Helioseismology and solar neutrinos

Jørgen Christensen-Dalsgaard
Stellar Astrophysics Centre
Aarhus University

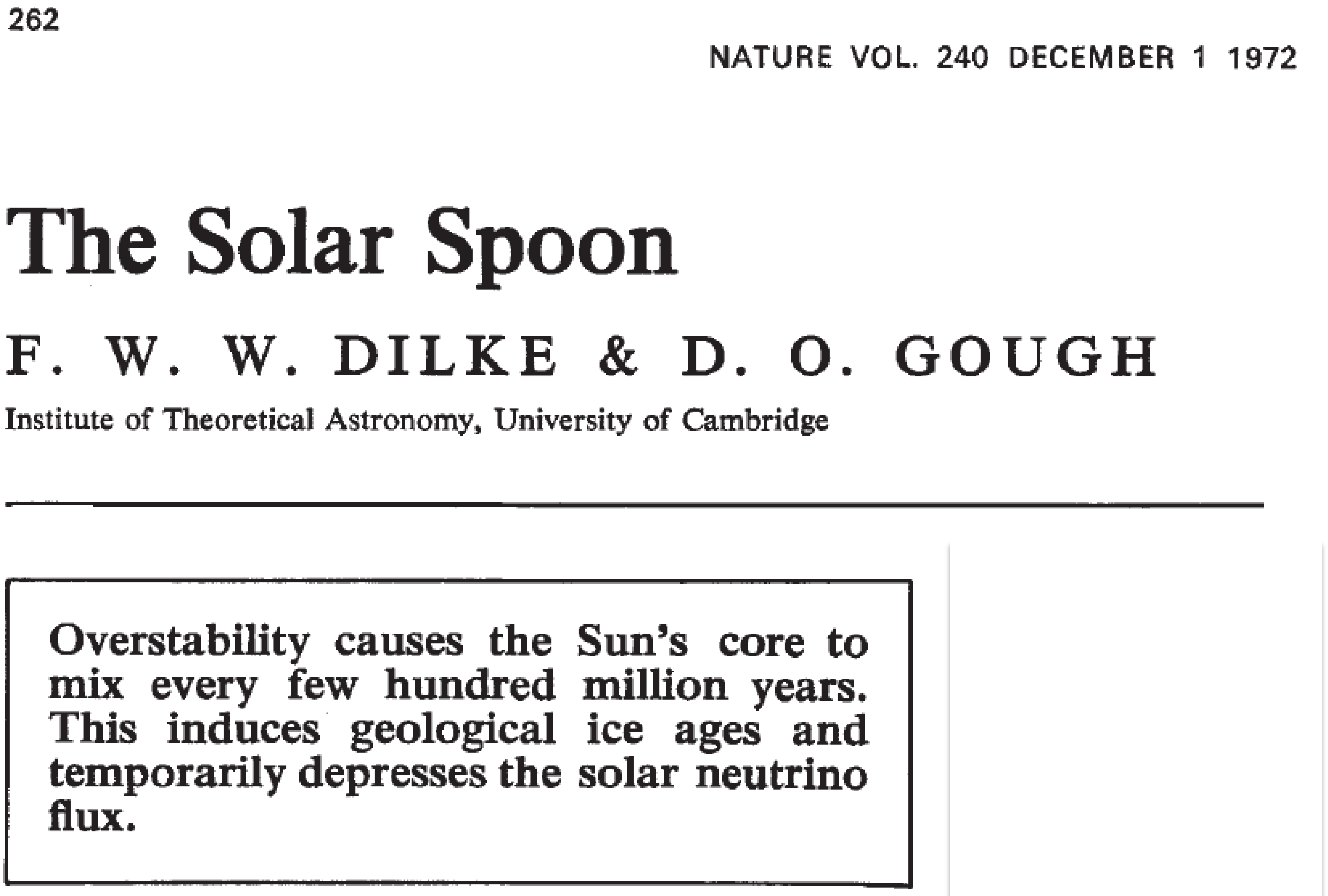
Danmarks
Grundforskningsfond
Danish National
Research Foundation

STELLAR ASTROPHYSICS CENTRE

Early model modifications

- ^{8}B strongly temperature sensitive $(\sim {T_c}^{20})$
- Reduce central temperature, maintain luminosity
  - Core mixing
  - Rapid rotation
  - ???

Intermittent mixing of the solar core?



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NATURE VOL. 240 DECEMBER 1 1972

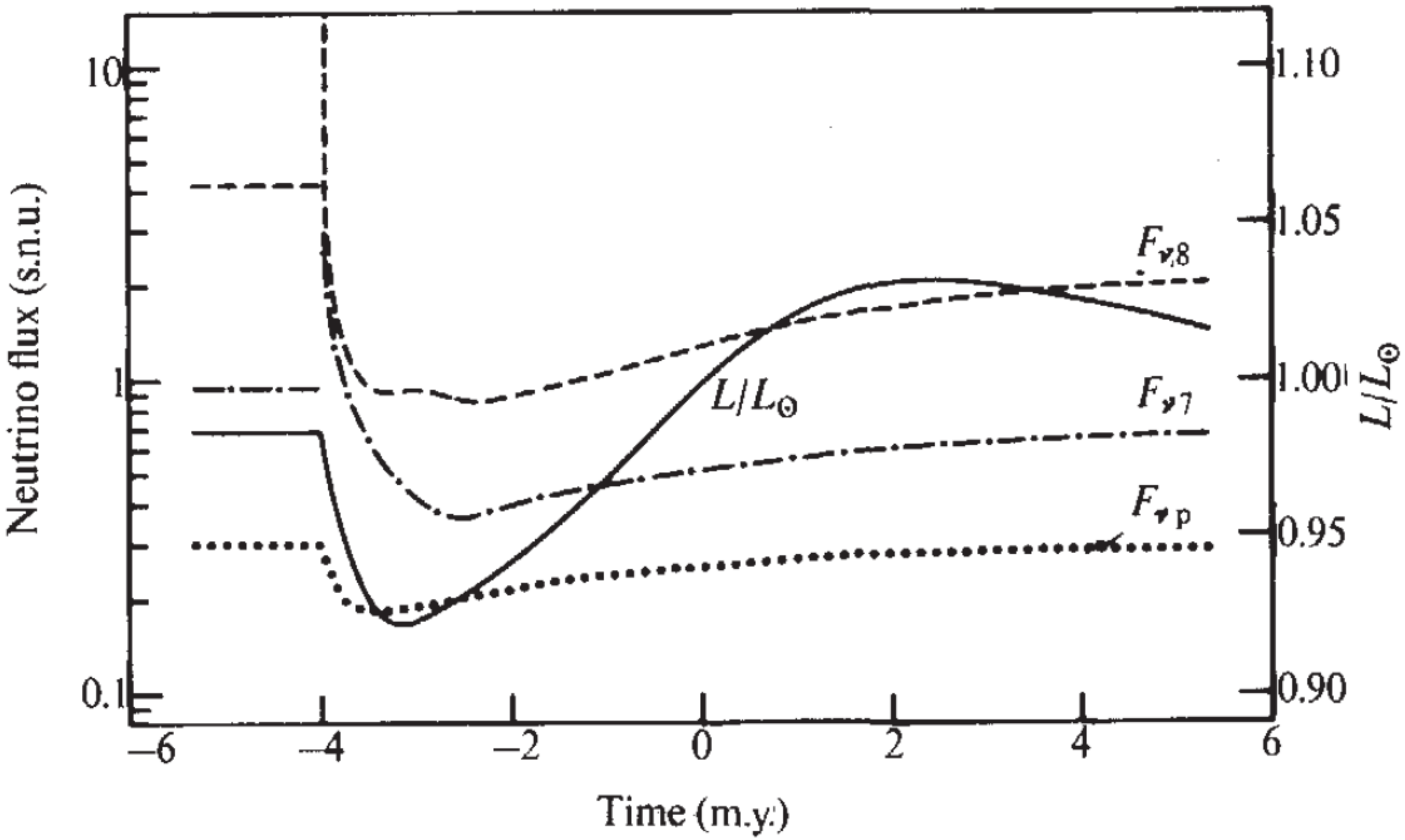
The Solar Spoon

F. W. W. DILKE & D. O. GOUGH

Institute of Theoretical Astronomy, University of Cambridge

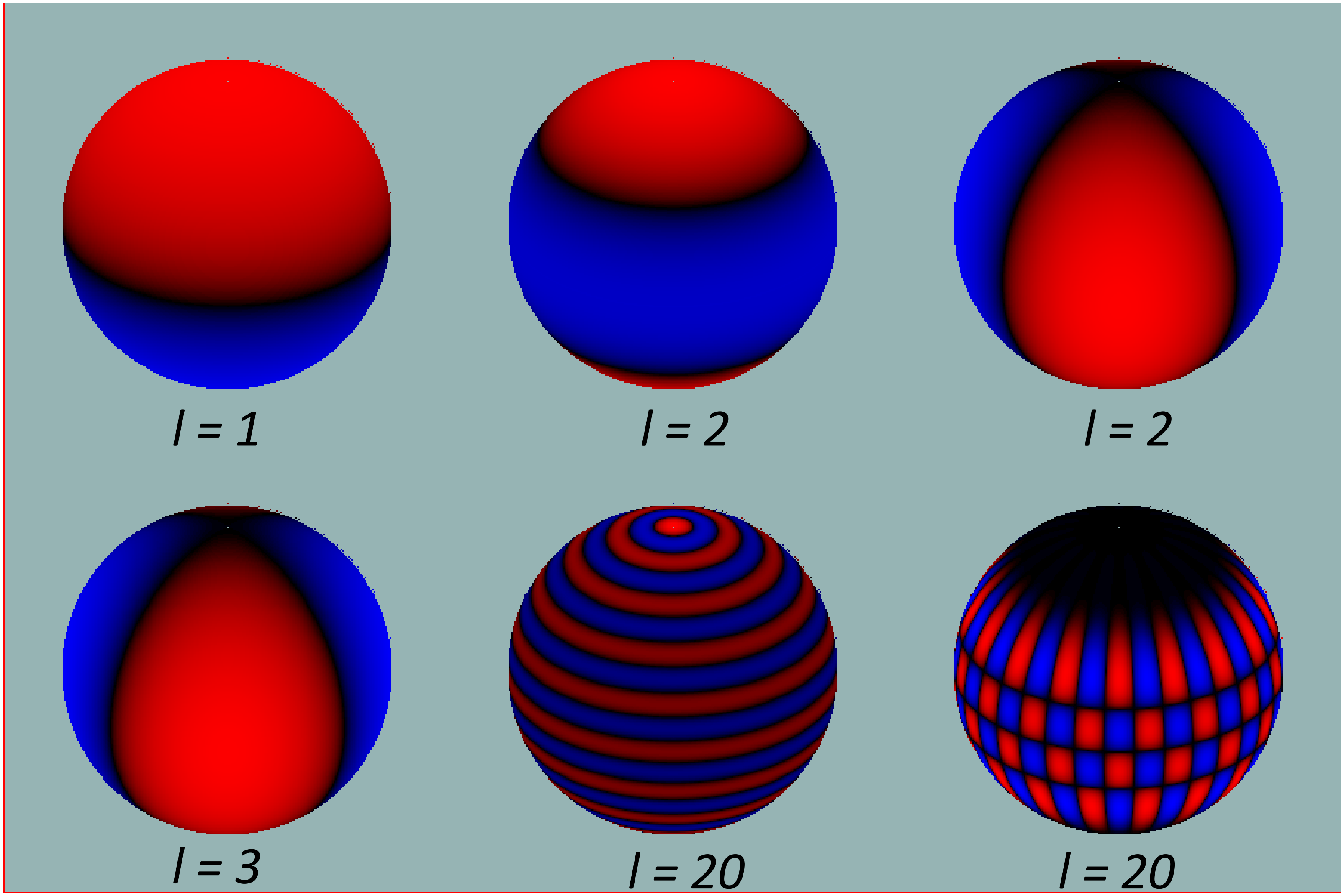
Overstability causes the Sun's core to mix every few hundred million years. This induces geological ice ages and temporarily depresses the solar neutrino flux.

