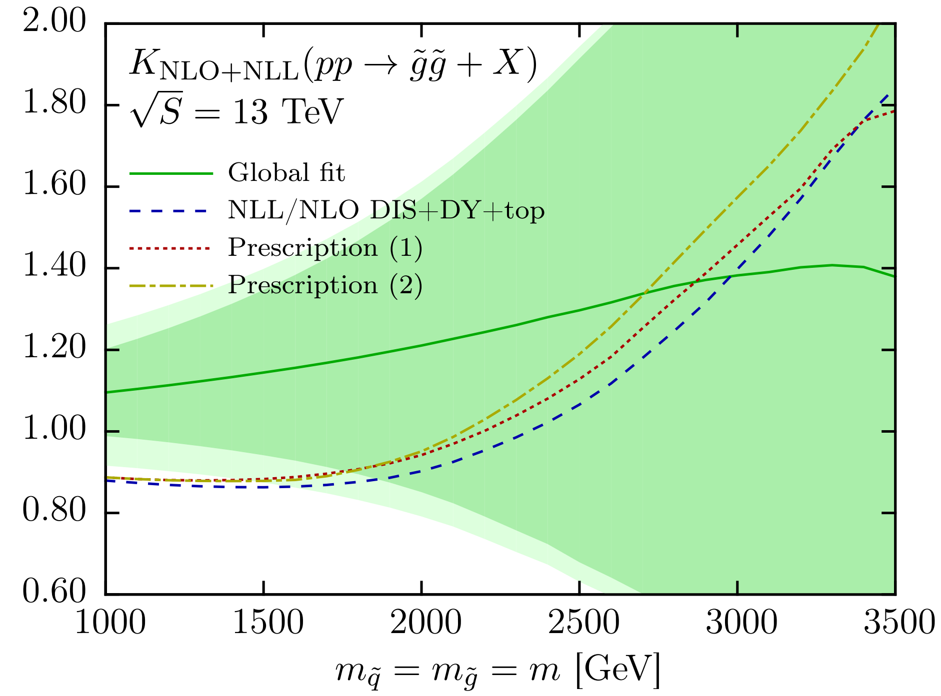
PARTONS WITH THRESHOLD RESUMMATION

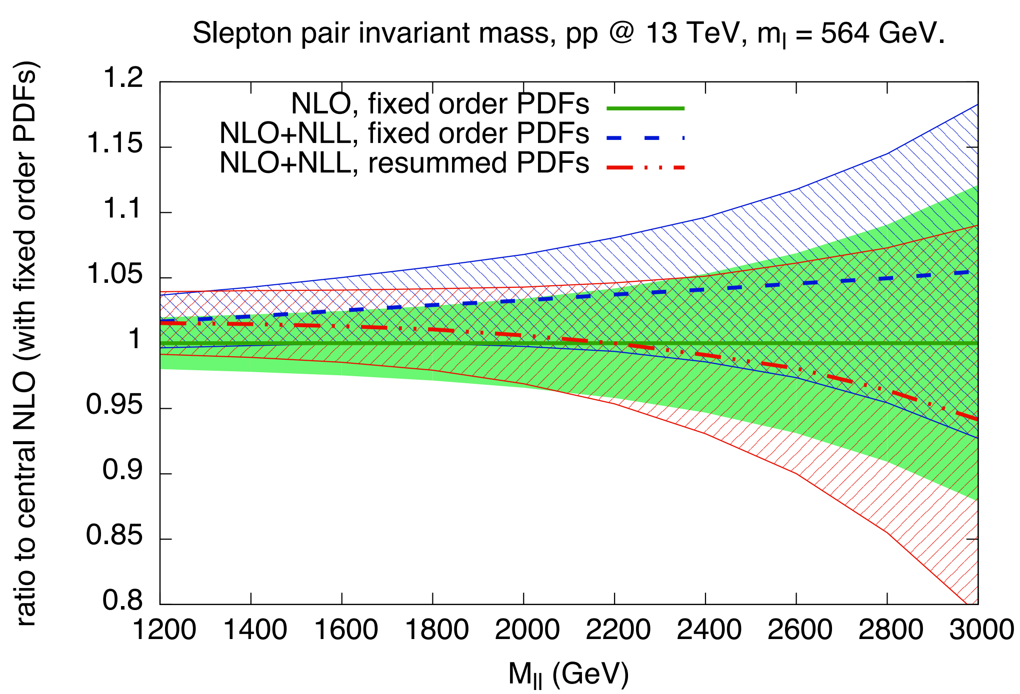


- comparison to global fit: larger uncertainties because of reduced dataset
- only “proof-of-concept” studies

PHENOMENOLOGY





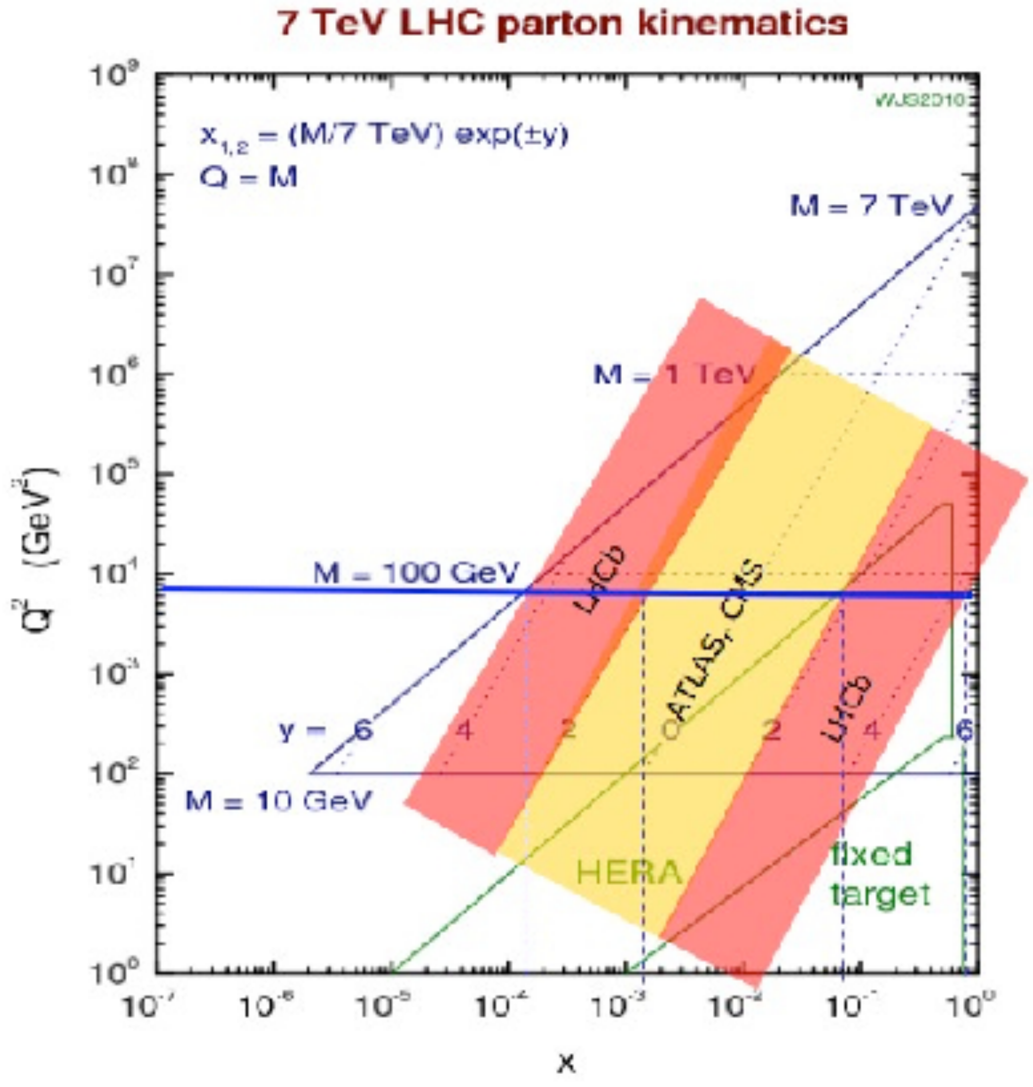


- effects on SM Higgs negligible
- more pronounced for high-mass states, still within PDF errors
- large-*x* PDFs not (yet) competitive because of missing jet data

HIGH-ENERGY (SMALL-X) RESUMMATION

LHC KINEMATICS

- PDFs are largely unconstrained at low *x*
- LHC does probe this region
- Is DGLAP enough to describe this region?
- Do we need to worry about small *x*? and saturation?



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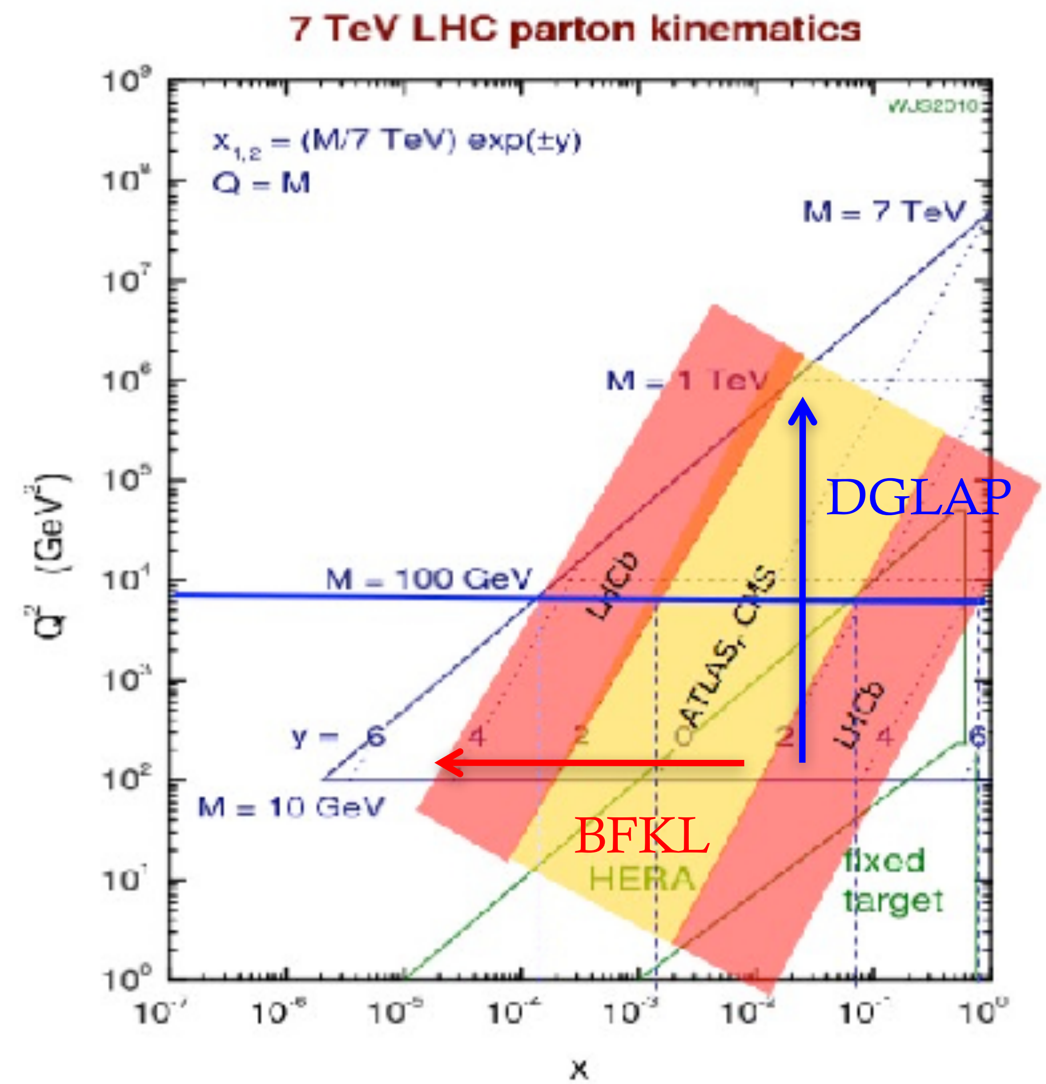


