Interactions at high gluon densities

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In the previous section we mentioned the subject of gluon saturation. In this section we continue the discussion of effects due to high gluon densities. First we look at what HERA can teach us about the density of gluons in the impact parameter plane and how this will affect our understanding of processes in pp collisions at LHC. Then, in Sec. 2 we go on to heavy ion collisions and discuss effects of a dense gluon medium there, concentrating on the description of Cherenkov gluons.

1 HERA constrains for LHC MC generators and probing high gluon densities in pp collisions using forward triggers

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In the high energy collisions the finite x component of the wave functions of the colliding hadrons is nearly frozen in transverse plane during the interaction process. Properties of produced final state depend strongly on whether hadrons collided at large impact parameter, b or head on. In particular for small b a chance for a parton to pass through high gluon density fields at a distance ρ from the center of the second nucleon (Fig. 1) is enhanced. The probability of multiple collisions parton collisions is enhanced as well.

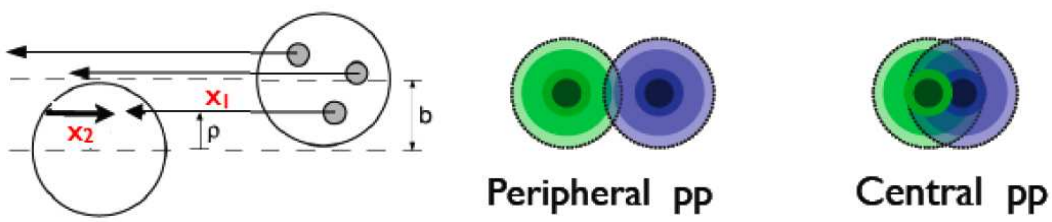

Fig. 1: Side and transverse views of pp collision.

The strength of the encounted gluon fields depends strongly on x of the parton - a parton with a given x_1 and resolution p_t is sensitive to the partons in the target with $x \geq x_2 = 4p_t^2/s_{NN}x_1$. For fixed x_1 characteristic x_2 decrease $\propto 1/s$. For example at the LHC a parton with $x_1 = 0.1, p_t = 2\text{GeV/c}$ resolves $x > 10^{-6}$ while at the GZK energies such parton resolves $x > 10^{-9}$ corresponding to huge gluon densities since a change of x by a factor of ten leads to an increase of gluon density by at least a factor of two.

Studies at HERA provided several important inputs which we discuss below: (i) transverse distribution of gluons in the nucleon, (ii) fluctuations of the strength of the gluon field in the nucleon, (iii) proximity to the black disk regime. When combined with information from the Tevatron collider they indicate also correlations of partons in the transverse plane.

These observations have a number of implications for the dynamics of pp collisions at LHC energies, which are most pronounced in the forward region. Hence we also discuss how to trigger on central pp collisions and how to use such collisions for study of the small x dynamics at very small x.

1.1 Exclusive hard diffraction at HERA - implications for MC at the LHC

The QCD factorization theorem [1, 2] allow to determine the generalized gluon distribution in nucleon for small x from the DIS exclusive meson production at small x as well as from the production of onium states. The t-dependence of these distributions is connected via Fourier transform to the transverse distribution of gluons in a nucleon for a given x. The data confirm our prediction of convergence of the t-slopes for different mesons with increase of Q^2 and weak dependence of the t-slope for the J/ψ-meson production on Q^2. Accordingly, this allows to determine the transverse distribution of gluons as a function of x (for review and references see [3]). It can be approximated as

$$F_g(x,\rho) = \frac{m_g^2}{2\pi}\left(\frac{m_g(x)\rho}{2}\right)K_1(m_g(x)\rho), \tag{1}$$

where K_1 denotes the modified Bessel function. We find $m_g^2(x = 0.05) \sim 1.1 GeV^2$ which corresponds to a much more narrow transverse distribution than given by the electro-magnetic form factors. The radius of the gluon distribution grows with decrease of x reaching the value comparable to the e.m. radius for $x \sim 10^{-4}$ ($m_g^2 \sim 0.7 GeV^2$).

Hence analysis of the HERA data suggests that the transverse gluon distribution, $F_g(x,\rho)$, significantly broadens with decrease of x. At the same time the current MC models of pp collisions assume that transverse parton distributions do not depend on x. Also, in the PYTHIA MC [4] it is assumed that two transverse scales are present in the ρ-dependence of F_g. It is not clear whether this assumption is consistent with Eq. (1) and correspondingly with the data on the exclusive J/ψ production.

Knowledge of $F_g(x,\rho)$ allows to calculate the rate of the production of four jets due to double parton collisions in the pp scattering assuming that the double parton distribution is given by a product of single parton distributions. Using Eq. (1) we find the rate which is a factor of two smaller than observed in the Tevatron experiment [5, 6]. This implies presence of the transverse correlations between partons.

