# RG flows in Non-Perturbative Gauge-Higgs Unification I

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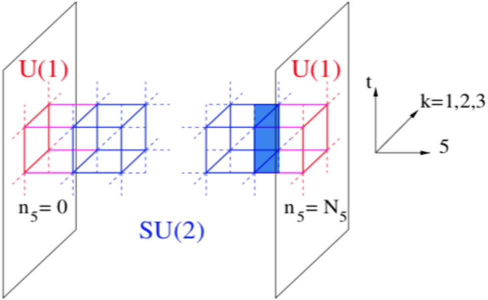
# Introduction

- Gauge-Higgs Unification (GHU) was first introduced by N. Manton [1], D. Fairlie[2] and Y. Hosotani [3].
- Our approach originates from a Non-perturbative description of GHU (NPGHU)[4]
- NPGHU: A 5$d$ Yang-Mills model defined on a hypercubic orbifold lattice, anisotropic in the fifth dimension. The anisotropy parameter then is $\gamma = a_4/a_5$.




[1] *A New Six-Dimensional Approach to the Weinberg-Salam Model*, Nucl.Phys. **B158** 141.

[2] *Fields and the Determination of the Weinberg Angle*, Phys.Lett. **B82** 97

[3] *Dynamical Gauge Symmetry Breaking as the Casimir Effect*, Phys.Lett. **B129** 193

[4] M. Alberti, N. Irges, F. Knechtli and G. Moir, *Five-Dimensional Gauge-Higgs Unification: A Standard Model-Like Spectrum*, JHEP **1509** 159.

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- The orbifold condition on the links read

$$\begin{aligned}(1-\Gamma)U(n_M, N) &= 0, \quad \Gamma \equiv \mathcal{R}\mathcal{T}_g \\ \mathcal{R} n_M &= \bar{n}_M \equiv (n_\mu, -n_5) \\ \mathcal{R} U(n_M, \nu) &= U(\bar{n}_M, \nu) \\ \mathcal{R} U(n_M, 5) &= U(\bar{n}_M - \hat{5}, 5) \\ \mathcal{T}_g U(n_M, N) &= g U(n_M, N) g^{-1}\end{aligned}$$

# Continuum limit

- The Euclidean anisotropic orbifold lattice action reads $S_{S^1/\mathbb{Z}_2} = S^{b-H}_{S^1/\mathbb{Z}_2} + S^{bulk}_{S^1/\mathbb{Z}_2}$ with

$$S^{b-H}_{S^1/\mathbb{Z}_2} = \frac{1}{2N_C}\sum_{n_\mu}\left[\beta_4\sum_{\mu<\nu}\frac{1}{2}\mathrm{tr}\left\{1 - U^{U(1)}_{\mu\nu}(n_\mu,0)\right\} + \beta_5\sum_\mu \mathrm{tr}\left\{1 - U^{H}_{\mu 5}(n_\mu,0)\right\}\right]$$

$$S^{bulk}_{S^1/\mathbb{Z}_2} = \frac{1}{2N_C}\sum_{n_\mu,n_5}\left[\beta_4\sum_{\mu<\nu}\mathrm{tr}\left\{1 - U_{\mu\nu}(n_\mu,n_5)\right\} + \beta_5\sum_\mu \mathrm{tr}\left\{1 - U_{\mu 5}(n_\mu,n_5)\right\}\right]$$

- $\beta_4 = \frac{2N_C a_5}{g_5^2} = \frac{\beta}{\gamma}$ and $\beta_5 = \frac{2N_C a_4^2}{a_5 g_5^2} = \beta\gamma$, are dimensionless lattice couplings given by
- Expanding the Wilson plaquettes to leading order in the lattice spacing we end up with

$$S^{b-H}_{S^1/\mathbb{Z}_2} = \sum_{n_\mu} a_4^4 a_5\left[\frac{1}{4}\sum_{\mu,\nu}F^3_{\mu\nu}F^3_{\mu\nu} + \sum_\mu |\hat{D}_\mu\phi|^2\right]$$

$$S^{bulk}_{S^1/\mathbb{Z}_2} = \sum_{n_\mu} a_4^4\sum_{n_5} a_5\sum_{M,N}\frac{1}{4}F^A_{MN}F^A_{MN}$$

# Leading order continuum action

- The gauge fixed $5d$ continuum orbifold action reads

$$S_{S^1/\mathbb{Z}_2} = \int d^5x \Bigg[ \Lambda_5 P(x_5) \Bigg\{ -\frac{1}{4} F^A_{MN} F^A_{MN} - \frac{1}{2\xi} (\partial_M A^A_M)^2 + \partial_M \bar{c}^C D^{CB}_M c^B \Bigg\}$$

$$+\delta(x_5) \Bigg\{ -\frac{1}{4} F^3_{\mu\nu} F^3_{\mu\nu} + |D_\mu \phi|^2 - \frac{1}{2\xi} (\partial_\mu A^3_\mu)^2 + \partial_\mu \bar{c}^3 \partial_\mu c^3 \Bigg\} \Bigg]$$

- We use the DR version of $\varepsilon$-expansion, $d = 4 - \varepsilon$. $\varepsilon = 0$ for the boundary, $\varepsilon = -1$ for the bulk.
- Boundary: auxiliary dimensionless coupling $\alpha_{4,0} = \frac{1}{(4\pi)^2} \mu^{d-4} g_0^2$. $\boldsymbol{\beta}_{\alpha_4} = \frac{2\gamma}{3} \alpha_4^2$, $\alpha_4(\mu) = \frac{3}{\gamma \ln \frac{\mu^2_{L4}}{\mu^2}}$.

