Event shapes and resummation

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1 Event shapes and resummation

For the sake of reliable measurements at present and future colliders, the use of precise QCD calculations is mandatory. Fixed-order calculations discussed in Sec. [1] are accurate enough to predict inclusive observables, such as total cross sections or widths, whereas more exclusive quantities, such as event-shape distributions, exhibit large logarithmic enhancements, corresponding to soft- or collinear-parton radiation, which need to be resummed to all orders to improve the perturbative prediction. Analytical resummation of soft/collinear-enhanced radiation can be performed following the general method in [2–4]. Such resummations are usually based on the approximation of multiple independent emissions, implying factorization of amplitudes and phase spaces, and resulting in the exponentiation of soft/collinear single-parton radiation.

In the following we describe recent progress in the understanding and development of such resummations, including a critical comparison of analytical resummations with partons shower resummations, a discussion of non-global logarithms and recent extraction of the strong coupling using newly available NLLA+NNLO matched predictions.

2 Parton showers and resummations for non-global QCD observables

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Resummation of soft and collinear logarithms are usually based on the approximation of multiple independent emissions, implying factorization of amplitudes and phase spaces, and resulting in the exponentiation of soft/collinear single-parton radiation. In fact, a resummed quantity $\Sigma(L)$, L being a large logarithm of soft or collinear origin, typically reads:

$$\Sigma(L) = \exp\left[Lg_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(L) + \dots\right], \tag{1}$$

where Lg_1 resums the double logarithms, *i.e.* both soft and collinear, $\mathcal{O}(\alpha_S^n L^{n+1})$, while g_2 resums single logarithms $\mathcal{O}(\alpha_S^n L^n)$, either soft or collinear, and so forth. Contributions $\sim \alpha_S^n L^{n+1}$ and $\sim \alpha_S^n L^n$ are typically classified as leading- (LL) and next-to-leading (NLL) logarithms. However, as we shall point out later on, if g_1 is zero, the LLs will be the ones contained in g_2.

As an alternative tool to resum large logarithms, one can employ Monte Carlo generators, such as HERWIG [5] or PYTHIA [6], which implement parton showers in the soft/collinear approximation and include models for hadronization and the underlying event. In particular, the evolution variable for the HERWIG showers is equivalent, for soft emissions, to angular ordering [7, 8], which is a reliable approximation in the large-N_C limit for azimuthally-averaged quantities. PYTHIA traditionally orders its cascades according to the virtuality of the splitting parton, with the possibility to reject non-angular-ordered showers. Lately, a new PYTHIA shower model [9] was released, ordering multiple emissions according to the transverse momentum of the radiated parton with respect to the emitter's direction. Monte Carlo algorithms are correct up to the double-logarithmic function g_1 and in some cases they can even account for g_2 (see, *e.g.*, [10] for some discussions on comparing parton showers and resummations).

In the following, we shall discuss the so-called non-global observables and compare the results of resummed calculations, with the possible inclusion of the angular-ordering approximation, with those given by Monte Carlo parton showers.

2.0.1 *Non-global observables*

It was recently found out [11] that for some quantities, called non-global observables, as they are sensitive to radiation in a limited region of the phase space, the independent-emission approximation is not sufficient any longer, even at LL level. As a case study, we consider e^+e^- annihilation into hadrons at the centre-of-mass energy Q and study the transverse-energy flow in an angular region Ω, a limited region in rapidity η and azimuth ϕ:

$$\Sigma(Q,Q_\Omega)=\frac{1}{\sigma}\int_0^{Q_\Omega}dE_t\frac{d\sigma}{dE_t}\ ;\ E_t=\sum_{i\in\Omega}E_{ti}. \tag{2}$$

Σ was computed in [12] and reads:

$$\Sigma(Q,Q_\Omega)=\exp(-4C_FA_\Omega t)S(t), \tag{3}$$

with

$$A_\Omega=\int d\eta\frac{d\phi}{2\pi}\ ;\ \ t=\frac{1}{2\pi}\int_{Q_\Omega}^{Q/2}\frac{dk}{k}\alpha_S(k). \tag{4}$$

In Eq. (3), the contribution $\sim\exp(-4C_FA_\Omega t)$ comes after exponentiating single-gluon radiation from the primary $q\bar{q}$ pair, which constitutes the Born event, whereas $S(t)$ includes non-global logarithms, due to correlated parton emission in the Ω region. The lowest-order contribution to $S(t)$ goes as $\alpha_S^2S_2\ln^2(Q/Q_\Omega)$, with $S_2\sim C_AC_F$. S_2 was calculated exactly, while the function $S(t)$ was computed at all orders in the LL approximation and in the large-N_C limit, by using the evolution algorithm presented in [11]. We point out that, for an observable like Σ, the function g_1 in Eq. (1) is zero, hence the leading logarithms are just $\sim\alpha_S^nL^n$: including the non-global function $S(t)$ is therefore necessary to fully account for LLs.

As in Ref. [13], we wish to investigate whether implementing angular ordering in the evolution algorithm of [11] still leads to acceptable results for $\Sigma(Q,Q_\Omega)$ and $S(t)$. In Fig. 1 we present the leading-order non-global coefficient, $-S_2/(C_FC_A)$, according to the full calculation and the angular-ordering approximation, in case Ω is a rapidity slice of width $\Delta\eta=2.5$. We also show the cross section $\Sigma(t)$ yielded by the full leading-log resummed calculation and in the angular-ordering (AO) approximation. For the sake of comparison, we also present the contribution coming from just exponentiating primary single-parton emission.

From Fig. 1 (left), we learn that for small gap sizes the full and AO results agree, while they start to differ once the gap is increased. In both cases, S_2 saturates for large $\Delta\eta$, with the AO result being about 10% lower than the full one. As for $\Sigma(t)$, the AO approximation is indeed able to include significant part of the full result, whereas the primary-emission contribution lies far above the two other predictions, thus giving unreliable spectra. Considering, *e.g.*, $t=0.15$, corresponding to $Q=100$ GeV and $Q_\Omega=1$ GeV, the AO and primary results are 10% and 75% above the full one, respectively. It was also shown in Ref. [13] that the results for the non-global function $S(t)$ are roughly independent of the size of the rapidity gap.

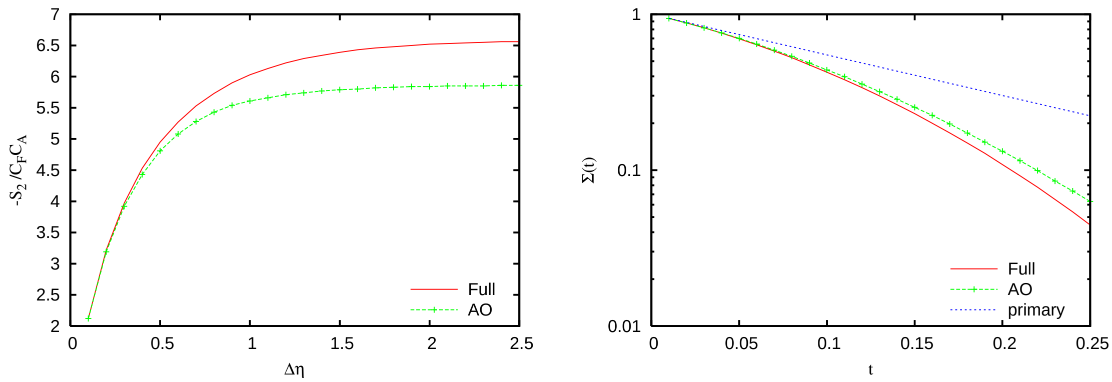

Fig. 1: Left: Function $S(t)$ at leading-order according to the full LL calculation and in the angular-ordering approximation, in terms of the rapidity gap $\Delta\eta$. Right: Function $\Sigma(t)$ according to the full resummed calculation and the angular-ordering approximation. Also shown is the result coming from the exponentiation of primary-emission contributions.