One of the sources of fluctuations is fluctuations of the overall size of the initial parton configurations. In the high energy scattering different initial configurations in the colliding nucleons can be considered as frozen. Studies of the soft inelastic diffraction indicate that the strength of the interaction for different configurations in nucleons fluctuates rather strongly. Presumably significant contribution to these fluctuations comes from the fluctuation of the size of these configurations. One also expects that parton distributions in different configurations should differ as well.

In ref. [7] we deduced the model–independent relation which allows one to infer the small x fluctuations of the gluon density from the observable ratio of inelastic ($\gamma *_L + p \rightarrow VM + X$) and elastic ($\gamma *_L + p \rightarrow VM + p$) diffractive vector meson production at $t = 0$:

$$\omega_g \equiv \frac{\langle G^2\rangle - \langle G\rangle^2}{\langle G\rangle^2} = \left[\frac{d\sigma_{\rm inel}}{dt}\Big/\frac{d\sigma_{\rm el}}{dt}\right]_{t=0}^{\gamma_L^* p\rightarrow VX}. \tag{2}$$

So far there have been no dedicated experimental studies of this ratio. Overall data suggest that $\omega_g \sim 0.2$ for Q^2 of few GeV^2 and $x \leq 10^{-3}$ which corresponds to rather large fluctuations of the gluon density. We also proposed a simple model based on information on the fluctuations of the strength of the strong interaction which allows to reproduce the magnitude of ω_g.

Correlations between fluctuations of the parton densities and the soft–interaction strength have numerous potential implications for high–energy $pp/\bar{p}p$ collisions with hard processes. One example is the relative probability of double binary parton–parton collisions.

The QCD evolution leads to a drop of the fluctuations with an increase of virtuality. As a result in the case of double scattering configurations, the main effect for the overall rate is due to fluctuations of the size of the transverse area of the configurations. The contribution of configurations of size smaller than average is enhanced leading to a a rather modest enhancement of the rate of four jet production $\sim 10-15\%$, which accounts for a small fraction of the remaining discrepancy with the CDF value[1]. However the size of configurations involved in the multijet double / triple scattering trigger is much smaller than the average size, leading to modification of the hadron product in the fragmentation region, long range fluctuations of multiplicity, etc.

Small effect from global fluctuations indicates that other dynamical mechanisms must be responsible for the enhancement of multi–parton collisions, *e.g.* local transverse correlations between partons as suggested by a "constituent quark" picture of the nucleon [3].

1.2 Onset of the black regime in the interaction of fast partons

Interactions of virtual photons with nucleons at HERA can be represented as superposition of the interaction of $q\bar{q}$ dipoles of sizes given by the square of the corresponding photon wave function. The cross section of the inelastic interaction of a $q\bar{q}$ or gluon dipole can be written as

$$\sigma^{\mathrm{q\bar{q}-hadron}}(x, d^2) \;=\; \frac{\pi^2}{4}\, F^2\, d^2\, \alpha_s(Q^2_{\mathrm{eff}})\, xG_T(x, Q^2_{\mathrm{eff}}). \tag{3}$$

Here $F^2 = 4/3$ is the Casimir operator of the fundamental representation of the $SU(3)$ gauge group. Furthermore, $\alpha_s(Q^2_{\mathrm{eff}})$ is the LO running coupling constant and $G_T(x, Q^2_{\mathrm{eff}})$ the LO gluon density in the target. They are evaluated at a scale $Q^2_{\mathrm{eff}} \approx \lambda d^{-2}$, where $\lambda = 5 \div 9$ can be determined from NLO calculations or from phenomenological considerations.

Since the gluon density rapidly increases with decrease of x while the transverse radius of the nucleon grows rather slowly, one expects based on Eq. (3) that interaction should approach the black disk regime of complete absorption at sufficiently large energies. To determine the proximity to this limit it is convenient to study the amplitude of the dipole - nucleon scattering, $A^{dp}(s,t)$ which can be inferred from analysis of the data on the total DIS cross section and data on exclusive production of vector mesons [8].

Introducing impact parameter representation of the amplitude

$$A^{dp}(s,t) \;=\; \frac{i\,s}{4\pi}\int d^2b\; e^{-i(\mathbf{\Delta}_\perp \mathbf{b})}\,\Gamma^{dp}(s,b) \qquad\qquad (t = -\mathbf{\Delta}^{\mathbf{2}}_\perp), \tag{4}$$

[1] Note that the CDF measurements correspond to relatively large x where the "radiative" model of the gluon density fluctuations we developed may not be applicable and where no data on the hard inelastic exclusive diffraction are available. However, if the gluon strength is larger for configurations of larger size, it would lead to reduction of already rather small enhancement of the rate of multiple collisions.

we can determine $\Gamma^{dp}(s,b)$ which is referred to as the profile function. In the situation when elastic scattering is the "shadow" of inelastic scattering, the profile function at a given impact parameter is restricted to $\left|\Gamma^{dp}(s,b)\right| \leq 1$. The probability of the inelastic interaction for given b

$$P_{inel}(b) = 1 - \left|1 - \Gamma^{dp}(s,b)\right|^2, \qquad (5)$$

is equal to one in the black-disc (BD) limit.

