

What is a Parton Shower?

Zoltan Nagy
DESY

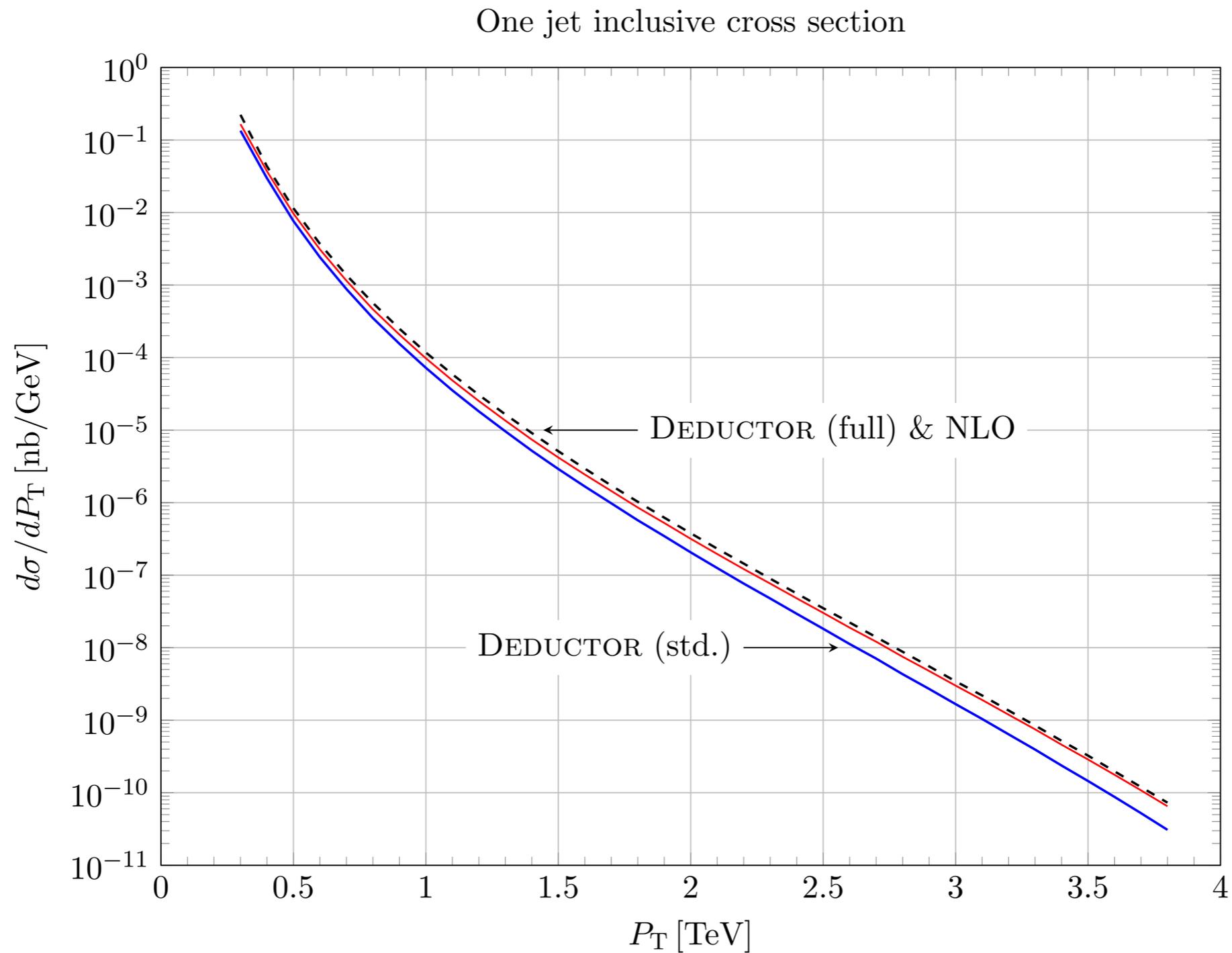
work with Dave Soper, University of Oregon

11th Annual Helmholtz Alliance Workshop on "Physics at the Terascale"
November 28, 2017, DESY-HH

Prequel

- Dave Soper and I have a parton shower event generator, DEDUCTOR.
 - Hadronization and underlying event models are not included.
 - Also, matching to NLO not included in the code.
- DEDUCTOR now includes summation of threshold logs

One jet cross section $d\sigma/dP_T$



Motivation for this talk

- Perturbative calculations at order α_s^k can be systematically improved by working to higher order.
- But what about parton showers?
- Are they just “QCD inspired” or do they fit into a scheme that can be systematically improved?

