Higher order QCD corrections to heavy quark form factors

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in collaboration with
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Physics at the Terascale, DESY, Hamburg, Germany

Plan of this talk

1. Preliminary

2. Computational details

3. Renormalization

4. Infrared structure

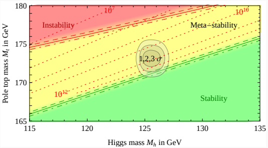
5. Results

6. Conclusion

Motivation

The heaviest SM particle
- probes the Higgs sector most

✓ - plays unique role in understanding
- the EW symmetry breaking



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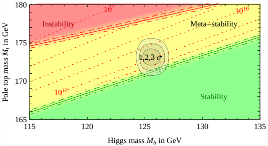

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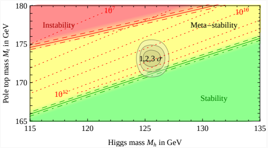

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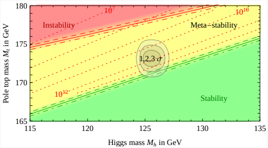

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✓ Decays almost exclusively to $t \to W^+ b$

$$t\bar{t} \to W^+ b W^- \bar{b} \quad \to \begin{cases} l\nu l\nu b\bar{b} \\ l\nu q\bar{q}' b\bar{b} \\ q\bar{q}' q\bar{q}' b\bar{b} \end{cases}$$

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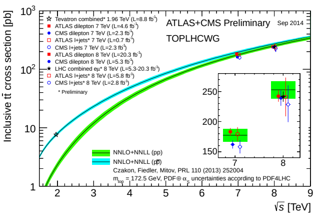
- ✓ Backgrounds also are known up to NNLO accuracy
- ✓ excellent agreement with NNLO+NNLL predictions at the LHC



ATLAS+CMS Sep'14

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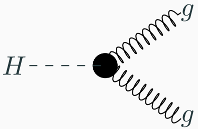
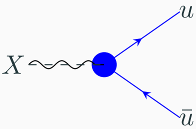
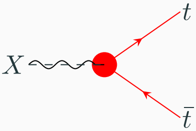
- ✓ One obtains many observables by proper analytic continuation of the total cross-section to different kinematical regions

Form factors

✓ The form factors are basic building blocks for many physical quantities







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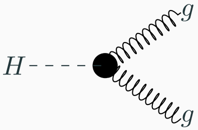

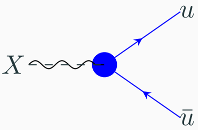

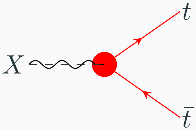

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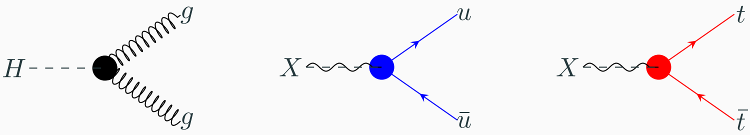

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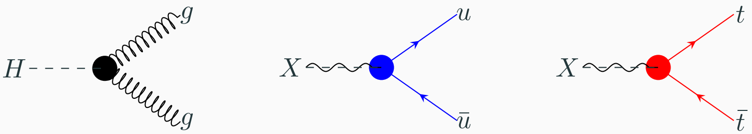
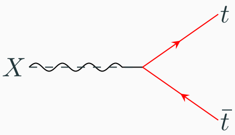
- ✓ They exhibit a universal infrared behavior - leads to information on anomalous dimensions
- ✓ The massive cusp anomalous dimension controls the infrared structure of massive form factors - studying the form factors helps in better understanding of the massive cusp
- ✓ Another important motive is to study high energy behavior of the massive form factors



Preliminary

The process

We consider decay of a color neutral massive particle to a pair of heavy quark of mass m.