2.0.2 Comparison with HERWIG and PYTHIA

In this section we compare the results of the resummed calculation with the ones yielded by the Monte Carlo programs HERWIG and PYTHIA. As in [13], we study e^+e^- annihilation at the centre-of-mass energy $Q = 10^5$ GeV. In fact, we chose such a high value of Q in order to kill subleading effects, weighted by $\alpha_S(Q)$ or suppressed by powers of $1/Q$, such as subleading soft/collinear logarithms, quark mass effects, hadronization corrections. Furthermore, we checked that our results depend only on the dimensionless variable t in Eq. (4), so that our findings for a given value of t can be easily translated to any value of the centre-of-mass energy.

In Fig. 2 we present the differential cross section $1/\sigma\ (d\sigma/dE_t)$ for the transverse-energy flow in a rapidity gap $\Delta\eta = 1$, according to the resummed result, matched to the exact NLO as in [12], and according to HERWIG and PYTHIA. In the resummation, we show the full result, the angular-ordering approximation and the primary-emission contribution. As for PYTHIA, we present the spectra obtained running the old and new models, with showers ordered in virtuality and transverse momentum, respectively. When using the old model, we shall always assume that non-AO radiation is vetoed.

As for the comparison with HERWIG, whose showers are ordered in angle, we observe good agreement with both AO and full results for $E_t > 10$ GeV, while the primary-radiation contribution exhibit relevant discrepancies. As for PYTHIA, the new model, ordered in transverse momentum, is in good agreement with the resummation, leading to results similar to HERWIG. On the contrary, a visible disagreement is present between the old PYTHIA model and the resummed curves. In fact, as discussed in [9], evolution in transverse momentum leads to a better treatment of angular ordering with respect to virtuality ordering. Comparing the spectra at $E_t = 10$ GeV, the discrepancies with respect to the full resummed result amount to -10% for HERWIG, $+7.5\%$ for the new PYTHIA model and -50% for the old PYTHIA.

In Fig. 3 we instead compare HERWIG, PYTHIA and the resummation for a rapidity slice $\Delta\eta = 3$. As in Fig. 2, HERWIG is in reasonable agreement with the resummed computation for $E_t > 10$ GeV and the old PYTHIA model lies quite far from the other curves throughout all

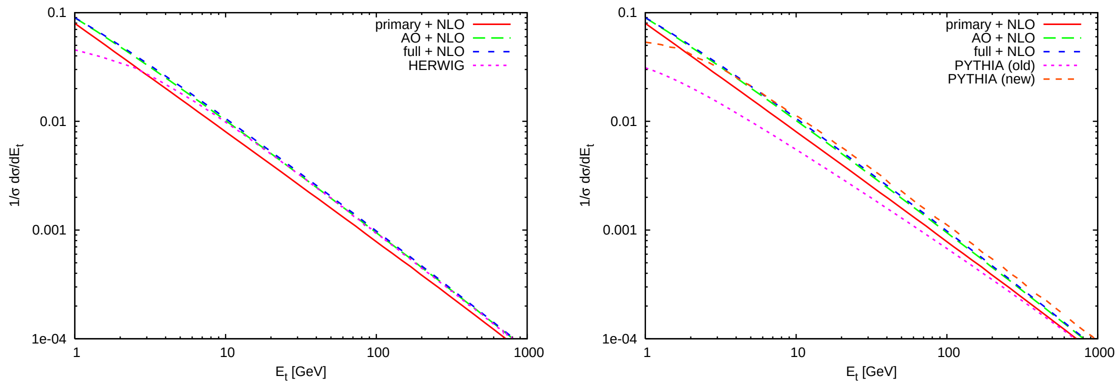

Fig. 2: Comparison of full, AO and primary resummed results with HERWIG (left) and PYTHIA (right) for $\Delta\eta = 1$ and $Q = 10^5$ GeV. As for PYTHIA, we show the spectra yielded by the old and new models, where parton showers are ordered in virtuality and transverse momentum, respectively.

E_t-range. However, unlike the $\Delta\eta = 1$ case, even the spectrum obtained with the new PYTHIA model exhibits a meaningful discrepancy for $E_t > 100$ GeV, which might signal that perhaps even the new PYTHIA ordering variable is not completely adequate to describe non-global observables at large rapidity slices. A more detailed investigation of this issue is mandatory.

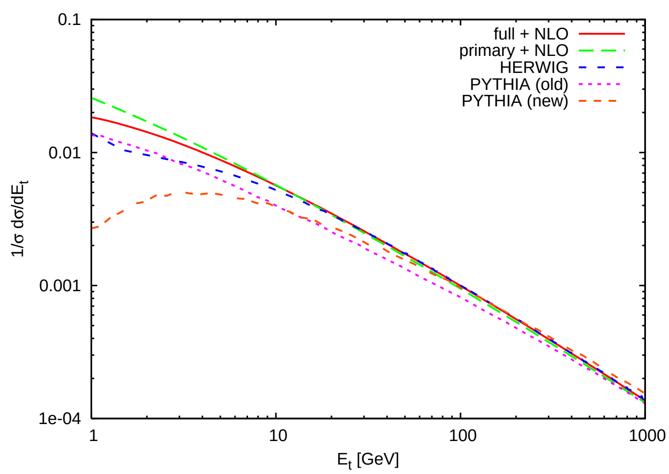

Fig. 3: Transverse-energy spectrum in a rapidity gap $\Delta\eta = 3$, according to the resummed calculation, HERWIG and PYTHIA (old and new models).

2.0.3 Conclusions

We studied non-global observables, namely the transverse-energy flow in a rapidity gap, and investigated the role played by angular ordering in the leading-logarithmic resummation. We found that the angular-ordering approximation indeed includes the bulk of the leading-logarithmic contribution, as the results are not too different with respect to the full resummed calculation.

The resummed spectra were compared with the results of the HERWIG and PYTHIA Monte Carlo generators. We found that HERWIG, whose evolution variable is equivalent to angular ordering in the soft limit, is in acceptable agreement with the resummation. As for PYTHIA, the old model, based on virtuality ordering, with an option to veto non-angular-ordered emissions, was found to be inadequate to describe non-global observables. The new model, ordered in transverse momentum and with an improved implementation of angular ordering, yields predictions qualitatively similar to HERWIG for relatively small rapidity gaps, whereas remarkable discrepancies are exhibited if the slice size is enlarged. In fact, as non-global observables are often used to tune Monte Carlo generators to data, we believe that such a discrepancy needs to be further investigated; otherwise, when fitting, *e.g.*, the old PYTHIA model to data, one would end up to include as much as 50% of perturbative leading logarithms in non-perturbative parameters, associated with hadronization or underlying event. A deeper understanding of the PYTHIA description of non-global observables, along with the application of the work here presented to hadron colliders, is in progress.

3 Azimuthal decorrelation between hard final state jets

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One of the most commonly measured jet observables in experimental QCD studies is the azimuthal decorrelation $\Delta\phi$ between hard final-state jets. When compared to theory this quantity is expected to provide valuable information both on QCD parameters (strong coupling, parton distribution functions – PDFs) as well as dynamics in the near back-to-back region sensitive to multiple soft and/or collinear emissions and non-perturbative effects. To this end it has been often examined in experimental QCD studies at HERA and the Tevatron [14, 15], used for the tuning of parameters of Monte Carlo event generator models and to constrain unintegrated PDFs (uPDFs) in conjunction with HERA data [16].

In this study we aim to provide a more accurate theoretical prediction for this observable by calculating a next-to-leading logarithm (NLL) resummed result which accounts for logarithmic terms enhanced in the region where jets are back-to-back in azimuthal angle – $\Delta\phi = \pi$. Such a resummation has not been carried out to date, the main complication being the application of a jet algorithm to define the final state which has non-trivial implications for the standard approximations that enable NLL resummation.

To be specific one is studying here an observable that is sensitive to energy flow outside well-defined jet regions which potentially means that it and similar observables fall into the category of non-global observables [17, 18]. Since it was shown that the resummation of non-global observables is substantially more complicated than that for “global” quantities such as most event-shape variables and in any case restricted to the large N_c approximation, the most accurate theoretical predictions can be obtained only for global observables. This appears to rule out the possibility of complete NLL estimates for many interesting jet observables including the azimuthal decorrelation we study here. As far as existing predictions for jet observables are concerned, the issue of non-global logarithms was not dealt with in Ref. [19] (published prior to the discovery of non-global effects) where they would arise in threshold resummation for one of the definitions ($M^2 = (p_1+p_2)^2$) of the dijet invariant mass studied there but would be absent for the definition $M^2 = 2p_1.p_2$. Further we should also mention here that the non-global component has been incorrectly treated in Ref. [20] where it is mentioned that such effects will vanish with jet radius when in fact one obtains a saturation in the small R limit.