Intermittent mixing of the solar core?

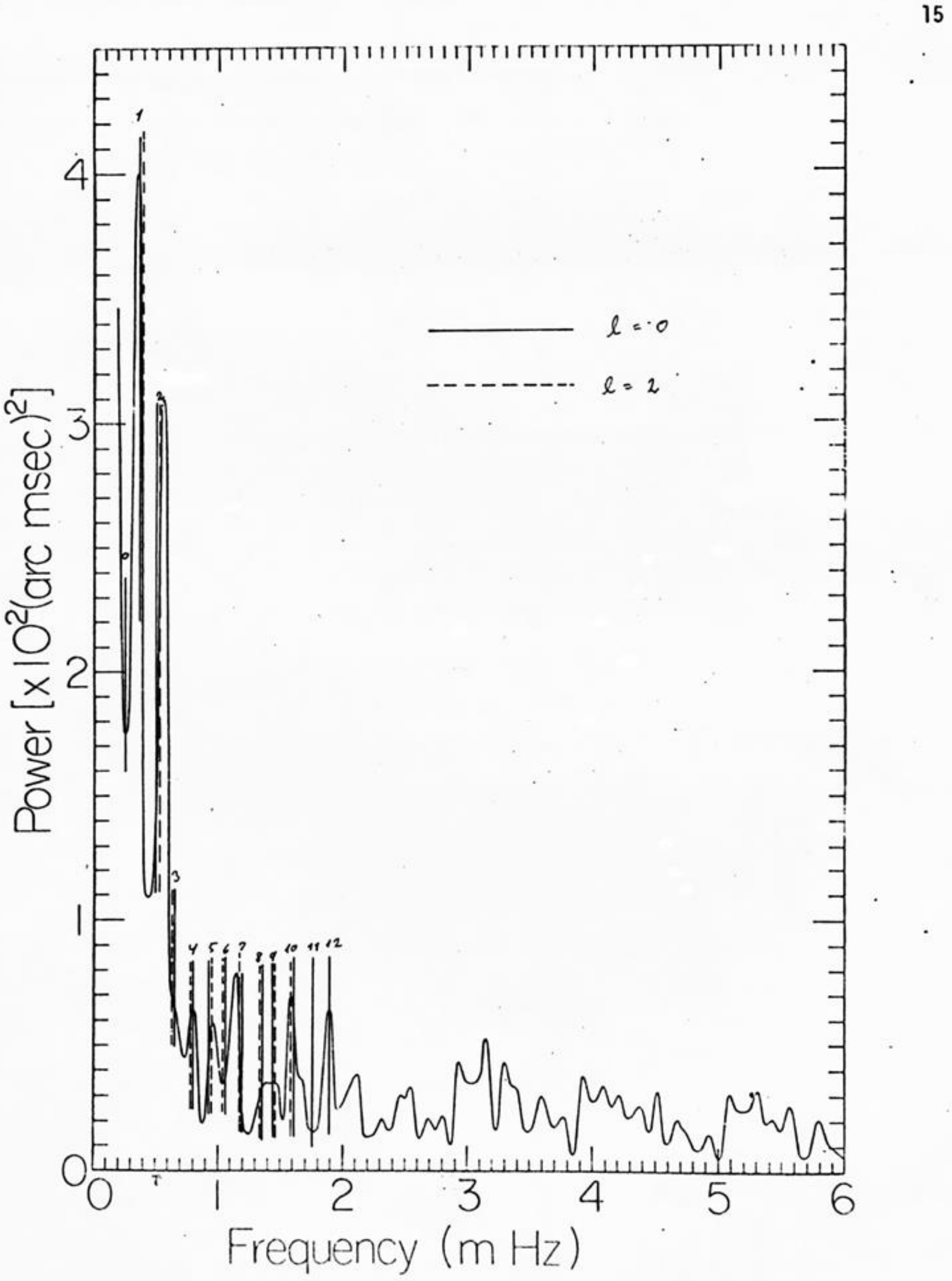


Dilke & Gough (1972; Nature 240, 262)

Oscillations on the solar surface



Global solar oscillations?

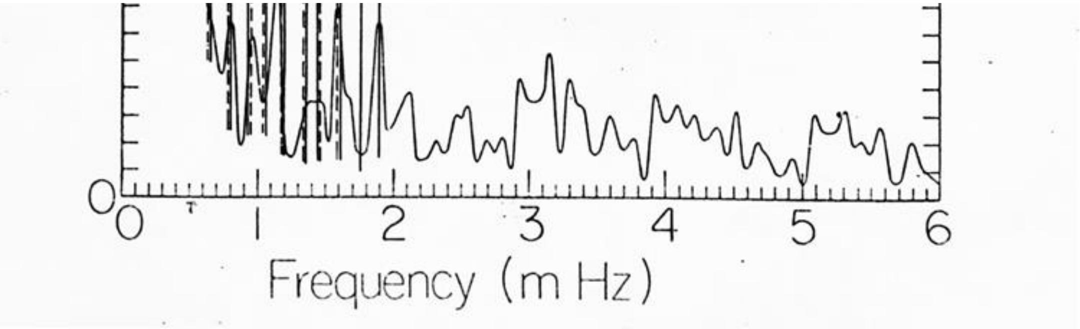


H. A. Hill (1975; IBM Conference on Astrophysical Fluid Dynamics, Cambridge).

Global solar oscillations?

Periods, in minutes

Hill		Theory		
1973	1975	$\ell = 0$	$\ell = 2$	$\ell = 4$
		p_0: 62.7		
52	48	p_1: 43.8	p^1: 42.2	
33		p_2: 32.6	p_2: 34.3	p_1: 38.0
	30			p_2: 29.3
24		p_3: 26.0	p_3: 26.7	p_3: 23.4
	21	p_4: 21.0	p_4: 21.5	
				p_4: 19.5
		p_5: 18.2	p_5: 17.9	
16.	17.	p_6: 15.8	p_6: 16.0	p_5: 16.8
13.3	14.6		p_7: 14.1	p_6: 14,6
			p_8: 12.6	p_7: 13,0
11.8	11.9	p_9: 11.5	p_9: 11.4	p_8: 11.7
10.4	10.5	p_{10}: 10.4	p_{10}: 10.4	
8.9	8.8	p_{12}: 8.84		



H. A. Hill (1975; IBM Conference on Astrophysical Fluid Dynamics, Cambridge).

Asymptotics of p modes

$$\nu_{nl} \sim \Delta\nu \left(n + \frac{l}{2} + \alpha\right) + \epsilon_{nl}$$

where

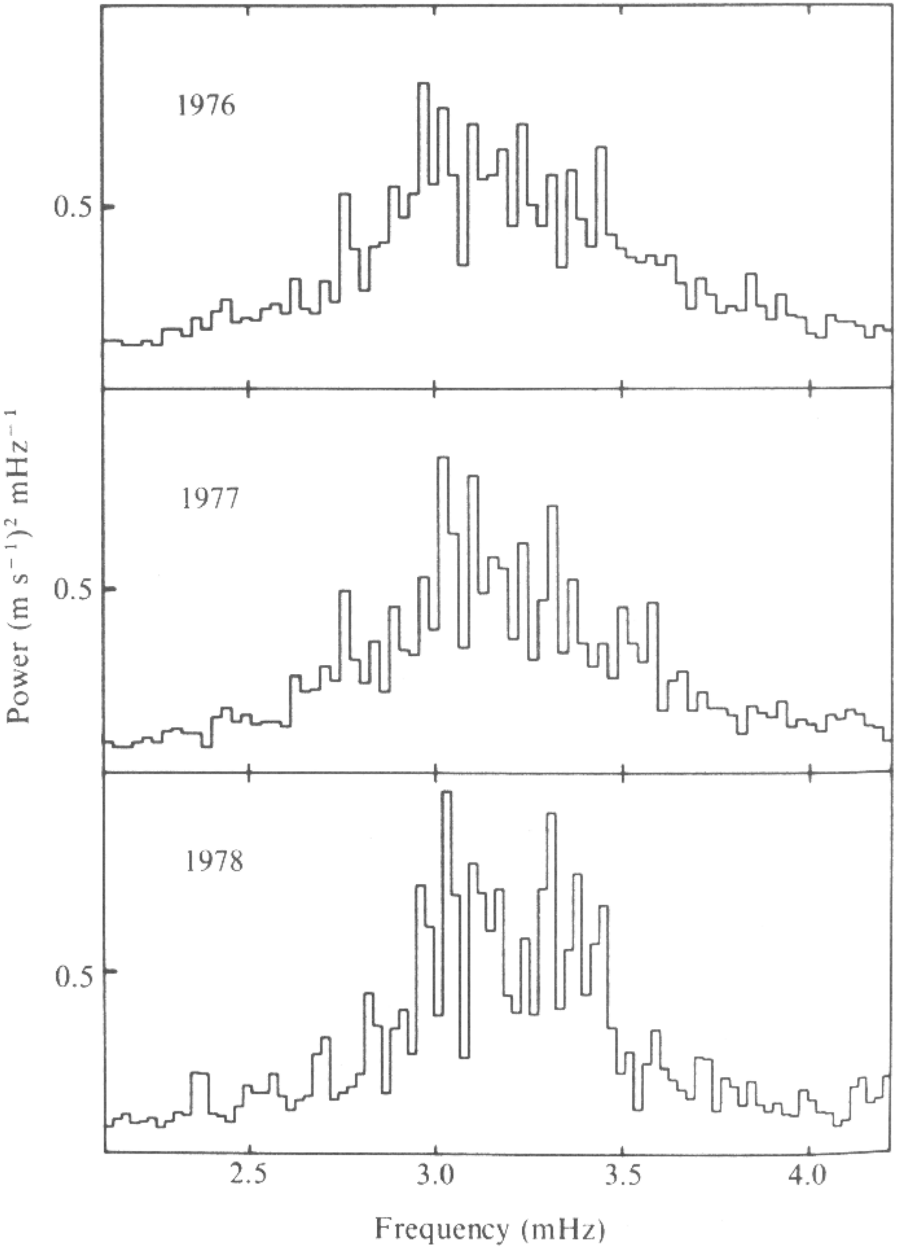
$$\Delta\nu = \left[2 \int_0^R \frac{\mathrm{d}r}{c}\right]^{-1}$$

$\alpha = \alpha(\nu)$ depends on surface properties.