Notation

$$X(q) \to t(q_1) + \bar{t}(q_2)$$

$$X = V, A, S, P$$

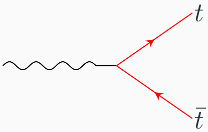
$$s = \frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

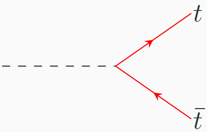
The general structure

Vector and Axial Vector

V: $-i\delta_{ij}v_Q\left(\gamma^\mu F_{V,1} + \frac{i}{2m}\sigma^{\mu\nu}q_\nu F_{V,2}\right)$

A: $-i\delta_{ij}a_Q\left(\gamma^\mu\gamma_5 F_{A,1} + \frac{1}{2m}q^\mu\gamma_5 F_{A,2}\right)$





Scalar and Pseudo Scalar

$-\frac{m}{v}\delta_{ij}\left[s_Q F_S + i p_Q \gamma_5 F_P\right]$

The form factors are expanded in the strong coupling constant as

$$F_I = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n F_I^{(n)}$$

To obtain $F_I^{(n)}$ $\Rightarrow$ appropriate projector on the amplitudes

$$P_{V,i} = \frac{i}{v_Q} \frac{\not{q}_2 - m}{m} \Big(\gamma_\mu g^1_{V,i} + \frac{1}{2m}(q_{2\mu} - q_{1\mu}) g^2_{V,i}\Big) \frac{\not{q}_1 + m}{m} \,,$$

$$P_{A,i} = \frac{i}{a_Q} \frac{\not{q}_2 - m}{m} \Big(\gamma_\mu \gamma_5 g^1_{A,i} + \frac{1}{2m}(q_{1\mu} + q_{2\mu}) \gamma_5 g^2_{A,i}\Big) \frac{\not{q}_1 + m}{m} \,,$$

$$P_S = \frac{v}{2ms_Q} \frac{\not{q}_2 - m}{m} \Big(g_S\Big) \frac{\not{q}_1 + m}{m} \,, \; P_P = \frac{v}{2mp_Q} \frac{\not{q}_2 - m}{m} \Big(i\gamma_5 g_P\Big) \frac{\not{q}_1 + m}{m} \,,$$

$g \equiv g(s,d)$ and are determined by demanding $F_I^{(0)} = 1$.

A study in literature

NLO

$F_{V,i}^{(1)}, F_{A,i}^{(1)}$ [Arbuzov, Bardin, Leike ’92; Djouadi, Lampe, Zerwas ’95]

$F_S^{(1)}, F_P^{(1)}$ [Braaten, Leveille ’80; Sakai ’80; Drees, Hikasa ’90]

partial NNLO

$F_{V,i}^{(2)}, F_{A,i}^{(2)}$ [Altarelli, Lampe ’93; Ravindran, van Neerven ’98; Catani, Seymour ’99]

$F_S^{(2)}, F_P^{(2)}$ [Gorishnii *et. al.* ’91; Chetyrkin, Kwiatkowski ’95; Harlander, Steinhauser ’97]

NNLO

$F_I^{(2)}$ [Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi ’04,’05]

$F_{V,i}^{(2)}(\mathcal{O}(\epsilon))$ [Gluza, Mitov, Moch, Riemann ’09]

Beyond NNLO

$F_{V,i}^{(3)}|_{\text{large N}}$ [Henn, Smirnov, Smirnov, Steinhauser ’16]

$F_{V,i}^{(4)}|_{\text{up to } \epsilon^{-2} \text{ poles at large N}}$ [Ahmed, Henn, Steinhauser ’17]

Goal

An important component of the N^3LO *contribution, is the* $\mathcal{O}(\epsilon)$ *piece at two-loop. In this talk, we present*

- *cross-check of the results available in the literature,*
- *computation of the integrals in different methods and one order higher in* ϵ*,*
- $F_I^{(2)}(\mathcal{O}(\epsilon^2))$ *for different currents and corresponding computational details.*

Computational details

The generic procedure

$d = 4 - 2\epsilon$

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- Decomposition of the dot products to obtain scalar integrals

$$\frac{2l.p}{l^2(l-p)^2} = \frac{l^2 - (l-p)^2 + p^2}{l^2(l-p)^2} = \frac{1}{(l-p)^2} - \frac{1}{l^2} + \frac{p^2}{l^2(l-p)^2}$$