DGLAP: Q^2 evolution for *N* moments of the parton density

$$\frac{d}{d\ln(Q^2/\mu^2)}G(N,Q^2) = \gamma(N,\alpha_s)G(N,Q^2)$$

BFKL: small-*x* evolution for *M* moments of the parton density

$$\frac{d}{d\ln(1/x)}G(x,M) = \chi(M,\alpha_s)G(x,M)$$

Mellin moments:

logs $\leftrightarrow$ poles

$$\ln^k \frac{Q^2}{\mu^2} \leftrightarrow \frac{1}{M^{k+1}}$$

$$\ln^k \frac{1}{x} \leftrightarrow \frac{1}{N^{k+1}}$$

DGLAP EVOLUTION AT SMALL-X

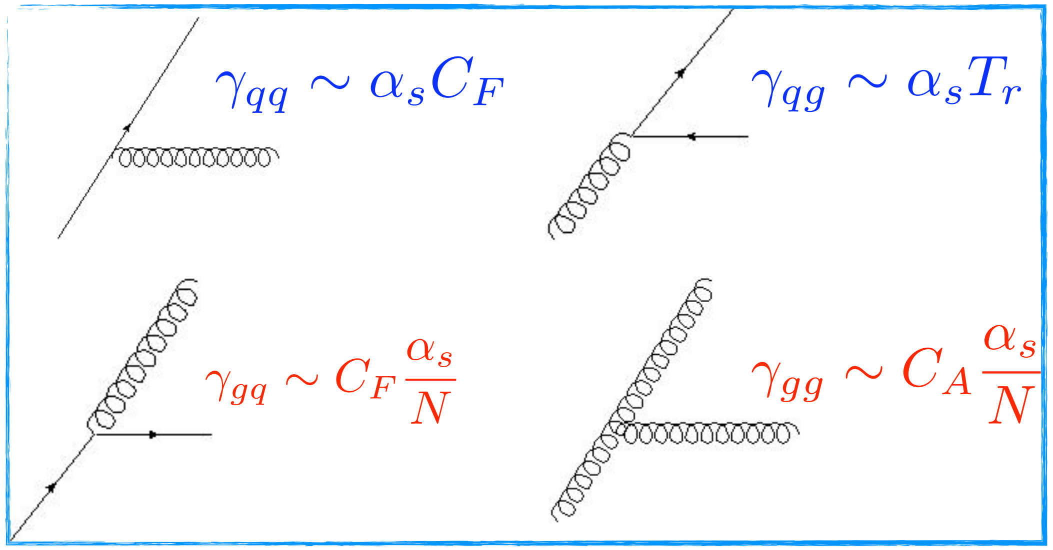
- DGLAP evolution in the singlet sector

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} f_g \\ f_q \end{pmatrix} = \Gamma(N, \alpha_s(Q^2)) \begin{pmatrix} f_g \\ f_q \end{pmatrix}, \qquad \Gamma(N, \alpha_s) \equiv \begin{pmatrix} \gamma_{gg} & \gamma_{gq} \\ \gamma_{qg} & \gamma_{qq} \end{pmatrix}$$

- the gluon splitting functions start at LL*x*

$$\gamma_{gg} \sim c_1 \frac{\alpha_s}{N} + c_2 \left(\frac{\alpha_s}{N}\right)^2 + \dots$$

$$\gamma_{gq} \sim \frac{C_F}{C_A} \gamma_{gg}$$



- while the quarks are NLL*x*

$$\gamma_{qg} \sim \alpha_s d_0 + d_1 \alpha_s \frac{\alpha_s}{N} + c_2 \alpha_s \left(\frac{\alpha_s}{N}\right)^2 + \dots$$

$$\gamma_{qq}^{(\mathrm{PS})} \sim \frac{C_F}{C_A} \left(\gamma_{gg} - \alpha_s d_0\right)$$

FIXED-ORDER CONSIDERATIONS

- Note that some of the coefficients can be zero because of accidental cancellations: most notably c_2 and c_3 in MS-like schemes

$$\gamma_{gg} \sim c_1 \left(\frac{\alpha_s}{N}\right) + c_2 \left(\frac{\alpha_s}{N}\right)^2 + c_3 \left(\frac{\alpha_s}{N}\right)^3 + c_4 \left(\frac{\alpha_s}{N}\right)^4 + \mathcal{O}(\alpha_s^5)$$

- NNLO is less stable than NLO (subleading logs survive)
- N^3LO (calculations underway) is likely to exhibit stronger instabilities

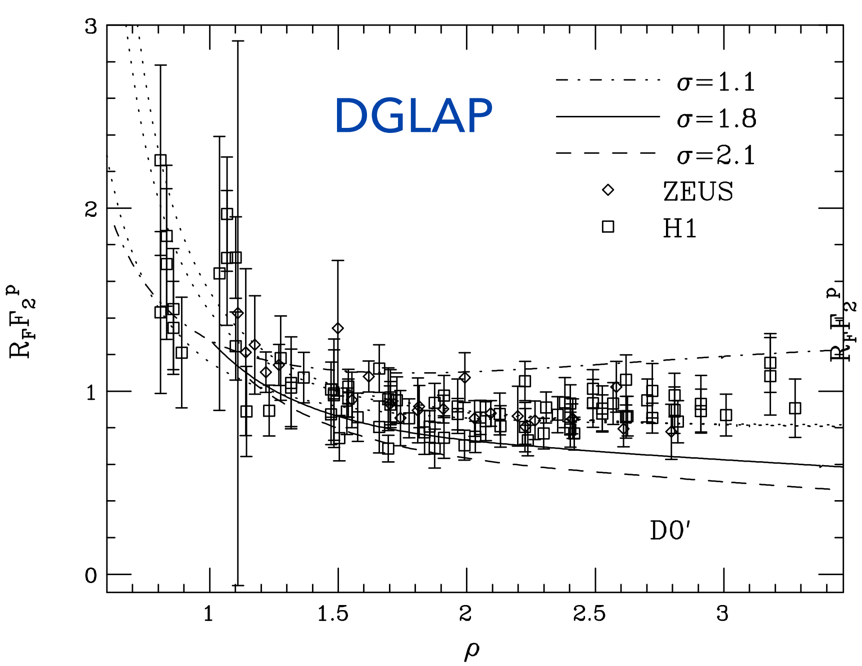
DGLAP-BFLK DUALITY

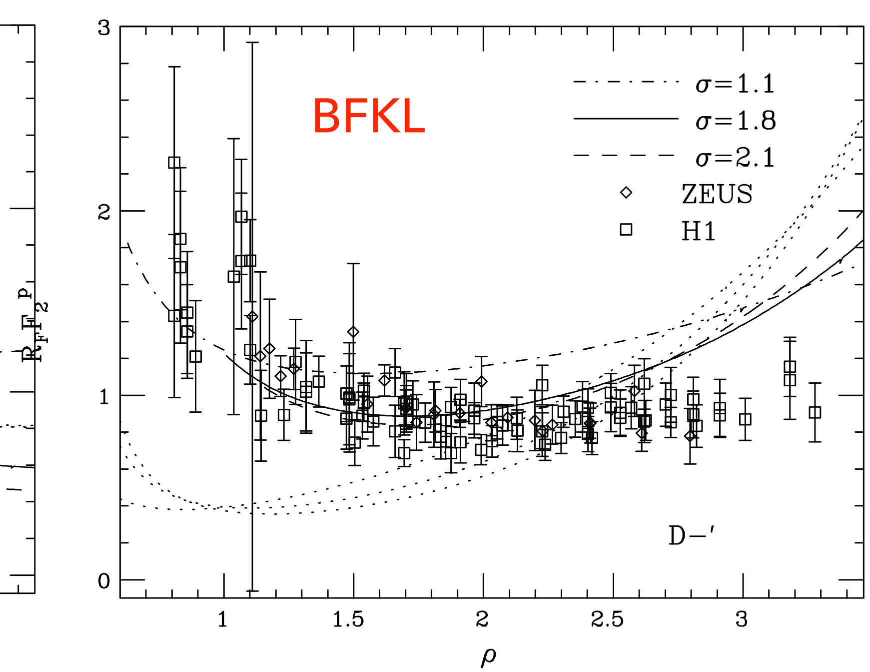
- (N)LLx behaviour can be determined from the (N)LO BFKL kernel

Jaroszewicz (1982)

$$G(N,M) = \underbrace{\frac{G_0(N)}{M-\gamma(\alpha_s,N)}}_{\text{DGLAP}} \underset{\text{and}}{=} \underbrace{\frac{\bar{G}_0(M)}{N-\chi(\alpha_s,M)}}_{\text{BFKL}} \longrightarrow \begin{array}{rcl} \chi(\gamma(N,\alpha_s),\alpha_s) & = & N \\ \gamma(\chi(M,\alpha_s),\alpha_s) & = & M \end{array}$$