Large frequency separation:

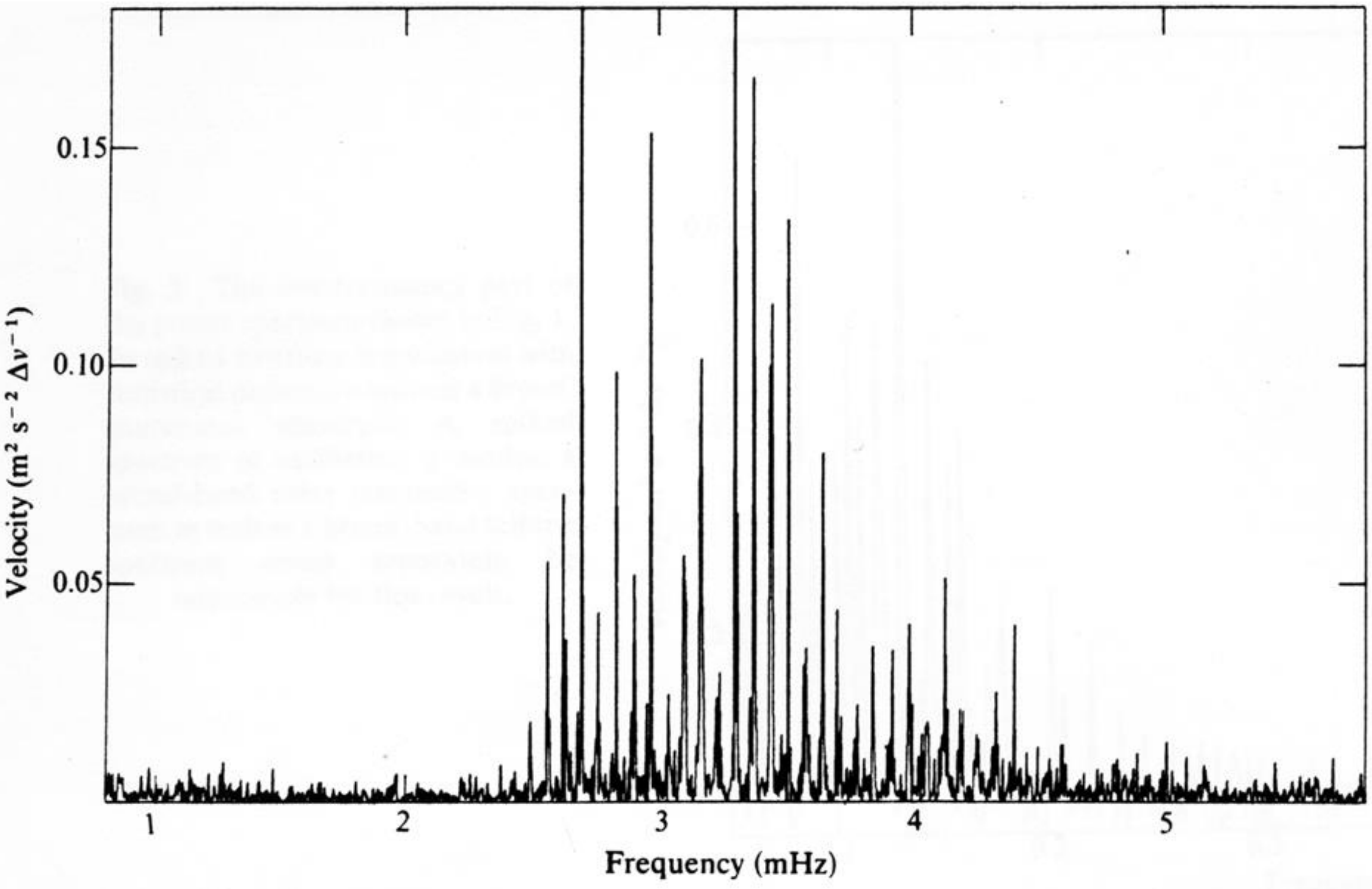
$$\Delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l} \simeq \Delta\nu$$

Global acoustic modes of the Sun



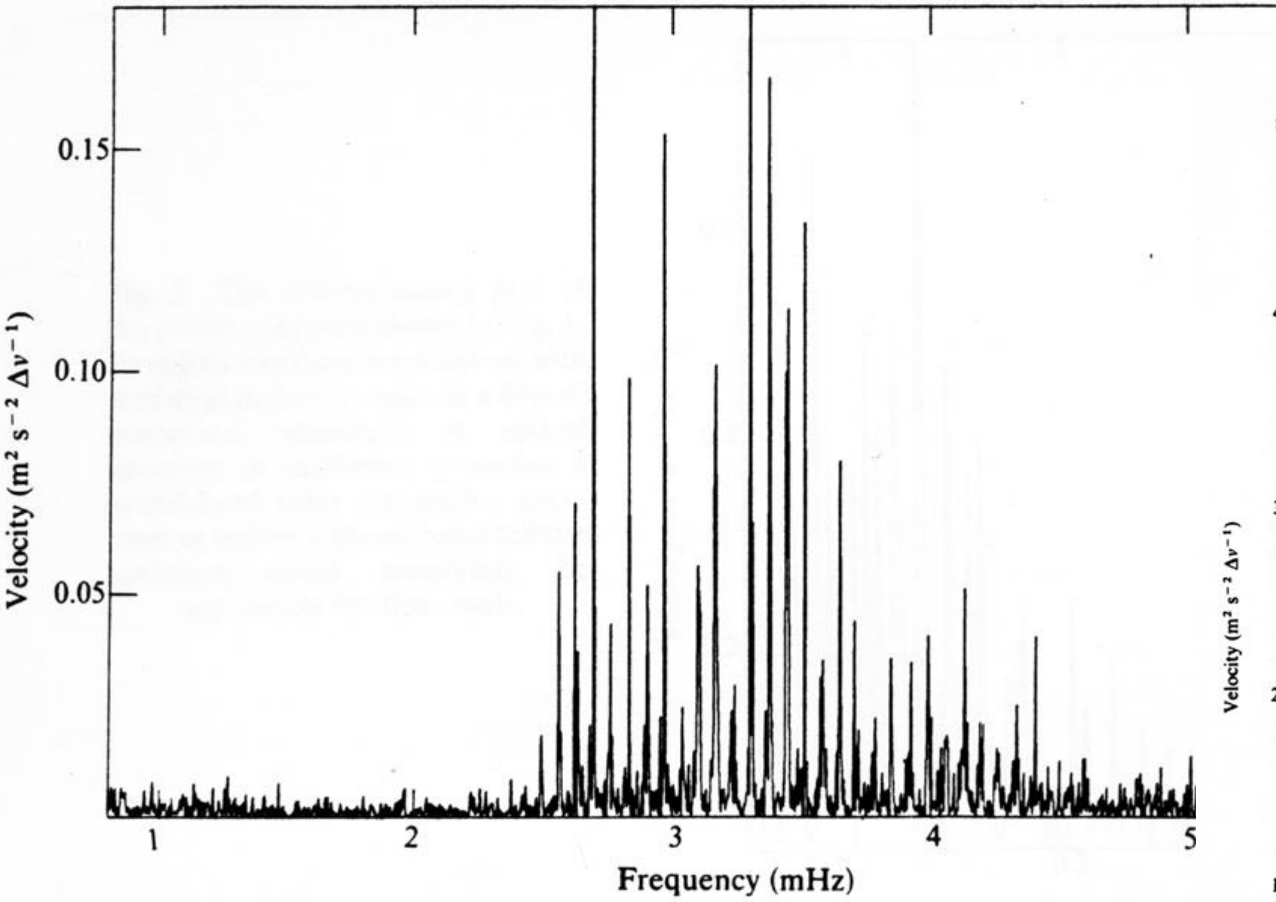
Claverie et al. (1979; Nature 282, 591)

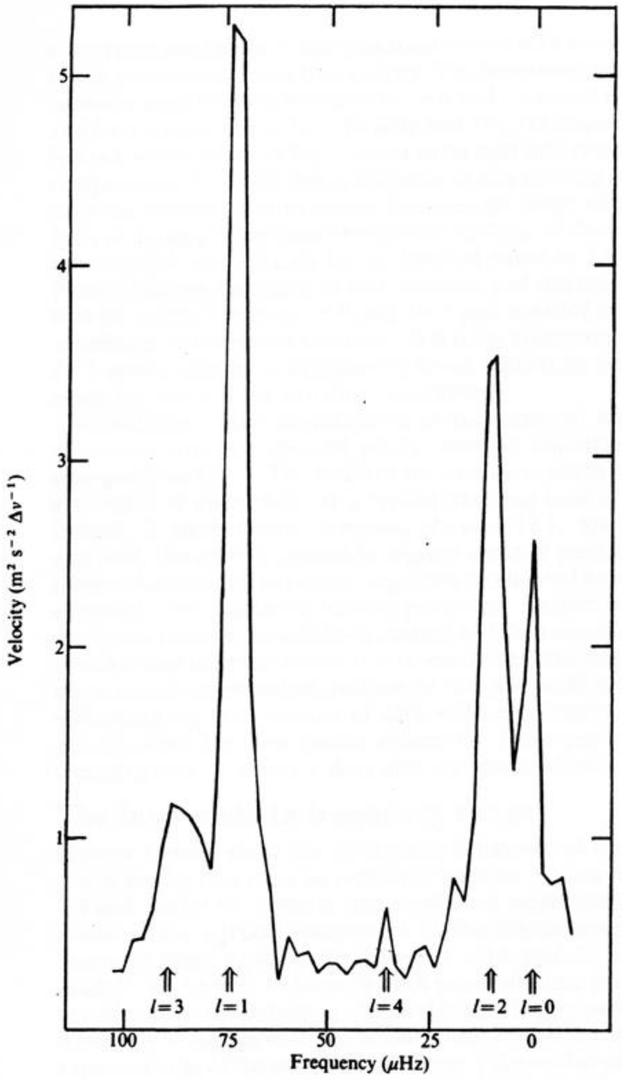
The solar core observed from the South Pole



Grec et al., Nature 288, 541; 1980

The solar core observed from the South Pole





Grec et al., Nature 288, 541; 1980

Small frequency separations

$$\nu_{nl} \sim \Delta\nu \left(n + \frac{l}{2} + \alpha \right) + \epsilon_{nl}$$