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Master integrals (MIs)

- CRUSHER [Marquard, Seidel] for reduction to master integrals
- Computation of MIs : *Differential eqns. and Difference eqns.*

Computing the master integrals

The master integrals can be expressed as

$$J(\nu_1, \ldots, \nu_n) = \left((4\pi)^{2-\epsilon} e^{\epsilon\gamma_E}\right)^2 \int \frac{d^D l_1 d^D l_2}{(2\pi)^{2D}} \frac{1}{D_1^{\nu_1} \ldots D_n^{\nu_n}}$$

where for non-singlet case ($n = 7$)

$$D_1 = (l_1 + q_1)^2 - m^2, \quad D_2 = (l_2 + q_1)^2 - m^2, \quad D_3 = (l_1 - q_2)^2 - m^2,$$
$$D_4 = (l_2 - q_2)^2 - m^2, \quad D_5 = l_1^2, \quad D_6 = (l_1 - l_2)^2, \quad D_7 = (l_1 - l_2 + q_2)^2 - m^2.$$

and for singlet case ($n = 6$)

$$D_1 = (l_1 + q_1)^2, \quad D_2 = (l_2 + q_1)^2 - m^2, \quad D_3 = (l_1 - q_2)^2,$$
$$D_4 = (l_2 - q_2)^2 - m^2, \quad D_5 = l_1^2, \quad D_6 = (l_1 - l_2)^2 - m^2,$$

Using differential equations

- We obtain systems of coupled differential equations of the MIs by taking derivative $w.r.t.\ x$ and using IBP relations - diff. eqns. depend on the integrals from the same sector or sub-sectors.
- The systems appeared mostly in a block-triangular form except a few 2×2 coupled systems.

$$\partial_x \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ 0 & \bullet & \bullet & \bullet & \cdots & \bullet \\ 0 & \bullet & \bullet & \bullet & \cdots & \bullet \\ 0 & 0 & 0 & \bullet & \cdots & \bullet \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \bullet \end{bmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix} + \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ \vdots \\ R_n \end{pmatrix}$$

- To solve them, we consider the bottom-up approach - first solve the simplest sectors and move up in the chain of subsystems.

An example of a 2×2 coupled system

$$\frac{d}{dx}\begin{pmatrix} J_{22} \\ J_{23} \end{pmatrix} = \begin{bmatrix} \frac{1+x^2}{x(1-x^2)} & \frac{1-x^2}{x^2} \\ -\frac{1}{1-x^2} & \frac{4(1+x^2)}{x(1-x^2)} \end{bmatrix} \begin{pmatrix} J_{22} \\ J_{23} \end{pmatrix} + \begin{pmatrix} R_1(\varepsilon, x) \\ R_2(\varepsilon, x) \end{pmatrix},$$

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Decouple it to obtain a 2^{nd} order non-homogeneous differential eqn.

$$\frac{d^2 J_{22}}{dx^2} + p(x)\frac{dJ_{22}}{dx} + q(x)J_{22} = r(x)\,; \quad J_{23} = p'(x)\frac{dJ_{22}}{dx} + q'(x)J_{22} + r'(x)$$

An example of a 2 × 2 coupled system

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Solve the homogeneous part : solutions $y_1(x)$ & $y_2(x)$

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Solve the homogeneous part : solutions $y_1(x)$ & $y_2(x)$

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Use variation of constant to obtain the solution

$$J_{22} = y_1(x)\left[C_1 - \int dx \frac{r(x)y_2(x)}{W(y_1, y_2)}\right] + y_2(x)\left[C_2 + \int dx \frac{r(x)y_1(x)}{W(y_1, y_2)}\right]$$

$W(y_1, y_2)$ is Wronskian of the system.

Boundary conditions are fixed by imposing regularity of the integrals in the limit of vanishing space-like momentum $q^2 \to 0$ $i.e.$ $x \to 1$.

