Resummation for transverse observables at hadron colliders

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Mainly based on 1604.02191 and 1705.09127
and ongoing work

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The quest for precision at the LHC

- The need for theory precision is twofold
    - As a precision machine, the LHC is providing us with %-accurate measurements of SM parameters/dynamics (couplings, PDFs, masses, …). A full exploitation of this data requires a deep understanding of the theory
    - Precision can allow for indirect constraints on new physics (NP) through mild distortions in kinematic distributions
        - Sensitivity is often improved by looking at exclusive regions of phase space where underlying QCD activity needs to be minimised (e.g. boosted kinematics, vetoes, …)
        - A careful assessment of the SM background is essential in most cases. This already reaches the few-percent level in some scenarios

Fixed-order vs. All-order

- Fixed-order calculations of radiative corrections are formulated in a well established way (technically very challenging, but well posed problem):
  - compute amplitudes at a given order
  - provide an effective subtraction of IRC divergences
  - compute any IRC-safe observable

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\rm Born}} \frac{d\sigma}{dv'} dv' \sim \overset{\text{LO}}{1} + \overset{\text{NLO}}{\alpha_s} + \overset{\text{NNLO}}{\alpha_s^2} + \dots$$

- All-order calculations are still at an earlier stage of evolution
  - Each different observable has its own type of sensitivity to IRC physics, it is hard to formulate a general method that works for all at a generic perturbative order
  - Higher-order resummations are therefore often formulated in an observable-dependent way, for few well-behaved collider observables

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\rm Born}} \frac{d\sigma}{dv'} dv' \underset{v \to 0}{\sim} e^{\overset{\text{LL}}{\alpha_s^n L^{n+1}} + \overset{\text{NLL}}{\alpha_s^n L^n} + \overset{\text{NNLL}}{\alpha_s^n L^{n-1}} + \dots}$$

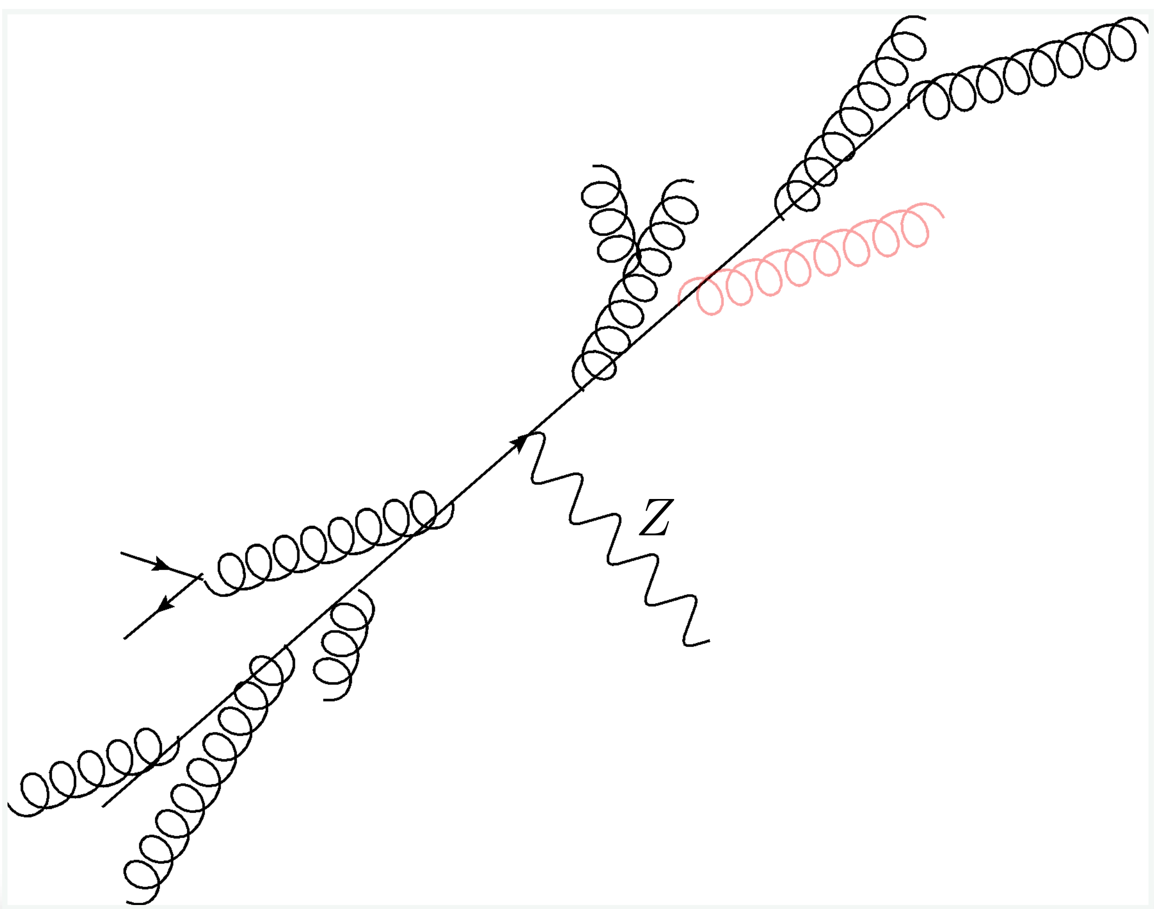
Path towards resummation: factorisation

- In the logarithmic regime, Born amplitudes receive radiative corrections from virtual diagrams (unconstrained), and soft/collinear real radiation

- The QCD amplitude (almost always) factorises in these kinematic limits

  - This is a necessary condition to formulate an all-order perturbative calculation (otherwise new structure would arise at each new order)

e.g. emission of a soft gluon



$$\mathcal{M}(k_1, k_2, \ldots, {\color{red}k}, \ldots, k_n)$$

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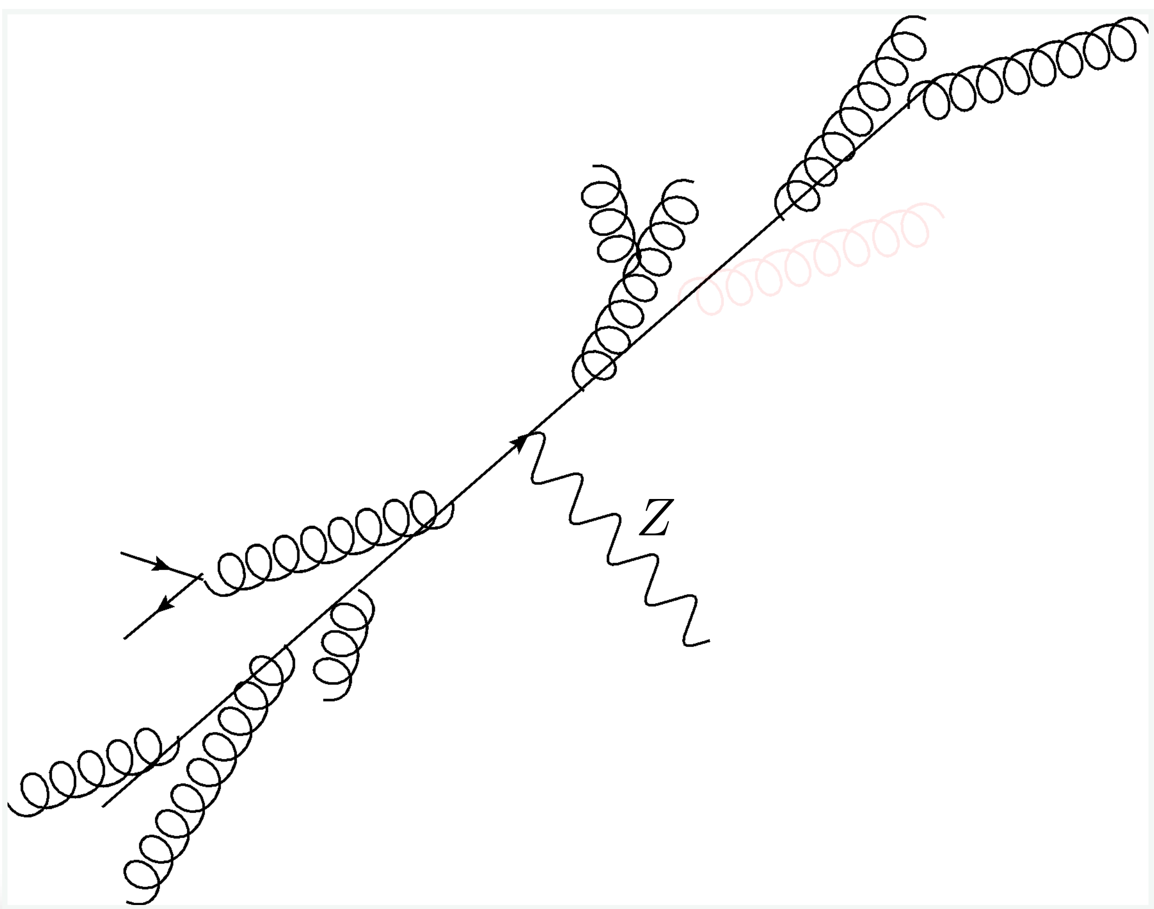


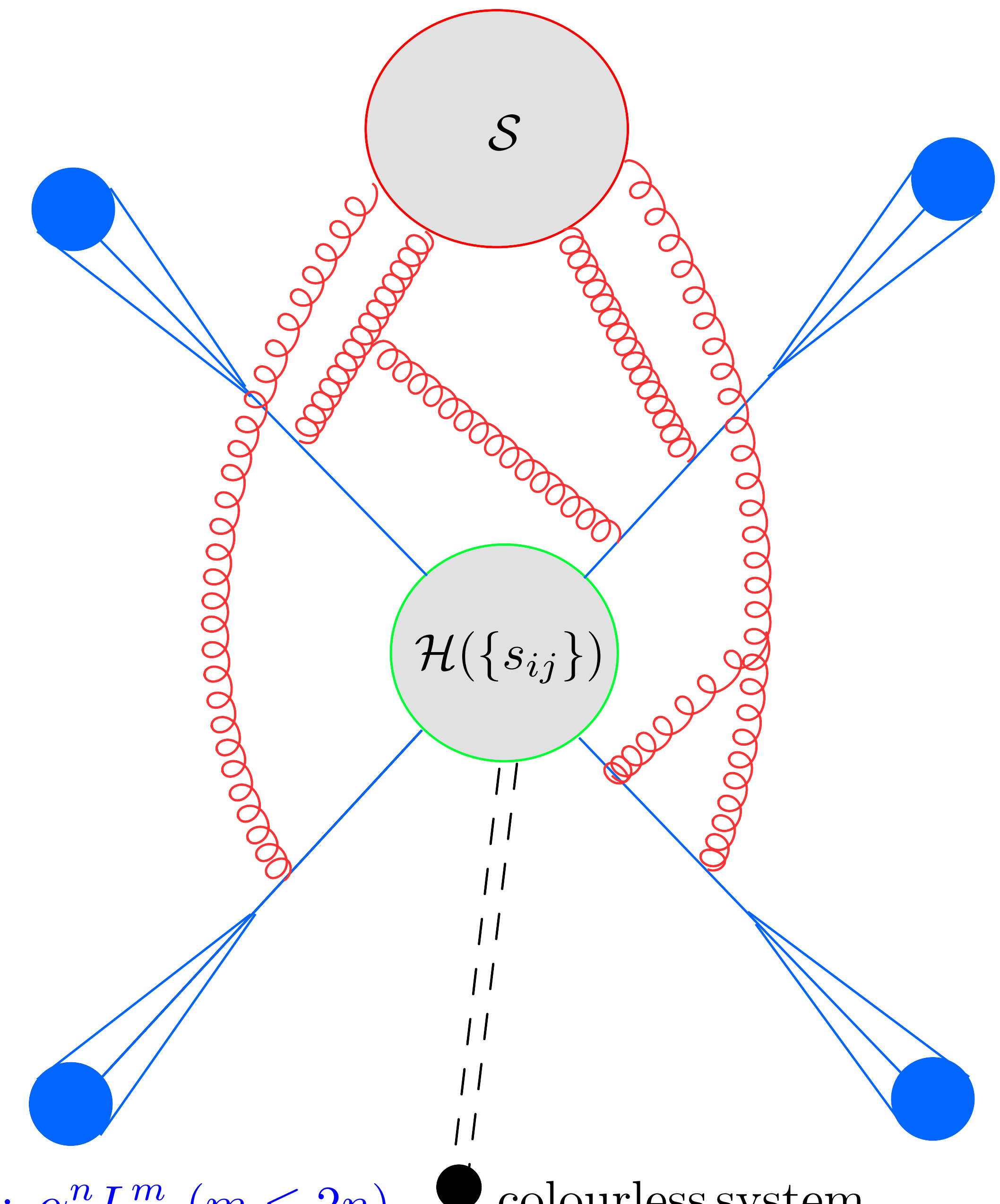
$$\mathcal{M}(k_1, k_2, \ldots, k, \ldots, k_n)$$
$$\simeq \mathcal{M}(k_1, k_2, \ldots\ldots, k_n)\mathcal{M}_{\mathrm{soft}}(k)$$

