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Non-perturbative corrections from an s-channel approach

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Abstract
We report on studies of multi-parton corrections from nonlocal operator expansion. We discuss relations between eikonal-line matrix elements and parton distributions, and present an illustration for initial-state collinear evolution.

1 Introduction

Non-perturbative dynamics affects the structure of LHC events even for high momentum transfer, through hadronization, soft underlying scattering, multiple hard interactions. Models for these processes are necessary, for instance, for Monte Carlo generators to produce realistic event simulations.

The treatment of multiple parton interactions in QCD will require methods that go beyond the local operator expansion, and likely involve fully unintegrated parton correlation functions [1]. Besides the relevance for event generators, this should also provide a natural framework for the investigation at the LHC of possible new strong-interaction effects at very high energies, including parton saturation [2].

This report is based on the analysis [3] of nonlocal operator expansion, investigating corrections from graphs with multiple gluon exchange. The point of view in this study is to connect the treatment of multi-gluon contributions with formulations in terms of standard partonic operators, and in this respect it can be seen as deriving from the approach of [4]. We present an illustration for the case of structure functions. This case is also treated in the analyses of [5]. More discussion may be found in [6]. The formulation discussed below trades parton distribution functions for moments of eikonal-line correlators. We expect this formulation to be useful also for the treatment of the associated final-state distributions.

2 From parton distribution functions to eikonal-line matrix elements

The analysis [3] starts with the quark distribution function, defined as

$$f_q(x,\mu) = \frac{1}{4\pi} \int dy^- e^{ixP^+y^-} \langle P|\bar{\psi}(0)Q(0)\gamma^+ Q^\dagger(y^-)\psi(0,y^-,\mathbf{0})|P\rangle_c \qquad (1)$$

where ψ is the quark field, Q is the gauge link, and the subscript c is the instruction to take connected graphs. The matrix element (1) can be rewritten as the real part of a forward scattering amplitude [3], in which we think of the operator $Q^\dagger\psi$ as creating an antiquark plus an eikonal line in the minus direction, starting at distance y^- from the position of the target.

Next, supposing that x is small, we treat the evolution of the antiquark-eikonal system in a hamiltonian framework (see [3] and references therein) which allows us to express the evolution operator in the high-energy approximation as an expansion in Wilson-line matrix elements. The leading term of this is ("dipole" term)

$$\Xi(\boldsymbol{z}, \boldsymbol{b}) = \int [dP'] \, \langle P' | \frac{1}{N_c} \, \mathrm{Tr}\{1 - F^{\dagger}(\boldsymbol{b} + \boldsymbol{z}/2) \, F(\boldsymbol{b} - \boldsymbol{z}/2)\} | P \rangle \quad , \tag{2}$$

where F is the eikonal operator

$$F(\boldsymbol{r}) = \mathcal{P} \exp \left\{ -ig \int_{-\infty}^{+\infty} dz^{-} \mathcal{A}_a^{+}(0, z^{-}, \boldsymbol{r}) \, t_a \right\} , \tag{3}$$

$\boldsymbol{z}$ is the transverse separation between the eikonals in (2), and $\boldsymbol{b}$ is the impact parameter.

In this representation the quark distribution (1) is given by the coordinate-space convolution

$$x f_q(x, \mu) = \int d\boldsymbol{b} \, d\boldsymbol{z} \; u(\mu, \boldsymbol{z}) \, \Xi(\boldsymbol{z}, \boldsymbol{b}) - UV \quad . \tag{4}$$

In [3] the explicit result is given for the function $u(\mu, \boldsymbol{z})$ at one loop in dimensional regularization and for the counterterm $-UV$ of $\overline{\mathrm{MS}}$ renormalization. The $\overline{\mathrm{MS}}$ result can also be recast in a physically more transparent form in terms of a cut-off on the $\boldsymbol{z}$ integration region, as long as the scale μ is sufficiently large compared to the inverse hadron radius:

$$x f_q(x, \mu) = \frac{N_c}{3\pi^4} \int d\boldsymbol{b} \, \frac{d\boldsymbol{z}}{\boldsymbol{z}^4} \, \theta(\boldsymbol{z}^2 \mu^2 > a^2) \, \Xi(\boldsymbol{z}, \boldsymbol{b}) \quad , \tag{5}$$

where a is a renormalization scheme dependent coefficient given in [3].

The Wilson-line matrix element $\Xi(\boldsymbol{z}, \boldsymbol{b})$ receives contribution from both long distances and short distances. At small $\boldsymbol{z}$ it may be treated by a short distance expansion. At large $\boldsymbol{z}$ it should be parameterized consistently with bounds from unitarity and saturation [2] and determined from data.