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$$\left. + \delta(x_5) \left\{ -\frac{1}{4} F^3_{\mu\nu} F^3_{\mu\nu} + |D_\mu \phi|^2 - \frac{1}{2\xi} (\partial_\mu A^3_\mu)^2 + \partial_\mu \bar{c}^3 \partial_\mu c^3 \right\} \right]$$

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- Only a Gaussian fixed point (G), at $\alpha_4 = 0$.

# Fixed Points and anomalous dimensions

- Bulk, in the $g_5$ basis: we use the auxiliary dimensionless coupling $\alpha_{5,0} = \frac{4}{(4\pi)^2}\mu^{d-4}g_{5,0}^2$. $\boldsymbol{\beta}_{\alpha_5} = \alpha_5 - \frac{11\alpha_5^2}{3}$, $\alpha_5(\mu) = \frac{3\mu}{11\alpha_{5,M}(\mu-M)+3M}\alpha_{5,M}$

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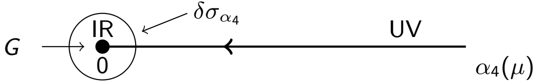
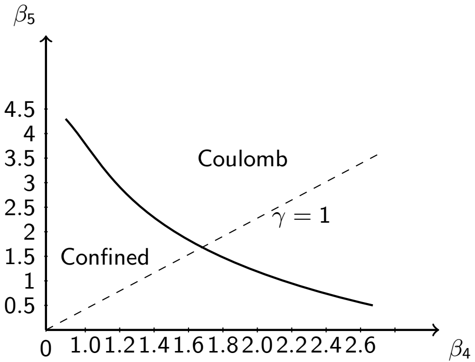
- The RG flow for the marginally irrelevant boundary coupling, $\alpha_4(\mu)$ reads



# Phase diagram of the bulk

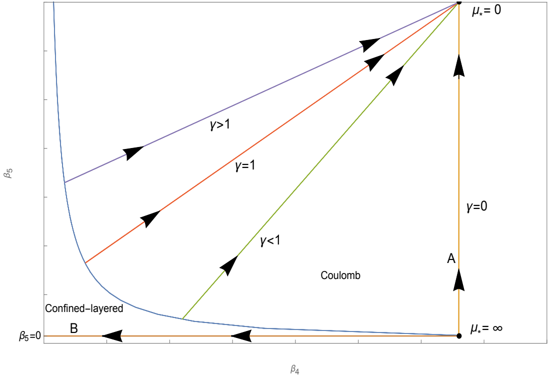
- Non-perturbatively, Bulk exhibits two possible phases[5], a Coulomb and a Confined phase, separated by a line of first order phase transitions.




[5] F. Knechtli, M. Luz and A. Rago, *On the phase structure of five-dimensional SU(2) gauge theories with anisotropic couplings*, Nucl. Phys. **B856** 74-94

# Phase diagram of the bulk

- We go to the space of $(\beta_4, \beta_5)$ couplings.
- We define a relation between $a_{4,5}$ and $\mu$ given by $\mu = \frac{F}{a_4} = \frac{1}{a_4}\left(f + f_q(\beta_4, \beta_5, \cdots)\right) \simeq \frac{f}{a_4}$
- In the Confined phase a special case is considered. For $\gamma < 1$ the $5d$ space breaks into approximately independently fluctuating $4d$ planes, called the layered phase.
- The perfectly layered phase is when $\beta_5 = \gamma = 0$.



# Conclusions

- The continuum action of an anisotropic 5$d$ orbifold was determined. Orbifold conditions imply a 4$d$ massless scalar QED on the boundary and a 5$d$ $SU(2)$ Yang-Mills model on the bulk. At the lowest non-trivial order in small lattice spacing expansion boundary and bulk are decoupled.
- The renormalization program was performed separately for the boundary and the bulk. $\boldsymbol{\beta}_g$ was obtained with the usual way. $\boldsymbol{\beta}_{g_5}$ was determined with $\varepsilon$-expansion procedure, expanding around $d = 4 - \varepsilon$ for $\varepsilon = -1$.
- The fixed points for both boundary and bulk couplings were determined. On the boundary there is only a Gaussian fixed point. On the bulk there are both a Gaussian and a Wilson-Fisher fixed point.
- The bulk phase diagram exhibits a Confined and a Coulomb phase. At the $(\beta_4, \beta_5)$ plane, for $\gamma = 0$ the 5$d$ space enters in layered phase. In that case, both a qualitative and a quantitative matching of the $\varepsilon$-expansion and of the non-perturbative phase diagrams is performed.

Thank You
For Your Attention
( I **BET** that now all of you want to go to the Farewell )