Factorisation of amplitudes in the IRC

- Consider a IRC observable $V = V(\{\tilde{p}\}, k_1, \ldots, k_n) \leq 1$ in the Born-like limit $V \to 0$

- In this limit radiative corrections are described exclusively by virtual corrections, and collinear and/or soft real emissions (singular limit) — QCD squared **amplitudes factorise** in these regimes w.r.t. the Born, up to regular corrections

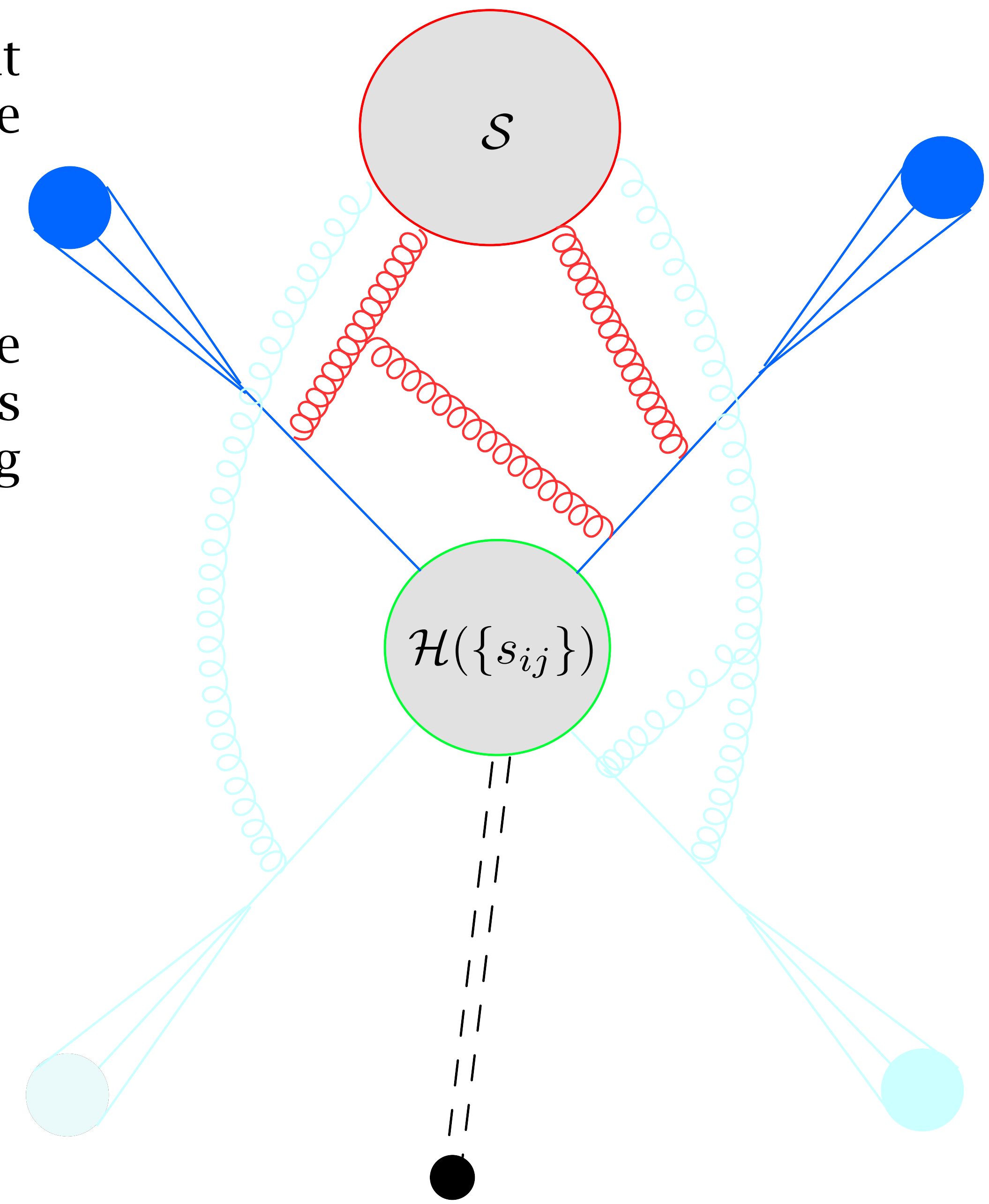
- Different observables are sensitive to different singular modes which determine the logarithmic structure of the perturbative expansion (e.g. (non) global, hard-collinear logarithms, …)



Two-emitter processes

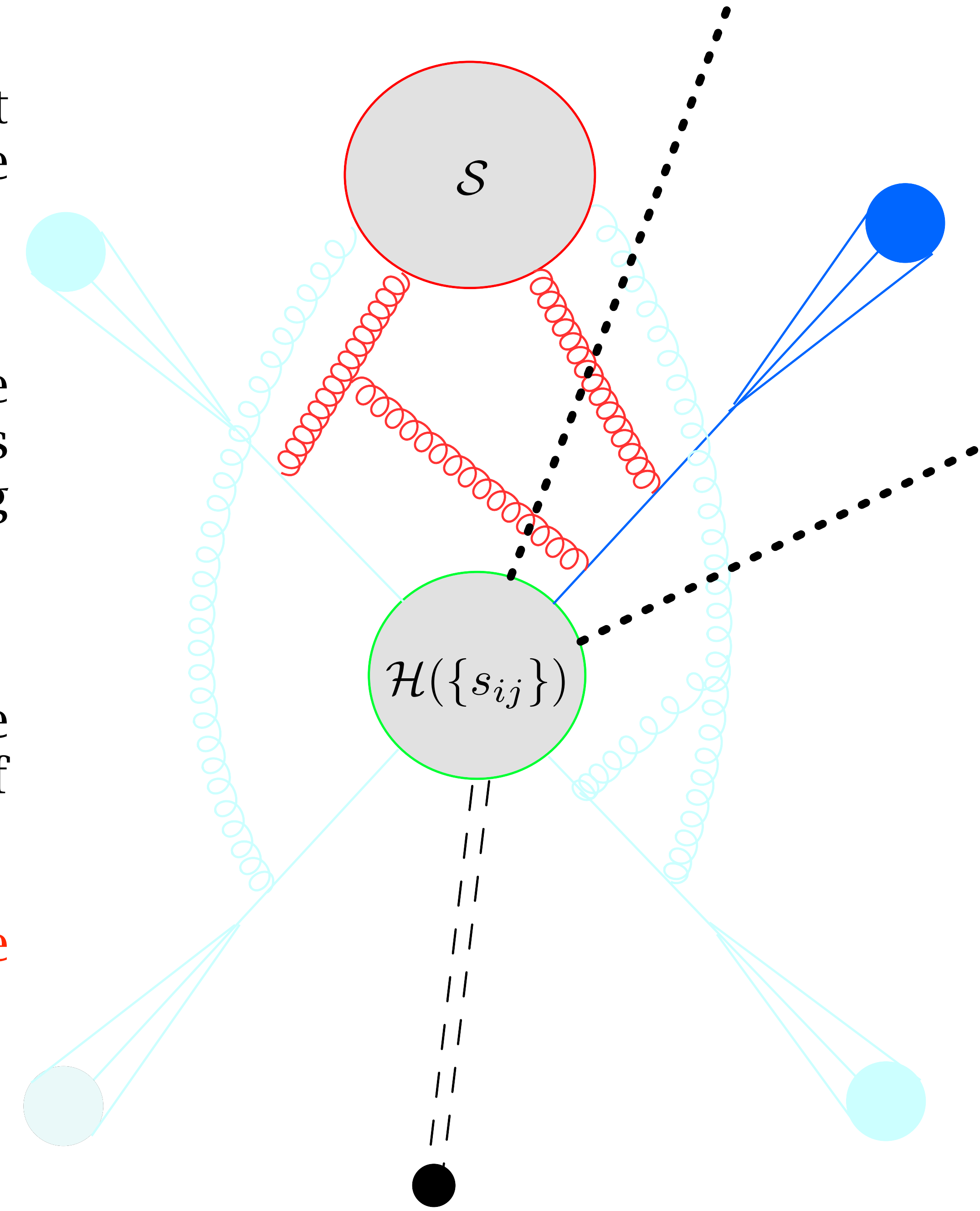
- The strong angular separation between different modes ensures they evolve independently at late times after the collision

- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs



Non-Global observables

- The strong angular separation between different modes ensures they evolve independently at late times after the collision
- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs
- For non-global observables one is always sensitive to the full evolution of the soft radiation outside of the resolved phase-space region
  - Both soft and collinear modes are present in the general case
  - Collinear modes can be absent



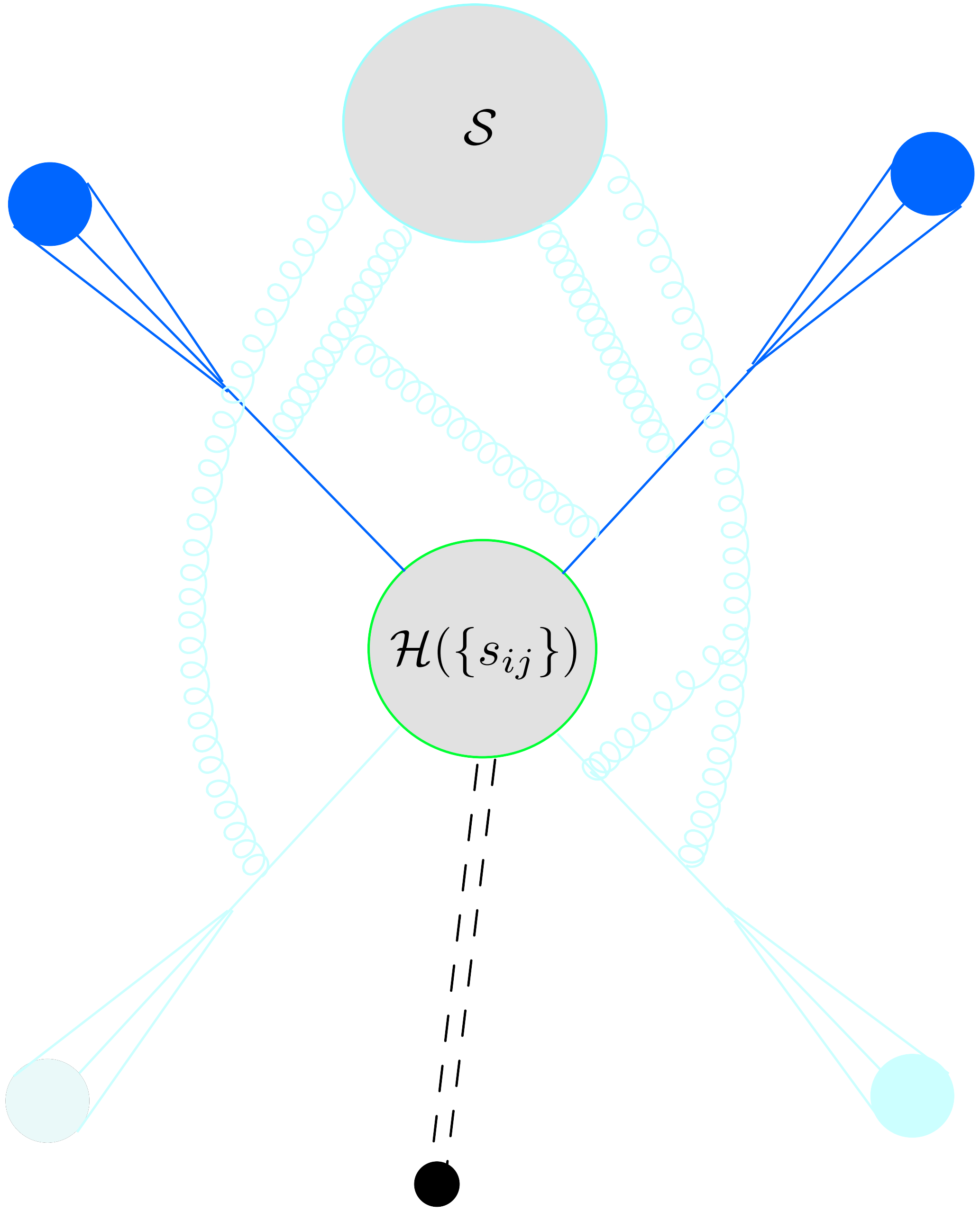
[Dasgupta, Salam ’01; Banfi, Marchesini, Smye ’02]
[Caron-Huot ’15-‘16; Larkoski, Moult, Neill ’15; Becher, Neubert, Rothen, Shao ’15-‘16]