However, for few integrals, there exists a branch cut at $x = 1$. In that case, we match the general solution with the asymptotic expansion around $x \to 1$.

Using difference equations

The idea : write integrals in series expansion & use differential eqns to obtain difference eqns for coefficients of the series expansions.

- In the non-singlet case, integrals are regular around $x = 1$, hence they can be expanded around $y = 1 - x$.

$$J_i(y) = \sum_{n=0}^{\infty} \sum_{j=-2}^{r} \epsilon^j C_{i,j}(n) y^n$$

- For singlet case, there are logarithms of the type $\ln(y)$

$$J_i(y) = \sum_{n=0}^{\infty} \sum_{k=0}^{3} \sum_{j=-2}^{r} \epsilon^j C_{i,j,k}(n) \ln^k(y) y^n$$

Algorithm

- Consider an integral J_1 and assume the integrals which belonged to it's sub-topology, are known.
- Insert generic expanded form of J_1 in corresponding differential eqn.
- One obtains a system of eqns. with $C_{1,j,k}(n), j, k, n$ after equating each power of $\epsilon^j \ln^k(y) y^n$ on both sides.
- $C_{1,j,k}(n)$ can be obtained by solving the system iteratively.

All of this are done automatically using

`Sigma` [Schneider '01-], `EvaluateMultiSums`,
`SumProduction` [Ablinger, Blümlein, Hasselhuhn, Schneider '10-]
and `HarmonicSums` [Ablinger, Blümlein, Schneider '10,'13]

The results are obtained in terms of harmonic sums and generalized harmonic sums and after performing the sums, in terms of HPLs.

Renormalization

We consider a hybrid scheme for UV renormalization.

Heavy quark mass and wave function $(Z_{m,\mathrm{OS}}, Z_{2,\mathrm{OS}})$:	On-shell
QCD strong coupling constant (Z_{a_s}) :	$\overline{MS}$

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The renormalization of $F_{V,i}$ and F_S is straightforward

$$F_{V,i} = Z_{2,\mathrm{OS}} \hat{F}_{V,i} \qquad F_S = Z_{m,\mathrm{OS}} Z_{2,\mathrm{OS}} \hat{F}_S$$

But, presence of γ_5 in $F_{A,i}$ and F_P, makes it complicated. Based on the appearance of γ_5 in the γ-chain, F_I can be of two types :

non-singlet : open fermion lines are attached to chiral vertex
singlet : a fermion loop is attached to chiral vertex

Renormalization of the non-singlet pieces of $F_{A,i}$ and F_P is similar

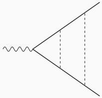
$$F^{ns}_{A,i} = Z_{2,\mathrm{OS}} \hat{F}^{ns}_{A,i} \qquad F^{ns}_P = Z_{m,\mathrm{OS}} Z_{2,\mathrm{OS}} \hat{F}^{ns}_P$$

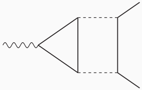
$F^{s,(2)}_{A,1}$ has a UV pole and can be removed by Z_J. The finite parts for singlet pieces are effected by the prescription used for γ_5. We use the prescription by Larin, and according to it, one can add a finite renormalization constant to maintain the anomalous Ward identity. But, to keep in mind, Ward identities are true for physical quantities and hence does not make sense to study them higher order in ϵ.

A note on γ_5

non-singlet : both γ_5 are in same γ-chain

We use an anti-commuting γ_5 in d-dimension with $\gamma_5^2 = 1$





singlet : two γ_5 are in different γ-chain

$$\gamma_5 = \frac{1}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$ [Larin '93]

The product of two ε tensors is expressed as determinant over metric tensors in d-dimensions.

