Fracture Functions at HERA and LHC

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Abstract
Developments of the fracture functions formalism in the context of DIS jet cross-sections and Semi-Inclusive Drell-Yan process at hadron colliders are briefly presented.

Fracture functions were introduced in Ref. [1] in order to give a QCD-based description of semi-inclusive Deep Inelastic Scattering in the target fragmentation region. The first analyses of HERA data [2] revealed a non-negligible contributions to the DIS cross-sections of events characterized by absence of hadronic activity in the remnant direction. Recent analyses of diffractive data collected by H1 and ZEUS collaborations have now confirmed substantial contributions of perturbative QCD effects in diffractive DIS cross-sections [3]. This experimental evidence strengthens the idea itself of fracture functions. These non-perturbative distributions, hereafter indicated by $M^i_{h/P}(x,z,Q^2)$, give the conditional probability of finding at a given scale Q^2 a parton i with momentum fraction x of the incoming hadron momentum P while a hadron h, with momentum fraction z, is detected in the target fragmentation region of P. In Ref. [4] it was shown within a fixed order $\mathcal{O}(\alpha_s)$ calculation that the additional collinear singularities occurring in the remnant direction can be properly renormalized only introducing fracture functions. An all-order proof of collinear and soft singularities factorization into $M^i_{h/P}(x,z,Q^2)$ was finally given in Refs. [5] and [6], respectively. This theoretical background offers the basis for an accurate analysis of diffractive data and the possibility to fully exploit factorization in order to extract diffractive parton distributions, *i.e.* fracture functions. In this brief contribution we will report on recent developments in this topic. In particular we will focus on the extension of fracture functions in the context of DIS jet cross-section and their possible applications to hadronic collisions.

As is well known, hadrons resulting from a hard interaction are often collimated in a definite portion of momentum space. Hadron jets are the highlighting signature of the dominant collinear branching of pQCD dynamics. For this reason jet cross-sections are the natural and, possibly, the most effective representation of hadronic final state. While jet cross-sections with a given, in general low, number of partons in the final state are calculable within pQCD, a description of the beam-jet in terms of pQCD is however precluded by its intrinsic soft and kinematical nature. It results from the fragmentation of the spectator partons of the hadron remnants plus, eventually, semi-hard radiation coming from the evolution of the active parton at low momentum transfer. Since at the forthcoming hadron collider topics as minimum bias and underlying event will play a central role and will probably plague the extraction of hard scattering events signals, we have proposed and introduced in Ref. [7] a new semi-inclusive jet-like distribution, here after indicated with $\mathcal{M}^i_{\triangleleft}(x,Q^2,z,t)$, referring to it as to a jet-like fracture function. $\mathcal{M}^i_{\triangleleft}(x,Q^2,z,t)$ expresses the probability of finding a parton i with fractional momentum x of the incoming hadron and virtuality Q^2, while a cluster of hadrons h_i is detected in a portion of phase space $\mathcal{R}$ specified by two variables, z and t. The region $\mathcal{R}$ is limited by the constraint

$$\mathcal{R}: \quad t_i = -(P - h_i)^2 < t, \qquad t_0 \leq t \ll Q^2 \,, \tag{1}$$

where the value of t is arbitrary chosen and can be conceived as the analogous of the clustering variable used in ordinary jet-algorithms. Once the clustering procedure is performed, the variable z is obtained by summing the fractional longitudinal momenta of all hadrons h_i satisfying the constraint in eq. (1):

$$z = \sum_i z_i, \quad h_i \in \mathcal{R}\,. \tag{2}$$

In analogy with the standard inclusive DIS, which makes use of parton distributions functions, we may write the beam-jet DIS cross-section as

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^{\mathcal{R},jet}}{dx dQ^2 dz dt} \propto x \sum_{i=q,\bar{q}} e_i^2 \, \mathcal{M}^i_{\triangleleft}(x, Q^2, z, t)\,. \tag{3}$$

In this framework, the parton initiating the space-like cascade is specified by the initial state radiation itself, *i.e.* the closest in rapidity to the hadron remnant. It has a fractional momentum $1 - z$, where z is overall fractional momentum taken away by the hadrons with $t_i \leq t$ and has the highest allowed virtuality, t, according to strong ordering. When t is chosen in the perturbative region, as shown in Ref. [7], jet-like fracture functions obey a standard DGLAP evolution equations:

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{M}^i_{\triangleleft}(x, Q^2, z, t) = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} \, P^i_j(u) \, \mathcal{M}^j_{\triangleleft}(x/u, Q^2, z, t)\,. \tag{4}$$

This equation describes how the virtual photon resolves the distributions $\mathcal{M}^i_{\triangleleft}$ when the virtuality of the latter is varied. In particular it resums potentially large collinear logarithms of the type $\alpha_s^n \log^n(Q^2/t)$. In real processes, strong t-ordering is only partially realized and one could in principle improve the theoretical description including higher order and coherence effects. As discussed in Ref. [7], the introduction of $\mathcal{M}^i_{\triangleleft}$ allows one to include the beam remnants jet in the perturbative treatment of DIS jet cross-sections. Moreover jet-like fracture functions could find applications also in hard diffractive processes. In events characterized by the absence of hadron activity in the remnant direction, this absence can be conceived as the *shadow* in the detector of the propagation of the exchanged object in the t-channel. The rapidity gap can then be considered as a *missing jet*. It can be defined in terms of a jet-like fracture functions specified by the value t of the measured particle at the edge of the gap, *i.e.* the one with the highest rapidity (a part from the proton itself). The study of *gap topology* might be important to investigate diffractive phenomena and jet-like fracture functions could be a useful tool in this context.