We found [8] that interaction of $q\bar{q}$ dipoles with transverse size ~ 0.3 fm corresponding to $Q^2 \sim 4GeV^2$ is still rather far from the BD regime for the range covered by HERA even for small impact parameters, b. At the same time a much stronger interaction in the gluon channel (a factor of 9/4 larger F^2 in Eq. (3)) leads to $\Gamma_{gg}(d \sim 0.3fm, x \sim 10^{-4})$ close to one in a large range of b, see Fig. 2. Proximity of Γ_{gg} to one in a wide range of b for $Q^2 \sim 4GeV^2$ naturally explains a large probability of diffraction ($\sim 30 \div 40\%$) in the gluon induced hard interactions which can be inferred from the HERA DGLAP analyses of the inclusive DIS diffractive data (see discussion and references in [3]).

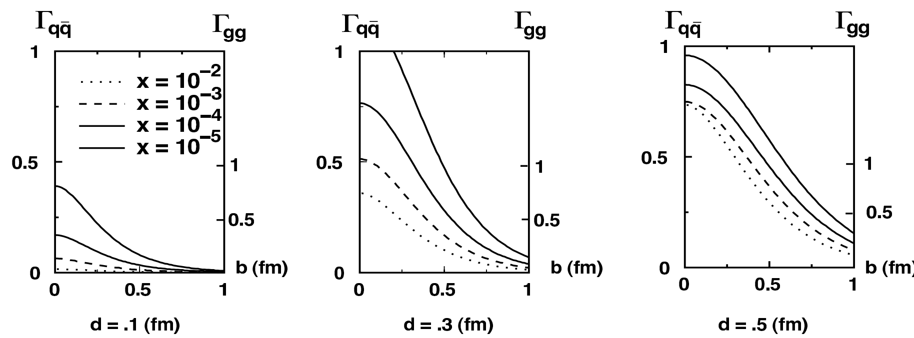

Fig. 2: The profile function of dipole-nucleon scattering, Γ^{dp}, as a function of the impact parameter, b, for various values of the dipole size, d, and x, as obtained from a phenomenological estimate outlined in the text. Shown are the results for $q\bar{q}$ (left scale) and gg dipoles (right scale)

In the BD regime parton obtains transverse momenta of the order of the maximal p_t scale at which interaction remains black and also looses a substantial fraction of its longitudinal momentum (one can also think of this as a post selection of configurations in the incoming wave function with large transverse momenta; the simplest example is scattering of virtual photon in the BD regime [9]). The analysis of the data obtained by the BRAHMS [10] and STAR collaborations [11] on the leading pion production in the deuteron - gold collisions including forward - central rapidity correlations supports presence of this phenomenon for gluon densities comparable to those encounted at HERA [12].

At the LHC energies for the fragmentation region BD regime extends to quite large p_t for the leading partons (especially for gluons) up to $\rho \sim 0.5\mathrm{fm}$ which give important contribution to the central pp collisions (see Fig. 3 adapted from [13]).

Hence, in the pp collisions large x partons of nucleon "1" passing at small transverse distances ρ from the nucleon "2" should get large transverse momenta and also loose significant

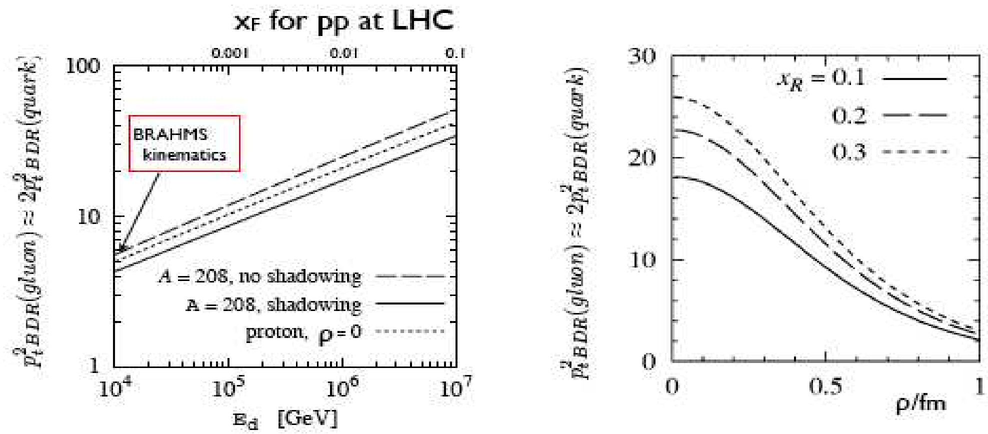

Fig. 3: Dependence of the maximum p_t^2 for gluon for which interaction is close to the BD regime as a function of x_F (energy of the parton) for $\rho = 0$ and as function of ρ for different x_R of the gluon for the LHC pp collisions.