Two-emitter processes

- The strong angular separation between different modes ensures they evolve independently at late times after the collision

- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs

- For continuously global observables in processes with two emitters, colour coherence forces the effect of soft modes exchanged with large angles to vanish
    - Only collinear (soft/hard) modes effectively remain
    - Soft modes can be absent in specific cases



Factorisation of the observable

- Factorisation of the amplitude is not enough as the all-order radiation is tangled by the observable

$$\Sigma(v) = \int d\Phi_{\rm rad} \sum_{n=0}^{\infty} |\mathcal{M}(k_1,\ldots,k_n)|^2 \Theta(v - V(k_1,\ldots,k_n))$$

- In order to perform an all-order calculation, one needs to ***break*** the observable too into hard, soft and collinear pieces. This can be done for some observables which treat the radiation rather inclusively
    - Resummation can be performed, e.g., by formulating a soft-collinear EFT of the singular modes (SCET)

[Bauer, Fleming, Pirjol, Stewart '01]
[Beneke, Chapovsky, Diehl, Feldmann '02]

e.g. the Thrust event shape $\tau \equiv 1 - T = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$

$$\Theta(Q^2\tau - \underbrace{\bar{k}^2}_{\bar{n}-\text{collinear}} - \underbrace{k^2}_{n-\text{collinear}} - \underbrace{wQ}_{\text{soft}}) = \frac{1}{2\pi i}\int_C \frac{d\nu}{\nu} \mathrm{e}^{\nu\tau Q^2} \mathrm{e}^{-\nu k^2} \mathrm{e}^{-\nu \bar{k}^2} \mathrm{e}^{-\nu w Q}$$

$$\Sigma(\tau) = |\mathcal{H}|^2 \frac{1}{2\pi i}\int_C \frac{d\nu}{\nu} e^{\nu\tau Q^2} S(wQ)\mathcal{J}_n(k^2)\mathcal{J}_{\bar{n}}(\bar{k}^2) + \mathcal{O}(\tau)$$

Eluding observable factorisation

- Factorisation is a powerful tool, but limited to observables that have a simple analytic expression in the relevant limits or do not mix soft and collinear radiation (e.g. jet rates)

- Ultimately, we want to use the modern knowledge of IRC dynamics to make more accurate generators. At present a general framework to assess the accuracy of Parton Showers is missing

  - It is of primary importance to formulate a link between higher-order resummation and PS

- Can we devise a formulation without a factorisation formula ?

  - ***recursive* IRC safety**: simple set of criteria for the observable that allows one to formulate the resummation at NLL for global observables without the need for an explicit factorisation.

    [Banfi, Salam, Zanderighi '01-'04]

  - Most of modern global observables fall into this category. Exceptions exist: e.g. rIRC unsafe observables (e.g. old JADE and Geneva algorithms), Sudakov-safe observables. No general structure beyond LL for these is known yet

  - The method can be reformulated and systematically extended at higher logarithmic orders

[Banfi, McAslan, PM, Zanderighi '14-'16]
[PM, Re, Torrielli '16]
[Bizon, PM, Re, Rottoli, Torrielli '17]

A case study: transverse observables

- Transverse observables in colour-singlet production offer a clean experimental and theoretical environment for precision physics:

$$\boxed{V(\{\tilde{p}\}, k) \equiv V(k) = d_\ell \, g_\ell(\phi) \left(\frac{k_t}{M}\right)^a}$$

  - SM measurements (e.g. W, Z, photon,…): parton distributions, strong coupling, W mass,…
  - BSM measurements/constraints (e.g. Higgs): light/heavy NP, Yukawa couplings,…

- Of this class, the family of inclusive observables probes directly the kinematics of the colour singlet:

$$\boxed{V(\{\tilde{p}\}, k_1, \ldots, k_n) = V(\{\tilde{p}\}, k_1 + \cdots + k_n)}$$

  - sensitive to non-perturbative effects (hadronisation, intrinsic kt) only through transverse recoil
  - very little/no sensitivity to multi-parton interactions
  - measured precisely at experiments

- Experimental uncertainty is already at the % level (or less) in some cases (e.g. Z production). Perturbation theory must be pushed to its limits

e.g. Z/H at small transverse momentum

- Study of small-pt region received a lot of attention in collider literature. Theoretically, it offers a clean environment to test/calibrate exclusive generators against more accurate predictions. Experimentally, shape is sensitive to light-quark Yukawa couplings

- Theoretically interesting observable. Two mechanisms compete in the $p_t \to 0$ limit
  - Sudakov (exponential) suppression when $k_{ti} \sim p_t$
  - Azimuthal cancellations (power suppression, dominant) when $k_{ti} \gg p_t$

- Standard solution obtained in impact-parameter space. Information on the radiation entirely lost

$$\delta^{(2)}(\vec{p_t} - (\vec{k}_{t1} + ... + \vec{k}_{tn})) = \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p_t}} \prod_{i=1}^{n} e^{i\vec{b}\cdot\vec{k}_{ti}},$$

$$\frac{d^2\Sigma(p_t)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\,db\,p_t J_0(p_t b)\,\mathbf{f}^T(b_0/b)\mathbf{C}^{c_1;T}_{N_1}(\alpha_S(b_0/b))H_{\rm CSS}(M)\mathbf{C}^{c_2}_{N_2}(\alpha_S(b_0/b))\mathbf{f}(b_0/b)$$
$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_{b_0/b}^{M}\frac{dk_t}{k_t}\mathbf{R}'_{{\rm CSS},\ell}(k_t)\right\}.$$

[Parisi, Petronzio '79]
[Collins et al. '85]
[Bozzi et al. '05]
[Becher et al. '10+'12]

- Coefficient functions and anomalous dimensions known up to N^{3}LL, except for four-loop cusp

[Catani, Grazzini '11][Catani et al. '12][Gehrmann, Luebbert, Yang '14][Davies, Stirling '84]
[De Florian, Grazzini '01][Becher, Neubert '10][Li, Zhu '16][Vladimirov '16]

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  - Su...

  - A...

- Stand... ...ely lost

Is it possible to obtain a more exclusive solution in momentum space ?

See also work in [Ebert, Tackmann ’16][Kang, Lee, Vaidya ’17]

$$\delta^{(2)}(p_t - (k_{t1} + \ldots + k_{tn})) = \int \frac{\cdots}{4\pi^2} e^{\cdots} \prod_{i=1} e^{\cdots},$$

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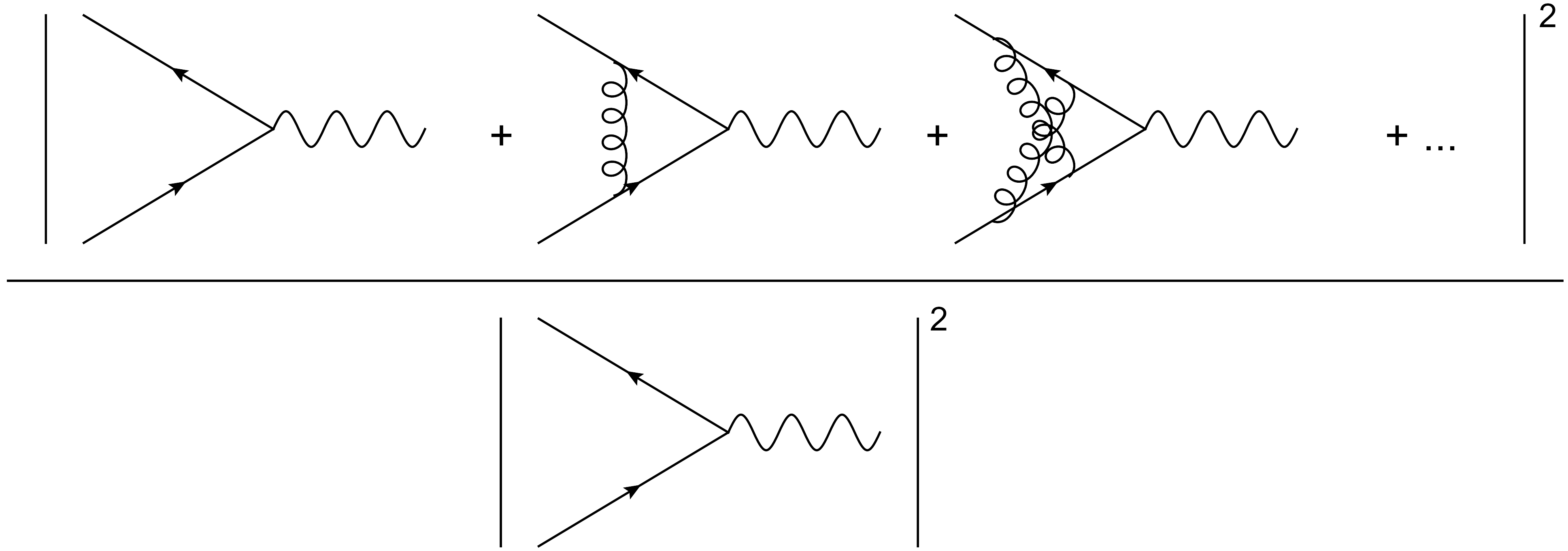
Direct space: virtual corrections

- Write all-order cross section as ($V(\{\tilde{p}\}, k_1, \ldots, k_n) = |\vec{k}_{t1} + \cdots + \vec{k}_{tn}|$)

$$\Sigma(v) = \int d\Phi_B \underline{\mathcal{V}(\Phi_B)} \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2 \, \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_n)\right)$$

All-order form factor

e.g. [Dixon, Magnea, Sterman '08]