The statistical space

- Consider production of a Higgs boson plus QCD partons.
- Describe momenta and flavors for m final state partons with

$$\{p, f\}_m = \{\eta_a, a, \eta_b, b, p_H, p_1, f_1, \dots, p_m, f_m\}$$

- Use amplitudes $|M(\{p, f\}_m)\rangle$.
- This is a vector in spin and color space.
- Expand in color \otimes spin basis vectors $|\{s, c\}_m\rangle$.
- Also need conjugate amplitudes $\langle M(\{p, f\}_m)|$.
- Expand these in basis vectors $\langle\{s', c'\}_m|$.

- Describe the statistical state of an ensemble of simulations of the scattering.
- The spins and colors are quantum.
- So we need quantum statistical mechanics.
- Use the density operator

$$\rho(\{p, f\}_m) = \sum_{\{s, s', c, c'\}_m} \rho(\{p, f, s, s', c, c'\}_m) |\{s, c\}_m\rangle \langle \{s', c'\}_m|$$

$$\rho(\{p, f\}_m) = \sum_{\{s, s', c, c'\}_m} \rho(\{p, f, s, s', c, c'\}_m) |\{s, c\}_m\rangle \langle \{s', c'\}_m|$$

- Think of this as a function named ρ .
- The space of such functions is a linear vector space.
- Call this the statistical space.
- Call a vector $|\rho\rangle$.
- Everything happens in the statistical space.

$$\rho(\{p, f\}_m) = \sum_{\{s, s', c, c'\}_m} \rho(\{p, f, s, s', c, c'\}_m) |\{s, c\}_m\rangle \langle \{s', c'\}_m|$$

- Basis vectors $|\{p, f, s, s', c, c'\}_m\rangle$:

$$(\{p, f, s, s', c, c'\}_m | \rho) = \rho(\{p, f, s, s', c, c'\}_m)$$

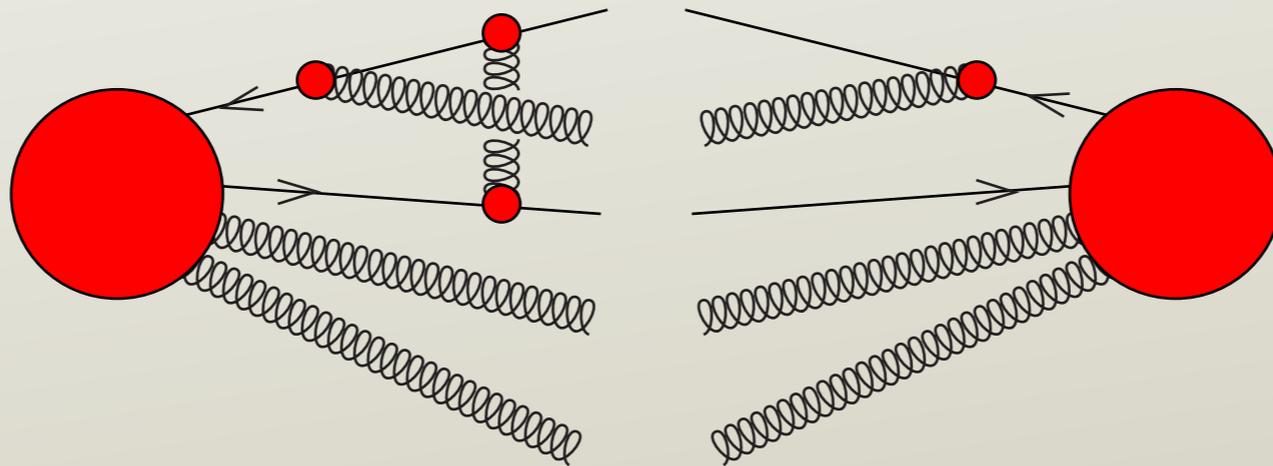
- Inclusive measurement

$$\begin{aligned} (1 | \rho) &= \sum_m \frac{1}{m!} \int [d\{p\}_m] \sum_{\{f\}_m} \sum_{\{s, s', c, c'\}_m} \langle \{s', c'\}_m | \{s, c\}_m \rangle \\ &\quad \times (\{p, f, s, s', c, c'\}_m | \rho) \end{aligned}$$

The infrared sensitive operator

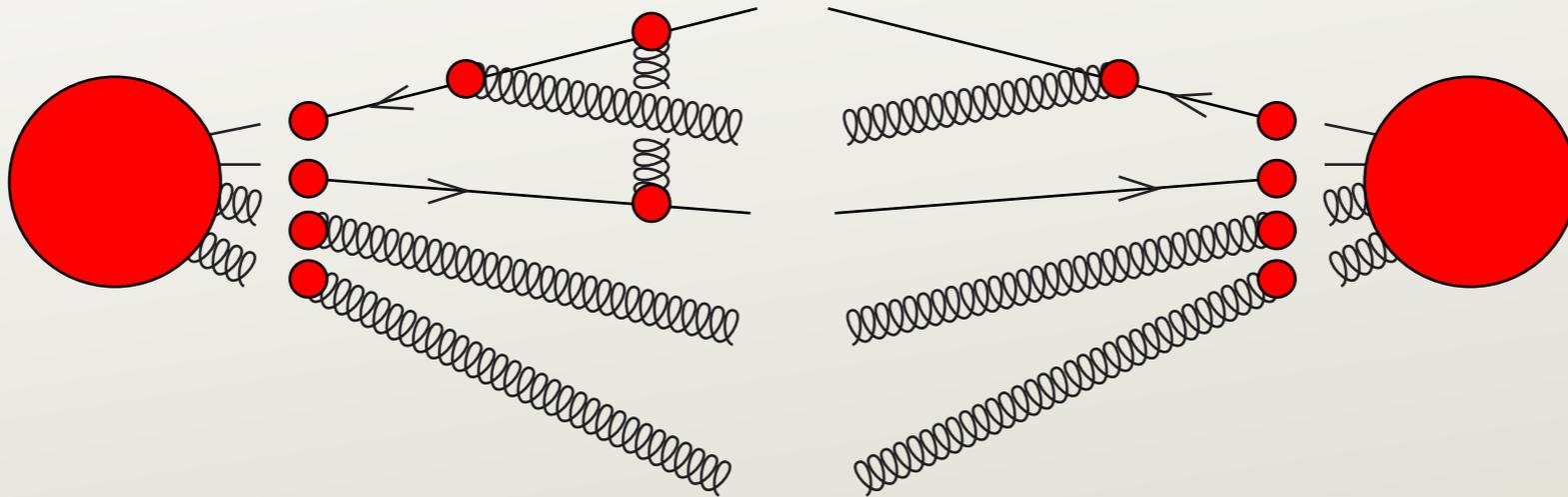
$$\mathcal{D}(\mu^2)$$

- Amplitudes have singularities when partons are soft or collinear.
- They have divergences $1/\epsilon$ from loops.



- We want to describe the singularity structure.
- Consider that everything inside the red subamplitudes is hard.

- Approximate the momenta coming from the hard part as fixed and on shell.