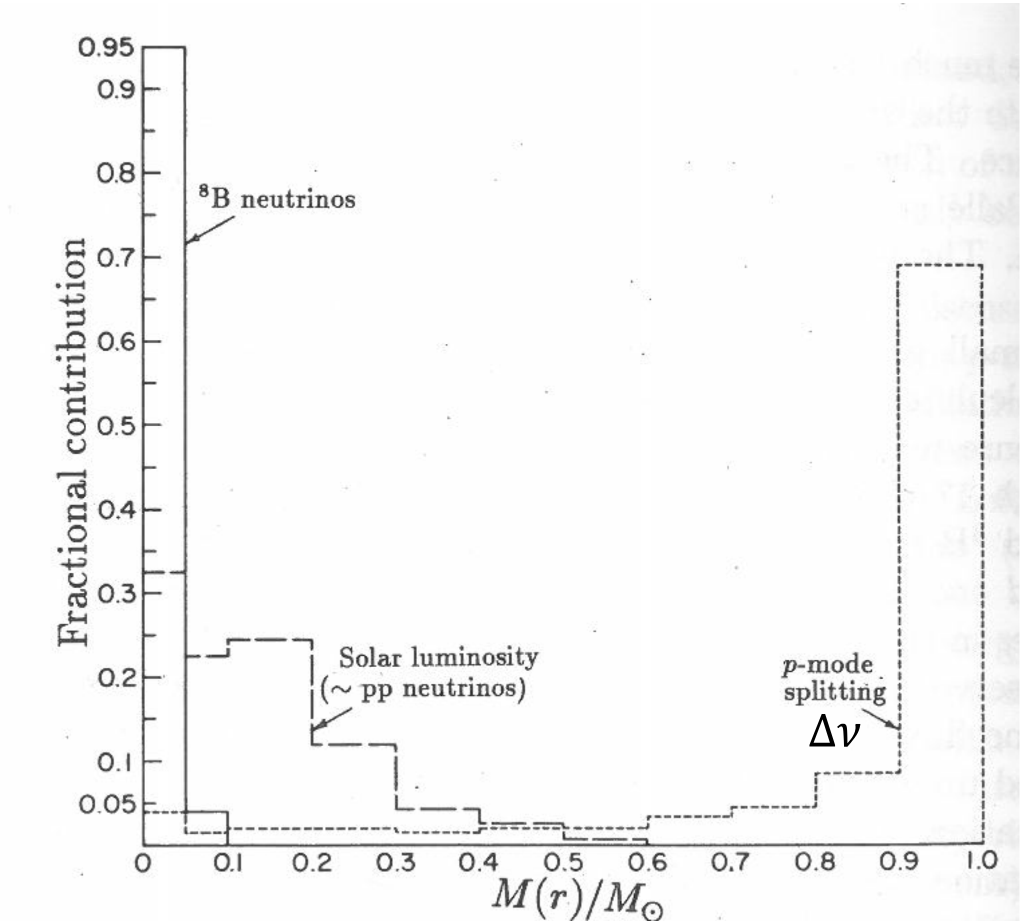
where

$$\epsilon_{nl} \simeq l(l+1) \frac{\Delta\nu}{4\pi^2 \nu_{nl}} \int_0^R \frac{\mathrm{d}c}{\mathrm{d}r} \frac{\mathrm{d}r}{r}$$

Frequency separations:

$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l+2} \simeq -(4l+6) \frac{\Delta\nu}{4\pi^2 \nu_{nl}} \int_0^R \frac{\mathrm{d}c}{\mathrm{d}r} \frac{\mathrm{d}r}{r}$$

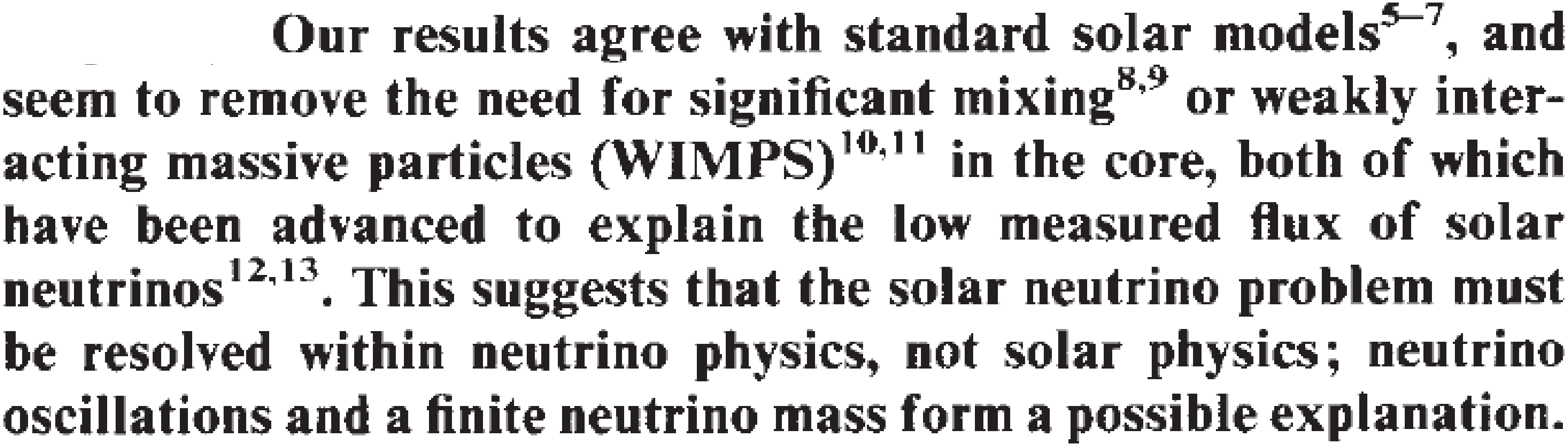
Neutrinos and helioseismology



Bahcall (1989; *Neutrino Astrophysics*)

The Birmingham Solar Oscillations Network (BiSON)

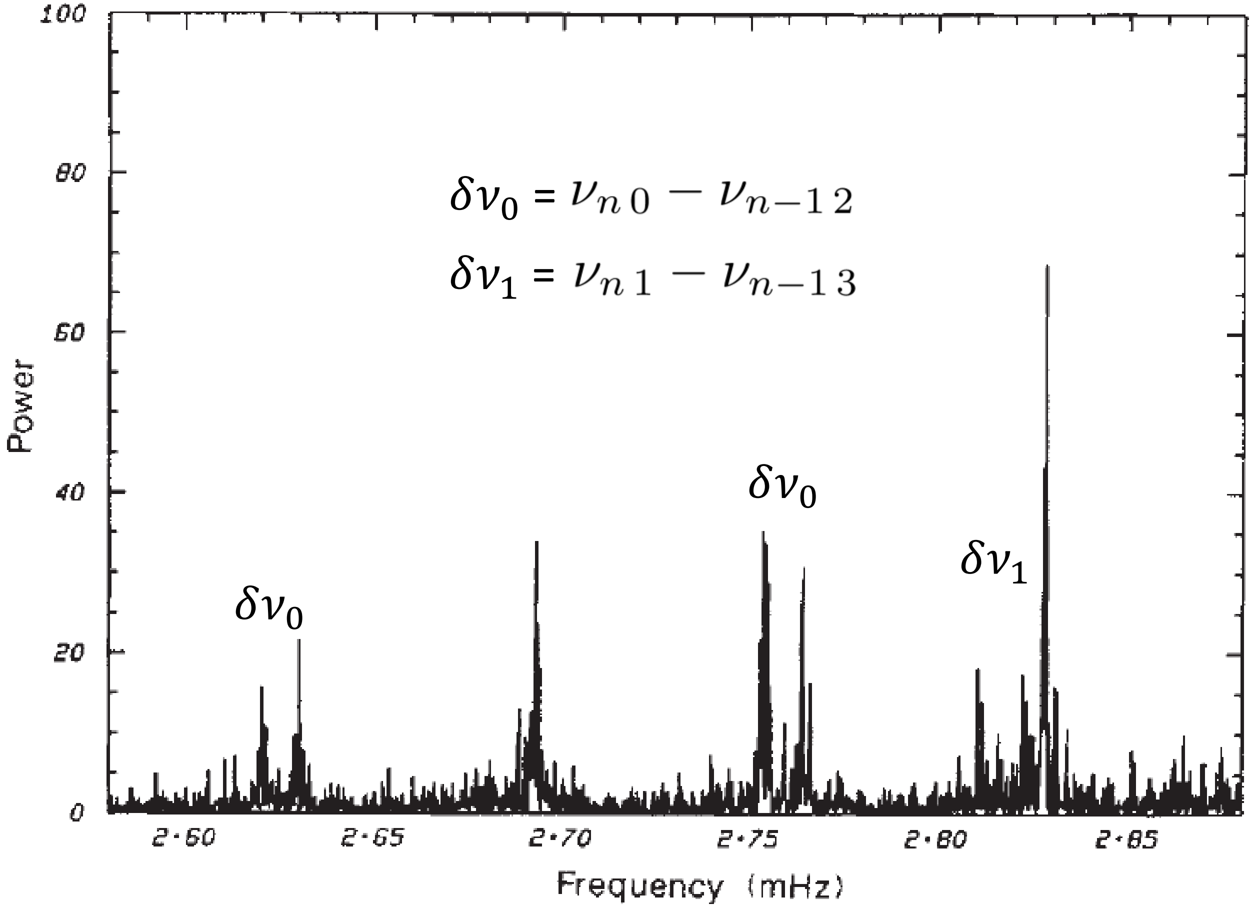
Neutrinos and low-degree helioseismology



Our results agree with standard solar models[5–7], and seem to remove the need for significant mixing[8,9] or weakly interacting massive particles (WIMPS)[10,11] in the core, both of which have been advanced to explain the low measured flux of solar neutrinos[12,13]. This suggests that the solar neutrino problem must be resolved within neutrino physics, not solar physics; neutrino oscillations and a finite neutrino mass form a possible explanation.

Elsworth et al. (1990; Nature 347, 536)

Neutrinos and low-degree helioseismology

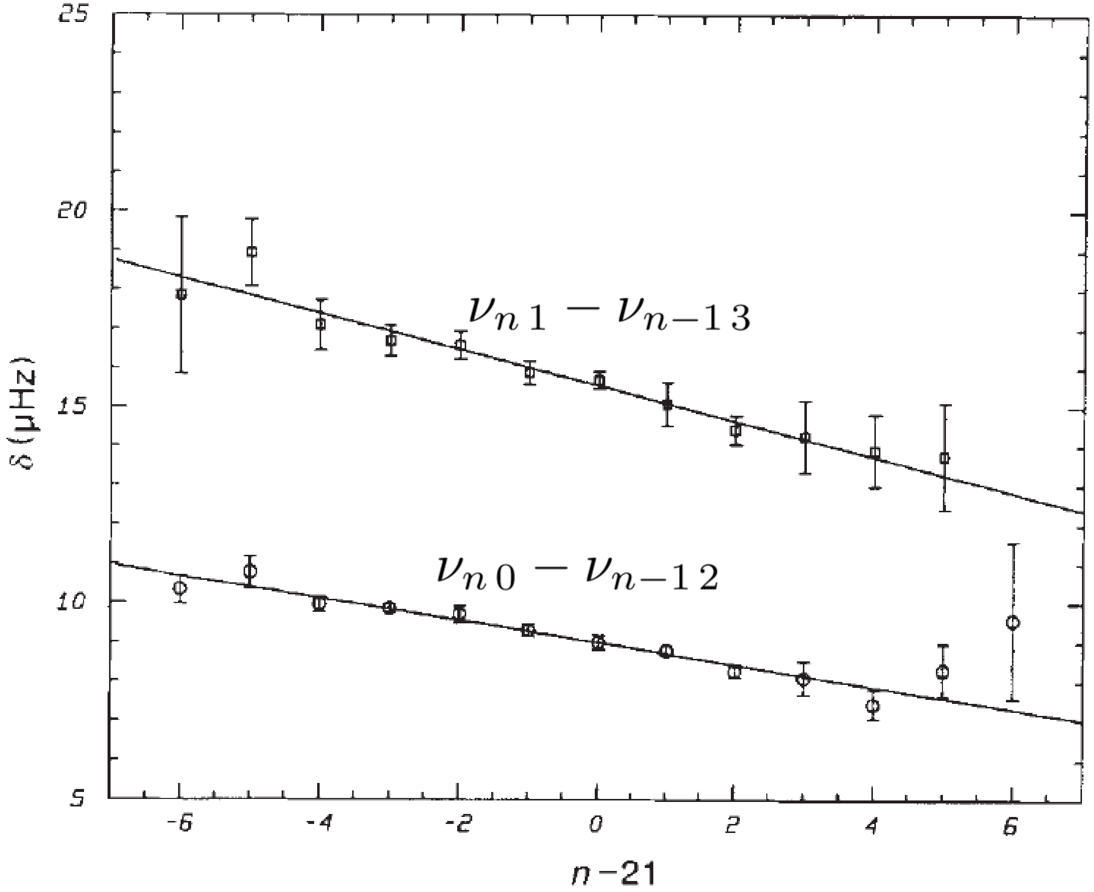


$\delta\nu_0 = \nu_{n\,0} - \nu_{n-1\,2}$

$\delta\nu_1 = \nu_{n\,1} - \nu_{n-1\,3}$

Elsworth et al. (1990; Nature 347, 536)