$\mathcal{V}(\Phi_B) =$ 

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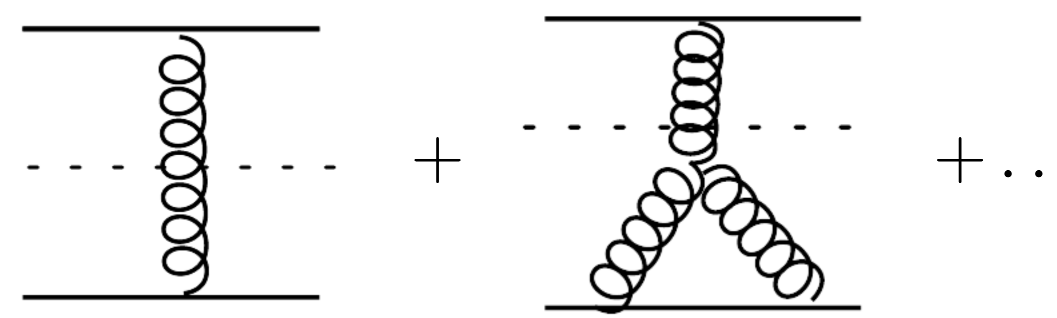
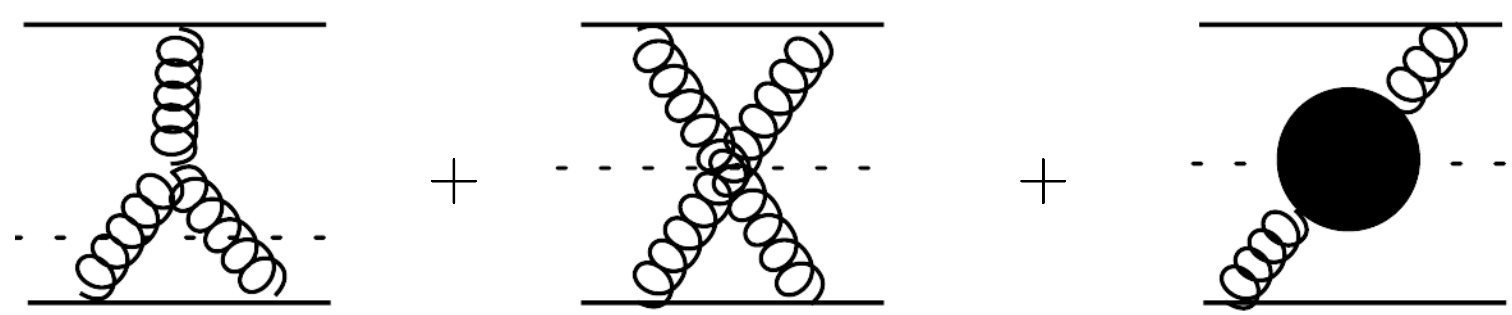
Real emissions

- Logarithmic counting: we need a logarithmic hierarchy in the squared amplitudes (resummation means iteration of lower-order amplitudes)

$$|\tilde{M}(k_a)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} = |M(k_a)|^2 \longrightarrow$$



$$|\tilde{M}(k_a, k_b)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a, k_b)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} - \frac{1}{2!}|M(k_a)|^2 |M(k_b)|^2 \longrightarrow$$



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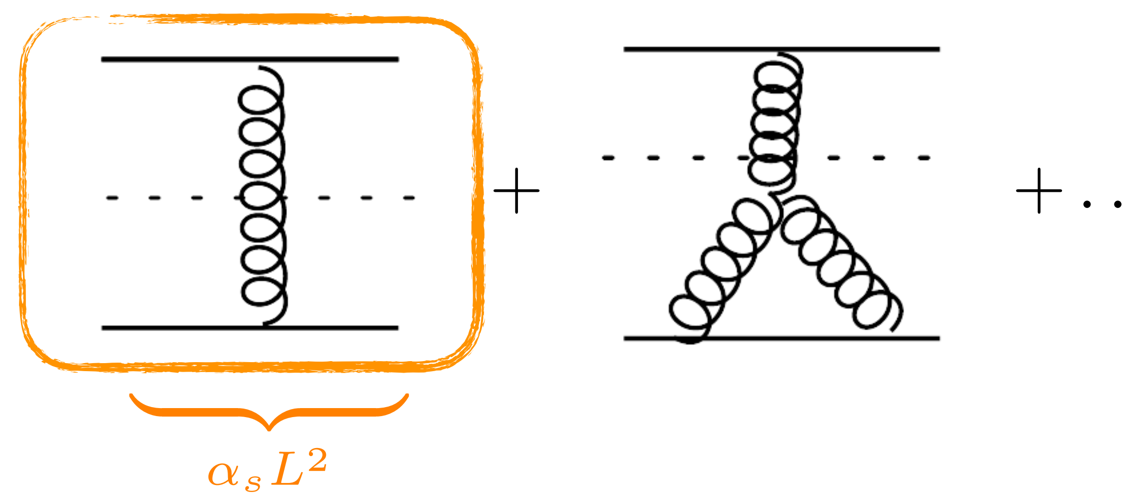

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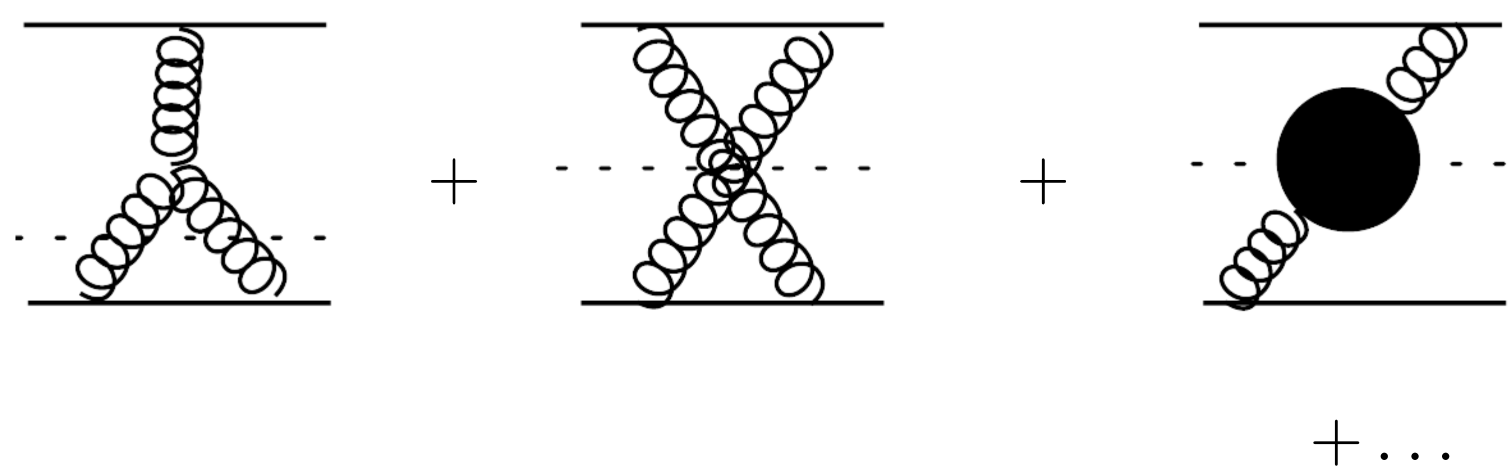

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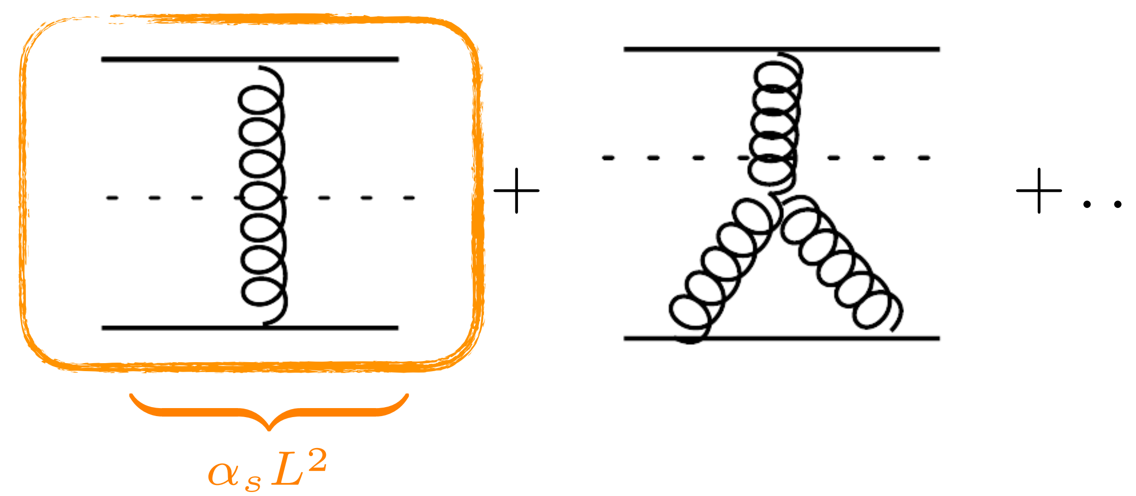

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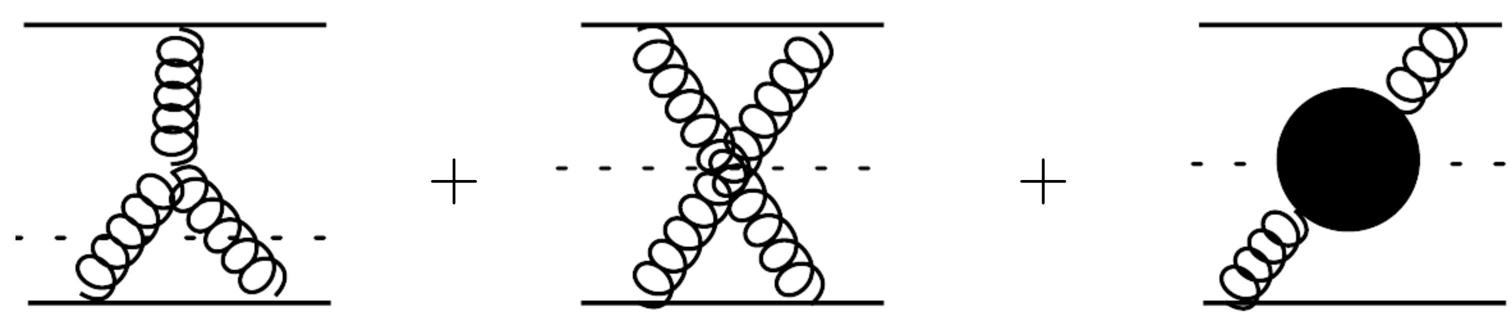

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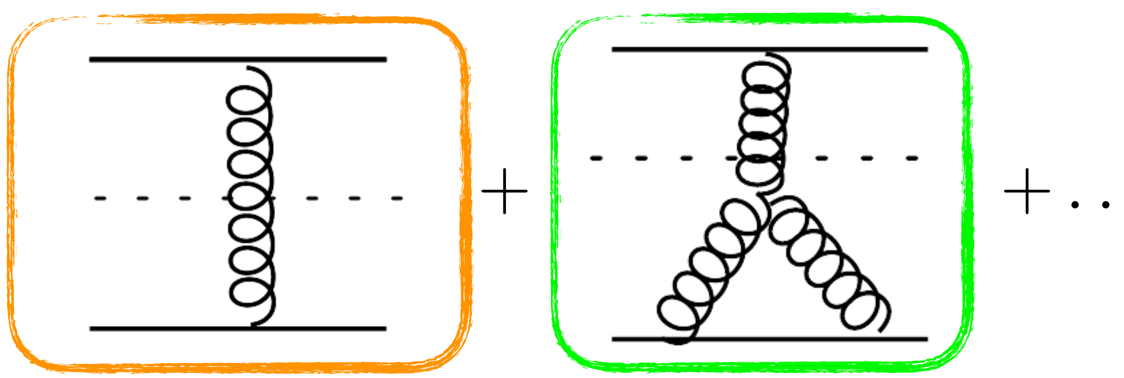

$\alpha_s L^2$

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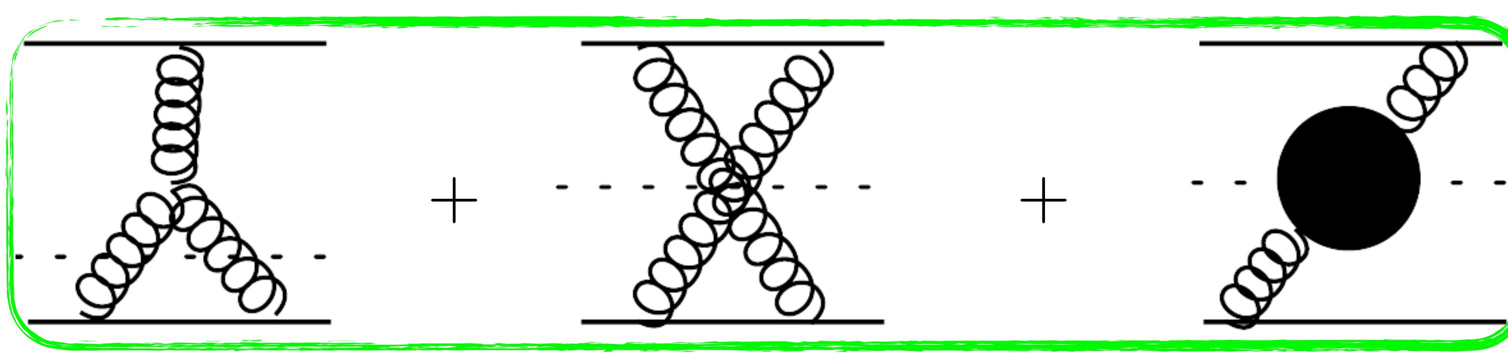