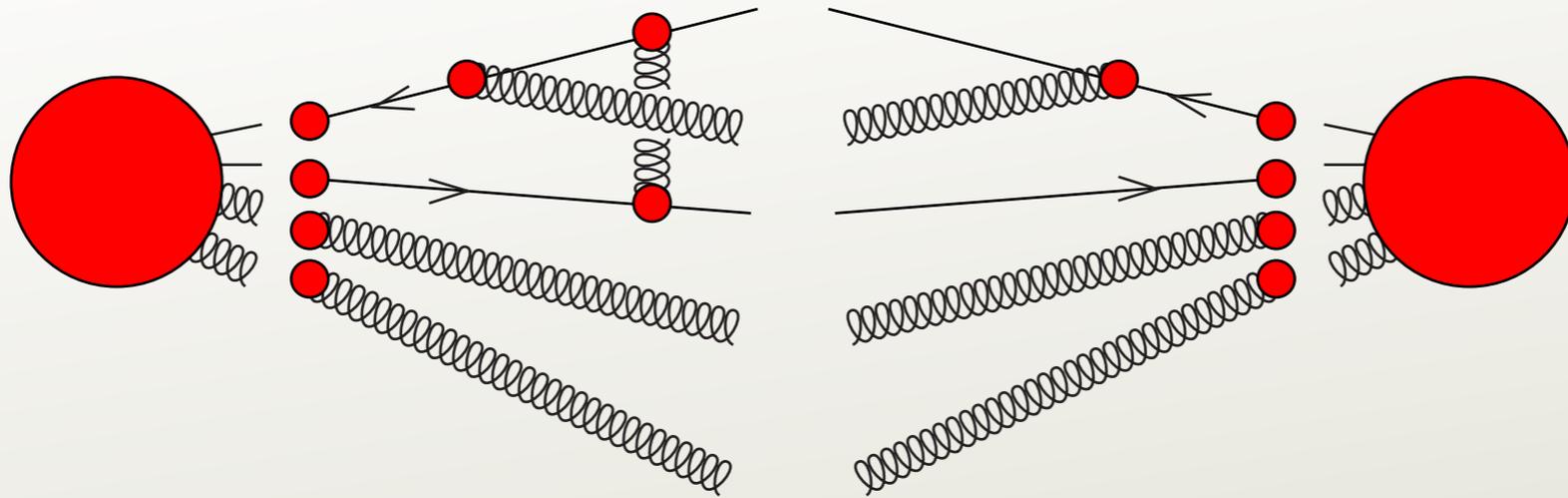
- This gives us an operator $\mathcal{D}(\mu^2)$.

$$\left(\{\hat{p}, \hat{f}, \hat{s}, \hat{s}', \hat{c}, \hat{c}'\}_{m+n} \mid \rho(\mu^2) \right)$$

$$\sim \frac{1}{m!} \int [d\{p\}_m] \sum_{\{f\}_m} \sum_{\{s, s', c, c'\}_m}$$

$$\times \left(\{\hat{p}, \hat{f}, \hat{s}, \hat{s}', \hat{c}, \hat{c}'\}_{m+n} \mid \mathcal{D}(\mu^2) \mid \{p, f, s, s', c, c'\}_m \right)$$

$$\times \left(\{p, f, s, s', c, c'\}_m \mid \rho_{\text{hard}}(\mu^2) \right)$$



- In $\mathcal{D}(\mu^2)$ we need an ultraviolet cutoff μ_s^2 .
- *e.g.* $\Lambda^2 < \mu_s^2$.
- We set $\mu_s^2 = \mu^2$.
- We also need a momentum mapping.
- We assume that $\mathcal{D}(\mu^2)$ is available and investigate what to do with it.

Standard perturbation theory

cancel initial state poles “Add back” the subtractions Subtractions

$$\sigma[J] = \left(1 \left| \left[\mathcal{F}_{\overline{\text{MS}}}(\mu_{\text{H}}^2) \circ \mathcal{Z}_F(\mu_{\text{H}}^2) \right] \mathcal{D}(\mu_{\text{H}}^2) \mathcal{D}^{-1}(\mu_{\text{H}}^2) \mathcal{O}_J \right| \rho(\mu_{\text{H}}^2) \right) + \mathcal{O}(\alpha_s^{k+1}) + \mathcal{O}(\mu_{\text{f}}^2/Q[J]^2)$$

Operator to measure desired cross section Feynman diagrams to α_s^k

- Normally $\mathcal{D}^{-1}(\mu_{\text{H}}^2)$ is constructed by hand and $\mathcal{D}(\mu_{\text{H}}^2)$ is its inverse.