- however: naive implementation of BFKL leads to results not supported by HERA data (too strong, too soon)





double-scaling variables

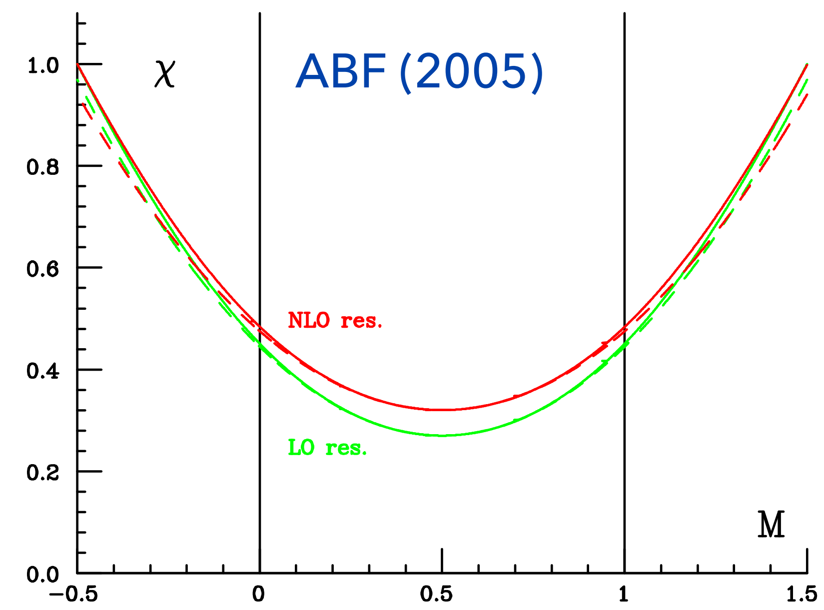
$$\sigma \equiv \sqrt{\ln\frac{x_0}{x}\ln\frac{t}{t_0}}, \qquad \rho \equiv \sqrt{\ln\frac{x_0}{x}\Big/\ln\frac{t}{t_0}}$$

models that naively implement BFKL are disfavoured by HERA data

Ball, Forte (1994)

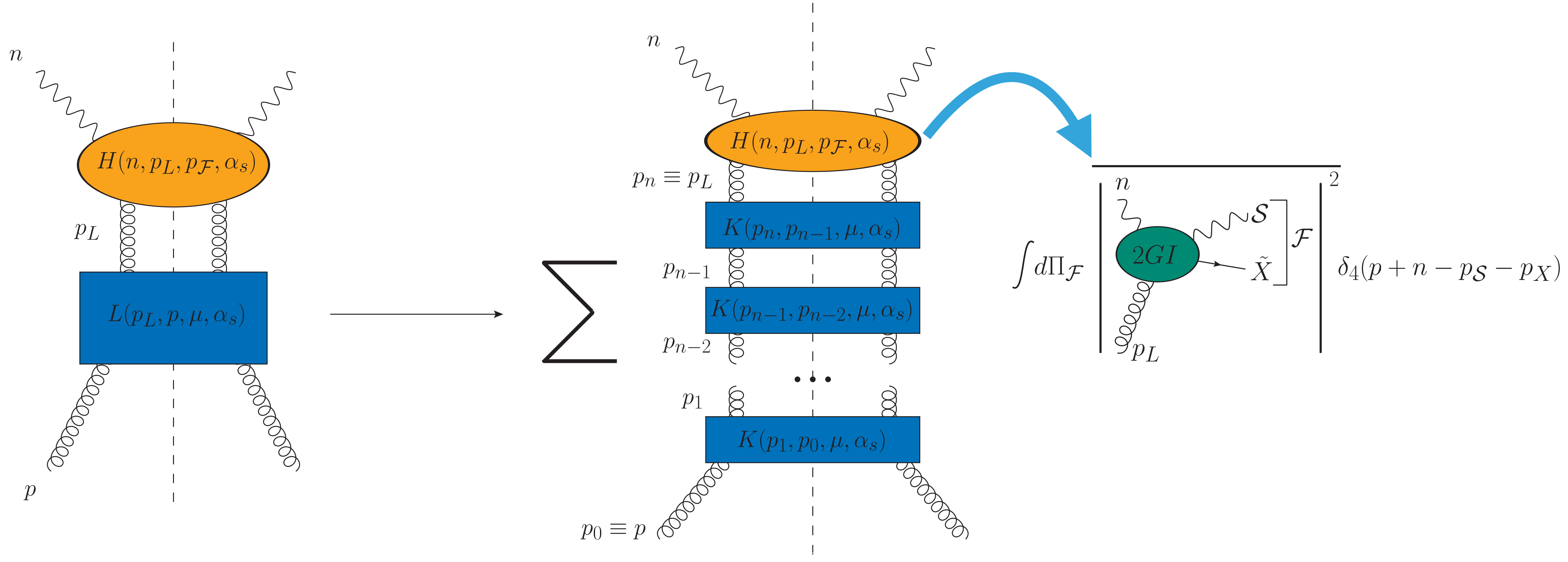
RESUMMATION OF DGLAP EVOLUTION

- Problem studied by different groups in late '90s /early '00s: Altarelli, Ball, Forte; Ciafaloni, Colferai, Salam, Stasto; Thorne, White
- for a comparative review see HERA-LHC Proc. arXiv:0903.3861
- recent progress in SCET Rothstein, Stewart (2016)
- we mostly follow the approach by ABF
- *key ingredients:*
  - duality between DGLAP and BFKL kernels
  - stable solution of the running coupling BFKL equation (important subleading effects)
  - match to standard DGLAP at large *N* *(x)*



COEFFICIENT FUNCTIONS AT SMALL X

- the high-energy behaviour of coefficient function is obtained using k_t-factorisation Catani, Ciafaloni, Hautmann (1991); Collins, Ellis (1991)
- derivation in terms of ladder expansion allowed for its generalisation to differential distributions Caola, Forte, SM (2010); Forte, Muselli (2016); Muselli (2017)



- for most processes of interest (DIS, DY) resummation starts at NLLx

RESUMMATION OF COEFFICIENT FUNCTIONS

- naive (i.e. fixed-log counting) resummation has same issues as evolution
- running coupling corrections are crucial
- elegant but complex treatment in Mellin space Ball (2008)
- our approach in a nutshell: resummation in momentum space

High-energy (k_T) factorization:

$$\sigma \propto \int \frac{dz}{z} \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(\frac{x}{z}, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s(Q^2)\right) \mathcal{F}_g(z, \boldsymbol{k}) \qquad \begin{cases} \mathcal{F}_g(x,\boldsymbol{k}) : \text{unintegrated PDF} \\ \hat{\sigma}_g\left(z, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) : \text{off-shell xs} \end{cases}$$

Defining

$$\mathcal{F}_g(N, \boldsymbol{k}) = U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) f_g(N, \mu^2)$$

we get

$$C_g(N, \alpha_s) = \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(N, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right)$$

At LLx accuracy, U has a simple form, in terms of small-x resummed anom dim γ

$$U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) \approx \boldsymbol{k}^2 \frac{d}{d\boldsymbol{k}^2} \exp \int_{\mu^2}^{\boldsymbol{k}^2} \frac{d\nu^2}{\nu^2} \gamma(N, \alpha_s(\nu^2))$$

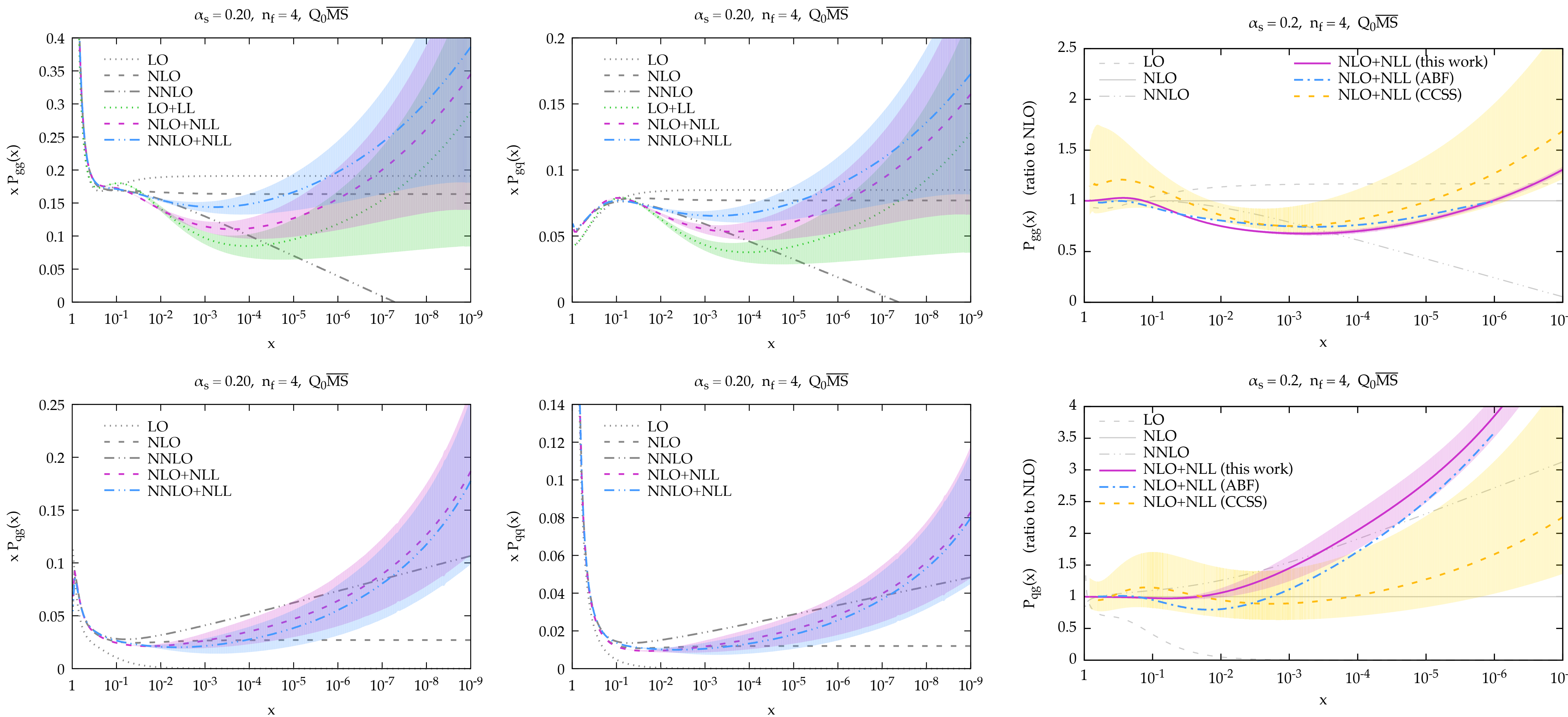
- until recent: very little phenomenology because a comprehensive code was missing

HIGH ENERGY LARGE LOGARITHMS

- public code that computes resummed splitting functions and perturbative coefficient functions
- HELL-x: pheno tool with pre-tabulated results, interfaced with evolution code APFEL
- in current HELL 2.0 version
  - DIS (both NC and CC)
  - heavy-quark matching conditions
- HELL 3.0 will appear soon (Higgs, DY)