fraction of energy. Note here that this effect is masked in many current MC event generators for pp collisions at the LHC, where a cutoff on minimal momentum transfer of the order 3 GeV is introduced.

One should note here that the necessity to tame intensity of hard collisions in pp scattering could be derived without invoking a study of the multiplicities of the produced hadrons as it is done e.g. in PYTHIA [4]. Instead, one can study the probability of inelastic interaction as a function of b which can be determined from unitarity - information on the elastic amplitude, and calculating the inelasticity due to hard parton-parton interactions. We found that for $b \sim 1.5 fm$ (where uncertainties due to the contribution of multiparton interactions appear to be small) one needs to introduce a cutoff of the order of three GeV to in order to avoid a contradiction with the S-channel unitarity [14]. The taming of the small x parton densities in the relevant $x \geq 10^{-4}$ range for $\rho \sim 0.7$ fm is very small. Hence, it is not clear so far what dynamical mechanism is responsible for resolving problems with S-channel unitarity.

Modifications of the pattern of the collisions due to the large scale of BD regime for small ρ should be pronounced most prominently in the collisions at small impact parameters. Therefore they are enhanced in the processes of production of new particles which correspond to significantly smaller impact parameters than the minimum bias inelastic collisions. Among the expected effects are suppression of the leading baryon production, energy flow from forward region to smaller rapidities, larger central multiplicity, etc.

1.3 Centrality trigger for pp collisions

To study effects of high gluon densities it is desirable to develop a trigger for centrality in pp collisions [15]. We explore the observation that the leading nucleons are usually produced when number of ”wounded” quarks, N_w is ≤ 1. If $N_w \geq 2$, at least two quarks receive large transverse momenta they cannot combine into a leading nucleon as they fragment independently, so the spectra for $N_w = 2$ and $N_w = 3$ should be rather similar and shifted to much smaller x_F than in soft interactions where the spectra of nucleons are known to be flat in x_F in a wide range of x_F.

We developed a MC event generator to quantify this observation. At the first step three quark configuration in one nucleon is generated with transverse coordinates given by the nucleon wave function. For given b we determine the gluon density encounted by each quark and if the gluon density corresponds to the BD regime, generate a transverse momentum for a quark using the model of [16] (we neglect the fractional energy losses expected in the BD regime [12]).

We implemented the fragmentation of the system produced in the first stage by constructing strings which decay using the LUND method. There are always two strings, drawn between a quark and a diquark from the interacting particles. When a quark of the diquark receives a high transverse momentum, the diquark becomes a system of two quarks and a junction. This has the nice property that one recovers the diquark when the invariant mass between the two quarks is small. The results are in good agreement with the qualitative expectation that spectra for $N_w = 0$ and $N_w = 1$ are similar and much harder than for $N_w = 2, 3$ which are very similar, see Fig. 3 in Ref. [15].

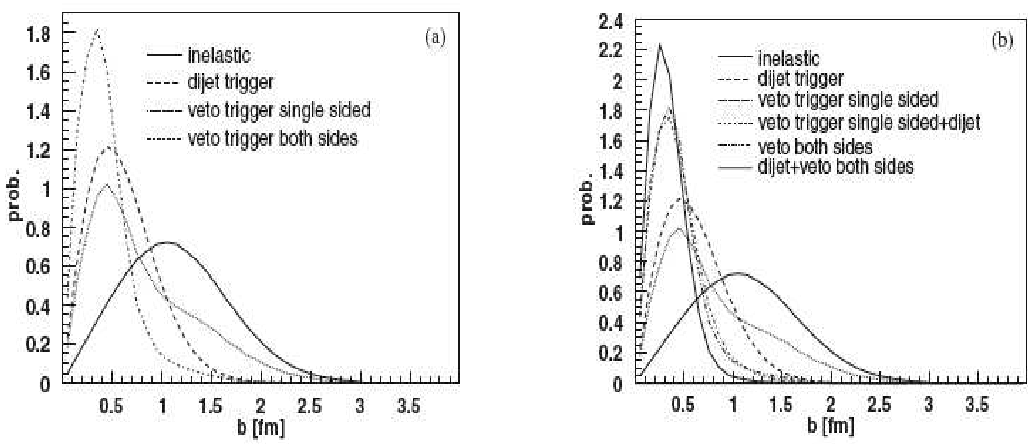

Fig. 4: (a):The combination of dijet and veto trigger gives the best constraints on central events in pp-collisions. (b):Impact parameter distributions for inelastic events, the dijet trigger and single and double sided veto-trigger (no baryon in the region $x_F > 0.1$).