Ward identities

Chiral Ward identity $$\boxed{q_\mu \Gamma_A^{\mu,ns} = 2m\Gamma_P^{ns}}$$

$$2F_{A,1}^{ns} + \frac{1}{2}\left(-\frac{(1-x)^2}{x}\right) F_{A,2}^{ns} = 2mF_P^{ns}$$

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Anomalous Ward identity $$q_\mu \Gamma_A^{\mu,s} = 2m\Gamma_P^{s} - i\frac{\alpha_s}{4\pi}T_F\langle G\tilde{G}\rangle_Q$$

$\langle G\tilde{G}\rangle_Q$ denotes the truncated matrix element of the gluonic operator $G\tilde{G}$ between the vacuum and an on-shell heavy quark pair ($Q\bar{Q}$).

Infrared structure

The infrared singularities factorize as a multiplicative factor

[Becher, Neubert '09]

$$F_I(\epsilon, x) = Z(\epsilon, x, \mu) F_I^{fin}(x, \mu)$$

$Z(\epsilon, x, \mu)$ is universal/independent of current
$F_I^{fin}(x, \mu)$ is finite as $\epsilon \to 0$

Renormalization group evolution of $Z(\epsilon, x, \mu)$ provides

$$Z(\epsilon, x, \mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left[\frac{\Gamma_0}{2\epsilon}\right] + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\frac{1}{\epsilon^2}\left(\frac{\Gamma_0^2}{8} - \frac{\beta_0\Gamma_0}{4}\right) + \frac{1}{\epsilon}\left(\frac{\Gamma_1}{4}\right)\right]$$

Γ_n is the n^{th} order massive cusp anomalous dimension.

[Korchemsky, Radyushkin '87, '92; Grozin, Henn, Korchemsky, Marquard '14, '15]

Results

Results & Checks

We have obtained $F_I^{(2)}$ up to $\mathcal{O}(\epsilon^2)$

$$F_{V,1}^{(2)}, F_{V,2}^{(2)}, F_{A,1}^{(2)}, F_{A,2}^{(2)}, F_{S}^{(2)}, F_{P}^{(2)}$$

- ✓ $F_I^{(2)}$ up to $\mathcal{O}(\epsilon^0)$ matches with the results from Bernreuther $et\,al.$
  (up to an overall factor due to different scheme)
- ✓ $F_{V,1}^{(2)}$ up to $\mathcal{O}(\epsilon)$ matches with the result from Gluza $et\,al.$
  (except a difference[1] of $-C_F C_A\left[\epsilon\left\{\frac{1037x^3}{(1+x)^6}\right\}\right]$)
- ✓ $F_{V,1}^{(2)}$ up to $\mathcal{O}(\epsilon^2)$ matches with the result from Henn $et\,al.$ (color-planar limit)
- ✓ Chiral Ward identity: relating non-singlet parts of Axial-vector and Pseudo-scalar
- ✓ Anomalous Ward identity: relating the singlet parts

[1]earlier noted by Henn $et\,al.$ for color-planar limit.

Interesting facts

★ Two constants appear in the two-loop result

$$c_1 = 12\zeta_2 \ln^2(2) + \ln^4(2) + 24\mathrm{Li}_4(1/2)$$
$$c_2 = 26\zeta_2^2 \ln(2) - 20\zeta_2 \ln^3(2) - \ln^5(2) + 120\mathrm{Li}_5(1/2)$$