We shall show an interplay between the potential non-global nature of the observable and the exact definition of the jet as provided by the choice of a recombination scheme. We show that in one of the resummation schemes employed in experimental studies of the azimuthal correlation the observable is in fact global and can be resummed to NLL accuracy. This may be taken as a general example of how carefully selecting the definition of the observable and the jets one may be able to render an exact NLL resummation possible, avoiding altogether the non-global issue and hence encourage future resummed studies for important regions of phase space in the context of jets.

3.0.4 Recombination scheme, kinematics and globalness

We wish to study the impact of two recombination schemes used to construct the angle $\Delta\phi$ between the final-state jets in dijet production. In the first scheme the jet azimuthal angle ϕ_j is given by a p_t-weighted sum over its hadronic constituents, $\phi_j = \sum_{i\in j} p_{t,i}\phi_i / \sum_{i\in j} p_{t,i}$, while in the second scheme one constructs the jet four-vector $p_j = \sum_{i\in j} p_i$, with the sum running over hadrons in the jet, and then parameterises $p_j = p_{t,j}\left(\cosh\eta_j, \cos\phi_j, \sin\phi_j, \sinh\eta_j\right)$ to obtain the jet azimuth ϕ_j. The first scheme is employed for instance by the H1 collaboration at HERA while the latter (E-scheme) is currently prefered by the Tevatron experiments.

The transverse momenta of final-state particles can be parameterised as below:[1]

$$\begin{aligned}
\vec{p}_{t,1} &= p_{t,1}(1,0), \\
\vec{p}_{t,2} &= p_{t,2}(\cos(\pi-\epsilon), \sin(\pi-\epsilon)), \\
&= p_{t,2}(-\cos\epsilon, \sin\epsilon), \\
\vec{k}_{t,i} &= k_{t,i}(\cos\phi_i, \sin\phi_i),
\end{aligned} \tag{5}$$

where the hard final-state partons are labeled by 1 and 2 and the soft gluons by the label i. For only soft emissions the hard partons are nearly back-to-back, $p_{t,1} = p_{t,2} = p_t$ and $|\epsilon| \ll 1$.

Using the above, in the scheme involving the p_t-weighted sum one obtains for $\Delta\phi = \phi_{j1} - \phi_{j2}$,:

$$|\pi - \Delta\phi| = \left|\sum_i \frac{k_{t,i}}{p_t}\left(\sin\phi_i - \theta_{i1}\phi_i - \theta_{i2}(\pi - \phi_i)\right)\right| + \mathcal{O}\left(k_t^2\right), \tag{6}$$

where $\theta_{ij} = 1$ if particle i is clustered to jet j and is zero otherwise. The definition above implies that the observable in question is global since it is sensitive to soft emissions in the whole phase-space, both in and outside the jets, and the dependence on soft emissions in either case is linear in k_t. This property ensures that it is possible to resum the large logarithms in the back-to-back region to next-to-leading (single) logarithmic accuracy without resorting to the large N_c approximation needed for non-global observables [17, 18].

Now turning to the E-scheme one obtains instead:

$$|\pi - \Delta\phi| = \left|\sum_{i\notin \text{jets}} \frac{k_{t,i}}{p_t}\sin\phi_i\right| + \mathcal{O}\left(k_t^2\right), \tag{7}$$

[1] Here one is looking at the projections of particle momenta in the plane perpendicular to the beam direction in hadron collisions or that perpendicular to the $\gamma^* P$ axis in the DIS Breit or hadronic centre-of-mass (HCM) frames.

where the sum extends only over all soft particles not recombined with the hard jets. Observables sensitive to soft emissions in such delimited angular intervals are of the non-global variety [17, 18], and hence in the E-scheme definition of jets the azimuthal decorrelation is a non-global observable.

3.0.5 Resummed Results

Having established that the observable at hand is a global observable in the p_t-weighted recombination scheme its resummation is now straightforward. We refer the reader to Ref. [21] for the details and just quote the results below.

Taking first the case of dijets produced in DIS, the integrated cross-section ie the integral of the distribution in $\pi - \Delta\phi$ up to some fixed value Δ is given by an integral over "impact parameter" b

$$\Sigma_a(\Delta) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{db}{b}\sin(b\Delta)e^{-R_a(b)} f_a\left(x, \mu_f^2/b^2\right). \tag{8}$$

The index a denotes the flavour of incoming parton and the function $R_a(b)$, known as the radiator, embodies the soft and/or collinear single-gluon result for emission from a three hard parton system while f denotes the PDF.

For the case of hadron collisions one can write a very similar formula to the one above except that in this case one has to account for two incoming partons and hence there are two PDFs while the relevant radiator now represents soft and collinear resummation from an ensemble of four hard partons.

The result for $R_a(\bar{b})$ for the DIS case can be expressed in terms of three pieces each with a distinct physical origin:

$$R_a(\bar{b}) = R^a_{\text{in}}(\bar{b}) + R^a_{\text{out}}(\bar{b}) - \ln S\left(\bar{b}, \{p\}\right), \tag{9}$$

with R^a_{in} and R^a_{out} being the contributions generated by emissions collinear to the incoming (excluding the set of single-logarithms already resummed in the parton densities) and outgoing legs respectively. In addition to these jet functions we have a soft function $S(\bar{b}, \{p\})$ which resums soft emission at large angles, and which depends on the geometry of the emitting hard ensemble expressed here as a dependence on the set of hard Born momenta $\{p\}$.

While our results eventually include the two-loop running of the coupling which is necessary to obtain full NLL accuracy (compute the full functions g_1 and g_2), for brevity and to illustrate the main features we report our results here in a fixed coupling approximation. In this case we simply obtain:

$$\begin{aligned}
R^a_{\text{out}}(\bar{b}) &= (C^a_1 + C^a_2)\frac{\alpha_s}{2\pi}\left(\frac{2}{3}L^2 + \frac{4}{3}L\left(-\ln 3 - 4\ln 2 + 3\ln\frac{Q}{p_t}\right)\right) + \\
&\quad + \frac{4}{3}\frac{\alpha_s}{2\pi}\left(C^a_1 B^a_1 + C^a_2 B^a_2\right) L, \qquad (10)\\
R^a_{\text{in}} &= C^a_i\frac{\alpha_s}{2\pi}\left(2L^2 + 4L\left(-\ln 2 + \ln\frac{Q}{p_t}\right)\right) + 4C^a_i\frac{\alpha_s}{2\pi}B^a_i L, \qquad (11)\\
\ln S(\bar{b}, \{p\}) &= -4L\left(2C_F\frac{\alpha_s}{2\pi}\ln\frac{Q_{qq'}}{Q} + C_A\frac{\alpha_s}{2\pi}\ln\frac{Q_{qg}Q_{gq'}}{Q_{qq'}Q}\right), \qquad (12)
\end{aligned}$$

with $L = \ln \bar{b}$. In the above C_i^a is the colour charge of the incoming parton in channel a, for instance $C_i^a = C_F$ for $a = q$, the incoming quark channel. Likewise $C_{1,2}^a$ are the colour charges of the partons initiating the outgoing jets 1 and 2 in channel a. The main aspect of the results for the collinear $R^a_{\mathrm{out,in}}$ jet functions is a leading double logarithmic behaviour, where one notes the unfamiliar coefficient $2/3$ (different from all commonly studied event shape variables for instance) associated to the double logs on the outgoing legs, *i.e.* in the function R^a_{out}. Additionally hard collinear radiation is described by single-logarithmic terms with the coefficients $C_\ell B_\ell$ for each leg, with the appropriate colour charge C_ℓ ($\ell = i, 1, 2$) and $B_{i,1,2}$ depending on the identities (spins) of the incoming and outgoing partons such that $B_\ell = -3/4$ for fermions and $B_\ell = -(11C_A - 4T_R n_f)/(12C_A)$ for a gluon.