$\alpha_s^2 L^3 + \alpha_s^2 L^2$

$+ \dots$

this LL is absorbed in the resummation of $|M(k)|^2$

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- Recast all-order squared ME for *n* real emissions as iteration of *correlated blocks*

- Scaling of the observable in the presence of radiation *must* preserve the above hierarchy

e.g. soft radiation (analogous considerations for hard-collinear)

$$|M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2 = |M_B(\tilde{p}_1, \tilde{p}_2)|^2 \left\{ \left(\frac{1}{n!} \prod_{i=1}^{n} |M(k_i)|^2 \right) + \right.$$

$$\left[\sum_{a>b} \frac{1}{(n-2)!} \left(\prod_{\substack{i=1 \\ i \neq a,b}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b) \right|^2 + \right.$$

$$\left. \sum_{a>b} \sum_{\substack{c>d \\ c,d \neq a,b}} \frac{1}{(n-4)!2!} \left(\prod_{\substack{i=1 \\ i \neq a,b,c,d}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b) \right|^2 \left| \tilde{M}(k_c, k_d) \right|^2 + \ldots \right]$$

$$\left. + \left[\sum_{a>b>c} \frac{1}{(n-3)!} \left(\prod_{\substack{i=1 \\ i \neq a,b,c}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b, k_c) \right|^2 + \ldots \right] + \ldots \right\},$$

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$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] \underbrace{|M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2}_{\text{Real emissions}} \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_n)\right)$$

- Recast all-order squared ME for *n* real emissions as iteration of *correlated blocks*
- Scaling of the observable in the presence of radiation *must* preserve the above hierarchy

e.g. soft radiation (analogous considerations for hard-collinear)

$$|M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2 = |M_B(\tilde{p}_1, \tilde{p}_2)|^2 \left\{ \underbrace{\boxed{\left(\frac{1}{n!} \prod_{i=1}^{n} |M(k_i)|^2 \right)}}_{\text{LL}} + \right.$$

$$\left[\sum_{a>b} \underbrace{\boxed{\frac{1}{(n-2)!} \left(\prod_{\substack{i=1 \\ i \neq a,b}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b) \right|^2}}_{\text{NLL}} + \right.$$

$$\left. \sum_{a>b} \sum_{\substack{c>d \\ c,d \neq a,b}} \underbrace{\boxed{\frac{1}{(n-4)!2!} \left(\prod_{\substack{i=1 \\ i \neq a,b,c,d}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b) \right|^2 \left| \tilde{M}(k_c, k_d) \right|^2 + \dots}}_{\text{NLL}} \right]$$

$$\left. + \left[\sum_{a>b>c} \frac{1}{(n-3)!} \left(\prod_{\substack{i=1 \\ i \neq a,b,c}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b, k_c) \right|^2 + \dots \right] + \dots \right\},$$

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$$\left[\sum_{a>b} \underbrace{\frac{1}{(n-2)!} \left(\prod_{\substack{i=1 \\ i \neq a,b}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b) \right|^2}_{\text{NLL}} + \right.$$

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$$+ \left[\sum_{a>b>c} \underbrace{\frac{1}{(n-3)!} \left(\prod_{\substack{i=1 \\ i \neq a,b,c}}^{n} |M(k_i)|^2 \right) \left| \tilde{M}(k_a, k_b, k_c) \right|^2 + \ldots}_{\text{NNLL}} \right] + \ldots \Bigg\},$$

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These requirements can be translated into simple scaling properties for the observables, known as recursive IRC safety

All-order subtraction of IRC singularities

- Subtraction of the IRC poles between $\sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2$ and $\mathcal{V}(\Phi_B)$:
  - introduce a phase-space resolution scale (slicing parameter) $Q_0 = \epsilon k_{t1}$
  - real correlated blocks with total transverse momentum $k_{ti} < \epsilon k_{t1}$(unresolved) do not modify the observable, and can be ***ignored*** in the measurement function
  - compute *unresolved* reals and *virtuals* analytically in D dimensions (*much* easier than full observable)

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$$\sum_{n=0}^{\infty} |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2 \longrightarrow |M_B(\tilde{p}_1, \tilde{p}_2)|^2$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|M(k_i)|^2 + \int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$

$$\left.\left. + \int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

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$$\prod_{i=1}^{n} \int [dk_i] \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|M(k_i)|^2 + \int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right. \right.$$
$$\left. \left. + \int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \Theta(\epsilon k_{t1} - k_{ti}) \right\}$$

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$$\prod_{i=1}^{n}\int[dk_i]\mathcal{V}(\Phi_B)\sum_{n=0}^{\infty}\frac{1}{n!}\left\{\prod_{i=1}^{n}\left(|M(k_i)|^2+\int[dk_a][dk_b]|\tilde{M}(k_a,k_b)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}-\vec{k}_{ti})\delta(Y_{ab}-Y_i)\right.\right.$$
$$\left.\left.+\int[dk_a][dk_b][dk_c]|\tilde{M}(k_a,k_b,k_c)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}+\vec{k}_{tc}-\vec{k}_{ti})\delta(Y_{abc}-Y_i)+\dots\right)\Theta(\epsilon k_{t1}-k_{ti})\right\}$$
$$\propto\int\frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(\epsilon k_{t1})}R'(k_{t1})$$

All-order subtraction of IRC singularities

- Subtraction of the IRC poles between $\sum_{n=0}^{\infty}\int\prod_{i=1}^{n}[dk_i]|M(\tilde{p}_1,\tilde{p}_2,k_1,\ldots,k_n)|^2$ and $\mathcal{V}(\Phi_B)$:
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  - compute *unresolved* reals and *virtuals* analytically in D dimensions (*much* easier than full observable)

$$R(\epsilon k_{t1}) \equiv \sum_{\ell=1}^{2}\int_{\epsilon k_{t1}}^{M}\frac{dk_t}{k_t}R'_\ell\,(k_t) = \sum_{\ell=1}^{2}\int_{\epsilon k_{t1}}^{M}\frac{dk_t}{k_t}\left(A_\ell(\alpha_s(k_t))\ln\frac{M^2}{k_t^2} + B_\ell(\alpha_s(k_t))\right)$$

Anomalous dimensions start differing from b-space ones at N³LL

$$\prod_{i=1}^{n}\int[dk_i]\mathcal{V}(\Phi_B)\sum_{n=0}^{\infty}\frac{1}{n!}\left\{\prod_{i=1}^{n}\left(|M(k_i)|^2 + \int[dk_a][dk_b]|\tilde{M}(k_a,k_b)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}-\vec{k}_{ti})\delta(Y_{ab}-Y_i)\right.\right.$$
$$\left.\left. + \int[dk_a][dk_b][dk_c]|\tilde{M}(k_a,k_b,k_c)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}+\vec{k}_{tc}-\vec{k}_{ti})\delta(Y_{abc}-Y_i)+\ldots\right)\Theta(\epsilon k_{t1}-k_{ti})\right\}$$

$$\propto \int\frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(\epsilon k_{t1})}R'(k_{t1})$$

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- Subtraction of the IRC poles between $\sum_{n=0}^{\infty}\int\prod_{i=1}^{n}[dk_i]|M(\tilde{p}_1,\tilde{p}_2,k_1,\ldots,k_n)|^2$ and $\mathcal{V}(\Phi_B)$:
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  - compute *unresolved* reals and *virtuals* analytically in D dimensions (*much* easier than full observable)

$$\hat{\boldsymbol{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)=\boxed{\begin{aligned}&\left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M\frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}\\&\times e^{-\mathbf{R}(\epsilon k_{t1})}\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t))+\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}\end{aligned}}$$

DGLAP anomalous dims

RGE evolution of coeff. functions

Sudakov radiator: integral of single inclusive block.

All-order subtraction of IRC singularities

- Subtraction of the IRC poles between $\sum_{n=0}^{\infty}\int\prod_{i=1}^{n}[dk_i]|M(\tilde{p}_1,\tilde{p}_2,k_1,\dots,k_n)|^2$ and $\mathcal{V}(\Phi_B)$:
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$$\times e^{-\mathbf{R}(\epsilon k_{t1})}\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t))+\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

DGLAP anomalous dims

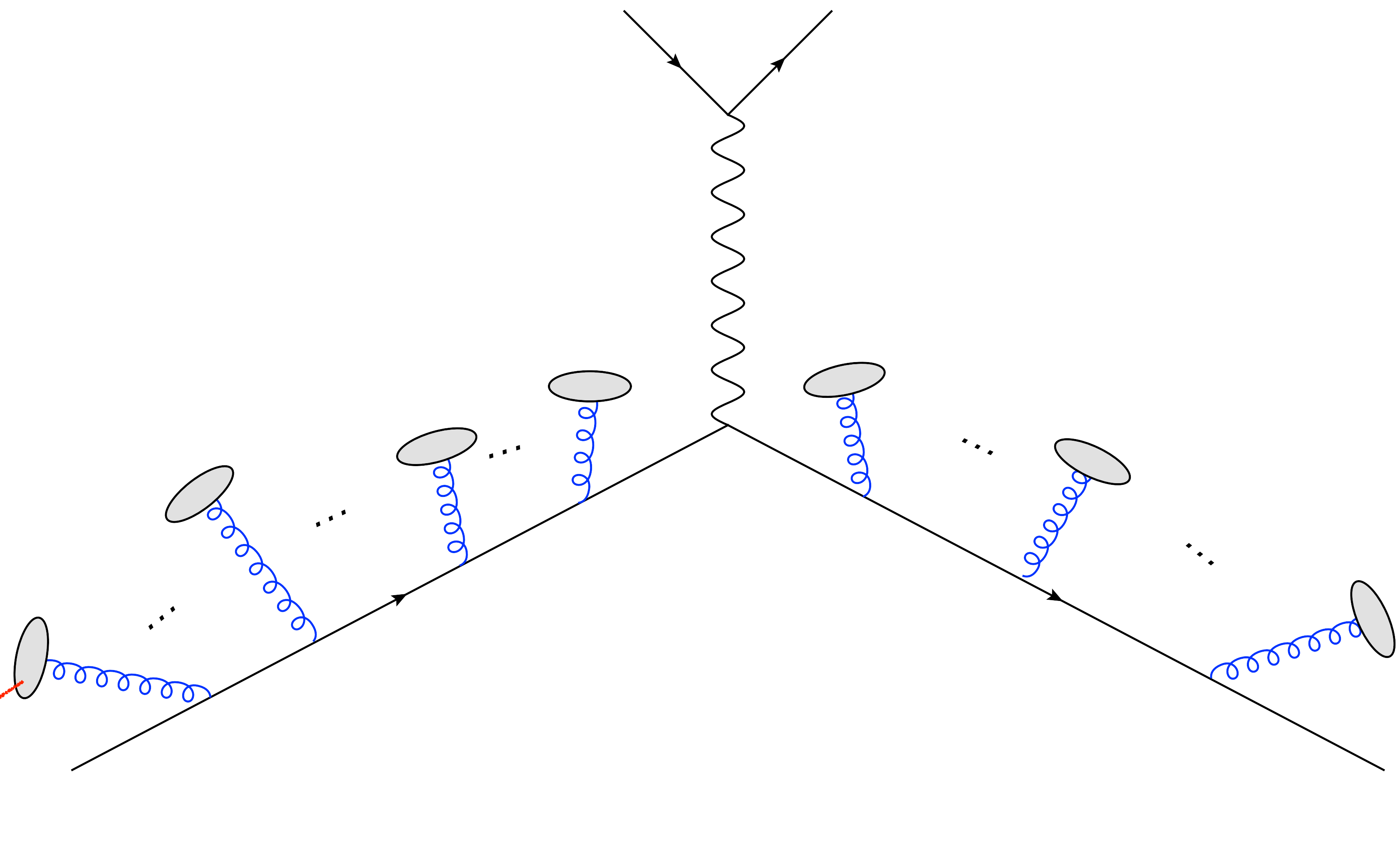
RGE evolution of coeff. functions

$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1})+\frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1}))+\mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$
$$\times\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti})+\frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti}))+\mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$
$$\times\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right),$$

- compute *resolved* (reals only) in 4 dim. with $\epsilon \to 0$ (MC events !)