We find that N_w strongly depends on b with $N_w \geq 2$ dominating for $b \leq 0.5\mathrm{fm}$. A strong correlation of N_w with the multiplicity of leading baryons allows one to determine the effectiveness of a centrality trigger based on a veto for the production of leading baryons with $x > x_{tr}$ as a function of x_{tr}. We find than an optimal value of x_{tr} is ~ 0.1. Current configurations of several LHC detectors allow to veto neutron production in this x-range. TOTEM, in addition, allows to veto production of protons with $x_F > 0.8$. Since neutron and proton multiplicities are similar, a one side veto for production of both charged and neutral baryons leads approximately to the same result as a two side veto for neutron production. Accordingly we will give results both for single side veto and for two side veto for both neutral and charged baryons (understanding that the full implementation of the latter option would require certain upgrades of the detectors some of which are currently under discussion). The results of the calculations are presented in Fig. 4a together with the distribution over b for generic inelastic events and the central dijet trigger [13]. We see that the single side veto trigger leads to a centrality similar to that of a the dijet trigger, while a double side veto leads to the most narrow distribution in b. An easy way to check this expectation would be to compare other characteristics of these types of events - one expects for example a progressive increase of the central multiplicity with a decrease of average b.

The most narrow distributions can be achieved by selecting events with dijets and without leading baryons, Fig. 4b in this case we reach the limit that $\langle \rho_{tr} \rangle = \left(\langle \rho^2 \rangle + \langle b^2 \rangle \right)^{1/2}$ becomes comparable to $\langle \rho \rangle$ which is the smallest possible average $\langle \rho \rangle$ for pp or DIS collisions.

1.4 Conclusions

Understanding of the complexity of the nucleon structure is gradually emerging from the studies of hard interactions at HERA and Tevatron collider. In addition to revealing a small transverse localization of the gluon field one finds a number of other pecularities: presence of significant fluctuations of the transverse size of the nucleon and the strength of the gluon fields, as well as indications of a lumpy structure of nucleon at low scale (constituent quarks).

Due to proximity of BD regime for a large range of virtualities the small x physics appears to be an unavoidable component of the new particle physics production at LHC.

One of the biggest challenges is to understand the mechanism and pattern of taming of parton interactions at transverse momenta of few GeV and how it affects spectra of leading partons in the central collisions. It maybe the best to study these phenomena using centrality triggers to amplify these phenomena. Among most sensitive tools are long range correlations in rapidity - central and forward hadron production, forward - backward correlations, transverse distribution in various hard processes with centrality trigger, etc. Large rapidity coverage of ATLAS and CMS / TOTEM allows to study correlations at much larger rapidity intervals than it was possible at previous colliders.

2 In-medium QCD and Cherenkov gluons vs. Mach waves at LHC

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The properties and evolution of the medium formed in ultrarelativistic heavy-ion collisions are widely debated. At the simplest level it is assumed to consist of a set of current quarks and gluons. The collective excitation modes of the medium may, however, play a crucial role. One of the ways to gain more knowledge about the excitation modes is to consider the propagation of relativistic partons through this matter. Phenomenologically their impact would be described by the nuclear permittivity of the matter corresponding to its response to passing partons. This approach is most successful for electrodynamical processes in matter. Therefore, it is reasonable to modify the QCD equations by taking into account collective properties of the quark-gluon medium [17]. Strangely enough, this was not done earlier. For the sake of simplicity we consider here the gluondynamics only.

The classical lowest-order solution of these equations coincides with Abelian electrodynamical results up to a trivial color factor. One of the most spectacular of them is Cherenkov radiation and its properties. Now, Cherenkov gluons take the place of Cherenkov photons [18–20]. Their emission in high-energy hadronic collisions is described by the same formulae but with the nuclear permittivity in place of the usual one. Actually, one considers them as quasiparticles, i.e. quanta of the medium excitations leading to shock waves with properties determined by the permittivity.

Another problem of this approach is related to the notion of the rest system of the medium. It results in some specific features of this effect at LHC energies.