Finally we have the soft wide-angle single-logarithmic contribution $\ln S$, which depends on the geometry of the hard three-jet system via the dependence on dipole invariant masses $Q_{ij} = 2(p_i.p_j)$. This structure is characteristic of soft inter-jet radiation for three-jet systems (see *e.g.* Ref. [22] for a detailed discussion). The result can be easily extended to the case of hadron collisions as shown in Ref. [21].

3.0.6 Results and Discussion

To provide a final resummed result for the $\Delta\phi$ distribution one still needs to carry out the b integration in Eq. (8). The b integral is not well behaved at small and large b. At small b one is outside the jurisdiction of resummation and hence free to modify the small b behaviour with a prescription that does not affect the next-to-leading logarithms (see Ref. [21]). At large b one has to regulate the effect of the Landau pole in the running coupling and introduce non-perturbative corrections which procedure is described in Ref. [21].

We plot the resummed result for the $\Delta\phi$ distribution in Fig. 4 along with the fixed order predictions for dijet production in DIS with $Q^2 = 67\ \mathrm{Gev}^2$ and $x = 2.86 \cdot 10^{-3}$. These values and other cuts on the jets have been taken from the H1 study to which we would eventually compare our results. As we can see the fixed order predictions diverge as expected near $\Delta\phi = \pi$. This divergence is cured by the resummation that goes to a fixed *non-zero* value at $\Delta\phi = \pi$. Of note here is the absence of a Sudakov peak since the Sudakov mechanism does not dominate the b integral at very small $\Delta = |\pi - \Delta\phi|$. The dominant mechanism to obtain back-to-back jets is thus a one-dimensional cancellation between emissions rather than a suppression of the k_t of each individual emission, leading to a washout of the Sudakov peak.

In order to obtain complete predictions which can be compared to data two further developments need to be made: matching to fixed-order NLO predictions and inclusion of non-perturbative effects. These issues will be addressed in forthcoming work.

4 Matching of NLLA to NNLO calculation for event shapes in e^+e^-

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Event shape distributions in e^+e^- annihilation processes are very popular hadronic observables. Their popularity is mainly due to the fact that they are well suited both for experimental measurement and for theoretical calculations because many of them are infrared and collinear safe.

The deviation from simple two-jet configurations, which are a limiting case in event shapes, is proportional to the strong coupling constant α_s, so that by comparing the measured

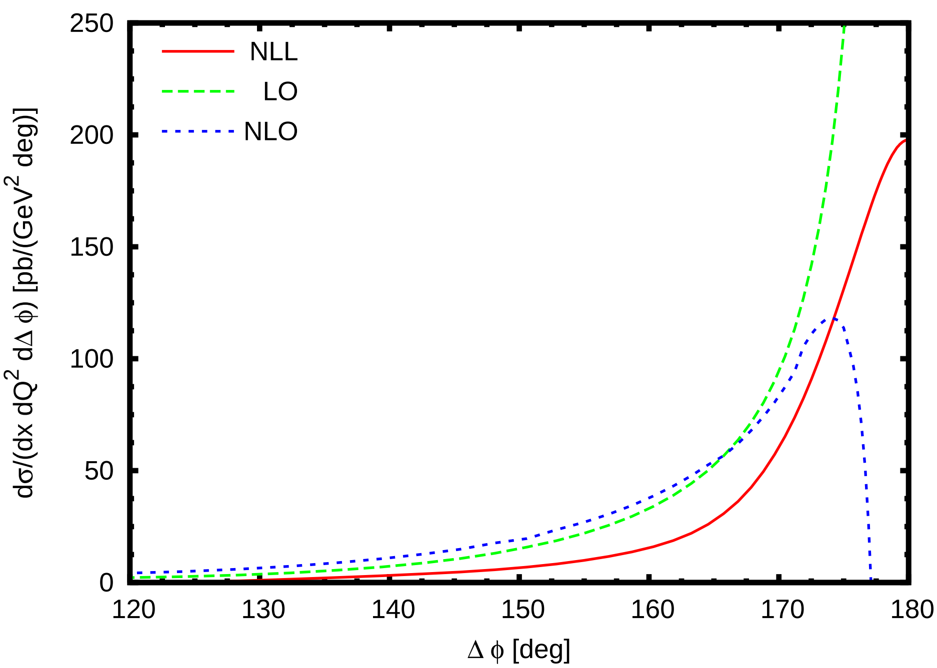

Fig. 4: The resummed $\Delta\phi$ distribution for dijets in DIS. Also shown for comparison are the leading order (LO) and next-to-leading order (NLO) predictions from NLOJET++.

$\bar{\alpha}_s \mathcal{A}(y)$	$\bar{\alpha}_s L$	$\bar{\alpha}_s L^2$				
$\bar{\alpha}_s^2 \mathcal{B}(y, x_\mu)$	$\bar{\alpha}_s^2 L$	$\bar{\alpha}_s^2 L^2$	$\bar{\alpha}_s^2 L^3$	$\bar{\alpha}_s^2 L^4$		
$\bar{\alpha}_s^3 \mathcal{C}(y, x_\mu)$	$\bar{\alpha}_s^3 L$	$\bar{\alpha}_s^3 L^2$	$\bar{\alpha}_s^3 L^3$	$\bar{\alpha}_s^3 L^4$	$\bar{\alpha}_s^3 L^5$	$\bar{\alpha}_s^3 L^6$

Table 1: Powers of the logarithms present at different orders in perturbation theory. The colour highlights the different orders in resummation: LL (red) and NLL (blue). The terms in green are contained in the LL and NLL contributions and exponentiate trivially with them.

event shape distribution with the theoretical prediction, one can determine α_s [23]. Below we will concentrate on this, using the newly available NNLO [24] and NLLA+NNLO results. At LEP, a standard set of event shapes was studied in great detail: thrust T (which is substituted here by $\tau = 1 - T$), heavy jet mass ρ, wide and total jet broadening B_W and B_T, C-parameter and two-to-three-jet transition parameter in the Durham algorithm Y_3. The definitions of these variables, which we denote collectively as y in the following, are summarized in [25]. The two-jet limit of each variable is $y \to 0$.

The theoretical state-of-the-art description of event shape distributions was based until very recently on the matching of the NLLA [26] onto the NLO [27–30] calculation. The newly available results of the NNLO corrections for the standard set of event shapes [24] introduced above, permits now to match them with resummed calculations, obtaining theoretical distributions at NLLA+NNLO.

At NNLO the integrated cross section

$$R(y, Q, \mu) \equiv \frac{1}{\sigma_{\rm had}} \int_0^y \frac{d\sigma(x, Q, \mu)}{dx} dx,$$

has the following fixed-order expansion:

$$R(y, Q, \mu) = 1 + \bar{\alpha}_s(\mu)\mathcal{A}(y) + \bar{\alpha}_s^2(\mu)\mathcal{B}(y, x_\mu) + \bar{\alpha}_s^3(\mu)\mathcal{C}(y, x_\mu).$$

where $\bar{\alpha}_s = \alpha_s/(2\pi)$ and $x_\mu = \mu/Q$. Approaching the two-jet region event shapes display large infrared logarithms which spoil the convergence of the series expansion. The main contribution in this case comes from the highest power of the logarithms which have to be resummed to all orders. For suitable observables resummation leads to exponentiation. At NLLA the resummed expression is given by

$$R(y, Q, \mu) = (1 + C_1 \bar{\alpha}_s)\, e^{(L\, g_1(\alpha_s L) + g_2(\alpha_s L))},$$

where the function $g_1(\alpha_s L)$ contains all leading-logarithms (LL), $g_2(\alpha_s L)$ all next-to-leading-logarithms (NLL) and $\mu = Q$ is used. Terms beyond NLL have been consistently omitted. The resummation functions $g_1(\alpha_s L)$ and $g_2(\alpha_s L)$ can be expanded as power series in $\bar{\alpha}_s L$

$$\begin{aligned} L\, g_1(\alpha_s L) &= G_{12} L^2 \bar{\alpha}_s + G_{23} L^3 \bar{\alpha}_s^2 + G_{34} L^4 \bar{\alpha}_s^3 + \dots \text{ (LL)}, \\ g_2(\alpha_s L) &= G_{11} L\, \bar{\alpha}_s + G_{22} L^2 \bar{\alpha}_s^2 + G_{33} L^3 \bar{\alpha}_s^3 + \dots \text{ (NLL)}. \end{aligned} \tag{13}$$

Table 1 shows the logarithmic terms present up to the third order in perturbation theory. From the expansion (13) of the exponentiated resummation functions it follows immediately, that at the fixed-order level, the LL are terms of the form $\alpha_S^n L^{n+1}$, the NLL terms go like $\alpha_S^n L^n$, and so on.