Monte Carlo formulation

- This is, essentially, a *non-fully-exclusive parton shower* with higher logarithmic accuracy



$$|M(k_i)|^2 + \int [dk_a][dk_b]|\tilde{M}(k_a,k_b)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}-\vec{k}_{ti})\delta(Y_{ab}-Y_i)$$
$$+\int [dk_a][dk_b][dk_c]|\tilde{M}(k_a,k_b,k_c)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}+\vec{k}_{tc}-\vec{k}_{ti})\delta(Y_{abc}-Y_i)+\dots$$

Monte Carlo formulation

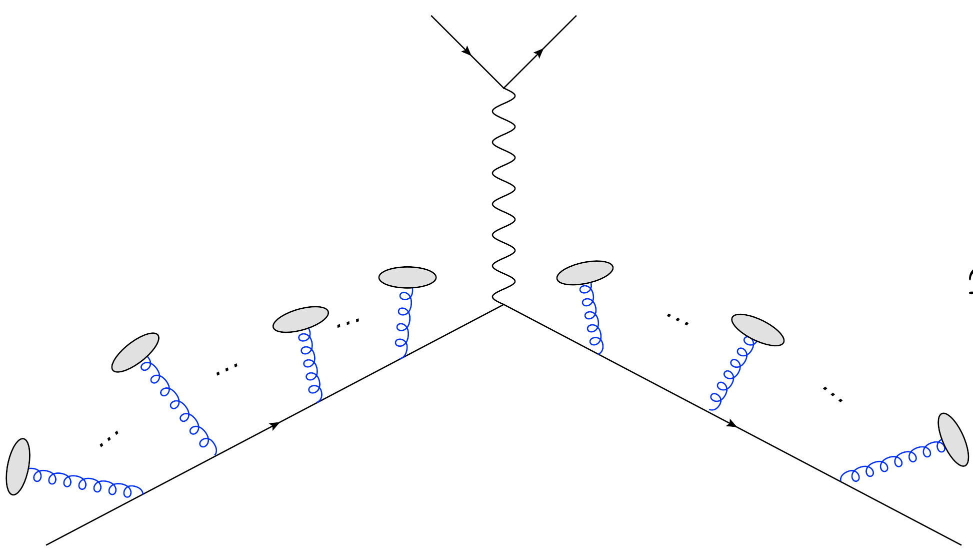
- One great simplification: choice of the resolution variable such that correlated blocks entering at N^{k}LL in the unresolved radiation only contribute at N^{k+1}LL in the resolved case
- i.e. we can expand out the cutoff dependence and retain in the Radiator only the terms necessary to cancel the singularities in the resolved radiation

$$R(\epsilon k_{t1}) = R(k_{t1}) + R'(k_{t1}) \ln \frac{1}{\epsilon} + \frac{1}{2} R''(k_{t1}) \ln^2 \frac{1}{\epsilon} + \ldots$$

$$R'(k_{ti}) = R'(k_{t1}) + R''(k_{t1}) \ln \frac{k_{t1}}{k_{ti}} + \ldots$$

Expansion is safe since in the resolved radiation $k_{t1}/k_{ti} \sim 1$

e.g. at NLL



$$\simeq \int \frac{dk_{t1}}{k_{t1}} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}(k_{t1}) \right) \int d\mathcal{Z}[\{R'(k_{t1}), k_i\}] \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}))$$

$$\int d\mathcal{Z}[\{R'(k_{t1}), k_i\}] = \epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1})$$

Monte Carlo formulation

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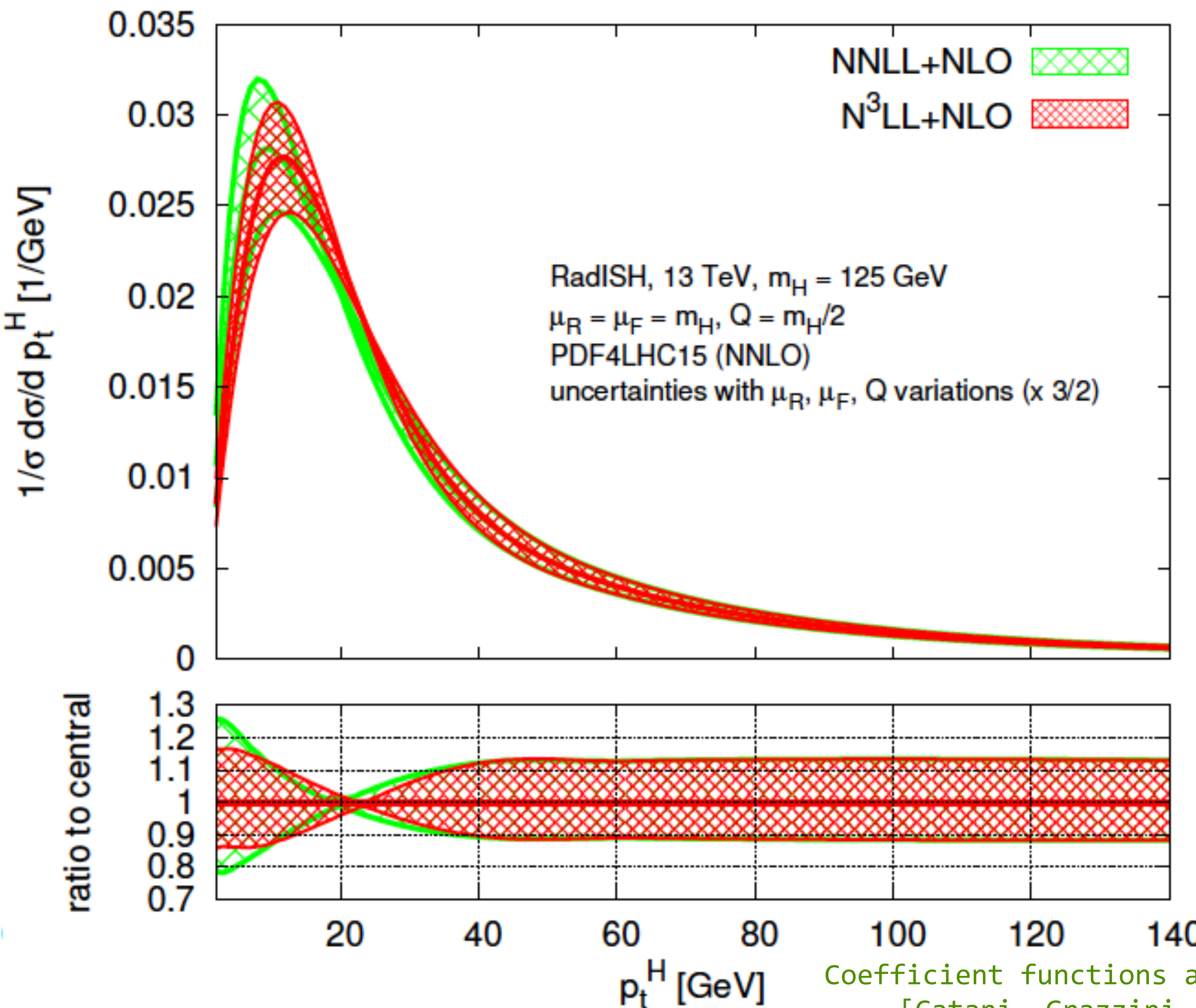
Expansion is safe since in the resolved radiation $k_{t1}/k_{ti} \sim 1$

- Corrections beyond NLL are obtained as follows
  - Add subleading effects in the Sudakov radiator and constants
  - Correct *a fixed number* of the NLL resolved emissions:
    - only one at NNLL
    - two at $\mathrm{N^3LL}$
    - …

An example: Higgs pT at N³LL+NNLO

- Implementation in a MC code (RadISH) up to N³LL



Coefficient functions and anomalous dimensions from:
[Catani, Grazzini '11; Gehrmann et al. '14]
[de Florian, Grazzini '01] [Becher, Neubert '10] [Li, Zhu '16]

An example: Higgs pT at N³LL+NNLO

- Implementation in a MC code (RadISH) up to N³LL

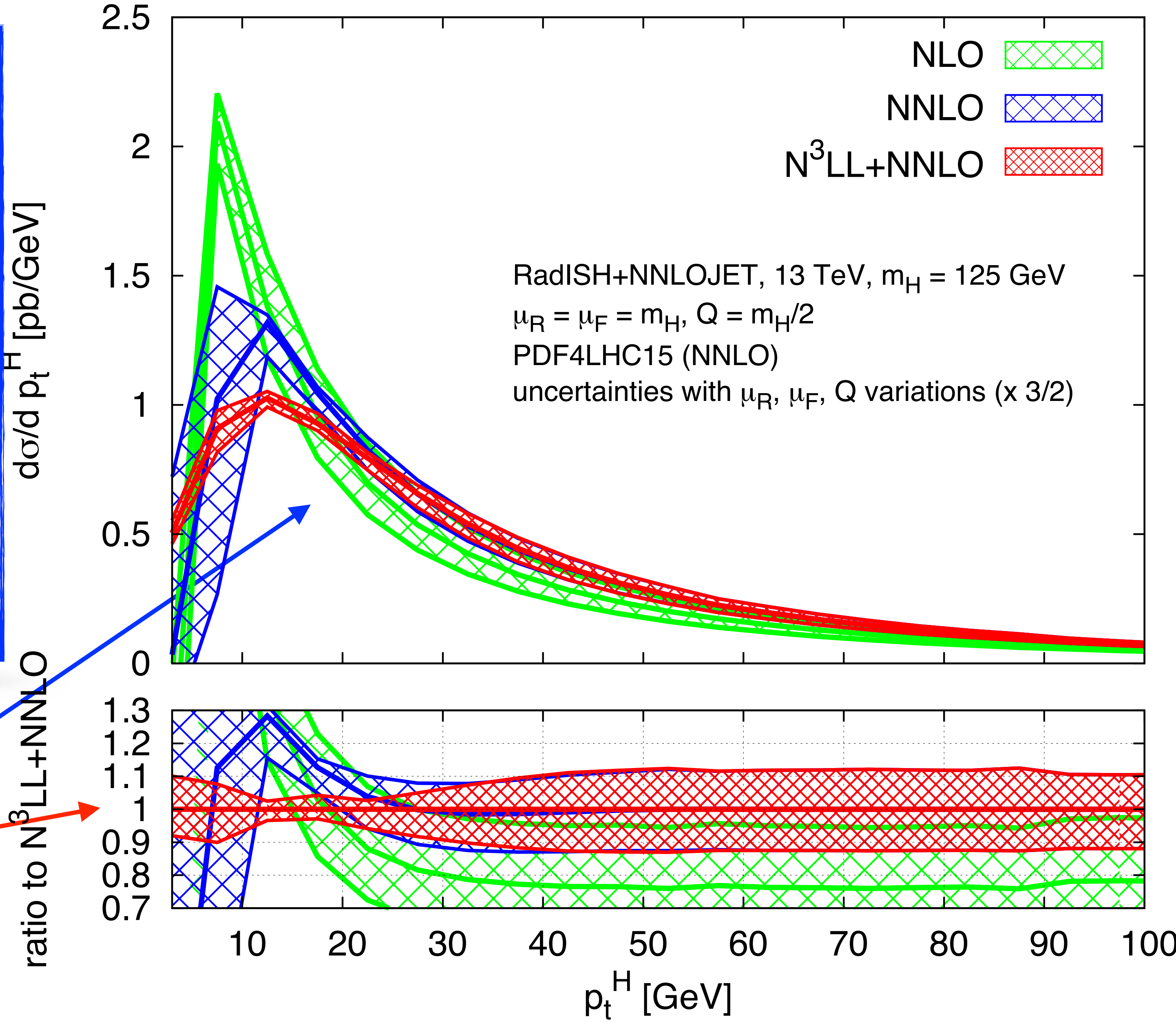
- Matching of the integrated distribution to N³LO via a multiplicative matching, i.e.