The knowledge acquired at HERA on Deep Inelastic process in the target fragmentation region is expected to be essential in the LHC diffractive physics program. Dedicated experiments as TOTEM will measure leading baryon production, while combined CMS-TOTEM measurements will trigger on a wide class of diffractive processes characterized by a large momentum transfer [8]. The fundamental step in transporting information from diffractive Deep Inelastic Scattering at HERA to LHC is to assume factorization to hold in hard diffractive hadron-hadron reactions. The Tevatron analysis has put, however, serious doubts on such an hypothesis. A non universality of diffractive parton distributions, as extracted from diffractive DIS, emerged when these distributions were used to predict hard scattering events cross-sections [9]. In such a reactions, at variance with diffractive DIS where factorization has been shown to hold in Ref. [6], theoretical arguments has been given such that the detection of particle in the target fragmentation region leads to a factorization breaking effect [6, 10]. For this reasons our understanding of the dynamics of diffractive processes is strongly correlated with the understanding of factorization.

Hard diffractive processes can be approached with pQCD techniques and the Drell-Yan process plays indeed a central role in this context. In particular it is the only hadrons-induced process for which factorization has been shown to hold at soft and collinear level [11]. Furthermore QCD corrections to this process have been calculated for inclusive and differential distributions in such a way that it constitutes a fundamental testing process of QCD at the hadron collider. For this reasons we have performed in Ref. [12] a pQCD analysis of the Semi-Inclusive Drell-Yan process

$$P_1 + P_2 \to \gamma^* + h + X \,. \tag{5}$$

In eq. (5) P_1 and P_2 stands for the incoming hadrons, γ^* the virtual photon of invariant mass Q^2 and h the additional hadron measured in the final state. If Q^2 is large enough so that perturbation theory applies, the factorization property of the considered cross-section should depend on the region of phase space in which the final hadron h is detected. In particular, if h is produced at sufficiently high transverse momentum, $p^2_{h\perp}$, then the relative cross-sections can be predicted by pQCD. On the contrary, if h is produced at low $p^2_{h\perp}$ and thus detected in the target fragmentation region, arguments against factorization have been already given in Refs. [6, 10]. The formalism of fracture functions allows one to performed a next-to-leading order QCD analysis of the Semi-Inclusive Drell-Yan process without introducing unphysical scale in order to separate the dominant production mechanisms in each region of phase space. The first step in order to perform consistently such a calculation is to provide a parton model formula for the considered process. Since in zero-th order QCD initial state radiation is absent, we assume the hadron h is "non-perturbatively" produced in the target fragmentation region of P_1 ($\mathcal{R}_{T_1}$) or P_2 ($\mathcal{R}_{T_2}$) by means of a "bare" (in the renormalization sense) fracture function $M^i_{h/P}(x,z)$. In the following we will consider the differential cross-sections for producing a lepton pair of invariant mass $Q^2 \gg \Lambda^2_{QCD}$, accompanied by an additional hadron h with fractional energy $z = 2E_h/\sqrt{S}$ (defined in the hadronic center of mass frame) and integrated over its transverse momentum, $p^2_{h\perp}$. By defining the combination $M^h_q(x,z) = M^{h/P_1}_q(x,z) + M^{h/P_2}_q(x,z)$, the parton model formula for the semi-inclusive Drell-Yan cross-sections reads:

$$\frac{d\sigma^{DY}(\tau)}{dQ^2 dz} = \frac{4\pi\alpha^2}{9SQ^2}\int_\tau^{1-z}\frac{dx_1}{x_1}\int_{\frac{\tau}{x_1}}^1\frac{dx_2}{x_2}\sum_q e_q^2\Big[M^h_q(x_1,z)f_{\bar q}(x_2) + (x_1 \leftrightarrow x_2)\Big]\,\delta\Big(1-\frac{\tau}{x_1x_2}\Big)\,. \tag{6}$$

A pictorial representation of this formula is drawn in Fig. (1). In the following we will restrict ourselves to the discussion of NLO corrections to the $q\bar q$ channel. The corrections to eq. (6) have

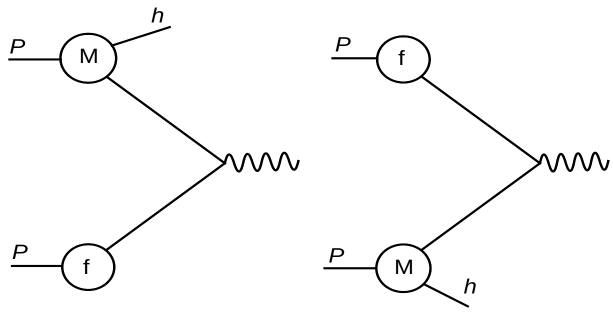

Fig. 1: A pictorial representation of the parton model formula for Semi-Inclusive Drell-Yan process, eq. (6).