Closed analytic forms for functions $g_1(\alpha_s L)$, $g_2(\alpha_s L)$ are available for τ and ρ [31], B_W and B_T [32, 33], C [34, 35] and Y_3 [36], and are collected in the appendix of [37]. For τ the $g_3(\alpha_s L)$ function is also known [38].

To obtain a reliable description of the event shape distributions over a wide range in y, it is mandatory to combine fixed-order and resummed predictions. To avoid the double counting of terms common to both, the two predictions have to be matched to each other. A number of different matching procedures have been proposed in the literature, see for example [25] for a review. We computed the matching in the so-called $\ln R$-matching [26] since in this particular scheme, all matching coefficients can be extracted analytically from the resummed calculation, while most other schemes require the numerical extraction of some of the matching coefficients from the distributions at fixed order. The $\ln R$-matching at NLO is described in detail in [26]. In the $\ln R$-matching scheme, the NLLA+NNLO expression is

$$\begin{aligned} \ln\left(R\left(y,\alpha_S\right)\right) \; &= \; L\, g_1\left(\alpha_s L\right) + g_2\left(\alpha_s L\right) + \bar{\alpha}_S\left(\mathcal{A}\left(y\right) - G_{11}L - G_{12}L^2\right) \\ &\quad + \bar{\alpha}_S^2\left(\mathcal{B}\left(y\right) - \frac{1}{2}\mathcal{A}^2\left(y\right) - G_{22}L^2 - G_{23}L^3\right) \\ &\quad + \bar{\alpha}_S^3\left(\mathcal{C}\left(y\right) - \mathcal{A}\left(y\right)\mathcal{B}\left(y\right) + \frac{1}{3}\mathcal{A}^3\left(y\right) - G_{33}L^3 - G_{34}L^4\right) . \end{aligned} \tag{14}$$

The matching coefficients appearing in this expression can be obtained from (13) and are listed in [37]. To ensure the vanishing of the matched expression at the kinematical boundary $y_{\max}$ a further shift of the logarithm is made [25].

The full renormalisation scale dependence of (14) is given by replacing the coupling constant, the fixed-order coefficients, the resummation functions and the matching coefficients as follows:

$$\begin{aligned} \alpha_s \;&\to\; \alpha_s(\mu)\,, \\ \mathcal{B}\left(y\right) \;&\to\; \mathcal{B}\left(y,\mu\right) = 2\,\beta_0\,\ln x_\mu\,\mathcal{A}\left(y\right) + \mathcal{B}\left(y\right)\,, \\ \mathcal{C}\left(y\right) \;&\to\; \mathcal{C}\left(y,\mu\right) = \left(2\,\beta_0\,\ln x_\mu\right)^2\mathcal{A}\left(y\right) + 2\,\ln x_\mu\,\left[2\,\beta_0\mathcal{B}\left(y\right) + 2\,\beta_1\,\mathcal{A}\left(y\right)\right] + \mathcal{C}\left(y\right)\,, \\ g_2\left(\alpha_S L\right) \;&\to\; g_2\left(\alpha_S L,\mu^2\right) = g_2\left(\alpha_S L\right) + \frac{\beta_0}{\pi}\left(\alpha_S L\right)^2\, g_1'\left(\alpha_S L\right)\,\ln x_\mu\,, \\ G_{22} \;&\to\; G_{22}\left(\mu\right) = G_{22} + 2\beta_0 G_{12}\ln x_\mu\,, \\ G_{33} \;&\to\; G_{33}\left(\mu\right) = G_{33} + 4\beta_0 G_{23}\ln x_\mu\,. \end{aligned}$$

In the above, g_1' denotes the derivative of g_1 with respect to its argument. The LO coefficient $\mathcal{A}$ and the LL resummation function g_1, as well as the matching coefficients $G_{i\,i+1}$ remain independent of μ.

In the two upper plots of Fig. 5 we compare the matched NLLA+NNLO predictions for the heavy jet mass with the fixed-order NNLO predictions, and the matched NLLA+NLO with fixed-order NLO. All distributions were weighted by the respective shape variables. We use $Q = M_Z$ and fix $x_\mu = 1$, the strong coupling constant is taken as $\alpha_s(M_Z) = 0.1189$. To quantify the renormalisation scale uncertainty, we have varied $1/2 < x_\mu < 2$, resulting in the error

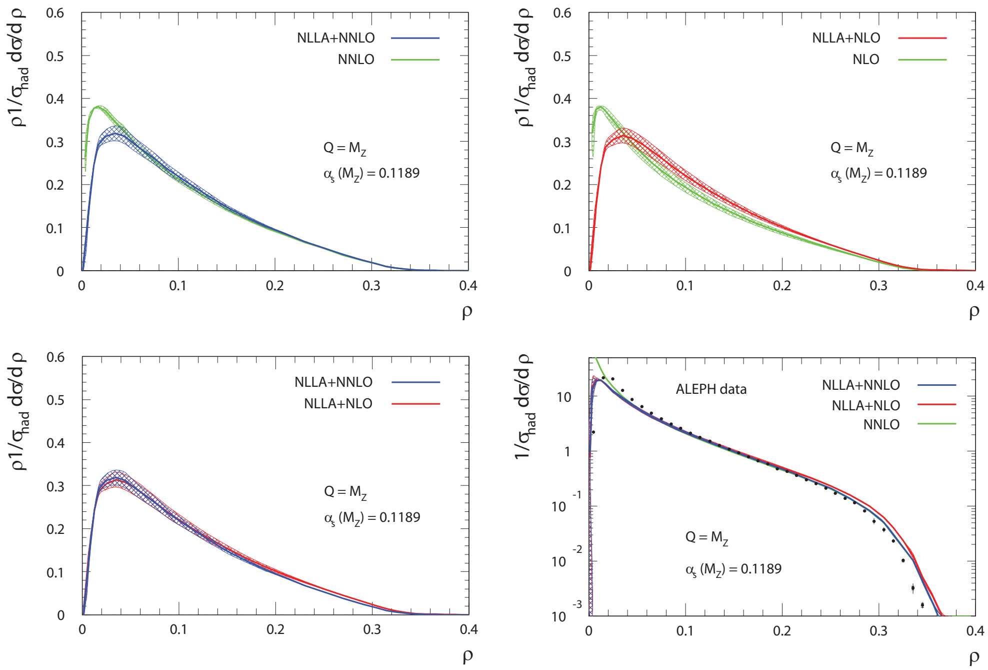

Fig. 5: Matched distributions of heavy jet mass ρ,.

band on these figures. The effects visible for the heavy jet mass are common to the whole set of observables which were analyzed. The most striking observation is that the difference between NLLA+NNLO and NNLO is largely restricted to the two-jet region, while NLLA+NLO and NLO differ in normalisation throughout the full kinematical range. This behaviour may serve as a first indication for the numerical smallness of corrections beyond NNLO in the three-jet region. In the approach to the two-jet region, the NLLA+NLO and NLLA+NNLO predictions agree by construction, since the matching suppresses any fixed-order terms. On the plot in the lower left corner we observe that the difference between NLLA+NNLO and NLLA+NLO is only moderate in the three-jet region. The renormalisation scale uncertainty in the three-jet region is reduced by 20-40% between NLLA+NLO and NLLA+NNLO. Finally the lower-right plot shows the parton-level fixed NNLO and the matched NLLA+NLO and NLLA+NNLO predictions are compared to hadron-level data taken by the ALEPH experiment. The description of the hadron-level data improves between parton-level NLLA+NLO and parton-level NLLA+NNLO, especially in the three-jet region. The behavior in the two-jet region is described better by the resummed predictions than by the fixed-order NNLO, although the agreement is far from perfect. This discrepancy can in part be attributed to hadronisation corrections, which become large in the approach to the two-jet limit. A very recent study of logarithmic corrections beyond NLLA for the thrust distribution [38] also shows that subleading logarithms in the two-jet region can account for about half of this discrepancy.