[Anastasiou et al. '15-'16] [Boughezal et al. '15]
[Caola et al. '15]
[Chen et al. '16]

$$\sigma^{\mathrm{N^3LO}}_{pp\to H} - \Sigma^{\mathrm{NNLO}}_{1-\mathrm{jet}}(p_t^H)$$

$$\Sigma_{\mathrm{MAT}}(p_t,\Phi_B) = (\Sigma_{\mathrm{RES}}(p_t,\Phi_B))^Z \frac{\Sigma_{\mathrm{FO}}(p_t,\Phi_B)}{(\Sigma_{\mathrm{EXP}}(p_t,\Phi_B))^Z}$$

$$Z = \left(1 - \left(\frac{p_t}{Q_{\mathrm{match}}}\right)\right)^h \Theta(Q_{\mathrm{match}} - p_t)$$



- Deviations from NNLO below 30 GeV

- Scale unc. ~10% down to very small transverse momentum

Small transverse momentum limit

- CSS result recovered by simply transforming observable into b-space

- Clear physical picture of the dynamics of azimuthal cancellations at small transverse momentum

e.g. NLL with $\mathcal{L}(\mathrm{k}_{t1}) = 1$ for simplicity

$$\frac{d^2\Sigma(v)}{d^2\vec{p}_t d\Phi_B} = \sigma^{(0)}(\Phi_B) \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} R'(k_{t1}) \int d\mathcal{Z}[\{R', k_i\}] \delta^{(2)} \left(\vec{p}_t - \left(\vec{k}_{t1} + \cdots + \vec{k}_{t(n+1)}\right)\right)$$

- Transition from exponential to a power-like suppression at small transverse momentum

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 4\sigma^{(0)}(\Phi_B)\, p_t \int_{\Lambda_{\rm QCD}}^{M} \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B) p_t \left(\frac{\Lambda_{\rm QCD}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

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as $\mathrm{p}_t \to 0$ Sudakov is "frozen" at $\mathrm{k}_{t1} \gg \mathrm{p}_t$
(no exponential suppression)

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$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 4\sigma^{(0)}(\Phi_B)\, p_t \int_{\Lambda_{\mathrm{QCD}}}^{M} \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B) p_t \left(\frac{\Lambda_{\mathrm{QCD}}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

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$$\frac{d^2\Sigma(v)}{d^2\vec{p}_t d\Phi_B} = \sigma^{(0)}(\Phi_B)\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\boxed{e^{-R(k_{t1})}R'(k_{t1})}\int \boxed{d\mathcal{Z}[\{R',k_i\}]\delta^{(2)}\left(\vec{p}_t - \left(\vec{k}_{t1} + \cdots + \vec{k}_{t(n+1)}\right)\right)}$$

as $\mathrm{p_t} \to 0$ Sudakov is "frozen" at $\mathrm{k_{t1}} \gg \mathrm{p_t}$
(no exponential suppression)

Random azimuthal orientation of momenta leads to scaling $\propto \mathrm{p_t}/\mathrm{k}_{\mathrm{t1}}^2$

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$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 4\sigma^{(0)}(\Phi_B)\,p_t\int_{\Lambda_{\mathrm{QCD}}}^{M}\frac{dk_{t1}}{k_{t1}^3}e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B)p_t\left(\frac{\Lambda_{\mathrm{QCD}}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

Generalisation to other observables

- Extension to non-inclusive observables:
  - Although the resummed formula obtained here is valid for inclusive observables, the Sudakov radiator is universal for all observables which feature the same scaling for a single, soft-collinear emission, i.e. the same LL structure

$$V_{\rm sc}(\{\tilde{p}\}, k) = \left(\frac{k_t}{M}\right)^a$$

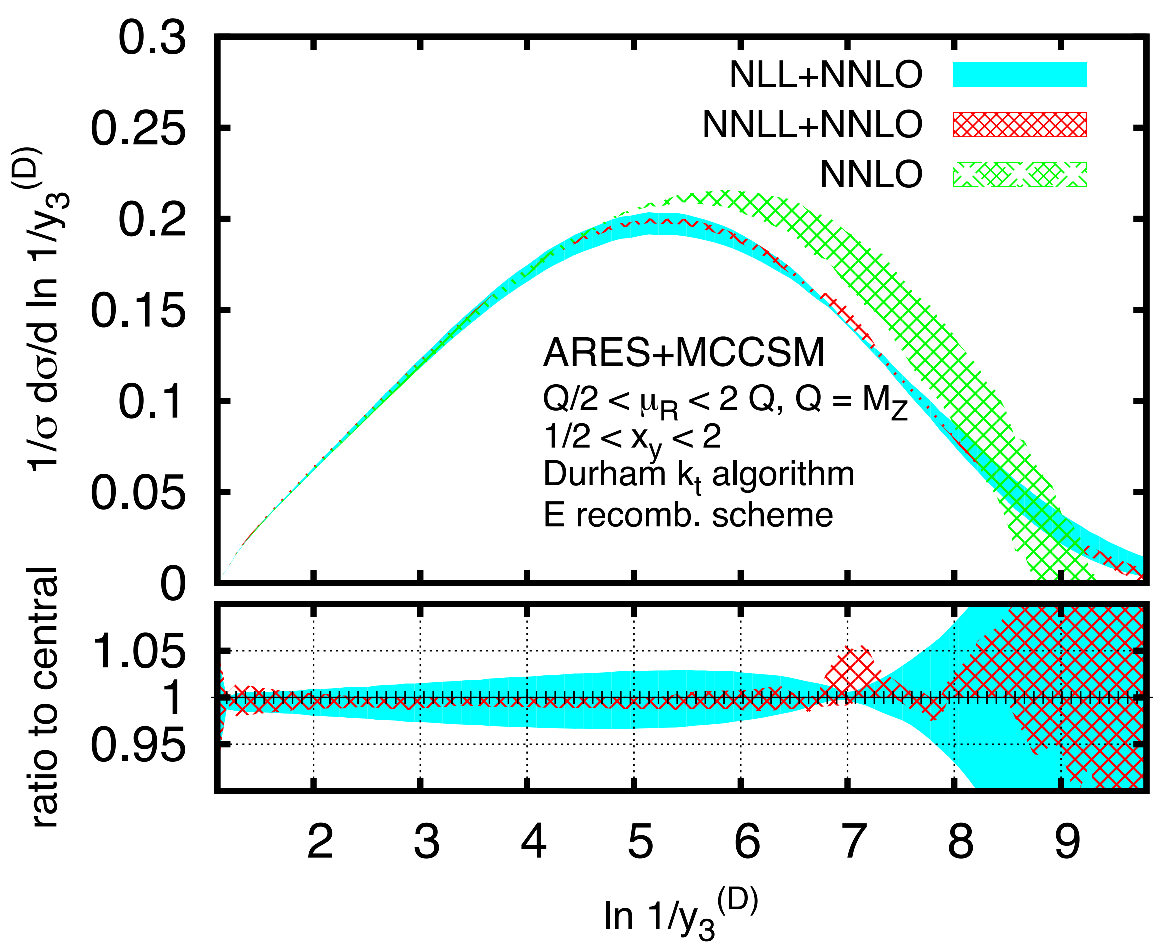
  - The exclusive treatment of resolved correlated blocks (n>1) is simplified by noticing that only a finite number of them must be included in the resolved radiation beyond NLL

[Banfi, PM, Salam, Zanderighi '12]
[Banfi, McAslan, PM, Zanderighi '14-'16]

- This leads to a general algorithm for all rIRC observables:

  e.g. k_t-algorithm 2-jet rate in e^+e^- at NNLL+NNLO

➡ Small residual perturbative uncertainty, and reduced sensitivity to hadronisation can be used for an extraction of the strong coupling



Generalisation to other observables

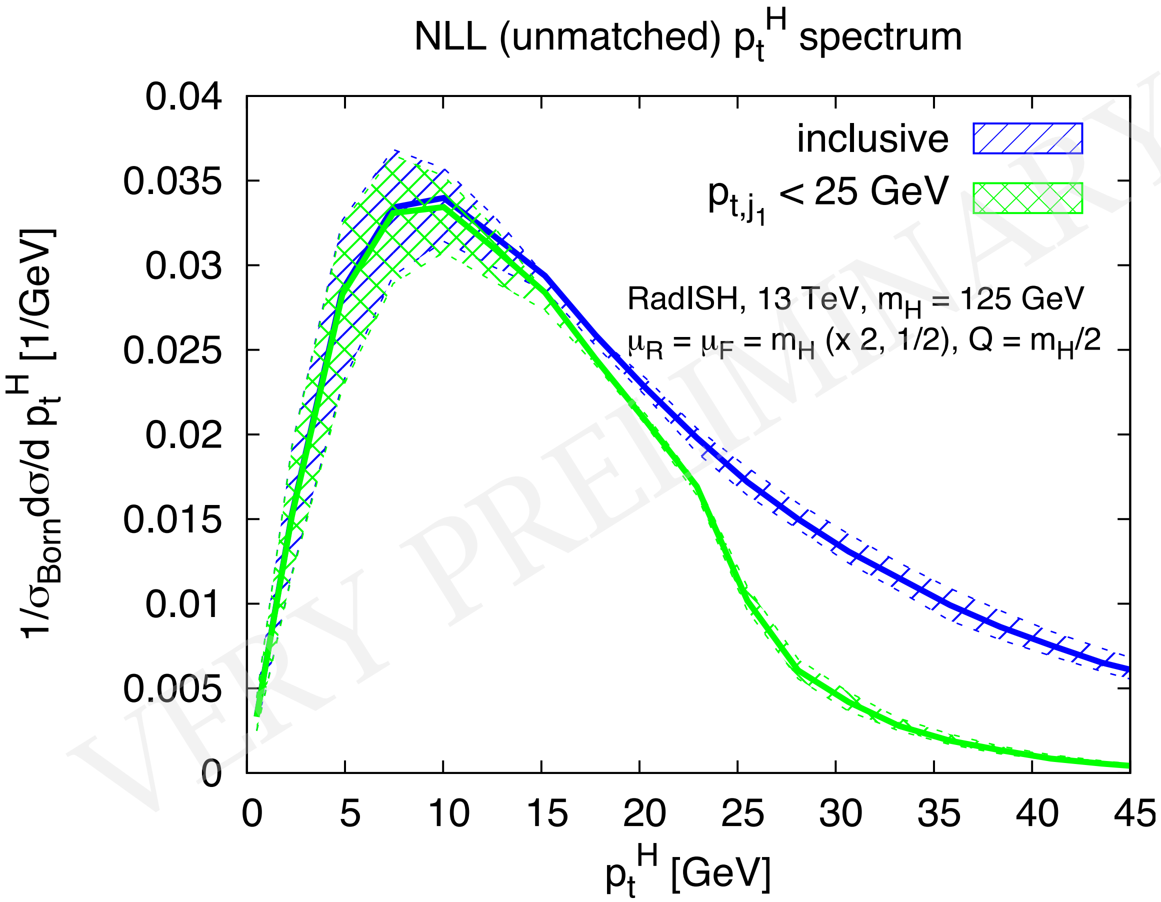
- Extension to non-inclusive observables:
  - Although the resummed formula obtained here is valid for inclusive observables, the Sudakov radiator is universal for all observables which feature the same scaling for a single, soft-collinear emission, i.e. the same LL structure

$$V_{\rm sc}(\{\tilde{p}\}, k) = \left(\frac{k_t}{M}\right)^a$$

  - The exclusive treatment of resolved correlated blocks (n>1) is simplified by noticing that only a finite number of them must be included in the resolved radiation beyond NLL

[Banfi, PM, Salam, Zanderighi '12]
[Banfi, McAslan, PM, Zanderighi '14-'16]

- Multi-differential cross sections:
  - Not being fully inclusive in the radiation allows one to have more exclusive cuts. The logarithmic accuracy can be easily spoiled (a lot of care is required!)
  - This makes it possible to access exclusive cross sections with higher logarithmic order (?)