- It is useful to rewrite our cross section a bit:

$$\sigma[J] = (1 | [\mathcal{F}_{\overline{\text{MS}}}(\mu_{\text{H}}^2) \circ \mathcal{Z}_F(\mu_{\text{H}}^2)] \mathcal{D}(\mu_{\text{H}}^2) \mathcal{D}^{-1}(\mu_{\text{H}}^2) \mathcal{O}_J | \rho(\mu_{\text{H}}^2))$$

becomes

$$\sigma[J] = (1 | [\mathcal{F}_{\overline{\text{MS}}}(\mu_{\text{H}}^2) \circ \mathcal{Z}_F(\mu_{\text{H}}^2)] \mathcal{O}_J \mathcal{D}(\mu_{\text{H}}^2) | \rho_{\text{H}})$$

where

$$|\rho_{\text{H}}) = \mathcal{D}^{-1}(\mu_{\text{H}}^2) |\rho(\mu_{\text{H}}^2))$$

Shower oriented parton distribution functions

- Standard $\overline{\text{MS}}$ parton distribution functions are not quite right for use in a shower.
- *E.g.* if you use k_T for the shower hardness parameter, $\overline{\text{MS}}$ is almost OK, but imposing a UV cut is not the same as subtracting a UV pole.
- Define

$$\mathcal{F}_{\overline{\text{MS}}}(\mu^2) = [\mathcal{F}(\mu^2) \circ \mathcal{K}(\mu^2)]$$

The inclusive infrared finite
operator $\mathcal{V}(\mu^2)$

- Define

$$\mathcal{X}(\mu^2) = [\mathcal{F}(\mu^2) \circ \mathcal{K}(\mu^2) \circ \mathcal{Z}_F(\mu^2)] \mathcal{D}(\mu^2) \mathcal{F}^{-1}(\mu^2)$$

- Then $(1|\mathcal{X}(\mu^2)|\{p, f, s, s', c, c'\}_m)$ is IR finite.
- Define an operator $\mathcal{V}(\mu^2)$ that leaves $\{p, f\}_m$ unchanged, with

$$(1|\mathcal{V}(\mu^2) = (1|\mathcal{X}(\mu^2)$$

- Then $\mathcal{V}(\mu^2)$ is IR finite.

The result

... without the derivation.

Operator to measure
desired cross section

parton distribution functions
and luminosity factor


$$\sigma[J] = (1 | \mathcal{O}_J \mathcal{U}(\mu_f^2, \mu_H^2) \mathcal{U}_V(\mu_f^2, \mu_H^2) \mathcal{F}(\mu_H^2) | \rho_H) \\ + \mathcal{O}(\alpha_s^{k+1}) + \mathcal{O}(\mu_f^2 / Q[J]^2)$$



Feynman diagrams to α_s^k
with subtractions

- The most important parts are $\mathcal{U}_V(\mu_f^2, \mu_H^2)$ and $\mathcal{U}(\mu_f^2, \mu_H^2)$.

$$\sigma[J] = (1 | \mathcal{O}_J \mathcal{U}(\mu_f^2, \mu_H^2) \mathcal{U}_V(\mu_f^2, \mu_H^2) \mathcal{F}(\mu_H^2) | \rho_H) \\ + \mathcal{O}(\alpha_s^{k+1}) + \mathcal{O}(\mu_f^2/Q[J]^2)$$

$$\mathcal{U}_V(\mu_f^2, \mu_H^2) = \mathcal{V}^{-1}(\mu_f^2) \mathcal{V}(\mu_H^2) \\ = \mathbb{T} \exp \left(\int_{\mu_f^2}^{\mu_H^2} \frac{d\mu^2}{\mu^2} \mathcal{S}_V(\mu^2) \right)$$

$$\mathcal{S}_V(\mu^2) = \mathcal{V}^{-1}(\mu^2) \mu^2 \frac{d}{d\mu^2} \mathcal{V}(\mu^2)$$