With the new NNLO and NLLA+NNLO results a new extraction of α_s can be performed. For this we used public ALEPH data at center-of-mass energies between 91 and 209 GeV [39]. The data are corrected to hadron level using Monte Carlo (MC) corrections and accounting for initial- and final-state-radiation (ISR/FSR) as well as background. They are fitted by NNLO respectively NLLA+NNLO predictions, including NLO quark mass corrections, folded to hadron level by means of MC generators. Finally, after estimating the missing higher orders using the uncertainty band method [25], the fits of 8 data sets and 6 different variables are combined together [23].

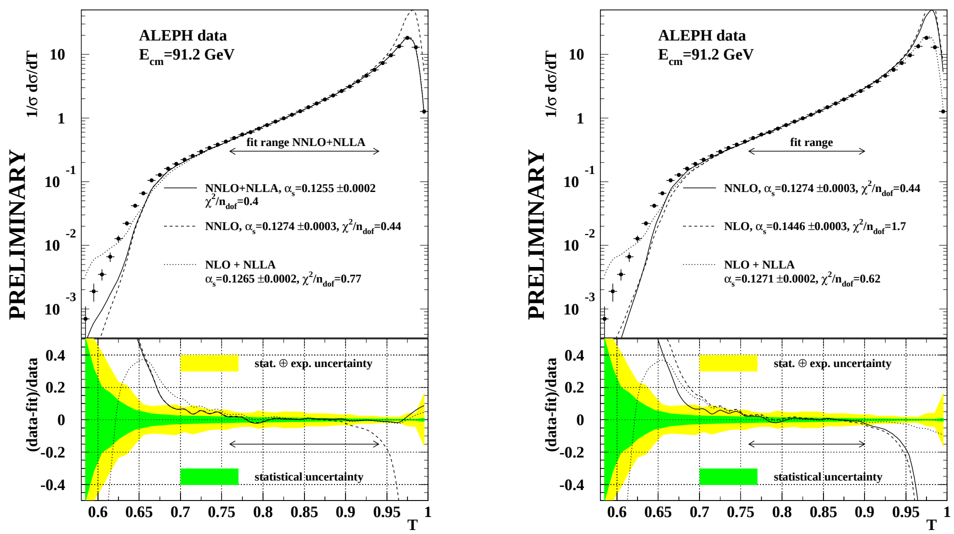

Fig. 6: Fit to ALEPH data for thrust.

The part of the distribution chosen for the fit (Fig. 6) is the one where the hadronizations and detector corrections are smaller than 25%. In the case of the NNLO distributions, the range was further reduced in the 2-jet region because of the divergence of the theoretical predictions. Only the statistical uncertainties are included in the χ^2.

At NNLO we see a clear improvement with respect to the old NLO results. The fit is of a good quality although it still includes large statistical uncertainties of the $\mathcal{C}$ coefficient and in the 2-jet region the NLLA+NLO predictions still yields a better result. The improvement between NNLO and NLLA+NNLO in visible especially in the 2-jet region. The fit range is also more extended in this direction. For the resulting α_s we observe that using fixed-order predictions leads basically to higher values, and that in both fixed-order and matched predictions there is a tendency for α_s to decrease passing from NLO to NNLO. Finally computing the weighted average for α_s from the 6 variables we obtain [23]:

$$\bar{\alpha}_s\,(M_Z)\;=\;0.1240\;\pm\;0.0008\,(\mathrm{stat})\;\pm\;0.0010\,(\mathrm{exp})\;\pm\;0.0011\,(\mathrm{had})\;\pm\;0.0029\,(\mathrm{theo}).$$

From Fig. 7 it is clearly visible that the results for the different variables are coherent and the scattering is much reduced. The improvement with respect to the NLO result in also remarkable.

The combined results for the NLLA+NLLO fits are still work-in-progress, but it can be anticipated that the improvement coming from the inclusion of resummed calculation will be less

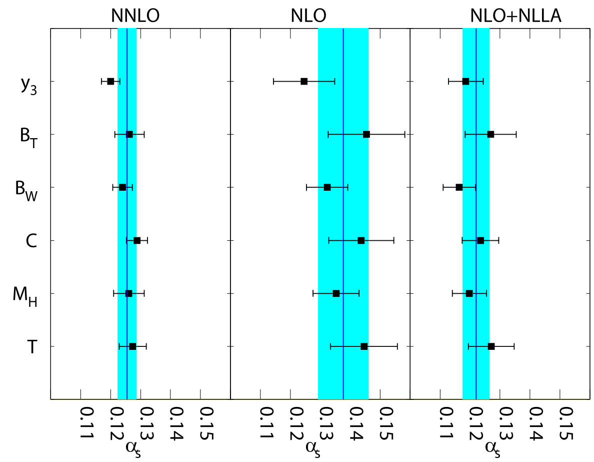

Fig. 7: Combination of α_s fits at NLO, NLLA+NLO and NNLO.

dramatic than the one obtained at NLO level. The reason for this is that the compensation for the two-loop running of the coupling constant is present only in the NNLO coefficient and not in the resummed part.

These results shows that there is space for further improvements, which could be obtained by resumming subleading logarithms similarly to what was done recently for thrust [38]. Improvements are also expected from the addition of electroweak corrections. Finally, a further step forward in the comparison of theoretical predictions with experimental data could be done by using modern MC tools based on NLO calculations matched with parton showers for the computation of the hadronizations corrections.

5 Precision resummed QEDxQCD theory for LHC physics: status and update

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With the advent of the LHC, we enter the era of precision QCD, by which we mean predictions for QCD processes at the total theoretical precision tag of 1% or better. The attendant requirement for this theoretical precision is control of the $\mathcal{O}(\alpha_s^2 L^{n_1}, \alpha_s \alpha L^{n_2}, \alpha^2 L^{n_3})$, $n_1 = 0, 1, 2,\ n_2 = 1, 2,\ n_3 = 2$ corrections in the presence of realistic parton showers, on an event-by-event basis – here, L is a generic big logarithm. This is the objective of our approach to precision QCD theory, which for example will be needed for the expected 2% experimental precision [40–42] at the LHC for processes such as $pp \to V + m(\gamma) + n(G) + X \to \bar{\ell}\ell' + m'(\gamma) + n(G) + X,\ \ V = W^\pm, Z,$ and $\ell = e, \mu,\ \ell' = \nu_e,\ \nu_\mu(e, \mu)$ for $V = W^+(Z)$ respectively, and $\ell = \nu_e,\ \nu_\mu,\ \ell' = e, \mu$ respectively for $V = W^-$. Here, we present the elements of our approach and its recent applications in Monte Carlo (MC) event generator studies, which are still preliminary.

At such a precision as we have as our goal, issues such as the role of QED are an integral part of the discussion and we deal with this by the simultaneous resummation of QED and QCD large infrared (IR) effects, $QED \otimes QCD$ resummation [43–49] in the presence of parton showers, to be realized on an event-by-event basis by MC methods. This is reviewed in the next section. Let us note already that in Refs. [50–55] it has been shown that QED evolution enters at the $\sim 0.3\%$ level for parton distributions and that in Refs. [56, 57] it has been shown that EW (large Sudakov logs, etc.) effects at LHC energies, as W's and Z's are almost massless on the TeV scale, can enter at the several percent level – such corrections must be treated systematically before any claim of 1% precision can be taken seriously. We are presenting a framework in which this can be done. The new amplitude-based resummation algebra then leads to a new scheme for calculating hard hadron-hadron scattering processes, IR-improved DGLAP-CS theory [58] for parton distributions, kernels, reduced cross sections with the appropriate shower/ME matching. This is summarized in Sec. 1.4.3. In this latter section, with an eye toward technical precision cross checks plus possible physical effects of heavy quark masses, we also deal with the issue of quark masses as collinear regulators [59–63] as an alternative [64] to the usual practice of setting all initial state quark masses to zero in calculating initial state radiation (ISR) effects in higher order QCD corrections. We also discuss in Sec. 1.4.3 the relationship between our resummation algebra and that of Refs. [65–69], as again such comparisons will be necessary in assessing the ultimate theoretical precision tag. In Sec. 1.4.4, we illustrate recent results we have obtained for the effects of our new approach on the parton showers as they are generated with the HERWIG6.5 MC [70]. Extensions of such studies to PYTHIA [71] and MC@NLO [72, 73] are in progress. Section 1.4.5 contains summary remarks.