★ Around 300 HPLs with alphabet $\{-1, 0, 1\}$ up to weight 6 appear

The independent HPLs up to $\mathcal{O}(\epsilon)$ are

H_{-1}, H_0, H_1, $H_{-1,1}$, $H_{0,-1}$, $H_{0,1}$, $H_{0,-1,-1}$, $H_{0,-1,1}$, $H_{0,0,-1}$, $H_{0,0,1}$, $H_{0,1,-1}$, $H_{0,1,1}$, $H_{0,-1,-1,-1}$, $H_{0,-1,-1,1}$, $H_{0,-1,0,1}$, $H_{0,-1,1,-1}$, $H_{0,-1,1,1}$, $H_{0,0,-1,-1}$, $H_{0,0,-1,1}$, $H_{0,0,0,-1}$, $H_{0,0,0,1}$, $H_{0,0,1,-1}$, $H_{0,0,1,1}$, $H_{0,1,-1,-1}$, $H_{0,1,-1,1}$, $H_{0,1,1,-1}$, $H_{0,1,1,1}$, $H_{0,-1,-1,0,1}$, $H_{0,-1,0,-1,-1}$, $H_{0,-1,0,-1,1}$, $H_{0,-1,0,1,-1}$, $H_{0,-1,0,1,1}$, $H_{0,-1,1,0,1}$, $H_{0,0,-1,-1,-1}$, $H_{0,0,-1,-1,1}$, $H_{0,0,-1,0,-1}$, $H_{0,0,-1,0,1}$, $H_{0,0,-1,1,-1}$, $H_{0,0,-1,1,1}$, $H_{0,0,0,-1,-1}$, $H_{0,0,0,-1,1}$, $H_{0,0,0,0,-1}$, $H_{0,0,0,0,1}$, $H_{0,0,0,1,-1}$, $H_{0,0,0,1,1}$, $H_{0,0,1,-1,-1}$, $H_{0,0,1,-1,1}$, $H_{0,0,1,0,-1}$, $H_{0,0,1,0,1}$, $H_{0,0,1,1,-1}$, $H_{0,0,1,1,1}$, $H_{0,1,0,1,-1}$, $H_{0,1,0,1,1}$

Form factors at various kinematical regions

Low energy region $q^2 \ll m^2$ or $x \to 1$

We redefine x as $x = e^{i\phi}$ and expand around $\phi = 0$ up to 4^{th} order.
Note that, for $\phi = 0$ $F_{V,1} = 1$, $F_{V,2} =$ Anomalous magnetic moment

High energy region $q^2 \gg m^2$ or $x \to 0$

We expand up to $\mathcal{O}(x^4)$. In the massless limit ($x = 0$),

- the chirality flipping form factors $F_{V,2}$ & $F_{A,2}$ vanishes.
- $F_{V,1}$ is equal to $F_{A,1}$, as expected
- F_S is equal to F_P too

Threshold region $q^2 \sim 4m^2$ or $x \to -1$

We define $\beta = \sqrt{1 - \frac{4m^2}{q^2}}$ and expand around $\beta = 0$ up to $\mathcal{O}(\beta^2)$
useful for applications *e.g.* $e^+e^- \to t\bar{t}$ near threshold

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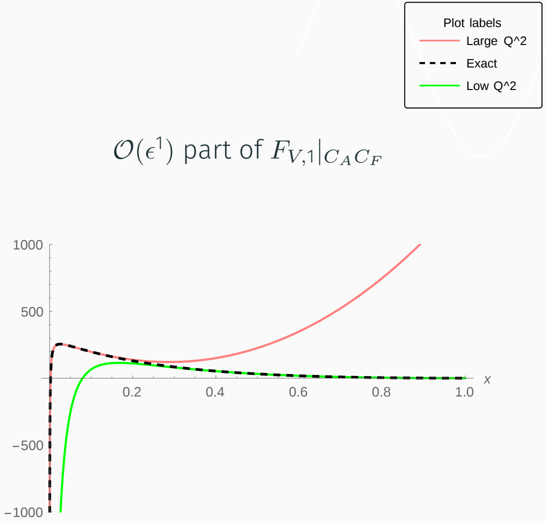
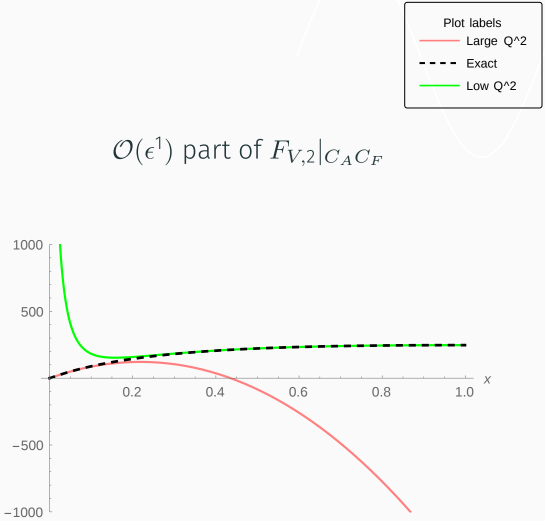
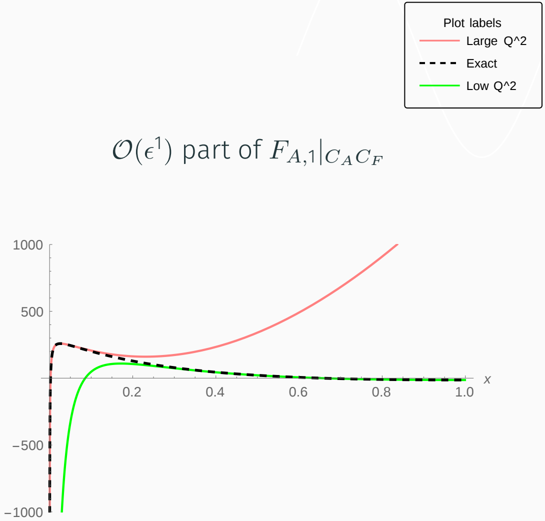