- Does not create new partons.
- Provides perturbative corrections to the hard scattering state $|\rho_H\rangle$.
- Sums threshold logarithms associated with $|\rho_H\rangle$.

$$\sigma[J] = (1 | \mathcal{O}_J \mathcal{U}(\mu_f^2, \mu_H^2) \mathcal{V}(\mu_f^2, \mu_H^2) \mathcal{F}(\mu_H^2) | \rho_H) \\ + \mathcal{O}(\alpha_s^{k+1}) + \mathcal{O}(\mu_f^2/Q[J]^2)$$

$$\mathcal{U}(\mu_f^2, \mu_H^2) = \mathcal{V}(\mu_f^2) \mathcal{X}^{-1}(\mu_f^2) \mathcal{X}(\mu_H^2) \mathcal{V}^{-1}(\mu_H^2) \\ = \mathbb{T} \exp \left(\int_{\mu_f^2}^{\mu_H^2} \frac{d\mu^2}{\mu^2} \mathcal{S}(\mu^2) \right)$$


$$\mathcal{S}(\mu^2) = \mathcal{V}(\mu^2) \mathcal{F}(\mu^2) \mathcal{D}^{-1}(\mu^2) \left[\mu^2 \frac{d}{d\mu^2} \mathcal{D}(\mu^2) \right] \mathcal{F}^{-1}(\mu^2) \mathcal{V}^{-1}(\mu^2) \\ - \left[\mu^2 \frac{d}{d\mu^2} \mathcal{F}(\mu^2) \right] \mathcal{F}^{-1}(\mu^2) - \left[\mu^2 \frac{d}{d\mu^2} \mathcal{V}(\mu^2) \right] \mathcal{V}^{-1}(\mu^2) .$$

- Creates new partons.
- Preserves probabilities: $(1 | \mathcal{U}(\mu_2^2, \mu_1^2) = (1 |$.

- What about the error estimate?

$$\sigma[J] = (1 | \mathcal{O}_J \mathcal{U}(\mu_f^2, \mu_H^2) \mathcal{U}_V(\mu_f^2, \mu_H^2) \mathcal{F}(\mu_H^2) | \rho_H) \\ + \mathcal{O}(\alpha_s^{k+1}) + \mathcal{O}(\mu_f^2 / Q[J]^2)$$

- There is a power suppressed error $\mathcal{O}(\mu_f^2 / Q[J]^2)$.
- Such an error is part of factorization.
- There is a perturbative error $\mathcal{O}(\alpha_s^{k+1})$.
- This is because we calculate only to order α_s^k .
- If \mathcal{O}_J involves different scales, say μ_H^2 and μ_L^2 we could have $[\alpha_s \log^2(\mu_H^2 / \mu_L^2)]^{k+1}$.
- We can hope that the shower sums the most important large logarithms and leaves us with a smaller error.
- Implementations will involve further approximations.

Summary

- Perturbative calculations

$$\begin{aligned}\sigma[J] = & \left(1 \left| \left[\mathcal{F}_{\overline{\text{MS}}}(\mu^2) \circ \mathcal{Z}_F(\mu^2) \right] \mathcal{D}(\mu^2) \mathcal{D}^{-1}(\mu^2) \mathcal{O}_J \right| \rho(\mu^2) \right) \\ & + \mathcal{O}(\alpha_s^{k+1}) + \mathcal{O}(\mu_f^2/Q[J]^2)\end{aligned}$$

can be systematically improved by working to higher order.

- Parton shower calculations

$$\begin{aligned}\sigma[J] = & \left(1 \left| \mathcal{O}_J \mathcal{U}(\mu_f^2, \mu_H^2) \mathcal{U}_V(\mu_f^2, \mu_H^2) \mathcal{F}(\mu_H^2) \right| \rho_H \right) \\ & + \mathcal{O}(\alpha_s^{k+1}) + \mathcal{O}(\mu_f^2/Q[J]^2)\end{aligned}$$

can be systematically improved by working to higher order.