Neutrinos and low-degree helioseismology



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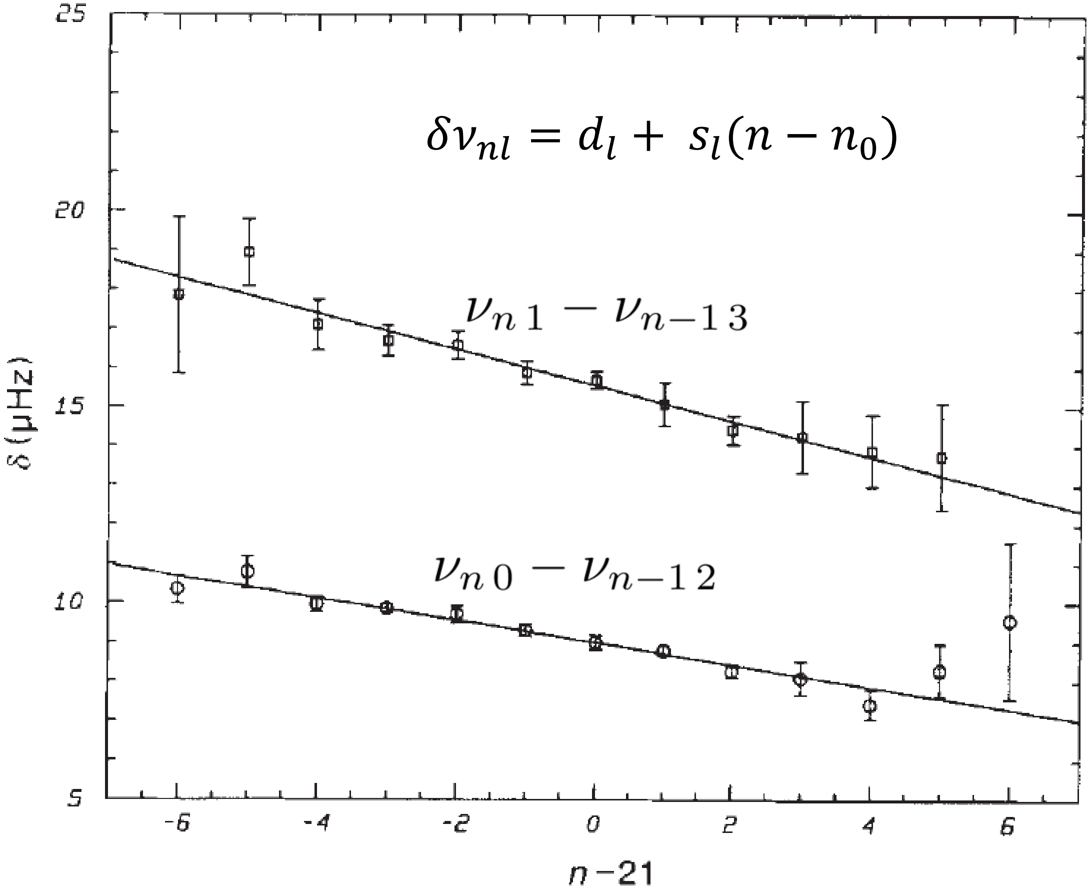
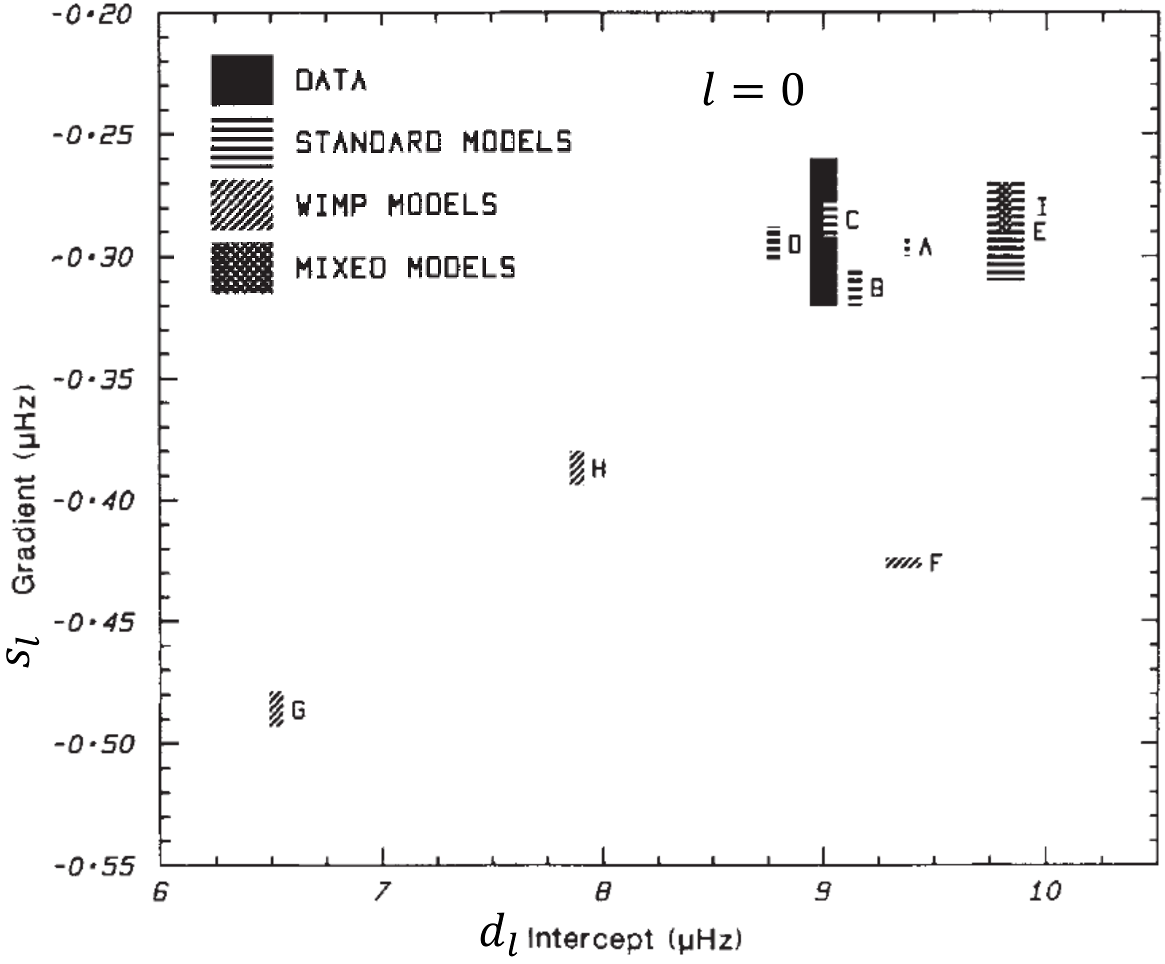


$$\delta\nu_{nl} = d_l + s_l(n - n_0)$$

Elsworth et al. (1990; Nature 347, 536)

Neutrinos and low-degree helioseismology

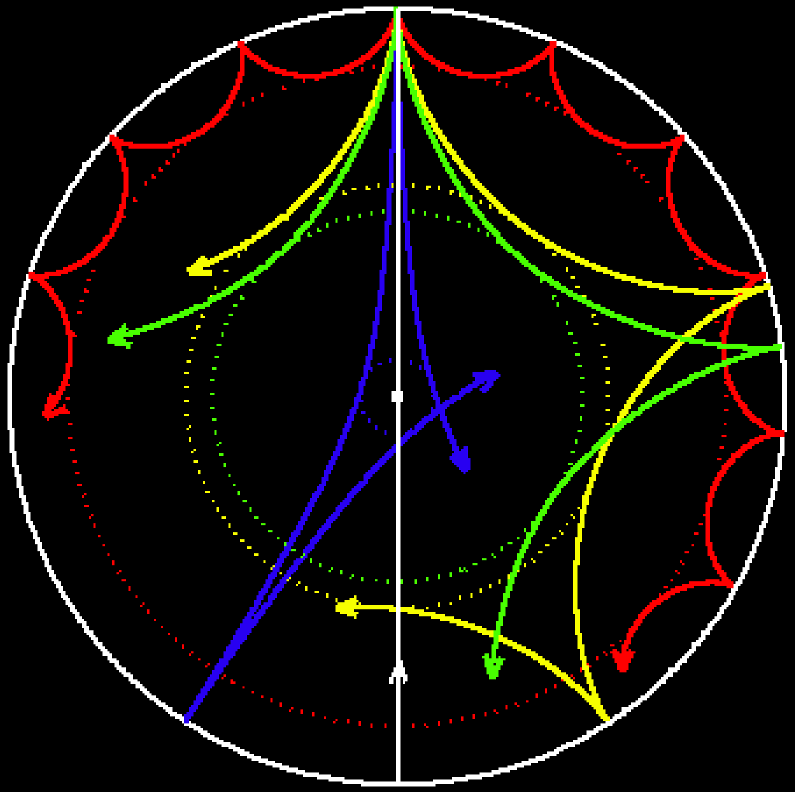


Elsworth et al. (1990; Nature 347, 536)