$\mathcal{O}(\epsilon^1)$ part of $F_{V,1}|_{C_A C_F}$

$\mathcal{O}(\epsilon^1)$ part of $F_{V,2}|_{C_A C_F}$



$\mathcal{O}(\epsilon^1)$ part of $F_{A,1}|_{C_A C_F}$



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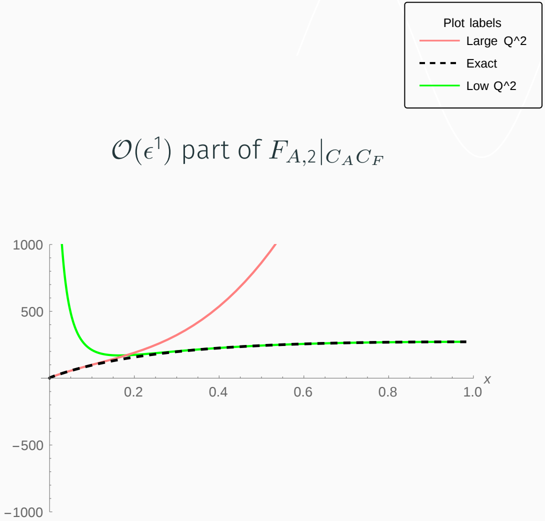
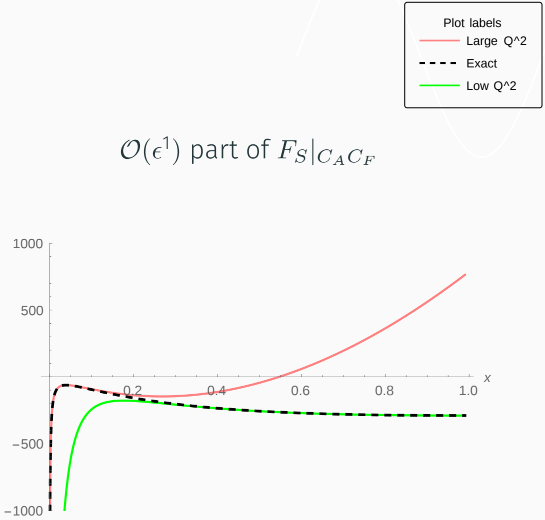


$\mathcal{O}(\epsilon^1)$ part of $F_S|_{C_A C_F}$



Conclusion

Summary

- We have obtained two-loop corrections to heavy quark form factors for different currents up to $\mathcal{O}(\epsilon^2)$. They are essential elements to higher order corrections.
- We computed the master integrals using two techniques - namely method of differential eqns and difference eqns.
- For the non-singlet contributions, γ_5 is implemented following the pragmatic approach (anti-commutation), whereas for the singlet contributions, we have followed the prescription by Larin (adapted from the 't Hooft Veltman prescription).

Thank You!