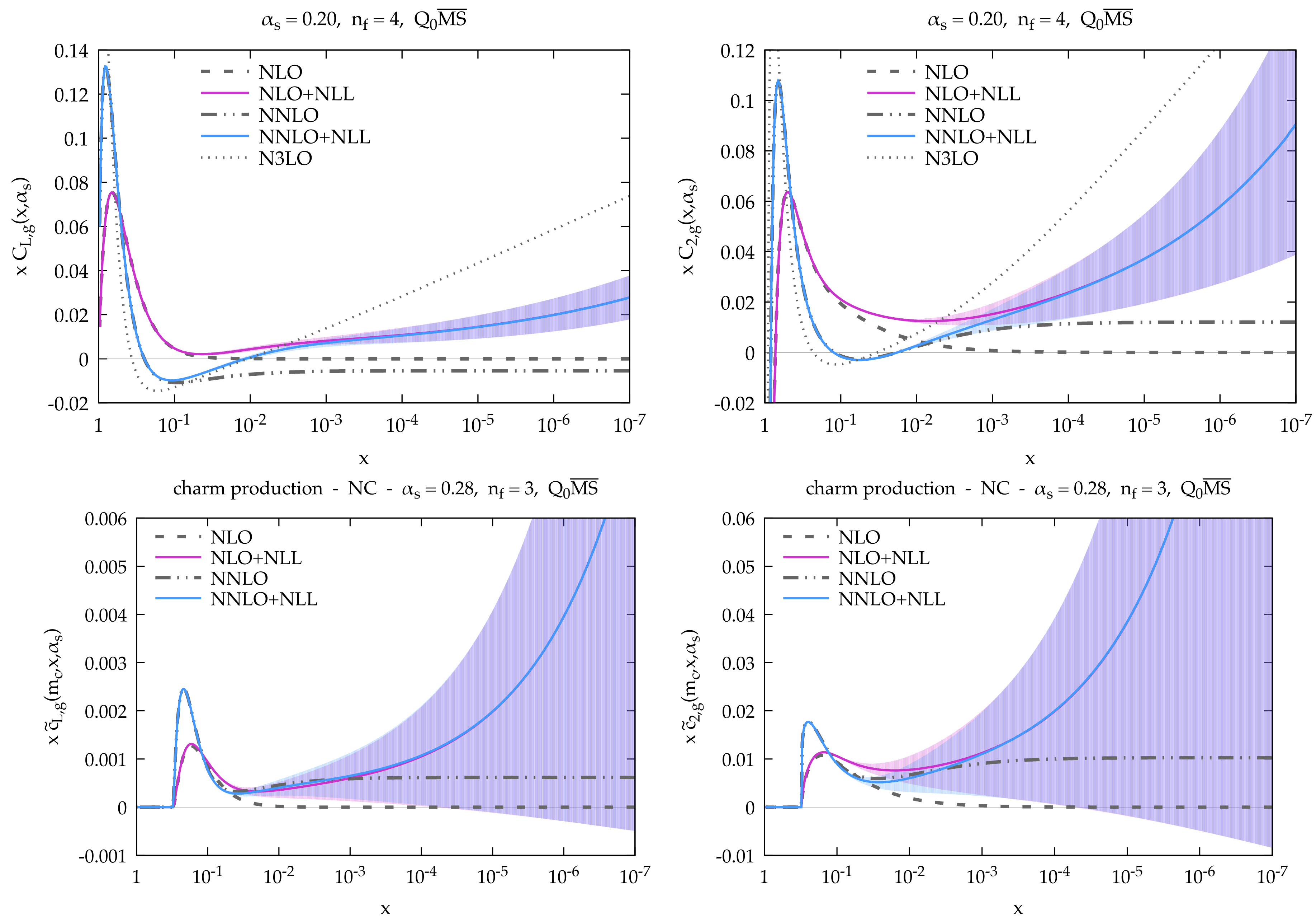
https://www.ge.infn.it/~bonvini/hell/

RESULTS FROM HELL: SPLITTING FUNCTIONS



- resummation matched up to NNLO
- uncertainty bands obtained by varying subleading corrections
- quark splitting functions under less control (they start at NLL)

RESULTS FROM HELL: DIS COEFFICIENT FUNCTIONS



we can already see N^3LO instabilities

- parton level results
- large theoretical uncertainty (they start at NLL)

A FIT WITH SMALL-X RESUMMATION: THE DATASET

- exploit NNPDF state-of-art technology to perform fits with small-*x* resummation
- for DIS with have a consistent implementation of small-*x* resummation (both evolution and coefficient functions)
- similar dataset as standard NNLO analysis (NNPDF 3.1)
- lower the initial scale of the fit to Q_0=1.64 GeV to include an extra bin of the HERA data (Q^2=2.7 GeV^2)
- what about hadronic data?

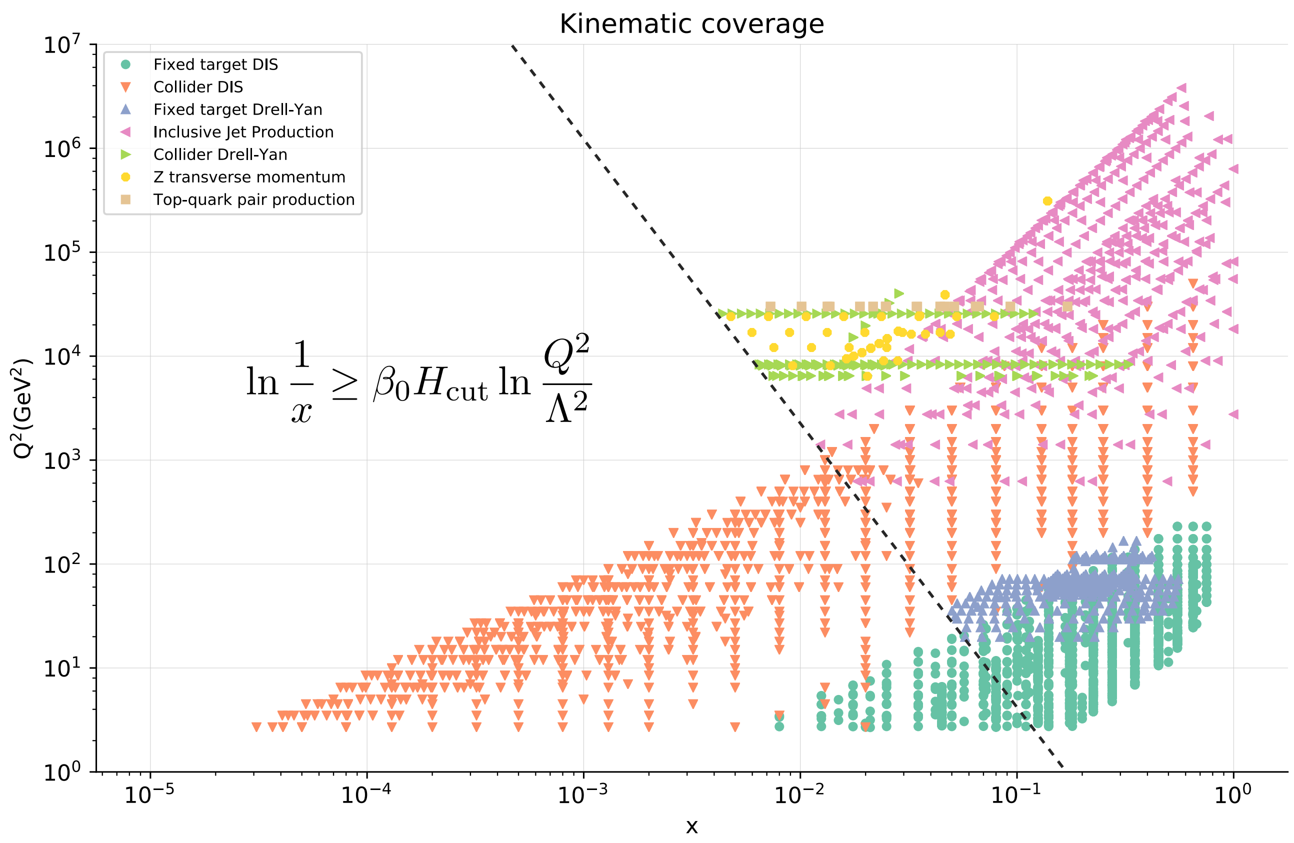
Experiment	N_{dat}
NMC	367
SLAC	80
BCDMS	581
CHORUS	886
NuTeV dimuon	79
HERA I+II incl. NC	1081
HERA I+II incl. CC	81
HERA σ_c^{NC}	47
HERA F_2^b	29
Total	**3231**

THE ISSUE WITH HADRONIC DATA

- resummation for coefficient functions in pp collisions is known but not yet implemented in HELL
- resummation only included in the evolution
- to avoid biases we cut away *hadronic* low-*x* data (mostly LHCb DY)
- we discard points for which (based on LO kinematics)

$$\alpha_s(Q^2) \ln \frac{1}{x} \geq H_{\rm cut}$$

- the smaller $H_{\rm cut}$, the tighter the cut
- we find $H_{\rm cut}$=0.6 to be a good compromise
- we keep ~70% of hadronic data



