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Kinetic theory of real scalar quantum field in curved spacetime

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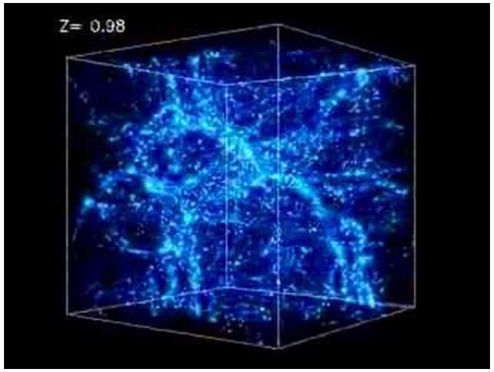
Based on

- Phys. Rev. D 96, 083504 (2017) with T. Prokopec
- Phys. Rev. D 98, 025010 (2018) with T. Prokopec

Cargèse, 20 July 2018

Classical particles in large-scale structure cosmology



- Dynamics for phase-space density of collisionless **cold dark matter** particles in GR (Vlasov equation)

$$\left[p^\mu \frac{\partial}{\partial X^\mu} + p^\mu p_\nu \Gamma^\nu{}_{\mu i}[f_{\text{cl}}] \frac{\partial}{\partial p_i}\right] f_{\text{cl}} = 0$$

$$G_{\mu\nu} \sim T_{\mu\nu}[f_{\text{cl}}]$$

- **Non-linear** equation with **no general solution**
  - perfect fluid truncation breaks down at shell-crossing
  - effective theory of large-scale structure as perturbation theory for fluid variables
  - studied with N-body simulations
- Another access to this problem based on QFT-techniques that contains a classical limit?

- Motivates **microscopic description** for **gravitationally interacting particles**, that
  - can systematically account for **relativistic corrections** and **scale separation**
  - reveals effects of **renormalization** (anomalies, UV noise)
  - goes **beyond condensates** (general Gaussian quantum state)

Real scalar field operator in semi-classical GR

$$\Box\hat{\phi} = \frac{m^2}{\hbar^2}\hat{\phi} + \xi R\hat{\phi} + \frac{1}{6}\frac{\lambda}{\hbar} : \hat{\phi}^3 :$$

$$G_{\mu\nu} \sim \left\langle : \hat{T}_{\mu\nu}[\hat{\phi}] : \right\rangle$$

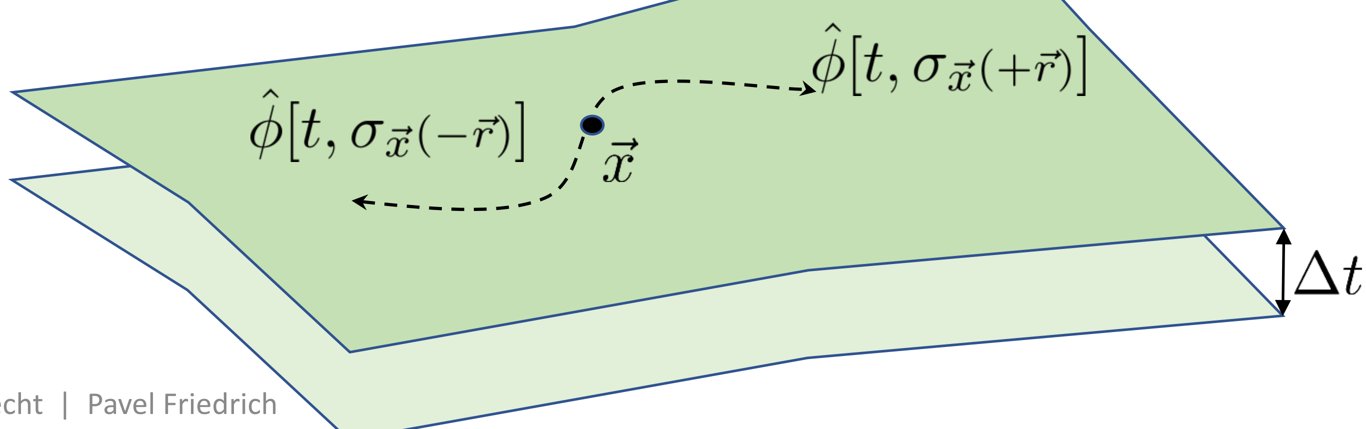
Phase-space densities via **Wigner transform** of 2P-functions of canonical operators

$$W_{\phi\phi}(t,\vec{x},\vec{p}) \sim \int d^3r e^{-\frac{i}{\hbar} r^k p_k} \left\langle \quad \ldots \quad \right\rangle$$

$$W_{\phi\Pi} \ldots$$

$$W_{\Pi\phi} \ldots$$

$$W_{\Pi\Pi} \ldots$$



Generalization of on-shell Vlasov equation for scalar dark matter

- Recombination and adjusting energy dimensions yields
  - a **particle density** whose dynamics has a **classical limit** with gradient corrections

$$f_1 \sim \frac{\omega_p}{\hbar} W_{\phi\phi} + \frac{\hbar}{\omega_p} W_{\Pi\Pi} + W_{\Pi\phi} - W_{\phi\Pi} \,,$$

$$\left[p^\mu \partial_\mu + p_\mu p^\nu \Gamma^\mu_{\ \nu i} \frac{\partial}{\partial p_i} + \mathcal{O}\big(\lambda\, \partial_k \int d^3 q f_1[q]\big) \frac{\partial}{\partial p_k} + \mathcal{O}\big(\hbar^2\big) \right] f_1(x^\mu, p_j) = \mathcal{O}\big(f_{2,3}\big)$$

$$G_{\mu\nu} \sim T_{\mu\nu}[f_{1,2,3}]$$

  - two additional densities (that represent oscillatory d.o.f. to leading order)

$$f_2 \sim \frac{\omega_p}{\hbar} W_{\phi\phi} - \frac{\hbar}{\omega_p} W_{\Pi\Pi} \,, \quad f_3 \sim W_{\Pi\phi} + W_{\phi\Pi}$$

- In **Phys. Rev. D 98, 025010 (2018)** we derive full coupled first-order system in time for $f_{1,2,3}$ with first-order corrections in spatial gradients, i.e. $\partial_t + \ldots + \hbar \frac{{}^{(3)}R^2}{\omega_p} + \hbar \frac{\Delta}{\omega_p} \ldots$, one-loop self-mass corrections in λ and terms including non-minimal coupling to gravity ξ

Summary

- Real scalar quantum field theory models general collisionless as well as self-interacting cold dark matter by taking into account connected two-point functions
- Two-point functions contain degrees of freedom to specify non-trivial moments in momentum space (e.g. anisotropic stress, vorticity)
  → goes beyond one-point function (classical field) approach
- Vlasov-Poisson system recovered to lowest order in gradient expansion and weak field limit of scalar longitudinal metric
- Plethora of correction terms whose influence needs to be studied carefully (e.g. correction of Newton potential due to one-loop self-mass correction)

Outlook

- Reformulation of theory only in terms of dynamical microscopic dofs might lead to new insights or computational techniques for non-linear cold dark matter evolution
- Quantify deviations induced by gradient expansion, self-mass corrections and the additional phase-space densities
- Formalism not only tied to dark matter particles (corrections for photon field?)