To begin, let us recall the classical in-vacuum Yang-Mills equations

$$D_\mu F^{\mu\nu} = J^\nu, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu], \tag{6}$$

where $A^\mu = iA_a^\mu T_a;\ A_a(A_a^0 \equiv \Phi_a, \mathbf{A}_a)$ are the gauge field (scalar and vector) potentials, the color matrices T_a satisfy the relation $[T_a, T_b] = if_{abc}T_c,\ \ D_\mu = \partial_\mu - ig[A_\mu, \cdot],\ J^\nu(\rho, \mathbf{j})$ is a classical source current, and the metric is given by $g^{\mu\nu}$=diag(+,–,–,–).

In the covariant gauge $\partial_\mu A^\mu = 0$ they are written

$$\square A^\mu = J^\mu + ig[A_\nu, \partial^\nu A^\mu + F^{\mu\nu}], \tag{7}$$

where $\square$ is the d'Alembertian operator.

The chromoelectric and chromomagnetic fields are $E^\mu = F^{\mu 0},\ B^\mu = -\frac{1}{2}\epsilon^{\mu ij}F^{ij}$ or, as functions of the gauge potentials in vector notation,

$$\mathbf{E}_a = -\mathrm{grad}\Phi_a - \frac{\partial \mathbf{A}_a}{\partial t} + gf_{abc}\mathbf{A}_b\Phi_c, \quad \mathbf{B}_a = \mathrm{curl}\mathbf{A}_a - \frac{1}{2}gf_{abc}[\mathbf{A}_b\mathbf{A}_c]. \tag{8}$$

Herefrom, one easily rewrites the in-vacuum equations of motion (6) in vector form. We do not show them explicitly here (see [17]) and write down the equations of the in-medium gluon dynamics using the same method as in electrodynamics. We introduce the nuclear permittivity and denote it also by ϵ, since this will not lead to any confusion. After that, one should replace $\mathbf{E}_a$ by $\epsilon\mathbf{E}_a$ and get

$$\epsilon(\mathrm{div}\mathbf{E}_a - gf_{abc}\mathbf{A}_b\mathbf{E}_c) = \rho_a, \quad \mathrm{curl}\mathbf{B}_a - \epsilon\frac{\partial \mathbf{E}_a}{\partial t} - gf_{abc}(\epsilon\Phi_b\mathbf{E}_c + [\mathbf{A}_b\mathbf{B}_c]) = \mathbf{j}_a. \tag{9}$$

The space-time dispersion of ϵ is neglected here.

In terms of potentials these equations are cast in the form

$$\begin{aligned} \triangle\mathbf{A}_a - \epsilon\frac{\partial^2\mathbf{A}_a}{\partial t^2} = -\mathbf{j}_a - gf_{abc}(\frac{1}{2}\mathrm{curl}[\mathbf{A}_b, \mathbf{A}_c] + \frac{\partial}{\partial t}(\mathbf{A}_b\Phi_c) + [\mathbf{A}_b\mathrm{curl}\mathbf{A}_c] - \\ \epsilon\Phi_b\frac{\partial\mathbf{A}_c}{\partial t} - \epsilon\Phi_b\mathrm{grad}\Phi_c - \frac{1}{2}gf_{cmn}[\mathbf{A}_b[\mathbf{A}_m\mathbf{A}_n]] + g\epsilon f_{cmn}\Phi_b\mathbf{A}_m\Phi_n), \end{aligned} \tag{10}$$

$$\begin{aligned} \triangle\Phi_a - \epsilon\frac{\partial^2\Phi_a}{\partial t^2} = -\frac{\rho_a}{\epsilon} + gf_{abc}(2\mathbf{A}_c\mathrm{grad}\Phi_b + \mathbf{A}_b\frac{\partial\mathbf{A}_c}{\partial t} + \frac{\partial\Phi_b}{\partial t}\Phi_c) - \\ g^2 f_{amn}f_{nlb}\mathbf{A}_m\mathbf{A}_l\Phi_b. \end{aligned} \tag{11}$$

If the terms with coupling constant g are omitted, one gets the set of Abelian equations, that differ from electrodynamical equations by the color index a only. The external current is due to a parton moving fast relative to partons ”at rest”.

The crucial distinction between (7) and (10), (11) is that there is no radiation (the field strength is zero in the forward light-cone and no gluons are produced) in the lowest order solution of (7), and it is admitted for (10), (11), because ϵ takes into account the collective response (color polarization) of the nuclear matter.

Cherenkov effects are especially suited for treating them by classical approach to (10), (11). Their unique feature is independence of the coherence of subsequent emissions on the time interval between these processes. The lack of balance of the phase $\Delta\phi$ between emissions with frequency $\omega = k/\sqrt{\epsilon}$ separated by the time interval Δt (or the length $\Delta z = v\Delta t$) is given by

$$\Delta\phi = \omega\Delta t - k\Delta z\cos\theta = k\Delta z(\frac{1}{v\sqrt{\epsilon}} - \cos\theta) \quad (12)$$

up to terms that vanish for large distances. For Cherenkov effects the angle θ is

$$\cos\theta = \frac{1}{v\sqrt{\epsilon}}. \quad (13)$$

The coherence condition $\Delta\phi = 0$ is valid independent of Δz. This is a crucial property specific for Cherenkov radiation only. The fields $(\Phi_a, \mathbf{A}_a)$ and the classical current for in-medium gluon dynamics can be represented by the product of the electrodynamical expressions $(\Phi, \mathbf{A})$ and the color matrix T_a.