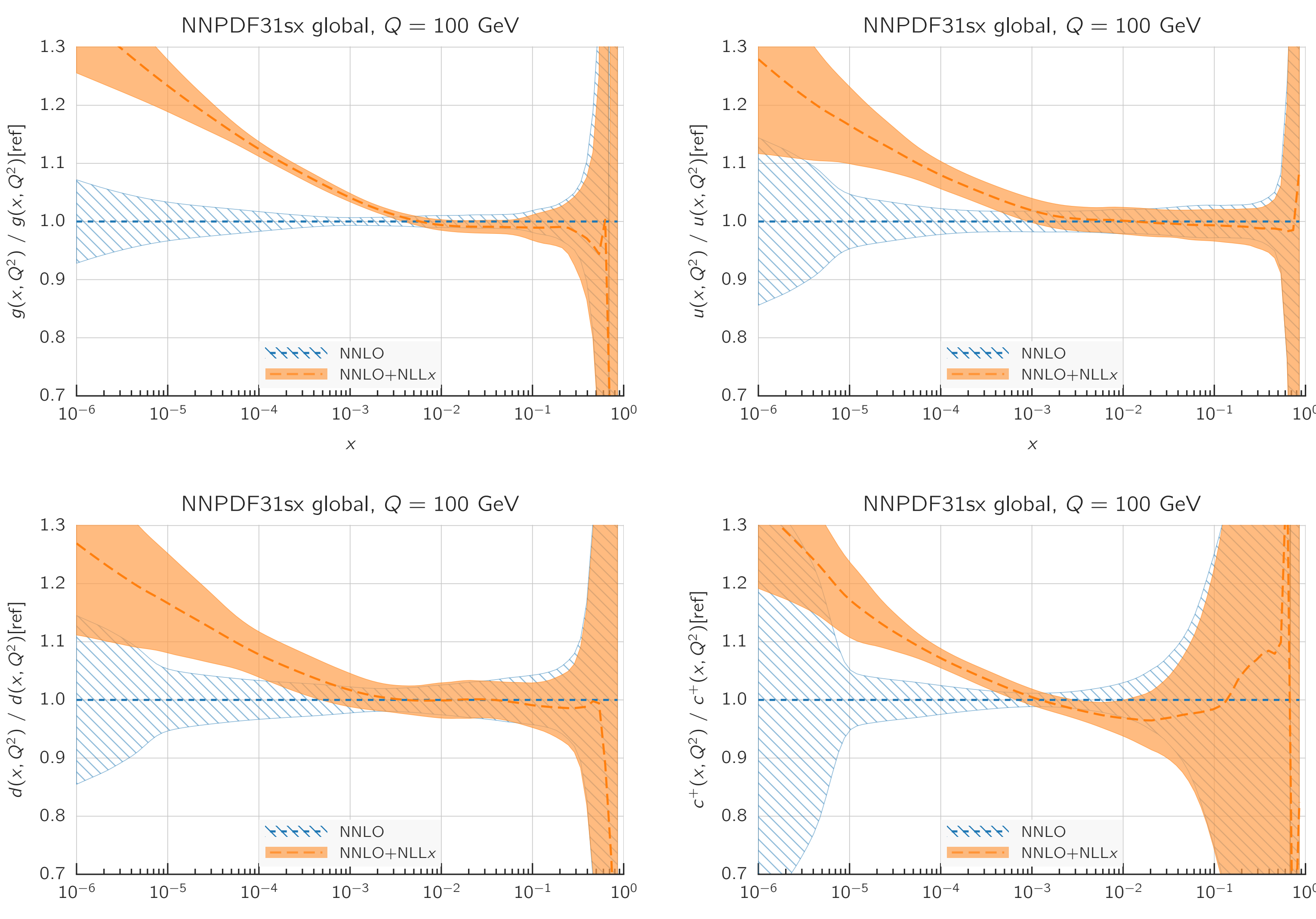
FIT RESULTS

	$\chi^2/N_{\rm dat}$ NLO	NLO+NLLx	$\Delta\chi^2$	$\chi^2/N_{\rm dat}$ NNLO	NNLO+NLLx	$\Delta\chi^2$
NMC	1.35	1.35	+1	1.30	1.33	+9
SLAC	1.16	1.14	−1	0.92	0.95	+2
BCDMS	1.13	1.15	+12	1.18	1.18	+3
CHORUS	1.07	1.10	+20	1.07	1.07	−2
NuTeV dimuon	0.90	0.84	−5	0.97	0.88	−7
HERA I+II incl. NC	1.12	1.12	-2	1.17	1.11	−62
HERA I+II incl. CC	1.24	1.24	-	1.25	1.24	−1
HERA $\sigma_c^{\rm NC}$	1.21	1.19	−1	2.33	1.14	−56
HERA F_2^b	1.07	1.16	+3	1.11	1.17	+2
DY E866 $\sigma^d_{\rm DY}/\sigma^p_{\rm DY}$	0.37	0.37	-	0.32	0.30	-
DY E886 σ^p	1.06	1.10	+3	1.31	1.32	-
DY E605 σ^p	0.89	0.92	+3	1.10	1.10	-
CDF Z rap	1.28	1.30	-	1.24	1.23	-
CDF Run II k_t jets	0.89	0.87	−2	0.85	0.80	−4
D0 Z rap	0.54	0.53	-	0.54	0.53	-
D0 $W \to e\nu$ asy	1.45	1.47	-	3.00	3.10	+1
D0 $W \to \mu\nu$ asy	1.46	1.42	-	1.59	1.56	-
ATLAS total	1.18	1.16	−7	0.99	0.98	−2
ATLAS W, Z 7 TeV 2010	1.52	1.47	-	1.36	1.21	−1
ATLAS HM DY 7 TeV	2.02	1.99	-	1.70	1.70	-
ATLAS W, Z 7 TeV 2011	3.80	3.73	−1	1.43	1.29	−1
ATLAS jets 2010 7 TeV	0.92	0.87	−4	0.86	0.83	−2
ATLAS jets 2.76 TeV	1.07	0.96	−6	0.96	0.96	-
ATLAS jets 2011 7 TeV	1.17	1.18	-	1.10	1.09	−1
ATLAS Z p_T 8 TeV (p_T^{ll}, M_{ll})	1.21	1.24	+2	0.94	0.98	+2
ATLAS Z p_T 8 TeV (p_T^{ll}, y_{ll})	3.89	4.26	+2	0.79	1.07	+2
ATLAS σ_{tt}^{tot}	2.11	2.79	+2	0.85	1.15	+1
ATLAS $t\bar{t}$ rap	1.48	1.49	-	1.61	1.64	-
CMS total	0.97	0.92	−13	0.86	0.85	−3
CMS Drell-Yan 2D 2011	0.77	0.77	-	0.58	0.57	-
CMS jets 7 TeV 2011	0.88	0.82	−9	0.84	0.81	−3
CMS jets 2.76 TeV	1.07	0.98	−7	1.00	1.00	-
CMS Z p_T 8 TeV (p_T^{ll}, y_{ll})	1.49	1.57	+1	0.73	0.77	-
CMS σ_{tt}^{tot}	0.74	1.28	+2	0.23	0.24	-
CMS $t\bar{t}$ rap	1.16	1.19	-	1.08	1.10	-
Total	**1.117**	**1.120**	**+11**	**1.130**	**1.100**	**−121**

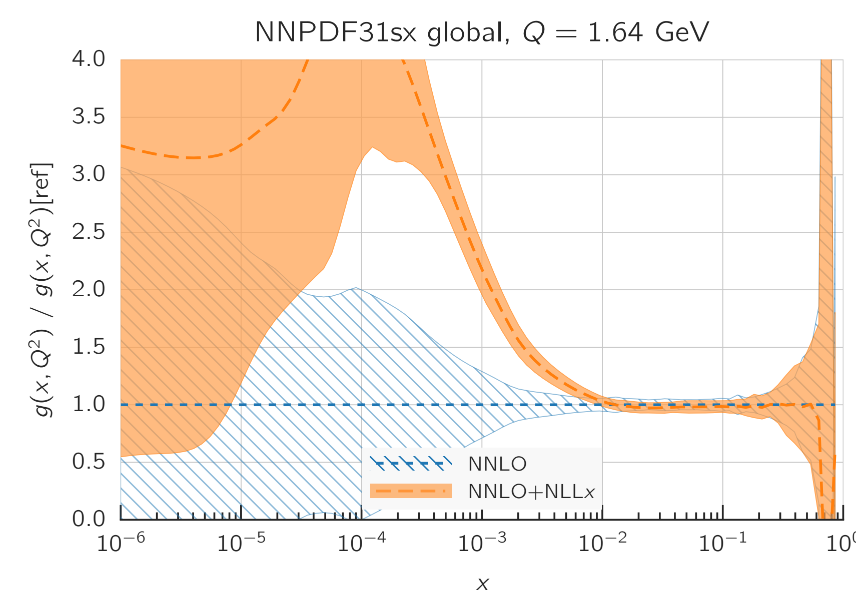
- the quality of NLO+NLL*x* and NLO fits is comparable
- it's expected because the two theories are rather similar
- situation changes dramatically at NNLO
- NNLO+NLL*x* provides the best fit
- the bulk of the improvement comes from HERA data

PARTON DENSITIES WITH SMALL-X RESUMMATION

- resulting PDFs show interesting features
- agreement at large *x* but they're much steeper at low *x*

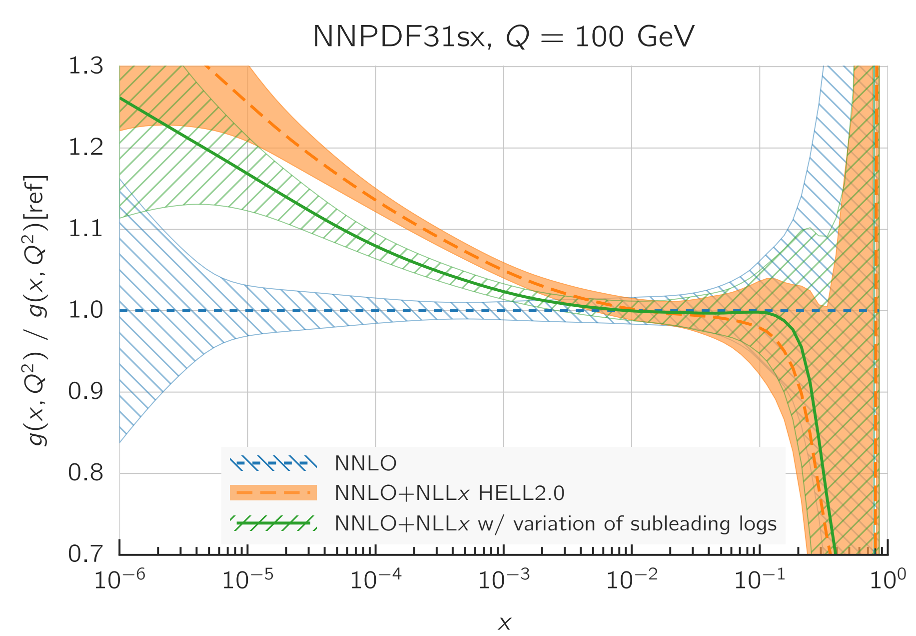


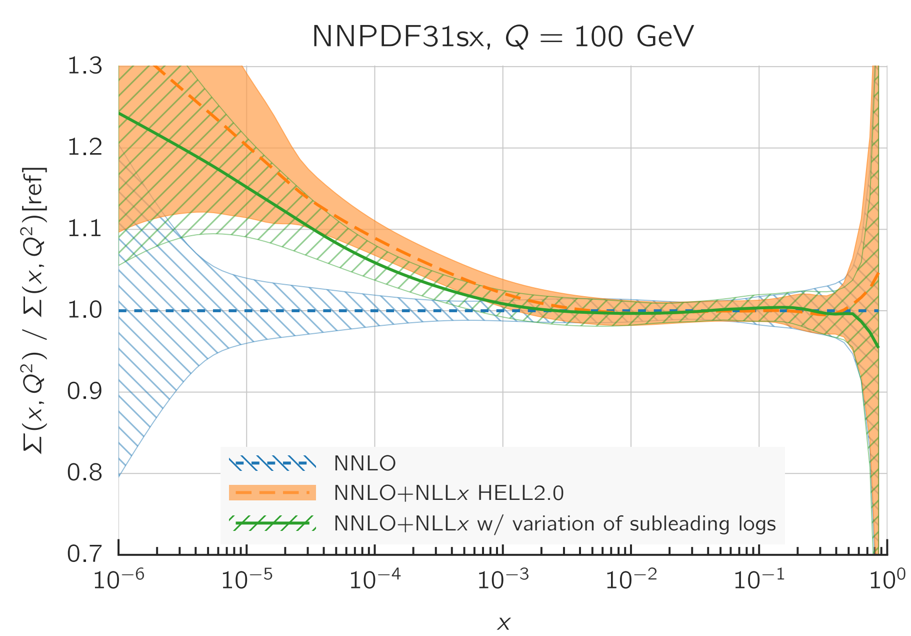
- how much is the initial condition modified?