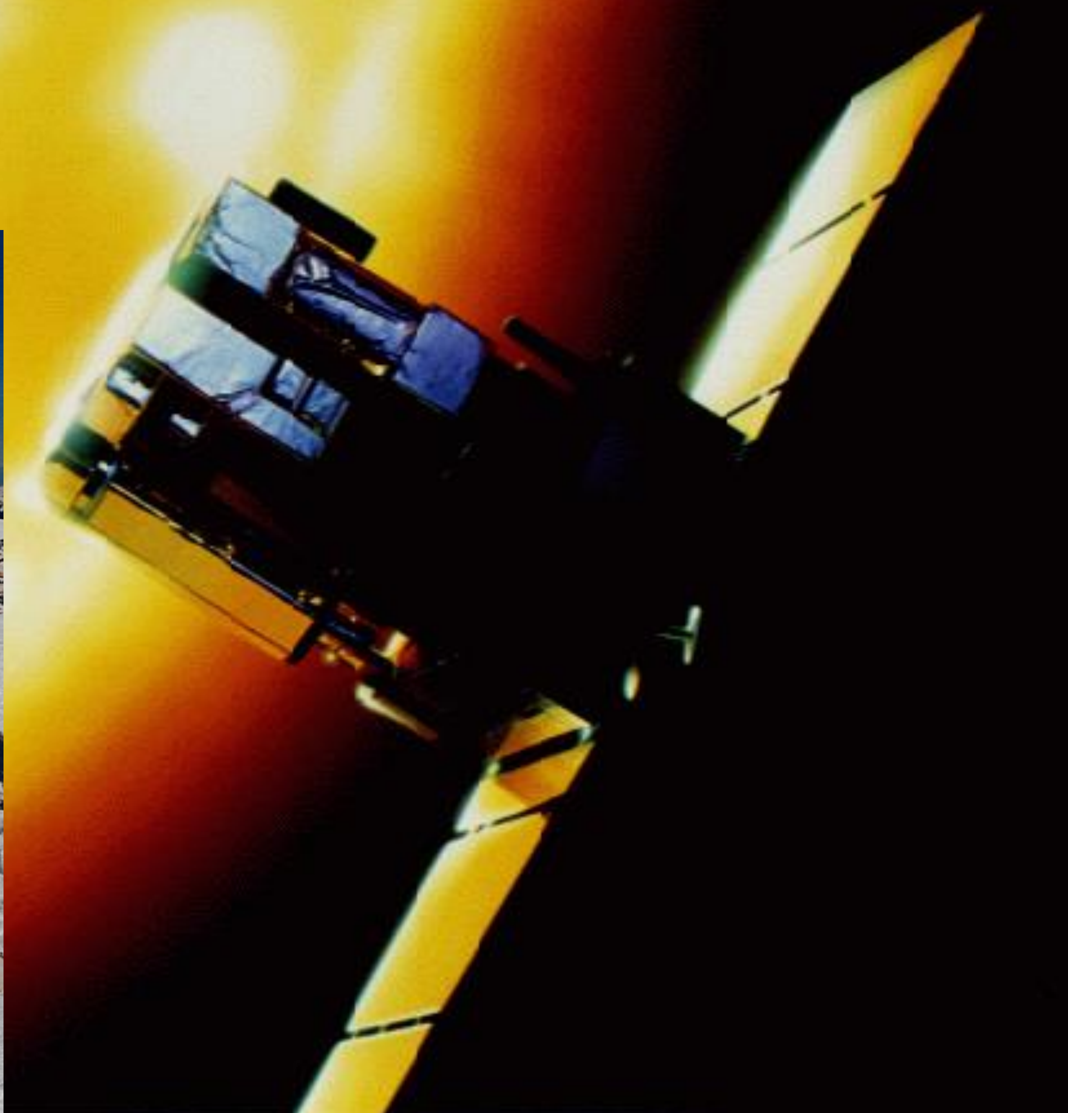
Caveat

- Oscillation frequencies depend on sound speed c and density ρ
- Neutrino flux depends mainly on temperature
- $c^2 \quad \approx \frac{5}{3}\frac{k_B T}{\mu m_u} \sim \frac{T}{\mu}$
- T and μ cannot be determined separately

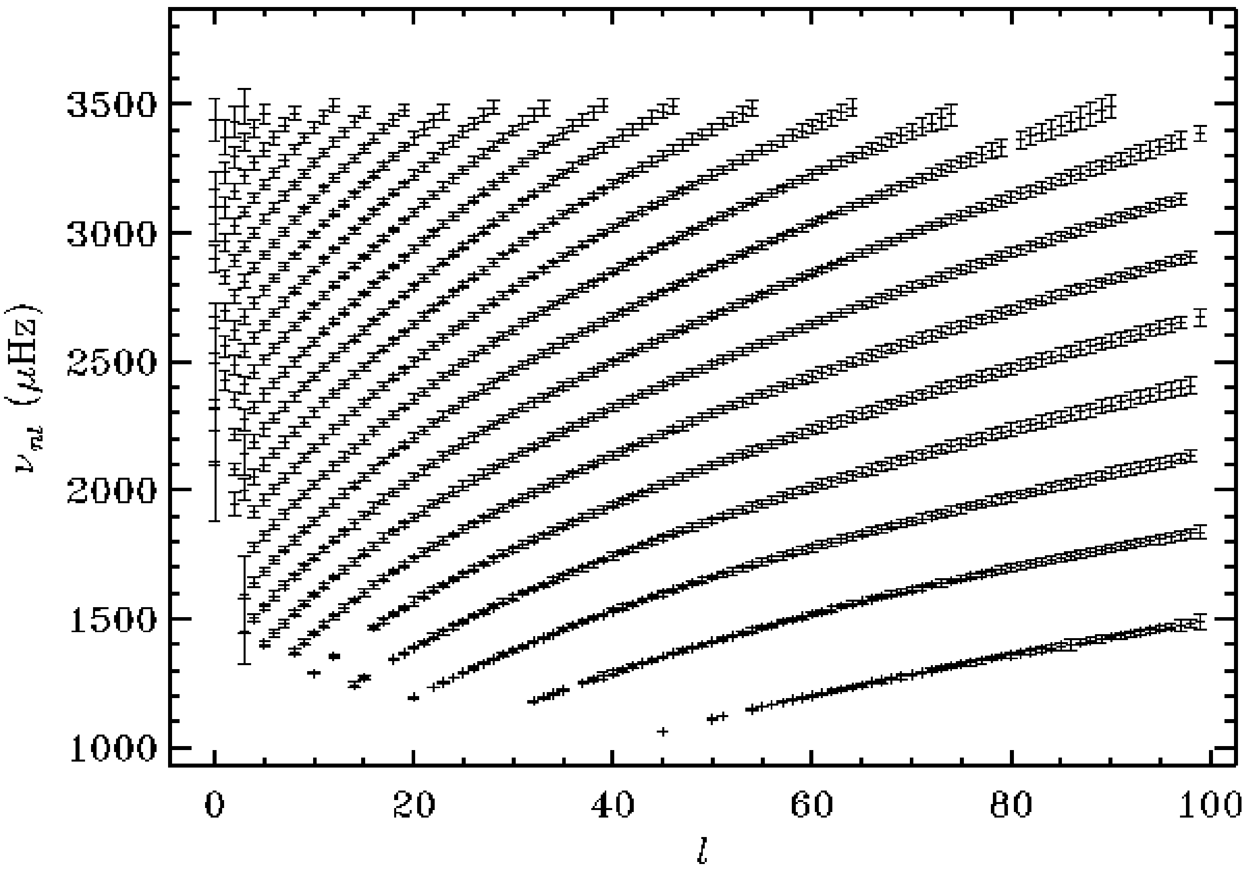
Rays of higher-degree modes



Probes of the solar interior
GONG and SOHO



Observed frequencies



m-averaged frequencies from MDI instrument on SOHO
1000 σ error bars

What is wrong with the solar model?

Structure inversion

Frequency differences between Sun and model:

$$\frac{\delta\omega_{nl}}{\omega_{nl}} = \int_0^R \left[K^{nl}_{c^2,\rho}(r) \frac{\delta_r c^2}{c^2}(r) + K^{nl}_{\rho,c^2}(r) \frac{\delta_r \rho}{\rho}(r) \right] \mathrm{d}r + Q_{nl}^{-1} \mathcal{G}(\omega_{nl}) + \epsilon_{nl} \ ,$$

$$\delta M = 4\pi \int_0^R \rho(r) r^2 \frac{\delta_r \rho(r)}{\rho(r)} \mathrm{d}r = 0 \ .$$

Combine the frequency differences

$$\begin{aligned}\sum_i c_i(r_0)\frac{\delta\omega_i}{\omega_i} \; &= \; \sum_i c_i(r_0)\int_0^R K^i_{c^2,\rho}(r)\frac{\delta_r c^2}{c^2}(r)\mathrm{d}r \\ &+ \sum_i c_i(r_0)\int_0^R K^i_{\rho,c^2}(r)\frac{\delta_r \rho}{\rho}(r)\mathrm{d}r \\ &+ \sum_i c_i(r_0)Q_i^{-1}\mathcal{G}(\omega_i) + \sum_i c_i(r_0)\epsilon_i \ ,\end{aligned}$$

Combine the frequency differences

Choose coefficients to isolate, as far as possible, $\delta_r c^2$

$$\sum_i c_i(r_0)\frac{\delta\omega_i}{\omega_i} = \sum_i c_i(r_0)\int_0^R K^i_{c^2,\rho}(r)\frac{\delta_r c^2}{c^2}(r)\mathrm{d}r$$

~~$+\sum_i c_i(r_0)\int_0^R K^i_{\rho,c^2}(r)\frac{\delta_r\rho}{\rho}(r)\mathrm{d}r$~~

~~$+\sum_i c_i(r_0)Q_i^{-1}\mathcal{G}(\omega_i)+\sum_i c_i(r_0)\epsilon_i$~~ ,

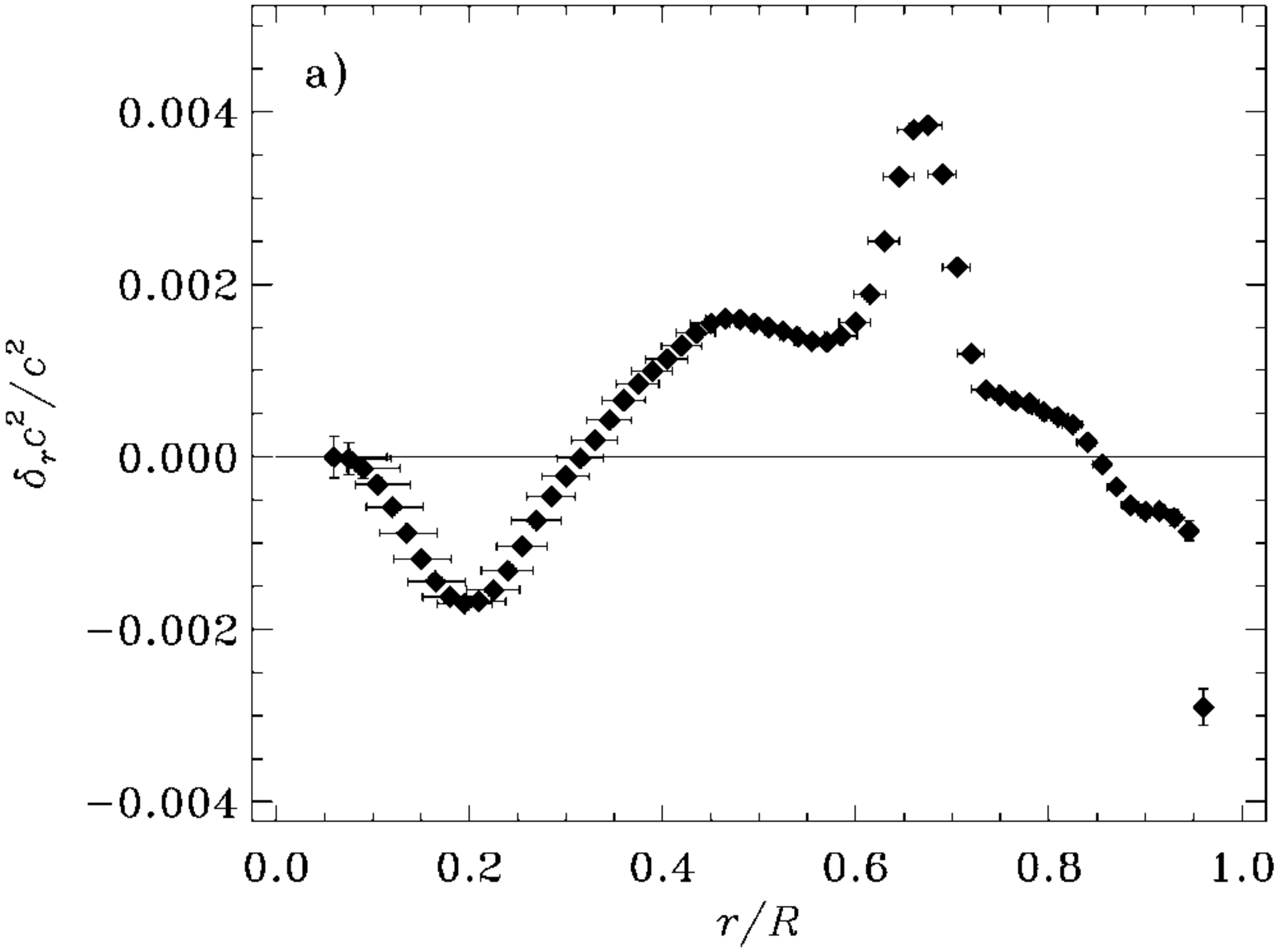
Estimate of (averaged) sound-speed correction

$$\overline{\left(\frac{\delta_r c^2}{c^2}\right)}(r_0) = \sum_i c_i(r_0)\frac{\delta\omega_i}{\omega_i} \simeq \int_0^R \mathcal{K}_{c^2,\rho}(r_0,r)\frac{\delta_r c^2}{c^2}\mathrm{d}r$$

$$\sigma^2\left[\overline{\left(\frac{\delta_r c^2}{c^2}\right)}(r_0)\right] = \sum_i \sigma_i^2 c_i(r_0)^2\ ,$$

$$\mathcal{K}_{c^2,\rho}(r_0,r) = \sum_i c_i(r_0)K^i_{c^2,\rho}(r)$$

Result: Sun - model



Model S: C-D et al. (1996; Science 272, 1286)
Including diffusion and settling of helium and heavy elements