Conclusions

- Higher-order resummation can be formulated directly in momentum space without the need for a factorisation for the considered observable
- Currently, two-scale problems in two-emitter processes are solved for all rIRC safe cases
  - Systematic extension to any logarithmic order
  - Efficient implementation in a computer code: automation possible
  - Connection between analytic resummation and parton showers one step closer
- Future directions
  - This method is not bound to the resummation in full QCD: formulation in SCET framework possible
  - Extension to processes with more than two legs requires a differential formulation of the soft resolved radiation with wide angles
  - Applicability to NG problems under study

Thank you for listening

Numerical implementation: RadISH

- Since the transverse momenta of the *resolved* reals are of the same order, we can expand the whole integrand about $k_{ti} \sim k_{t1}$ up to the desired logarithmic accuracy
- This expansion allows us to compute higher-order corrections to the NLL *resolved* reals by simply including one correction at a time

e.g. expansion up to NLL

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}}(k_{t1}) \right) \int d\mathcal{Z}[\{R', k_i\}] \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{N^3LL}}(k_{t1}) = & \sum_{c,c'} \frac{d|M_B|^2_{cc'}}{d\Phi_B} \sum_{i,j} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} f_i\left(k_{t1}, \frac{x_1}{z_1}\right) f_j\left(k_{t1}, \frac{x_2}{z_2}\right) \\
& \Bigg\{ \delta_{ci}\delta_{c'j}\delta(1-z_1)\delta(1-z_2) \left(1 + \frac{\alpha_s(\mu_R)}{2\pi} H^{(1)}(\mu_R) + \frac{\alpha_s^2(\mu_R)}{(2\pi)^2} H^{(2)}(\mu_R)\right) \\
& + \frac{\alpha_s(\mu_R)}{2\pi} \frac{1}{1-2\alpha_s(\mu_R)\beta_0 L} \left(1 - \alpha_s(\mu_R)\frac{\beta_1}{\beta_0} \frac{\ln(1-2\alpha_s(\mu_R)\beta_0 L)}{1-2\alpha_s(\mu_R)\beta_0 L}\right) \\
& \times \left(C_{ci}^{(1)}(z_1)\delta(1-z_2)\delta_{c'j} + \{z_1 \leftrightarrow z_2; c, i \leftrightarrow c', j\}\right) \\
& + \frac{\alpha_s^2(\mu_R)}{(2\pi)^2} \frac{1}{(1-2\alpha_s(\mu_R)\beta_0 L)^2} \left(\left(C_{ci}^{(2)}(z_1) - 2\pi\beta_0 C_{ci}^{(1)}(z_1) \ln\frac{M^2}{\mu_R^2} \right) \delta(1-z_2)\delta_{c'j} \right. \\
& \left. + \{z_1 \leftrightarrow z_2; c, i \leftrightarrow c', j\} \right) + \frac{\alpha_s^2(\mu_R)}{(2\pi)^2} \frac{1}{(1-2\alpha_s(\mu_R)\beta_0 L)^2} \left(C_{ci}^{(1)}(z_1) C_{c'j}^{(1)}(z_2) + G_{ci}^{(1)}(z_1) G_{c'j}^{(1)}(z_2) \right) \\
& + \frac{\alpha_s^2(\mu_R)}{(2\pi)^2} H^{(1)}(\mu_R) \frac{1}{1-2\alpha_s(\mu_R)\beta_0 L} \left(C_{ci}^{(1)}(z_1)\delta(1-z_2)\delta_{c'j} + \{z_1 \leftrightarrow z_2; c, i \leftrightarrow c', j\} \right) \Bigg\}
\end{aligned}$$

- Coefficient functions and hard-virtual corrections absorbed into effective parton luminosities

Numerical implementation: RadISH

- Since the transverse momenta of the *resolved* reals are of the same order, we can expand the whole integrand about $k_{ti} \sim k_{t1}$ up to the desired logarithmic accuracy
- This expansion allows us to compute higher-order corrections to the NLL *resolved* reals by simply including one correction at a time

e.g. expansion up to NLL

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}}(k_{t1}) \right) \int d\mathcal{Z}[\{R', k_i\}] \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

$$k_{ti}/k_{t1} = \zeta_i = \mathcal{O}(1)$$

$$\boxed{\int d\mathcal{Z}[\{R', k_i\}] G(\{\tilde{p}\}, \{k_i\}) = \epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) G(\{\tilde{p}\}, k_1, \ldots, k_{n+1})}$$

- Coefficient functions and hard-virtual corrections absorbed into effective parton luminosities
- The ensemble of NLL real emissions dZ is generated as a parton shower. Fast numerical evaluation with Monte-Carlo methods.

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- Since the transverse momenta of the *resolved* reals are of the same order, we can expand the whole integrand about $k_{ti} \sim k_{t1}$ up to the desired logarithmic accuracy
- This expansion allows us to compute higher-order corrections to the NLL *resolved* reals by simply including one correction at a time

e.g. expansion up to NNLL

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right) \int d\mathcal{Z}[\{R', k_i\}] \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

$$+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \left\{ \left(R'(k_{t1}) \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) \right) \right.$$
$$\times \left(R''(k_{t1}) \ln\frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2\frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \ln\frac{1}{\zeta_s} \right)$$
$$\left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \right\} \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \right\}$$

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e.g. expansion up to N³LL

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right) \int d\mathcal{Z}[\{R', k_i\}] \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

$$+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \Bigg\{ \Big(R'(k_{t1}) \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) \Big)$$
$$\times \left(R''(k_{t1}) \ln \frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2 \frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \ln \frac{1}{\zeta_s} \right)$$
$$+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \Bigg\} \Big\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \Big\}$$

$$+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1})$$
$$\times \Bigg\{ \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} - \partial_L \mathcal{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right)$$
$$+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \Bigg\}$$
$$\times \Big\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1})\right) -$$
$$\Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \Big\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-6} \frac{1}{v}\right)$$

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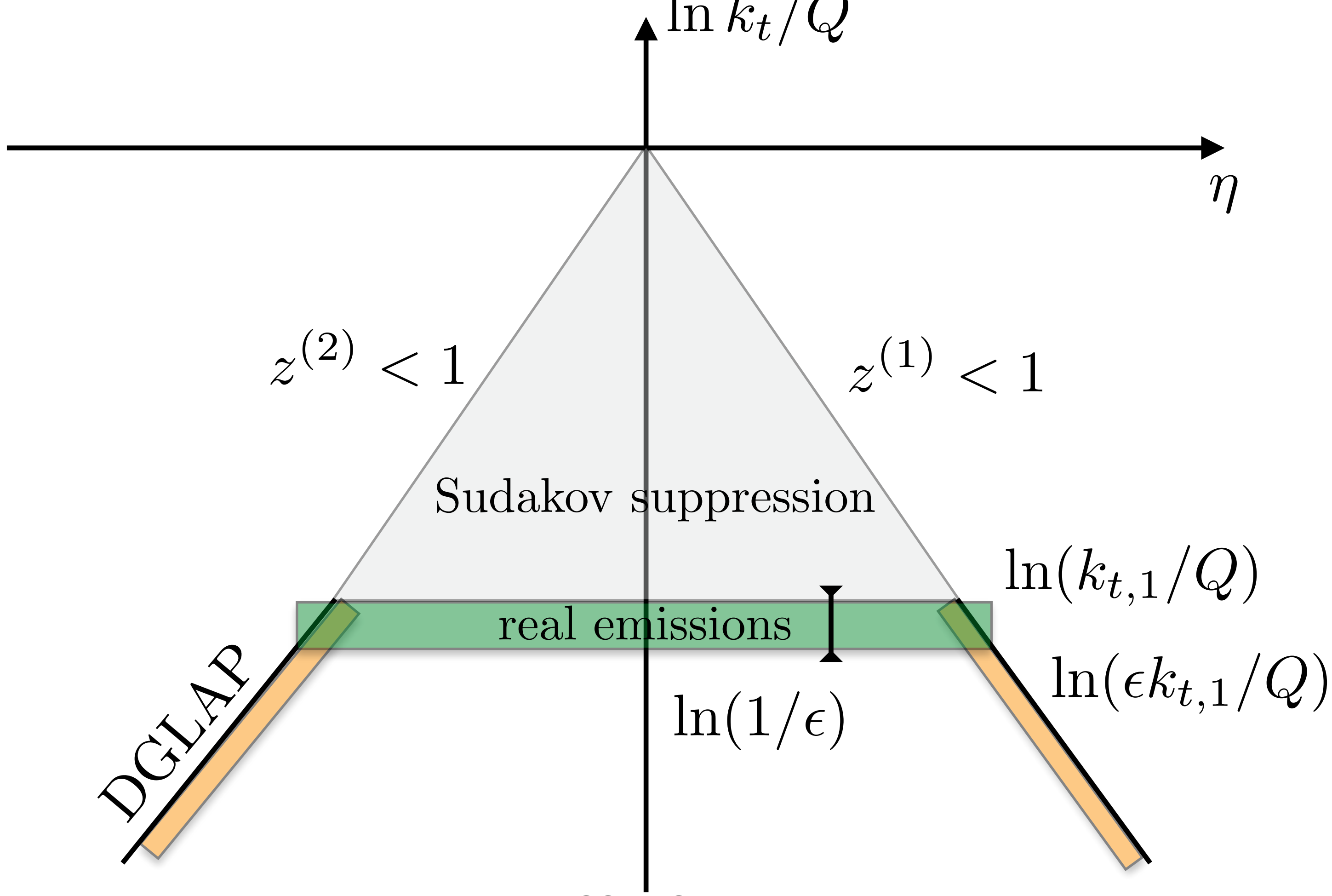
Treatment of initial state radiation

- At NLL *resolved* real radiation is soft and collinear, therefore there's no overlapping with the DGLAP evolution (PDFs can be evaluated at kt1)
- Beyond NLL a *resolved* real hard-collinear radiation is allowed; need to perform of DGLAP evolution exclusively for a fixed number of collinear emissions



- e.g. at NNLL expand around the IR cutoff of the last resolved emission

$$q(x, \epsilon k_{t,1}) = q(x, k_{t,1}) - \frac{\alpha_s(k_{t,1})}{\pi} P(z) \otimes q(x, k_{t,1}) \ln \frac{1}{\epsilon} + \mathcal{O}(\mathrm{N^3LL})$$

cutoff dependence cancels against the real counterpart

Equivalence to CSS formula

- Hard-collinear emissions off initial-state legs require some care in the treatment of kinematics. Final result reads

$$\frac{d\Sigma(v)}{dp_t d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \frac{d\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)}{dp_t} \mathbf{f}_{N_2}(\mu_0)$$

$$\begin{aligned}
\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = {} & \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \\
& \times e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2} \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\} \\
& \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right) \\
& \times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1}^{2} \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right) \\
& \times \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),
\end{aligned}$$

- Formulation equivalent to b-space result, up to a scheme change. Using the delta representation for the distribution one finds

$$\boxed{\delta^{(2)}(\vec{p_t} - (\vec{k}_{t1} + \ldots + \vec{k}_{tn})) = \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p_t}} \prod_{i=1}^{n} e^{i\vec{b}\cdot\vec{k}_{ti}}}$$

$$\frac{d\Sigma(v)}{dp_t d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \frac{d\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)}{dp_t} \mathbf{f}_{N_2}(\mu_0) =$$

$$\boxed{(1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12} \frac{\partial^3}{\partial \ln(Mb/b_0)^3} \Theta(k_t - \frac{b_0}{b}) + \ldots}$$

$$\begin{aligned}
= {} & \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b) \\
& \times \exp\left\{-\sum_{\ell=1}^{2} \int_0^M \frac{dk_t}{k_t} \mathbf{R}'_\ell(k_t) (1 - J_0(bk_t))\right\}.
\end{aligned}$$