IMPACT OF THEORETICAL UNCERTAINTIES

- we have seen that quark splitting functions and coefficient functions suffer from large theoretical uncertainties
- the inclusion of theory errors in PDF fit is currently an active area of research
- we can use a setting that varies from the standard one beyond NLL*x*
- a DIS-only study shows that the fit quality is unchanged
- qualitative behaviour on solid grounds, however quantitative results do change

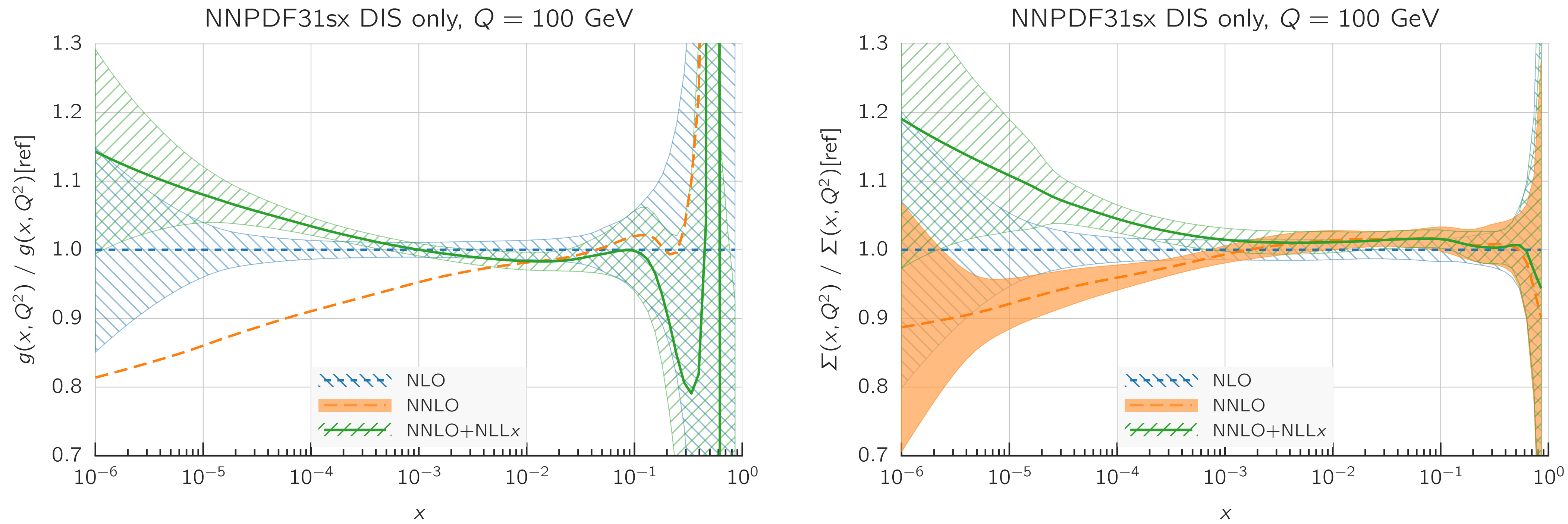




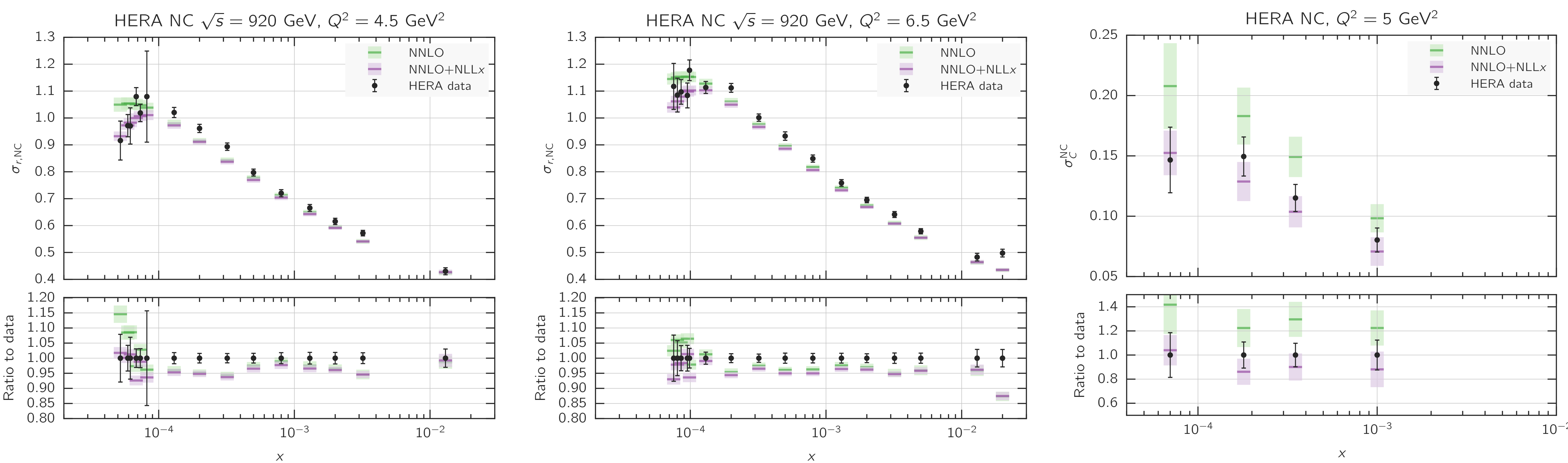
this shows the need for NNLL*x* resummation (at least in the quark sector)

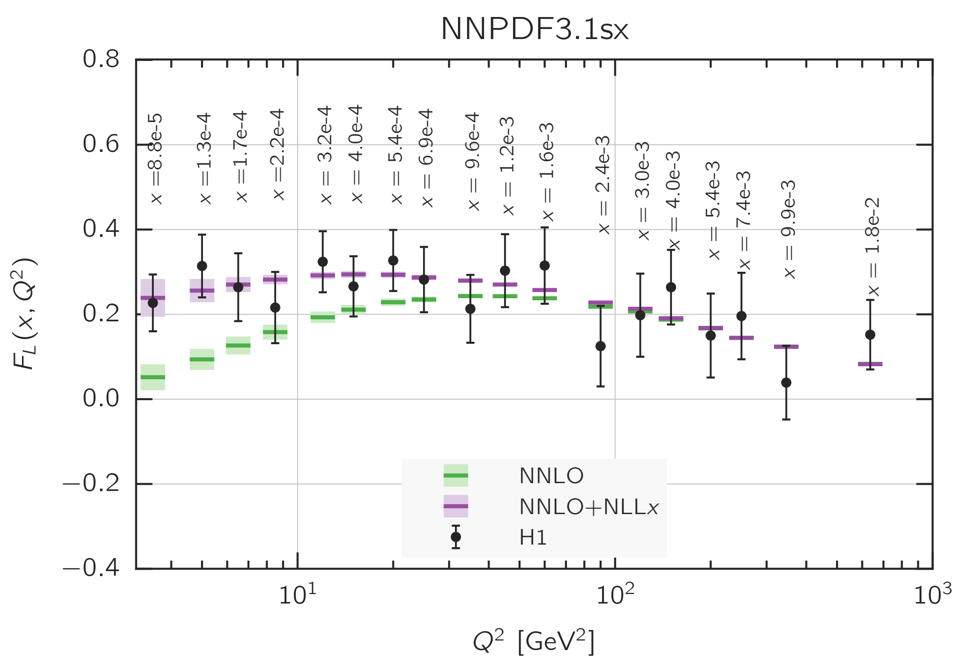
PERTURBATIVE STABILITY

- NNLO and NNLO+NLLx differ quite dramatically
- one could question the reliability of the resummed procedure
- what gives us confidence we're not talking rubbish?
- resummation cures perturbative instability of NNLO