VOLUME 78, NUMBER 2 PHYSICAL REVIEW LETTERS 13 JANUARY 1997

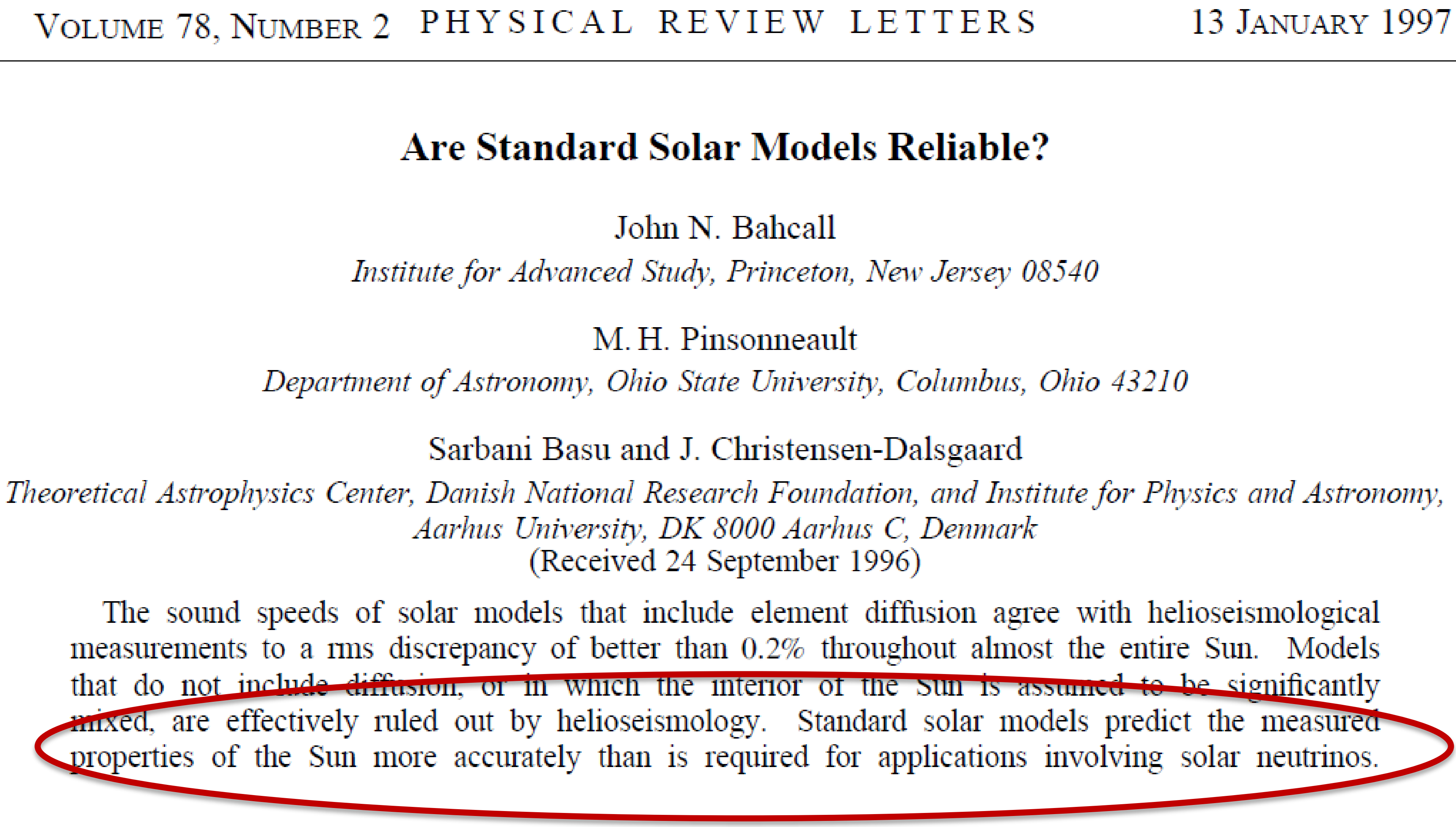
Are Standard Solar Models Reliable?

John N. Bahcall
Institute for Advanced Study, Princeton, New Jersey 08540

M. H. Pinsonneault
Department of Astronomy, Ohio State University, Columbus, Ohio 43210

Sarbani Basu and J. Christensen-Dalsgaard
Theoretical Astrophysics Center, Danish National Research Foundation, and Institute for Physics and Astronomy, Aarhus University, DK 8000 Aarhus C, Denmark
(Received 24 September 1996)

The sound speeds of solar models that include element diffusion agree with helioseismological measurements to a rms discrepancy of better than 0.2% throughout almost the entire Sun. Models that do not include diffusion, or in which the interior of the Sun is assumed to be significantly mixed, are effectively ruled out by helioseismology. Standard solar models predict the measured properties of the Sun more accurately than is required for applications involving solar neutrinos.



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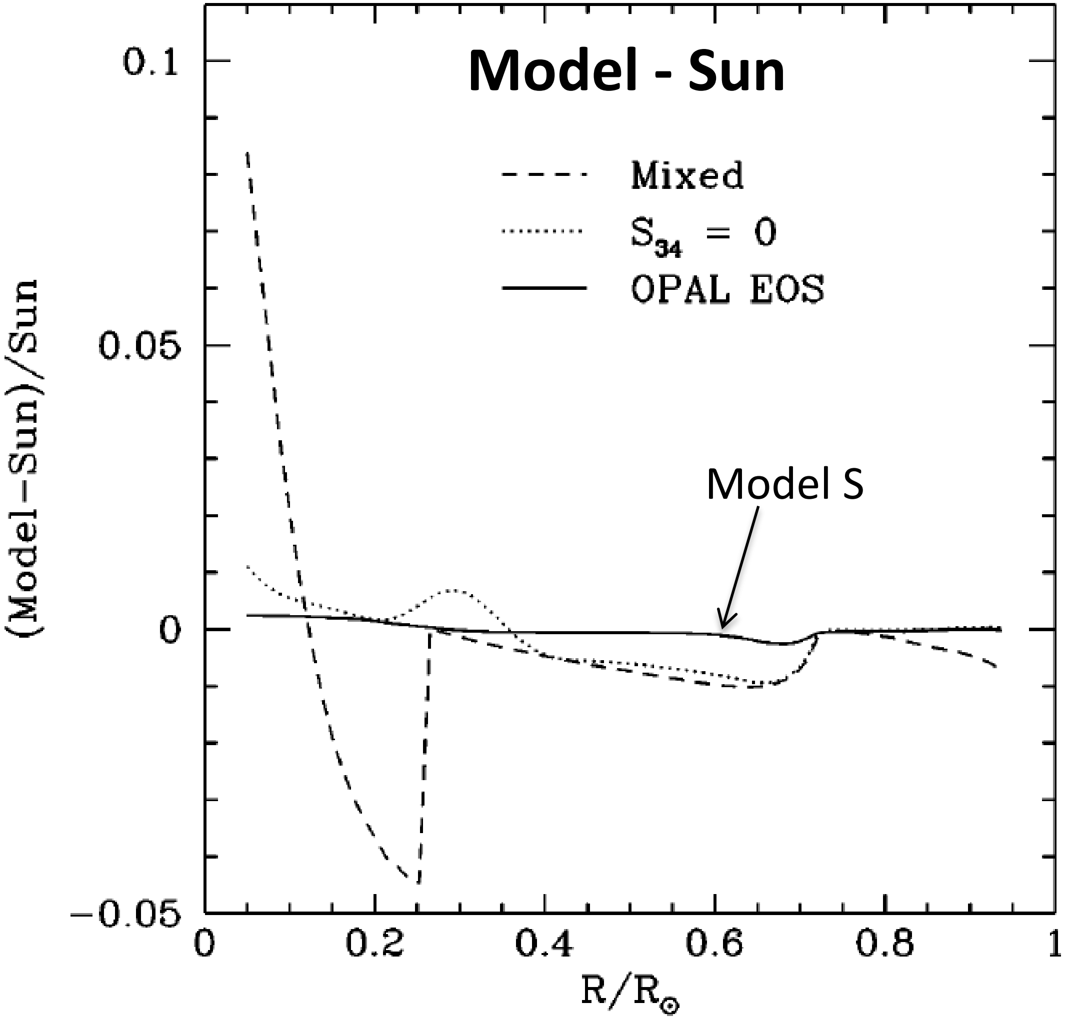
Mixing or switching off $^3He + {}^4He$



Bahcall et al. (1997; PRL 78, 171)

Errors in helioseismic inferences in determinations of solar structure

1. Observational errors
2. Departures from linearity in the relation between frequency and structure differences
3. Dependence on reference model
   - Systematic errors in reference model