Let us recall the Abelian solution for the current with velocity $\mathbf{v}$ along z-axis:

$$\mathbf{j}(\mathbf{r}, t) = \mathbf{v}\rho(\mathbf{r}, t) = 4\pi g\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t). \quad (14)$$

In the lowest order the solutions for the scalar and vector potentials are related $\mathbf{A}^{(1)}(\mathbf{r}, t) = \epsilon\mathbf{v}\Phi^{(1)}(\mathbf{r}, t)$ and

$$\Phi^{(1)}(\mathbf{r}, t) = \frac{2g}{\epsilon}\frac{\theta(vt - z - r_\perp\sqrt{\epsilon v^2 - 1})}{\sqrt{(vt - z)^2 - r_\perp^2(\epsilon v^2 - 1)}}. \quad (15)$$

Here $r_\perp = \sqrt{x^2 + y^2}$ is the cylindrical coordinate; z symmetry axis. The cone

$$z = vt - r_\perp\sqrt{\epsilon v^2 - 1} \quad (16)$$

determines the position of the shock wave due to the θ-function in (15). The field is localized within this cone and decreases with time as $1/t$ at any fixed point. The gluons emission is perpendicular to the cone (16) at the Cherenkov angle (13).

Due to the antisymmetry of f_{abc}, the higher order terms (g^3,...) are equal to zero for any solution multiplicative in space-time and color as seen from (10), (11).

The expression for the intensity of the radiation is given by the Tamm-Frank formula (up to Casimir operators) that leads to infinity for constant ϵ. The ω-dependence of ϵ (dispersion), its imaginary part (absorption) and chromomagnetic permeability can be taken into account [17].

The attempts to calculate the nuclear permittivity from first principles are not very convincing. It can be obtained from the polarization operator. The corresponding dispersion branches have been computed in the lowest order perturbation theory [21, 22]. The properties of collective excitations have been studied in the framework of the thermal field theories (see, e.g., [23]). The results with an additional phenomenological ad hoc assumption about the role of resonances were used in a simplified model of scalar fields [20] to show that the nuclear permittivity can be larger than 1, i.e. admits Cherenkov gluons. Extensive studies were performed in [24]. No final decision about the nuclear permittivity is yet obtained from these approaches. It must be notrivial problem because we know that, e.g., the energy dependence of the refractive index of water [25] (especially, its imaginary part) is so complicated that it is not described quantitatively in electrodynamics.

Therefore, we prefer to use the general formulae of the scattering theory to estimate the nuclear permittivity. It is related to the refractive index n of the medium $\epsilon = n^2$ and the latter one is expressed through the real part of the forward scattering amplitude of the refracted quanta $\mathrm{Re}F(0^{\mathrm{o}}, E)$ by

$$\mathrm{Re}n(E) = 1 + \Delta n_R = 1 + \frac{6m_\pi^3\nu}{E^2}\mathrm{Re}F(E) = 1 + \frac{3m_\pi^3\nu}{4\pi E}\sigma(E)\rho(E). \tag{17}$$

Here E denotes the energy, ν the number of scatterers within a single nucleon, m_π the pion mass, $\sigma(E)$ the cross section and $\rho(E)$ the ratio of real to imaginary parts of the forward scattering amplitude $F(E)$.

Thus the emission of Cherenkov gluons is possible only for processes with positive $\mathrm{Re}F(E)$ or $\rho(E)$. Unfortunately, we are unable to calculate directly in QCD these characteristics of gluons and have to rely on analogies and our knowledge of the properties of hadrons. The only experimental facts we get for this medium are brought about by particles registered at the final stage. They have some features in common, which (one may hope!) are also relevant for gluons as the carriers of the strong forces. Those are the resonant behavior of amplitudes at rather low energies and the positive real part of the forward scattering amplitudes at very high energies for hadron-hadron and photon-hadron processes as measured from the interference of the Coulomb and hadronic parts of the amplitudes. $\mathrm{Re}F(0^{\mathrm{o}}, E)$ is always positive (i.e., $n > 1$) within the low-mass wings of the Breit-Wigner resonances. This shows that the necessary condition for Cherenkov effects $n > 1$ is satisfied at least within these two energy intervals. This fact was used to describe experimental observations at SPS, RHIC and cosmic ray energies. The asymmetry of the ρ-meson shape at SPS [26] and azimuthal correlations of in-medium jets at RHIC [27–30] were explained by emission of comparatively low-energy Cherenkov gluons [31, 32]. The parton density and intensity of the radiation were estimated. In its turn, cosmic ray data [33] at energies corresponding to LHC require very high-energy gluons to be emitted by the ultrarelativistic partons moving along the collision axis [18, 19]. Let us note the important difference from electrodynamics, where $n < 1$ at high frequencies.