HERA STRUCTURE FUNCTIONS

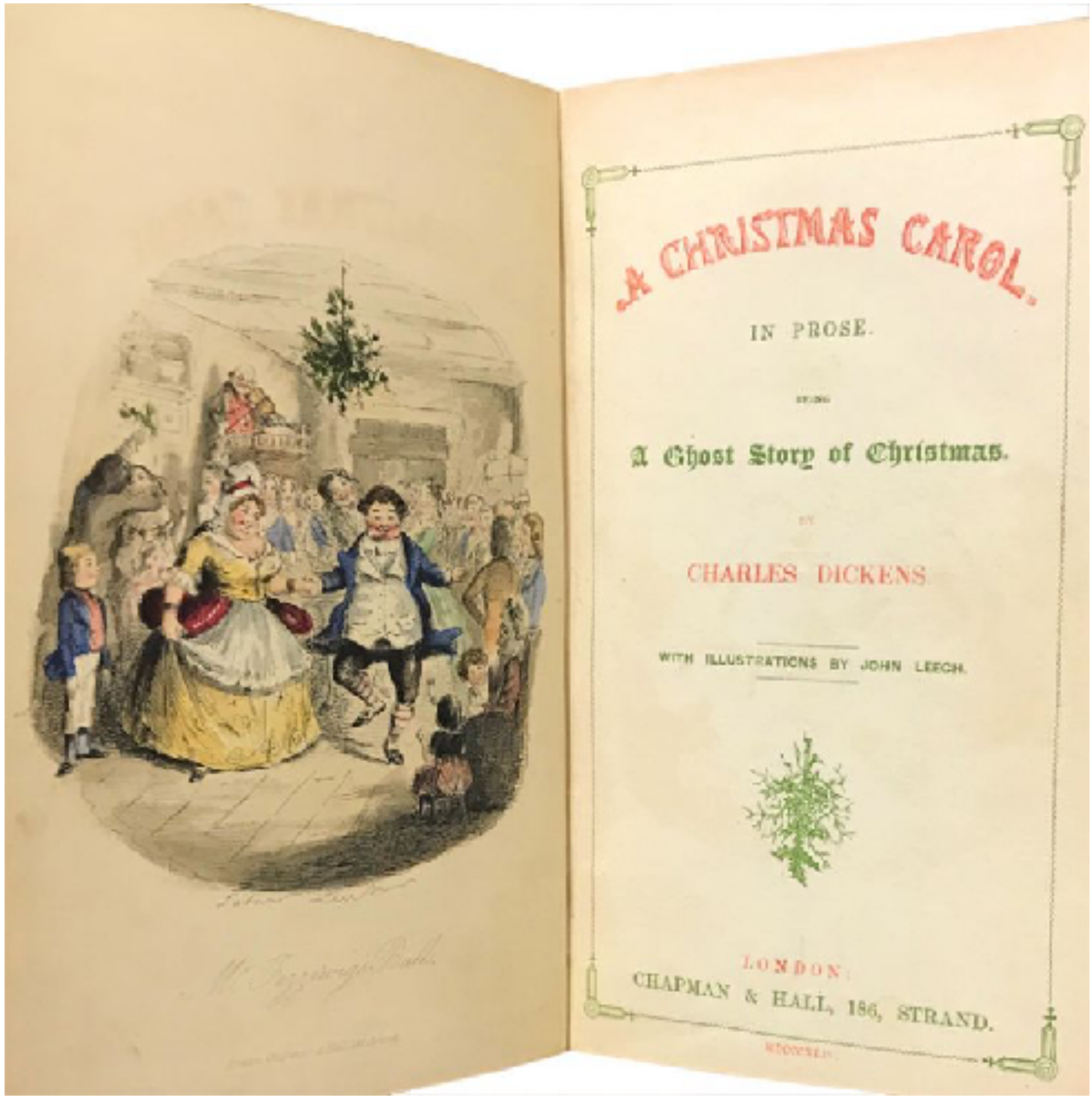




- the improved description of DIS structure functions is clearly visible
- this is particularly true for F_L where resummation effects starts at its LO

A DICKENSIAN TAKE ON THE RESULTS

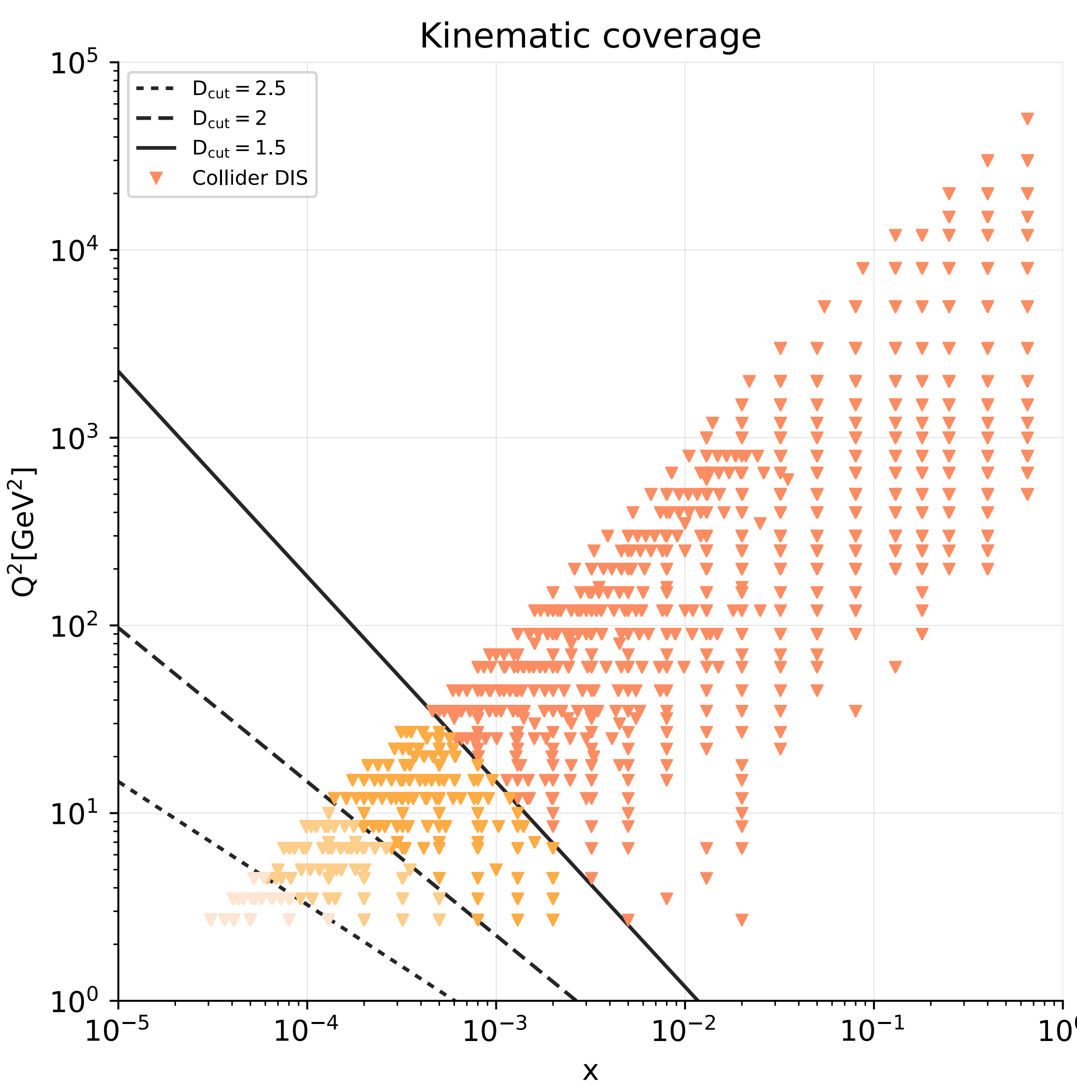
- BFKL is a beautiful but tough framework
- QCD at low x is hard work
- it seems appropriate to humbly borrow Charles Dickens images to describe it
- as in the novel: in the end, we'll do better (physics)

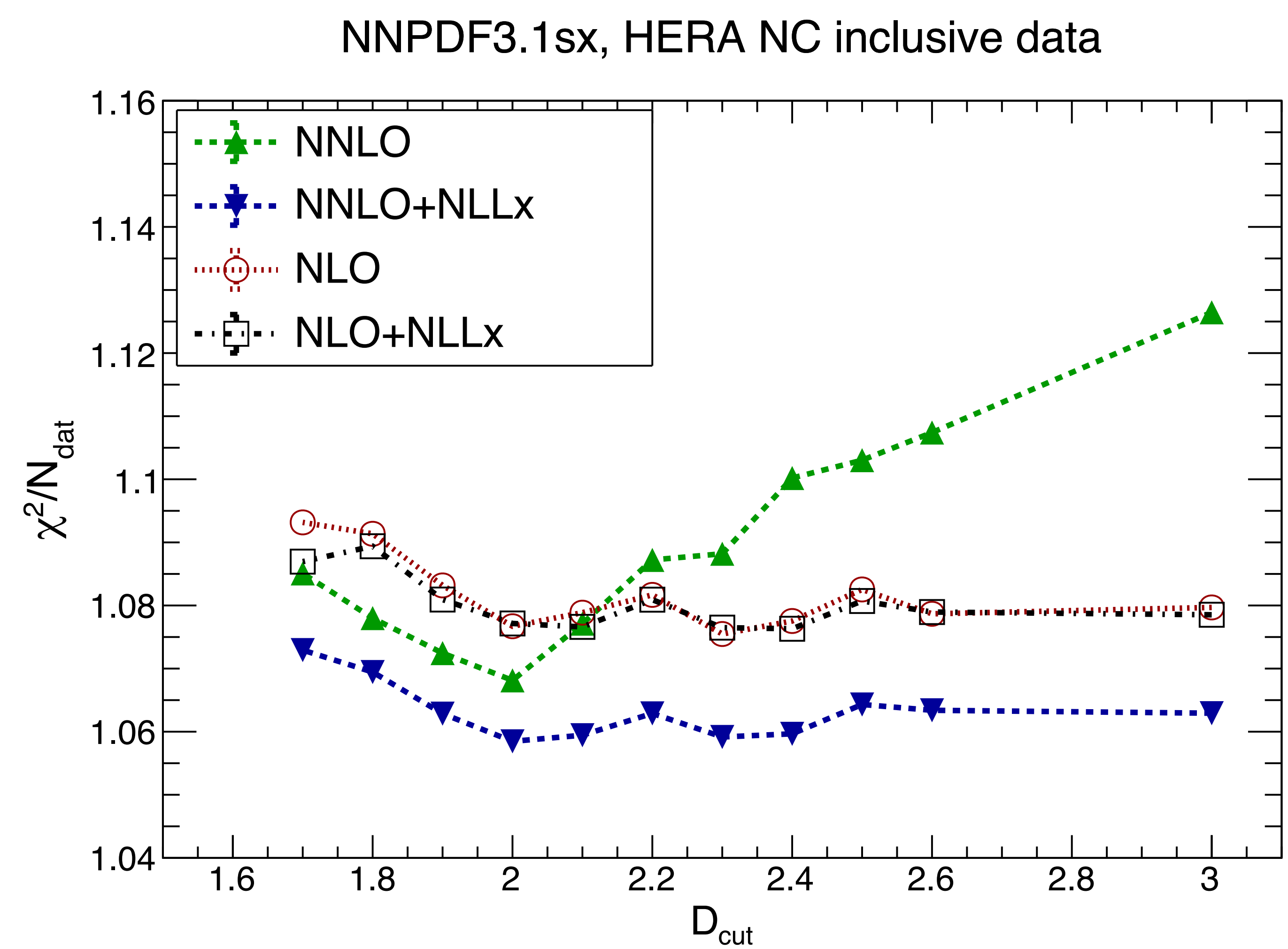


BFKL: THE GHOST OF CHRISTMAS PAST

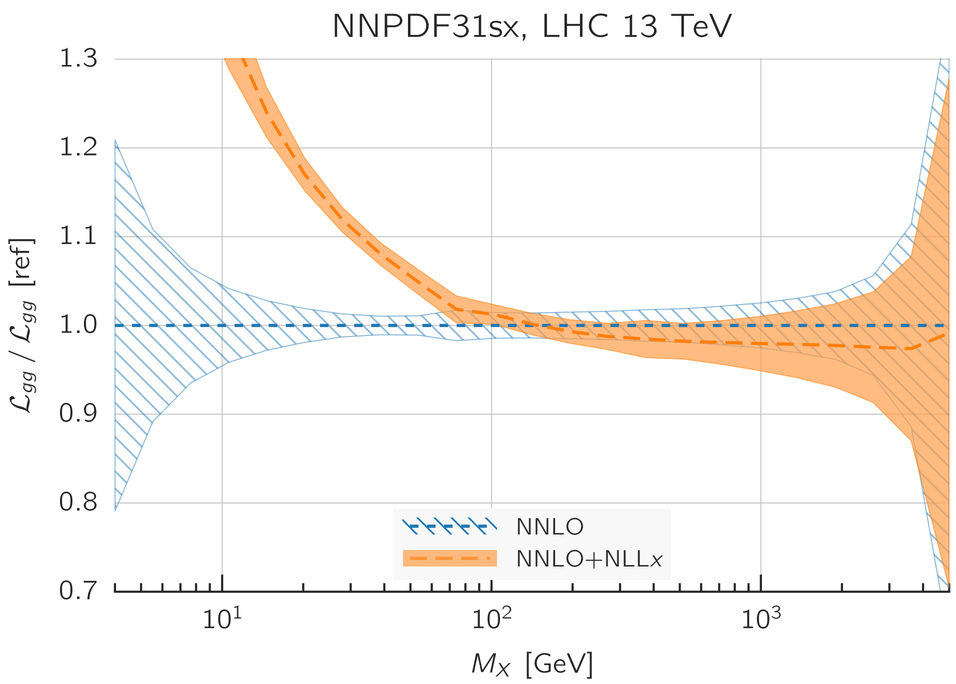
- How does the fit-quality change if we include data at smaller and smaller x?
- similar strategy as for hadronic data