3 An algebraic relation for eikonal operators

A general relation between fundamental and adjoint representation for Ξ, valid for any distance $\boldsymbol{z}$, is given in [3], based on the algebraic relation

$$\begin{aligned} \frac{1}{N_c^2 - 1} \mathrm{Tr} \left[1 - U^{\dagger}(\boldsymbol{z}) U(\boldsymbol{0}) \right] &= \frac{C_A}{C_F} \frac{1}{N_c} \, \mathrm{Re\ Tr} \left[1 - V^{\dagger}(\boldsymbol{z}) V(\boldsymbol{0}) \right] \\ &- \frac{1}{2} \frac{C_A}{C_F} \frac{1}{N_c^2} \, |\mathrm{Tr} \left[1 - V^{\dagger}(\boldsymbol{z}) V(\boldsymbol{0}) \right] |^2 \end{aligned} \tag{6}$$

with $V = F_{\mathrm{fund.}}, U = F_{\mathrm{adj.}}$.

From this one can obtain small-$\boldsymbol{z}$ relations connecting Ξ to the gluon distribution. For instance, for the fundamental representation at small $\boldsymbol{z}$ this yields

$$\Xi(\boldsymbol{b}, \boldsymbol{z}) = \boldsymbol{z}^2 \, \frac{\pi^2 \alpha_s}{2 N_c} \, x G(x, \mu) \, \phi(\boldsymbol{b}), \tag{7}$$

where by xG we denote the gluon distribution (either the x_c-scale or weighted-average expressions in [3]), and $\phi(\boldsymbol{b})$ obeys

$$\int d\boldsymbol{b}\ \phi(\boldsymbol{b}) = 1. \tag{8}$$

The result for Ξ in the fundamental representation corresponds directly to the one for the dipole cross section in the saturation model [2]. Results in the fundamental and adjoint cases are relevant to discuss quark saturation and gluon saturation.

4 Power-suppressed contributions

In the s-channel framework of [3] contributions to hard processes suppressed by powers of the hard scale are controlled by moments of Ξ,

$$\mathcal{M}_p = \frac{2^{2p}\ p}{\Gamma(1-p)} \int \frac{d\boldsymbol{z}}{\pi \boldsymbol{z}^2}\ (\boldsymbol{z}^2)^{-p} \int d\boldsymbol{b}\ \Xi(\boldsymbol{z},\boldsymbol{b}) \ \ , \tag{9}$$

analytically continued for $p > 1$. Models for the dipole scattering function including saturation are reviewed in [2]. In this case the moments (9) are proportional to integrals over impact parameter of powers of the saturation scale. Higher moments are obtained from derivatives with respect to p,

$$\mathcal{M}_{p,0} \simeq \int d\boldsymbol{b}\ [Q_s^2(\boldsymbol{b})]^p \quad , \quad \mathcal{M}_{p,k} \simeq (-1)^k\ \frac{d^k}{dp^k} \mathcal{M}_{p,0} \ \ . \tag{10}$$

As an illustration, we determine the C_A/x part of the coefficients of the first subleading power correction from the s-channel for transverse and longitudinal structure functions F_T, F_L. Denoting the Q^2 derivative by $\dot{F}_j = dF_j/d\ln Q^2$ for $j = T, L$, and its leading-power contribution by $\dot{F}_{j,lead.}$, one has

$$\dot{F}_j - \dot{F}_{j,lead.} = b_{j,0}\ \mathcal{M}_{2,0}/Q^2 + b_{j,1}\ \mathcal{M}_{2,1}/Q^2 + \cdots \tag{11}$$

Structure functions can be analyzed in the same way [3] as described in Sec. 2 for the quark distribution function. The main difference compared to the case of the quark distribution (1) is that the ultraviolet region of small $\boldsymbol{z}$ is now regulated by the physical scale Q^2 rather than requiring, e.g., $\overline{\text{MS}}$ renormalization. Saturation is reobtained [3] within the dipole approximation [2]. By the analysis based on (6),(7) the saturation scale $Q_s(\boldsymbol{b})$ for a dipole in the fundamental representation is

$$Q_s^2(\boldsymbol{b}) = \frac{2\pi^2 \alpha_s}{N_c}\ xG(x,\mu)\ \phi(\boldsymbol{b}). \tag{12}$$

To study the expansion in powers of $1/Q^2$ it is convenient to go to Mellin moment space by representing Ξ via the Mellin transform

$$\Xi(\boldsymbol{z},\boldsymbol{b}) = \boldsymbol{z}^2 \int_{a-i\infty}^{a+i\infty} \frac{du}{2\pi i}\ (\boldsymbol{z}^2)^{-u}\ \widetilde{\Xi}(u,\mathbf{b}) \ \ , \tag{13}$$