The in-medium equations are not Lorentz-invariant. There is no problem in macroscopic electrodynamics, because the rest system of the macroscopic matter is well defined and its permittivity is considered there. For collisions of two nuclei (or hadrons) it requires special discussion.

Let us consider a particular parton that radiates in the nuclear matter. It would ”feel” the surrounding medium at rest if the momenta of all other partons, with which this parton can interact, are smaller and sum to zero. In RHIC experiments the triggers, that registered the jets (created by partons), were positioned at 90^o to the collision axis. Such partons should be produced by two initial forward-backward moving partons scattered at 90^o. The total momentum of the other partons (medium spectators) is balanced, because for such a geometry the partons from both nuclei play the role of spectators forming the medium. Thus the center of mass system is the proper one to consider the nuclear matter at rest in this experiment. The permittivity must be defined there. The Cherenkov rings consisting of hadrons have been registered around the away-side jet, which traversed the nuclear medium. This geometry requires, however, high statistics, because the rare process of scattering at 90^o has been chosen.

The forward (backward) moving partons are much more numerous and have higher energies. However, one cannot treat the radiation of such a primary parton in the c.m.s. in a similar way, because the momentum of the spectators is different from zero, i.e. the matter is not at rest. Now the spectators (the medium) are formed from the partons of another nucleus only. Then the rest system of the medium coincides with the rest system of that nucleus and the permittivity should refer to this system. The Cherenkov radiation of such highly energetic partons must be considered there. That is what was done for interpretation of the cosmic ray event in [18, 19]. This discussion shows that one must carefully define the rest system for other geometries of the experiment with triggers positioned at different angles.

Thus our conclusion is that the definition of ϵ depends on the geometry of the experiment. Its corollary is that partons moving in different directions with different energies can "feel" different states of matter in the $\mathbf{same}$ collision of two nuclei because of the permittivity dispersion. The transversely scattered partons with comparatively low energies can analyze the matter with rather large permittivity corresponding to the resonance region, while the forward moving partons with high energies would "observe" a low permittivity in the same collision. This peculiar feature can help scan the $(\ln x, Q^2)$-plane as discussed in [34]. It explains also the different values of ϵ needed for the description of the RHIC and cosmic ray data.

These conclusions can be checked at LHC, because both RHIC and cosmic ray geometry will become available there. The energy of the forward moving partons would exceed the thresholds above which $n > 1$. Then both types of experiments can be done, i.e. the 90^o-trigger and non-trigger forward-backward partons experiments. The predicted results for 90^o-trigger geometry are similar to those at RHIC. The non-trigger Cherenkov gluons should be emitted within the rings at polar angles of tens degrees in c.m.s. at LHC by the forward moving partons (and symmetrically by the backward ones) according to some events observed in cosmic rays [32, 33].

Let us compare the conclusions for Cherenkov and Mach shock waves. The Cherenkov gluons are described as the transverse waves while the Mach waves are longitudinal. Up to now, no experimental signatures of these features were proposed.

The most important experimental fact is the position of the maxima of humps in two-particle correlations. They are displaced from the away-side jet by 1.05-1.23 radian [35–38]. This requires rather large values of $\mathrm{Re}\epsilon \sim 2 - 3$ and indicates high density of the medium [32] that agrees with other conclusions. The fits of the humps with complex permittivity are in progress. The maxima due to Mach shock waves should be shifted by the smaller value 0.955 if the relativistic equation of state is used ($\cos\theta = 1/\sqrt{3}$). To fit experimental values one must consider different equation of state. In three-particle correlations, this displacement is about 1.38 [27–29].

There are some claims [27–30] that Cherenkov effect contradicts to experimental observations because it predicts the shift of these maxima to smaller angles for larger momenta. They refer to the prediction made in [20]. However, the conclusions of this paper about the momentum dependence of the refractive index can hardly be considered as quantitative ones because the oversimplified scalar Φ^3-model with simplest resonance insertions was used for computing the refractive index. In view of difficult task of its calculation discussed above, the fits of maxima seem to be more important for our conclusions about the validity of the two schemes.

Mach waves should appear for forward moving partons at RHIC but were not found. The energy threshold of ϵ explains this phenomenon for Cherenkov gluons.

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