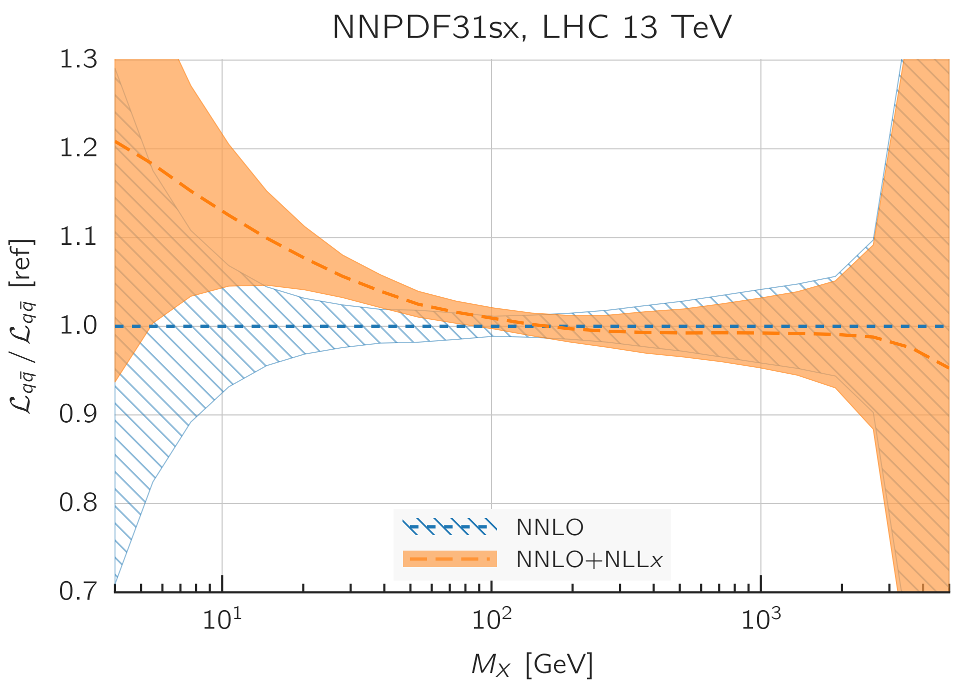
$$\alpha_s(Q^2)\ln\frac{1}{x} \geq D_{\rm cut}$$



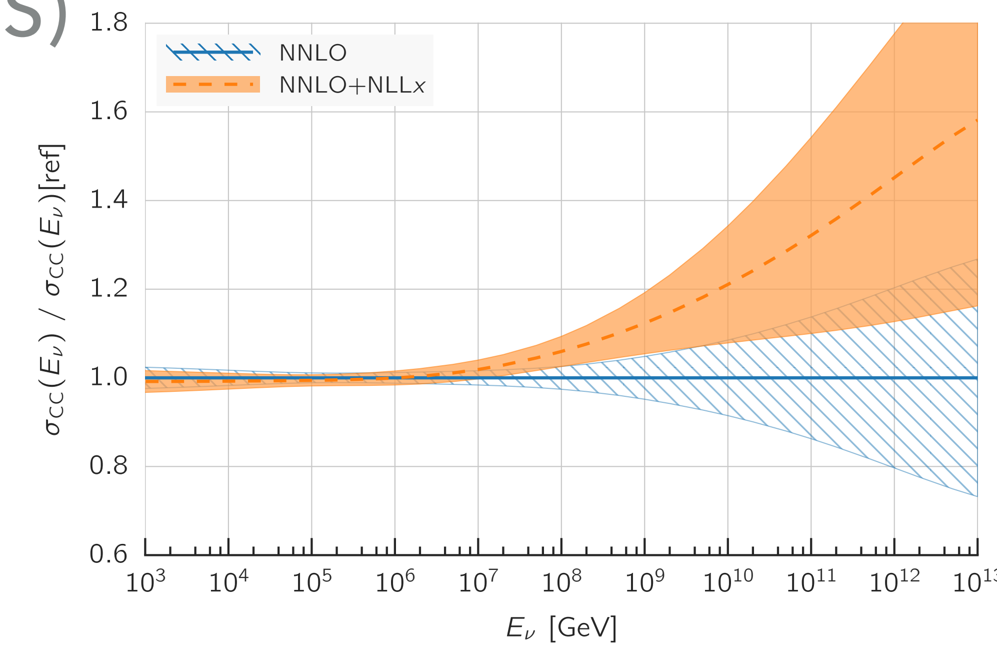


BFKL: THE GHOST OF CHRISTMAS PRESENT

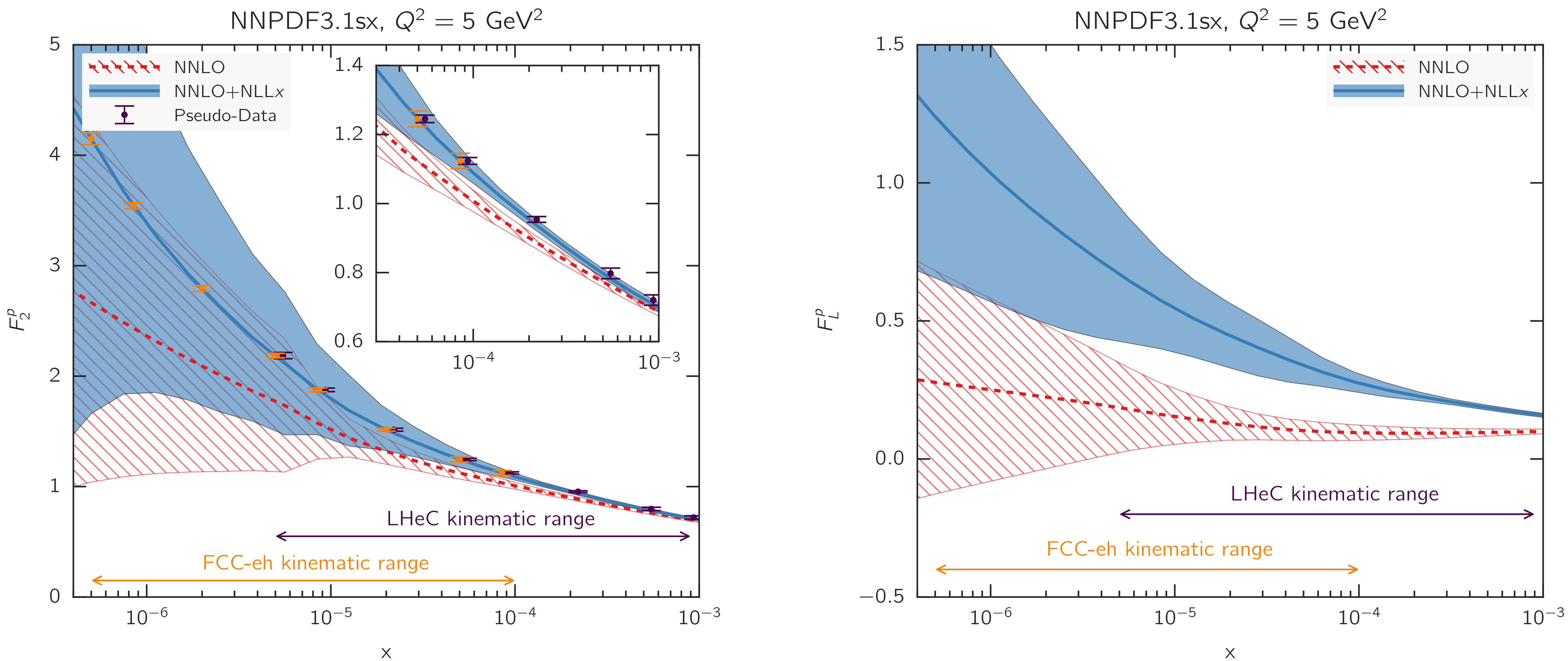




- to investigate LHC phenomenology we need resummed coefficient functions
- we can have a look at parton luminosities: qqbar doesn't change much but the change in gg is striking!
- consistent phenomenology for cosmic ray neutrinos (CC-DIS)
- unique "lab" for low-x physics



BFKL: THE GHOST OF CHRISTMAS YET-TO-COME



- small-x physics will be crucial at future circular colliders
- *e* (60 GeV) - *p* (7 TeV or 50 TeV) collisions
- to gauge the impact: fits including (resummed) pseudo-data

	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ NNLO	$\chi^2/N_{\rm dat}$ NNLO+NLLx	$\Delta\chi^2$
HERA I+II incl. NC	922	1.22	1.07	-138
LHeC incl. NC	148	1.71	1.22	-73
FCC-eh incl. NC	98	2.72	1.34	-135
Total	**1168**	**1.407**	**1.110**	**-346**

CONCLUSIONS & OUTLOOK

- Better determinations of PDFs require both data and theory
- resummation offers a complementary direction
- large-x resummed fits performed with restrict data set
- small-x resummed fit shows evidence of BFKL dynamics in HERa inclusive data
- towards truly global resummed fits:
  - Higgs and DY at small x are the next items on the agenda
  - then jets, both at large- and small-x
- Theory uncertainties ??? (I leave this one to real PDFs experts)

THANK YOU!

if we have seen further it is only by standing on the shoulders of giants