$0 < a < 1$. Then the structure functions $F_{T,L}$ have the representation

$$xF_{T,L} = \int d\boldsymbol{b} \int_{a-i\infty}^{a+i\infty} \frac{du}{2\pi i}\ \widetilde{\Xi}(u,\mathbf{b})\ \Phi_{T,L}(u) \ \ , \tag{14}$$

where $\Phi_{T,L}(u)$ can be read from [7] and are given by

$$\Phi_T(u) = \langle e_a^2 \rangle \frac{N_c}{4^{u+2}\pi^2} (Q^2)^u \frac{\Gamma(3-u)\Gamma(2-u)\Gamma(1-u)}{\Gamma(5/2-u)\Gamma(3/2+u)} \, (1+u) \, \Gamma(u), \tag{15}$$

$$\Phi_L(u) = \langle e_a^2 \rangle \frac{N_c}{4^{u+2}\pi^2} (Q^2)^u \frac{[\Gamma(2-u)]^3}{\Gamma(5/2-u)\Gamma(3/2+u)} \, 2\,\Gamma(1+u), \tag{16}$$

with Γ the Euler gamma function. The expansion in $1/Q^2$ of (14) is controlled by the singularity structure of the integrand in the u-plane [3, 5, 6]. Eqs. (15),(16) show that longitudinal Φ_L has no pole at $u = 0$, so that the leading singularity is given by the $u = 0$ pole in $\widetilde{\Xi}$, while the first subleading pole $u = -1$ is absent in transverse Φ_T due to the numerator factor $(1 + u)$, so that the answer for the transverse case at next-to-leading level is determined by the singularity in $\widetilde{\Xi}$, with Φ contributing to the coefficient of the residue.

It can be verified that contributions to (14) in the lowest $p = 1$ moments in Eq. (10) correctly reproduce the small-x gluon part of renormalization-group evolution,

$$\begin{aligned} \dot{F}_{T,lead.} &= \langle e_a^2 \rangle \, \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \, \frac{[z^2 + (1-z)^2]}{2} \, f_g\Big(\frac{x}{z}, Q\Big) + \text{quark term} \\ &\simeq \langle e_a^2 \rangle \, \frac{\alpha_s}{2\pi} \, \frac{1}{3} \, G + \text{quark term} \ , \end{aligned} \tag{17}$$

using (8),(12) and the gluon distribution G evaluated at the average [3] $x \simeq x_c$, with the lowest x-moment of the gluon $\to$ quark splitting function

$$\int_0^1 dz \, P_{qg}(z) = \int_0^1 dz \, [z^2 + (1-z)^2]/2 = 1/3 \ . \tag{18}$$

Beyond leading power, the first subleading corrections read

$$\dot{F}_T - \dot{F}_{T,lead.} = - \langle e_a^2 \rangle \frac{C_A}{20\pi^3 x} \frac{1}{Q^2} \int d\boldsymbol{b} \, [Q_s^2(\boldsymbol{b})]^2 + \dots \ , \tag{19}$$

$$\begin{aligned} \dot{F}_L - \dot{F}_{L,lead.} &= - \langle e_a^2 \rangle \frac{C_A}{15\pi^3 x} [\frac{14}{15} + \psi(1)] \frac{1}{Q^2} \int d\boldsymbol{b} \, [Q_s^2(\boldsymbol{b})]^2 \\ &+ \langle e_a^2 \rangle \frac{C_A}{15\pi^3 x} \frac{1}{Q^2} \int d\boldsymbol{b} \, [Q_s^2(\boldsymbol{b})]^2 \ln[Q^2/Q_s^2(\boldsymbol{b})] + \dots \ . \end{aligned} \tag{20}$$

That is, the b coefficients in (11) are given by

$$\begin{aligned} b_{T,0} = - \langle e_a^2 \rangle \, C_A/(20\pi^3 x) \quad &, \quad b_{T,1} = 0 \ , \\ b_{L,0} = - \langle e_a^2 \rangle \, C_A \, [14/225 + \psi(1)/15]/(\pi^3 x) \quad &, \quad b_{L,1} = \langle e_a^2 \rangle \, C_A/(15\pi^3 x) \ , \end{aligned} \tag{21}$$

with ψ the Euler psi function.

Via process-dependent coefficients analogous to those in (11), the eikonal-operator moments (9) will also control power-like contributions to the associated jet cross sections due to multi-parton interactions in the initial state. At present these processes are modeled by Monte Carlo, which point to their quantitative significance for the proper simulation of hard events at the LHC. The above discussion also suggests the potential usefulness in this context of analyzing jet and structure function data by trading parton distribution functions for s-channel correlators defined according to the method of Sec. 2.

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