As a point of reference, in Ref. [74] it has been argued that the current state-of-the-art theoretical precision tag on single Z production at the LHC is $(4.1 \pm 0.3)\% = (1.51 \pm 0.75)\%(QCD) \oplus 3.79(PDF) \oplus 0.38 \pm 0.26(EW)\%$, where the results of Refs. [72, 73, 75–86] have been used in this precision tag determination.[2]

5.0.7 QED⊗QCD Resummation

In Refs. [43–49], we have extended the YFS theory to the simultaneous exponentiation of the large IR terms in QCD and the exact IR divergent terms in QED, so that for the prototypical subprocesses $\bar{Q}'Q \to \bar{Q}'''Q'' + m(G) + n(\gamma)$ we arrive at the new result

$$\begin{aligned} d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} & \\ & \sum_{m,n=0}^{\infty} \frac{1}{m!n!} \int \prod_{j_1=1}^{m} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{n} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} \\ & e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\ & \tilde{\bar{\beta}}_{m,n}(k_1,\ldots,k_m;k'_1,\ldots,k'_n) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0}, \end{aligned} \tag{15}$$

where the new YFS [88–98] residuals, defined in Refs. [43–49], $\tilde{\bar{\beta}}_{m,n}(k_1,\ldots,k_m;k'_1,\ldots,k'_n)$, with m hard gluons and n hard photons, represent the successive application of the YFS expansion first for QCD and subsequently for QED. The functions $\rm SUM_{IR}(QCED), D_{\rm QCED}$ are determined from their analogues $\rm SUM_{IR}(QCD), D_{\rm QCD}$ in Refs. [99–104] via the substitutions

$$\begin{aligned} B^{nls}_{QCD} &\to B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED}, \\ \tilde{B}^{nls}_{QCD} &\to \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED}, \\ \tilde{S}^{nls}_{QCD} &\to \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED} \end{aligned} \tag{16}$$

[2]Recently, the analogous estimate for single W production has been given in Ref. [87] – it is $\sim 5.7\%$.

everywhere in expressions for the latter functions given in Refs. [99–104] – see Refs. [43–49] for the details of this substitution. It can be readily established [43–49] that the QCD dominant corrections happen an order of magnitude earlier in time compared to those of QED so that the leading term $\bar{\tilde{\beta}}_{0,0}$ already gives us a good estimate of the size of the effects we study.

Important in any total theoretical prediction is knowledge of possible systematic issues associated with one's methods. This entails the relationship between different approaches to the same classes of corrections and moves us to the relationship between our approach to QCD resummation and the more familiar approach in Refs. [65–67]. It has been shown in Ref. [105] that the latter approach is entirely equivalent to the approach in Refs. [68, 69]. Establishing the relationship between our approach and that in Refs. [65–67] will then suffice to relate all three approaches.

In Ref. [106] the more familiar resummation for soft gluons in Refs. [65–67] is applied to a general $2 \to n$ parton process [f] at hard scale Q, $f_1(p_1, r_1) + f_2(p_2, r_2) \to f_3(p_3, r_3) + f_4(p_4, r_4) + \cdots + f_{n+2}(p_{n+2}, r_{n+2})$, where the p_i, r_i label 4-momenta and color indices respectively, with all parton masses set to zero to get

$$\begin{aligned} \mathcal{M}^{[f]}_{\{r_i\}} &= \sum_L^C \mathcal{M}^{[f]}_L (c_L)_{\{r_i\}} \\ &= J^{[f]} \sum_L^C S_{LI} H^{[f]}_I (c_L)_{\{r_i\}}, \end{aligned} \tag{17}$$

where repeated indices are summed, $J^{[f]}$ is the jet function, S_{LI} is the soft function which describes the exchange of soft gluons between the external lines, and $H^{[f]}_I$ is the hard coefficient function. The attendant IR and collinear poles are calculated to 2-loop order. To make contact with our approach, identify in $\bar{Q}'Q \to \bar{Q}'''Q'' + m(G)$ in (15) $f_1 = Q, \bar{Q}', f_2 = \bar{Q}', f_3 = Q'', f_4 = \bar{Q}''', \{f_5, \cdots, f_{n+2}\} = \{G_1, \cdots, G_m\}$ so that $n = m+2$ here. Observe the following:

- By its definition in Eq.(2.23) of Ref. [106], the anomalous dimension of the matrix S_{LI} does not contain any of the diagonal effects described by our infrared functions $\Sigma_{IR}(QCD)$ and D_{QCD}.
- By its definition in Eqs.(2.5) and (2.7) of Ref. [106], the jet function $J^{[f]}$ contains the exponential of the virtual infrared function $\alpha_s \Re B_{QCD}$, so that we have to take care that we do not double count when we use (17) in (15) and the equations that lead thereto.

It follows that, referring to our analysis in Ref. [107], we identify $\bar{\rho}^{(m)}$ in Eq.(73) in this latter reference in our theory as

$$\begin{aligned} &\bar{\rho}^{(m)}(p_1, q_1, p_2, q_2, k_1, \cdots, k_m) = \overline{\sum}_{colors,spin} |\mathcal{M}'^{[f]}_{\{r_i\}}|^2 \\ &\equiv \sum_{spins,\{r_i\},\{r'_i\}} \mathfrak{h}^{cs}_{\{r_i\}\{r'_i\}} |\bar{J}^{[f]}|^2 \sum_{L=1}^C \sum_{L'=1}^C S^{[f]}_{LI} H^{[f]}_I (c_L)_{\{r_i\}} \left(S^{[f]}_{L'I'} H^{[f]}_{I'} (c_{L'})_{\{r'_i\}} \right)^\dagger, \end{aligned} \tag{18}$$

where here we defined $\bar{J}^{[f]} = e^{-\alpha_s \Re B_{QCD}} J^{[f]}$, and we introduced the color-spin density matrix for the initial state, $\mathfrak{h}^{cs}$. Here, we recall (see Refs. [58, 107], for example) that in our theory, we have

$$d\hat{\sigma}^n = \frac{e^{2\alpha_s ReB_{QCD}}}{n!} \int \prod_{m=1}^{n} \frac{d^3 k_m}{(k_m^2 + \lambda^2)^{1/2}} \delta(p_1 + q_1 - p_2 - q_2 - \sum_{i=1}^{n} k_i)$$
$$\bar{\rho}^{(n)}(p_1, q_1, p_2, q_2, k_1, \cdots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}, \quad (19)$$

for n-gluon emission. It follows that we can repeat thus our usual steps (see Refs. [58, 107]) to get the QCD corrections in our formula (15), without any double counting of effects. This use of the results in Ref. [106] is in progress.

5.0.8 IR-Improved DGLAP-CS Theory: Applications

In Refs. [58,107] it has been shown that application of the result (15) to all aspects of the standard formula for hard hadron-hadron scattering processes,

$$\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) \hat{\sigma}(x_1 x_2 s) \quad (20)$$

where the $\{F_i(x)\}$ and $\hat{\sigma}$ denote the parton densities and reduced cross section, respectively, leads one to its application to the DGLAP-CS theory itself for the kernels which govern the evolution of the parton densities in addition to the the implied application to the respective hard scattering reduced cross section. The result is a new set of IR-improved kernels [58],

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right], \quad (21)$$

$$P_{Gq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}, \quad (22)$$

$$P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G}$$
$$+ \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z)\}, \quad (23)$$

$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \}. \quad (24)$$

in the standard notation, where

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0} \quad (25)$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}) \quad (26)$$

$$\gamma_G = C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0} \quad (27)$$

$$\delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}) \quad (28)$$

and

$$F_{YFS}(\gamma_q) = \frac{e^{-C_E\gamma_q}}{\Gamma(1+\gamma_q)}, \tag{29}$$

so that

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q+1} + \frac{1}{\gamma_q+2} \tag{30}$$

$$f_G(\gamma_G) = \frac{n_f}{C_G}\frac{1}{(1+\gamma_G)(2+\gamma_G)(3+\gamma_G)} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} \tag{31}$$

$$+ \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} \tag{32}$$

$$+ \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}. \tag{33}$$

Here, $C_E = 0.5772...$ is Euler's constant and $\Gamma(w)$ is the Euler Gamma function. We see that the kernels are integrable at the IR end-points and this admits a more friendly MC implementation, which is in progress.