Errors in helioseismic inferences in determinations of solar structure

1. Observational errors
2. Departures from linearity in the relation between frequency and structure differences
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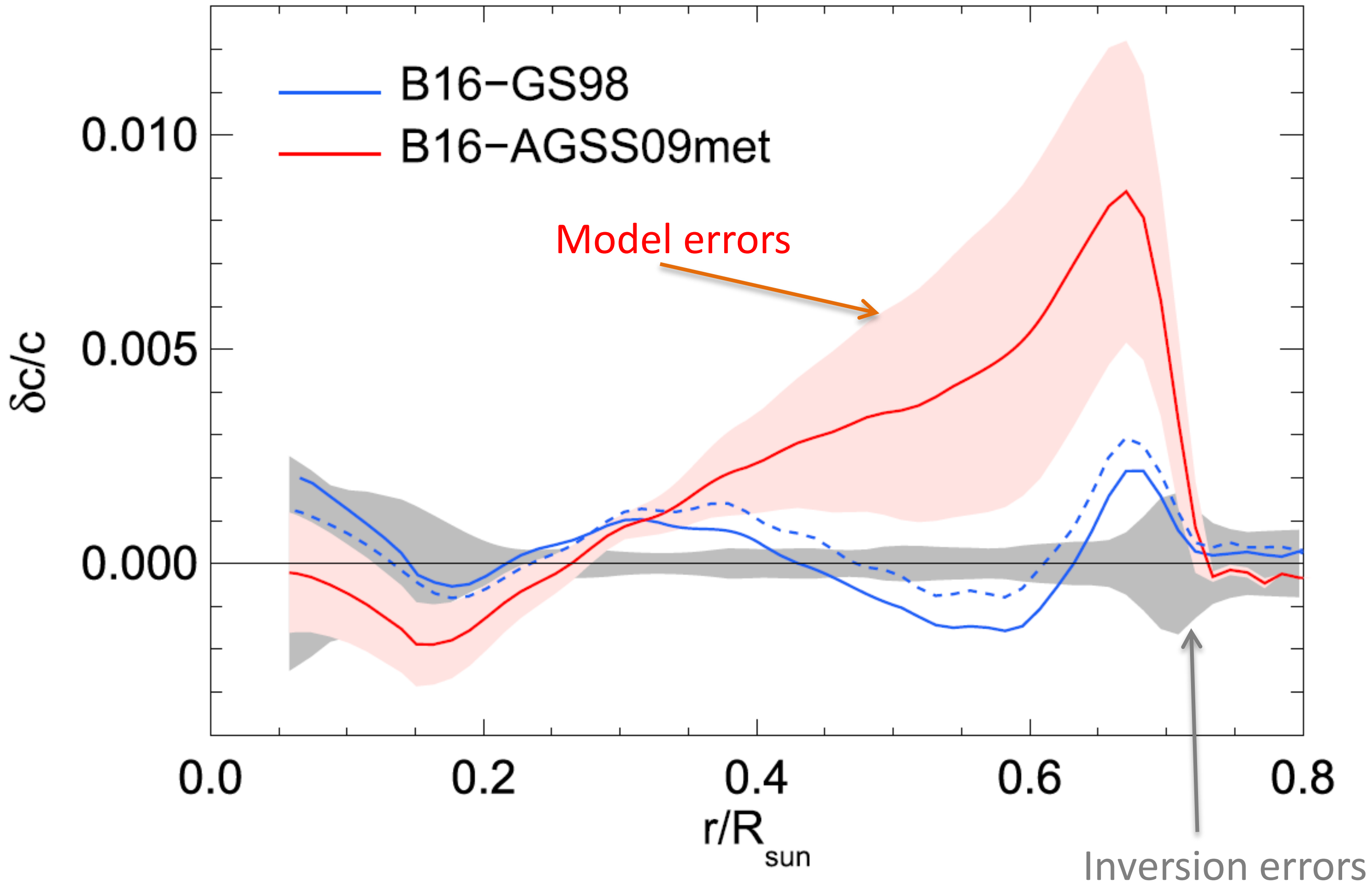
Note: in differences inferred from inversion only 1. and 2. matter

Errors in structure differences



Model errors

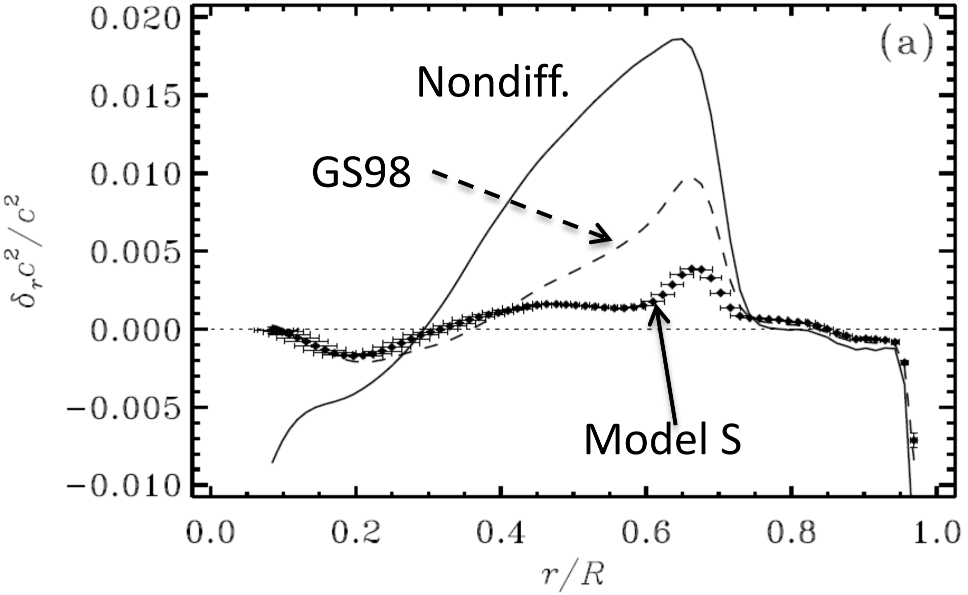
Inversion errors

Vinyoles et al. (2017; ApJ 835, 202)

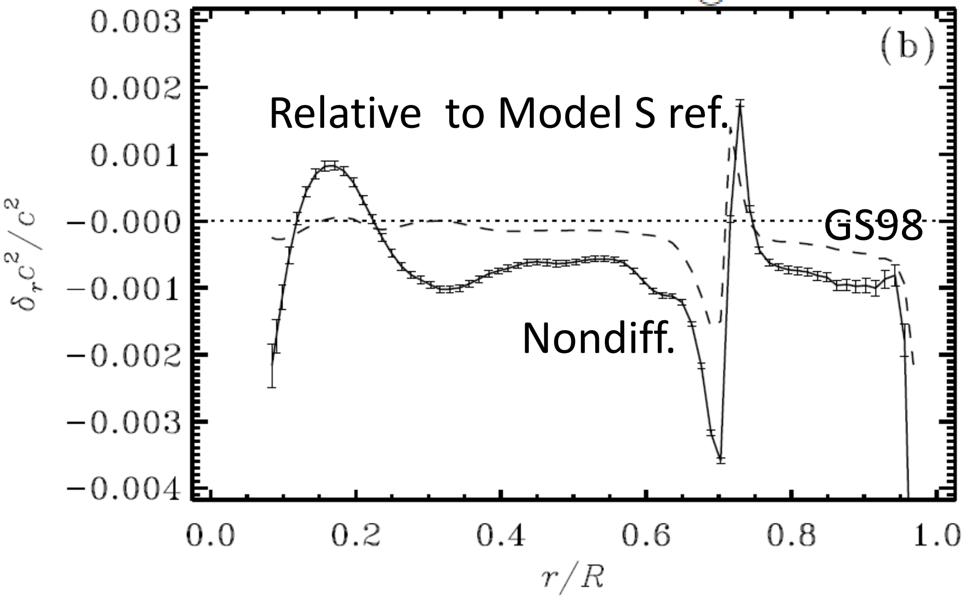
Effects of reference model on inferred solar sound speed

$$c_{\odot}^2 = c_{\mathrm{mod}}^2\left(1+\frac{\delta_r c^2}{c^2}\right)$$

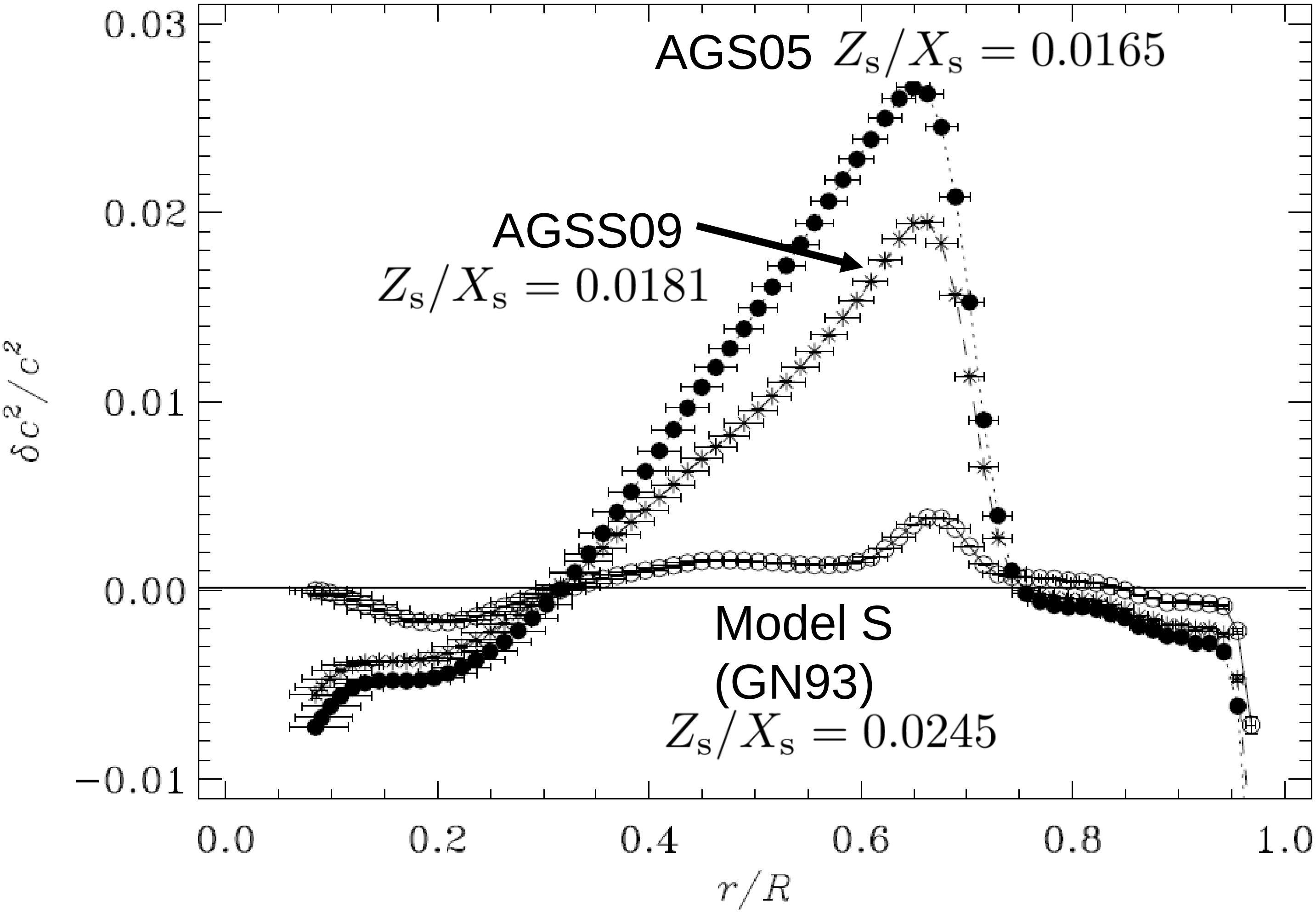
Inversion results



Differences in $c_{\odot}^2$



A new problem: revised solar composition



Left to Francesco Villante

Summary

- 1990 →: Models modified to reduce neutrino flux essentially ruled out by helioseismology
- 1995 → : ‘Standard’ solar models are confirmed by helioseismology, particularly for the core structure
- Status: With well-determined neutrino properties helioseismology and neutrinos are truly complementary

Conclusion, by Haxton

- “Effectively, the recent progress made on neutrino mixing angles and mass differences has turned the neutrino into a well-understood probe of the Sun.
- We now have two precise tools, helioseismology and neutrinos, that can be used to see into the solar interior.
- We have come full circle: The Homestake experiment was to have been a measurement of the solar core temperature, until the solar neutrino problem intervened.”

Haxton et al. (2013; ARAA 51, 21)