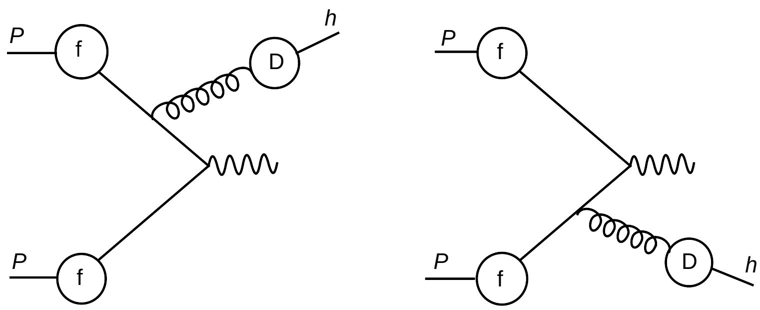

Fig. 2: A pictorial representation of the second term on r.h.s. in eq. (7). The observed hadron h results from the hadronization of initial state radiation (gluon).

the following formal structure

$$d\sigma_{q\bar{q}}^{DY,(1)} \simeq M_q^h \otimes f_{\bar{q}} \otimes \left[1 + \frac{\alpha_s}{2\pi} C_{q\bar{q}}\right] + \frac{\alpha_s}{2\pi} f_q \otimes f_{\bar{q}} \otimes D_g^h \otimes K_{q\bar{q}}^g \,, \tag{7}$$

where the symbol $\otimes$ stands for the convolution on the momentum fraction of the participating partons. The more involved part of the calculation does consist in evaluating next-to-leading order diagrams in which the final state parton hadronize into the observed hadron h. These diagrams are at the origin of the second term on the right hand side of eq. (7). An example of such a diagram is shown in Fig. (2). The coefficient functions $C_{q\bar{q}}$ and $K_{q\bar{q}}^g$ at this level still present poles due to collinear singularities. It is however possible to show, see Ref. [12] for details, that all collinear singularities can be subtracted from the coefficient functions by the same factorization procedure firstly used in Ref. [4] in the context of Deep Inelastic Scattering. We consider this result as a direct evidence of collinear factorization for the Semi-Inclusive Drell-Yan cross-sections. The present QCD-based calculation deals however only with standard soft gluon exchange between active partons but it is blind to soft gluon exchange between spectators. Since our findings support factorization at the collinear level, we implicitly confirm the general widespread idea indicating soft exchanges between spectators partons as responsible for factorization breaking in semi-inclusive hadronic collisions. When diffractive parton distribution, as obtained from HERA data, are used in the present calculation, the resulting predictions would be valid only in the case that factorization hypothesis holds. As a consequence, any deviation observed in the data not accounted for by the present NLO calculation, could be interpreted as a manifestation of factorization breaking. A comparison with data would also establish whether a factorization breaking shows up only in a diffractive kinematic regime or if it manifests itself also in processes with a gapless final state containing, as well, a single hadron in the target fragmentation region. At the same time it would be interesting to study, within the proposed approach, light mesons production which is sensitive to the soft, high multiplicity, fragmentation process. For this reason, in Ref. [12], we address the Semi-Inclusive Drell-Yan process as a prototype of *factorization analyzer*. Since we expect that the factorizing properties of the cross-sections to be extremely sensible to the $p_{h\perp}^2$ of the measured hadron h, we guess that a more efficient observable in this context would be the triple-differential cross-sections:

$$\frac{d\sigma^{DY}}{dQ^2 dp_{h\perp}^2 dz}\,, \tag{8}$$

for which an analog of the present calculation is still not available. The possible identification of an intermediate scale or range of scales at which the factorization breaking effects start to manifest themselves would constitute an important insight into the dynamics of the factorization mechanism.

Let us conclude by listing some further possible developments of the formalism. The present work can be generalized to double hadron production. The evaluation of a double hadron production cross-section needs a full $\mathcal{O}(\alpha_s^2)$ QCD calculation. However, as discussed in Ref. [12], an approximate result could be obtained if one considers the production of two hadrons at low $p_{h\perp}^2$ observed in opposite fragmentation regions with respect to the incoming hadrons. In this case higher order corrections for this process should be the same as for inclusive Drell-Yan process, when the proper kinematics is taken into account. Finally we are thinking to a generalization of the present approach to include gluon initiated hard processes [13] whose relevance in diffractive Higgs production was first suggested in Ref. [14].

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