Some observations are in order. First, we note that the connection of (24) with the higher-order kernel results in Refs. [108–117] is immediate and has been shown in Refs. [58, 107]. Second, there is no contradiction with the standard Wilson expansion, as the terms we resum are not in that expansion by its usual definition. Third, we do not change the predicted cross section: we have a new scheme such that the cross section in (20) becomes

$$\sigma = \sum_{i,j}\int dx_1 dx_2 F'_i(x_1)F'_j(x_2)\hat{\sigma}'(x_1x_2s) \tag{34}$$

order by order in perturbation theory, where $\{P^{exp}\}$ factorize $\hat{\sigma}_{\text{unfactorized}}$ to yield $\hat{\sigma}'$ and its attendant parton densities $\{F'_i\}$. Fourth, when one solves for the effects of the exponentiation in (24) on the actual evolution of the parton densities from the typical reference scale of $Q_0 \sim$ 2GeV to $Q = 100$ GeV one finds [58, 107] shifts of $\sim 5\%$ for the NS n=2 moment for example, which is thus of some phenomenological interest– see for example Ref. [118]. Finally, we note that we have used [43–49] the result (15) for single Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness and we find agreement with the literature in Refs. [119–123] for exact $\mathcal{O}(\alpha)$ results and Refs. [124–126] for exact $\mathcal{O}(\alpha_s^2)$ results, with a threshold QED effect of 0.3%, similar to that found for the parton evolution itself from QED in Refs. [50–55]. Evidently, any 1% precision tag must account for all such effects.

5.0.9 Shower/ME Matching

In using (15) in (34) for $\hat{\sigma}'(x_ix_j)$, we intend to combine our exact extended YFS calculus with HERWIG [70] and PYTHIA [71] as follows: they generate a parton shower starting from (x_1, x_2) at the factorization scale μ after this point is provided by the $\{F'_i\}$ and we may use [43–49] either a p_T-matching scheme or a shower-subtracted residual scheme where the respective new residuals $\{\hat{\hat{\bar{\beta}}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)\}$ are obtained by expanding the shower formula and the result in (15) on product and requiring the agreement with exact results to the specified order.[3]

[3]See Ref. [127, 128] for a realization of the shower subtracted residual scheme in the context of QED parton showers.

This combination of theoretical constructs can be systematically improved with exact results order-by-order in α_s, α, with exact phase space. [4] The recently developed new parton evolution algorithms in Refs. [129, 130] may also be used here.

The issue of the non-zero quark masses in the ISR is present when one wants 1% precision, as we know that the parton densities for the heavy quarks are all different and the generic size of mass corrections for bremsstrahlung is α_s/π for cross sections [131], so that one would like to know whether regularizing a zero-mass ISR radiation result with dimensional methods, carrying through the factorization procedure gives the same result as doing the same calculation with the physical, non-zero mass of the quark and again carrying through the factorization procedure to the accuracy α_s^2/π^2, for example. Until the analysis in Ref. [64], this cross check was not possible because in Refs. [59–62] it was shown that there is a lack of Bloch-Nordsieck cancellation in the ISR at $\mathcal{O}(\alpha_s^2)$ unless the radiating quarks are massless. The QCD resummation algebra, as used in (15), allows us to obviate [64] this theorem, so that now such cross checks are possible and they are in progress.

5.0.10 Sample MC data: IR-Improved Kernels in HERWIG6.5

We have preliminary results on IR-improved showers in HERWIG6.5: we compare the z - distributions and the p_T of the IR-improved and usual DGLAP-CS showers in the Figs. 8-10. As we would expect, the IR-improved shower re-populates the soft region in both variables. The details of the implementation procedure and the respective new version of HERWIG6.5, HERWIG6.5-YFS, will appear elsewhere [132]. The analogous implementations in PYTHIA and MC@NLO are in progress, as are comparisons with IR-safe observables.

5.0.11 Conclusions

The theory of Refs. [88, 89] extends to the joint resummation of QED and QCD with proper shower/ME matching built-in. For the simultaneous QED$\otimes$QCD resummed theory, full MC event generator realization is open: a firm basis for the complete $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC results needed for precision LHC physics has been demonstrated and all the latter are in progress – see Refs. [133–137] for new results on ϵ expansions for the higher-order Feynman integrals needed to isolate the residuals in our approach, for example. This allows cross check between residuals isolated with the quark masses as regulators, something now allowed by the result in Ref. [64], and those isolated in dimensional regularization for the massless quark limit. Such cross checks are relevant for precision QCD theory. The first MC data have been shown with IR-improved showers in HERWIG6.5. The spectra are softer as expected. We look forward to the detailed comparison with IR-safe observables as generated with IR-improved and with the usual showers – this will appear elsewhere. [132]. Already, semi-analytical results at the $\tilde{\bar{\beta}}^{0,0}_{0,0}$ are consistent with the literature on single Z production, while a cross check for the analogous W production is near. As the QED is at 0.3% at threshold, it is needed for 1% precision.

References

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[4] The current state of the art for such shower/ME matching is given in Refs. [72, 73], which realizes exactness at $\mathcal{O}(\alpha_s)$.

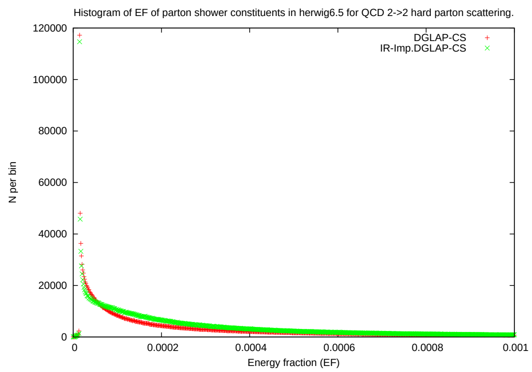

Fig. 8: The z-distribution shower comparison in HERWIG6.5 – preliminary results.

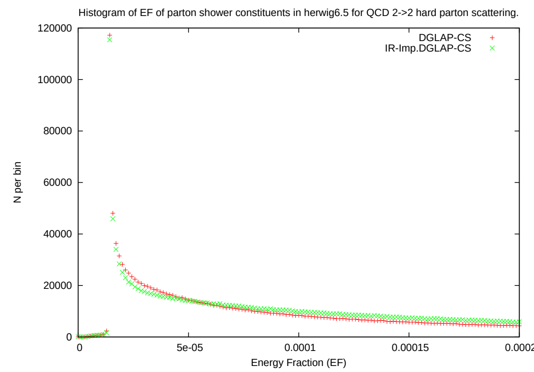

Fig. 9: The z-distribution shower comparison in HERWIG6.5 at small z – preliminary results.

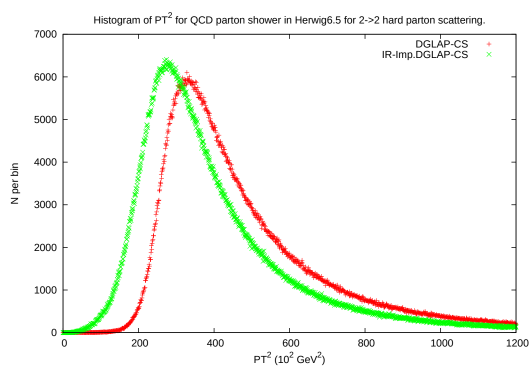

Fig. 10: The p_T-distribution shower comparison in HERWIG6.5 – preliminary results.

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