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(A) Physical significance of EWSB

A1. The standard (textbook) picture

The discussion which can be found on textbooks on ElectroWeak Symmetry Breaking (EWSB) goes along this scheme:

i) the classical scalar potential is used to define the vacuum

ii) One identifies the global symmetry obtained in the limit of vanishing gauge couplings (ex $SU(2) \times U(1)$ or $SU(2) \times SU(2)$) and its spontaneous breaking in the vacuum

iii) The failure of the Goldstone theorem is explained as the consequence of the local symmetry (Brout-Englert-Higgs mechanism)

iv) The spectrum and properties of the asymptotic states are derived from perturbation theory using the (gauge-dependent) fields in the Lagrangian

For example, in the context of the Standard Model the classical scalar potential has the form

$$V(H) = -\mu^2 H^\dagger H + \lambda \, (H^\dagger H)^2$$

where H is the Higgs electroweak doublet (with hyperchange $+\frac{1}{2}$):

$$H = \begin{bmatrix} \phi_1 + i\,\phi_2 \\ \phi_3 + i\,\phi_4 \end{bmatrix} \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

The potential $V(H)$ is certainly invariant under local $SU(2)_L \times U(1)_Y$ transformations

$$H(x) \xrightarrow{SU(2)_L} e^{i\alpha^a(x) T^a} H(x) \qquad T^a = SU(2)_L \text{ generators}$$

$$H(x) \xrightarrow{U(1)_Y} e^{\frac{i}{2}\alpha_Y(x)} H(x)$$

but if one neglects the gauging it enjoys a larger global $SO(4)$ symmetry under which the four real components ϕ_i of H are rotated into each other

$$\phi_i \to R_{ij}\,\phi_j \qquad RR^T = \mathbb{1} \qquad R \in SO(4)$$

The global part of the $SU(2)_L \times U(1)_Y$ gauge group is contained in $SO(4)$ and this can be better understood by noticing that at the level of the algebra

$$SO(4) \sim SU(2)_L \times SU(2)_R$$

$\uparrow$
same algebra as

where $U(1)_Y \subset SU(2)_R$ $\qquad Y = T_{3R}$

The action of $SU(2)_R$ on the Higgs field becomes more clear if one defines

$$\mathcal{H} \equiv \left[(H) \, (H^c) \right] \qquad H^c \equiv i\sigma^2 H^* = \begin{bmatrix} \phi_3 - i\phi_4 \\ -\phi_1 + i\phi_2 \end{bmatrix} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

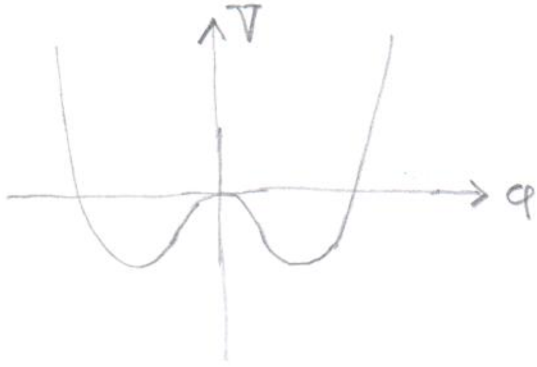
$$\mathcal{H} \to e^{i\alpha_L^a T_L^a} \, \mathcal{H} \, e^{-i\alpha_R^a T_R^a} \equiv L \, \mathcal{H} \, R^\dagger$$

$$\mathcal{H}^\dagger \mathcal{H} = (H^\dagger H) \cdot \mathbb{1} \qquad \mathrm{Tr}(\mathcal{H}^\dagger \mathcal{H}) = 2\,(H^\dagger H)$$

(Notice : $H^\dagger H^c = H^{c\dagger} H = 0$)

Hence $SU(2)_R$ acts by mixing (rotating) the two $SU(2)_L$ doublets H and H^c.

The (classical) potential $V(H)$ has the famous mexican-hat shape (for $\mu^2 > 0$)



$\varphi^2 \equiv H^\dagger H$

and does not depend on the $SO(4) \sim SU(2)_L \times SU(2)_R$ "phase" of H.

Hence vacuum field configurations are characterized by

$$\langle H^\dagger H \rangle = \frac{v^2}{2} \qquad v^2 = \mu^2/\lambda \qquad v = 246\ \text{GeV}$$

If the SO(4) symmetry was global, there would be a spontaneous breaking SO(4) → SO(3) and the vacuum would be identified by a given SO(4) phase. Different vacua (with different phases) would generate different (i.e. physically disjoint) Hilbert spaces.

With an abuse of language, the same terminology is also used in the case of a local $SU(2) \times U(1)$ theory.
It is said that in the vacuum the field H develops a vev

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

and that consequently the $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken to $U(1)_{em}$ (SO(4) is broken to SO(3))

Notice that the vacuum preserves a global SO(3) "custodial" invariance, which corresponds to rotation of the real components ϕ_1, ϕ_2, ϕ_4 of H :

$$\langle \phi_3 \rangle = v/\sqrt{2} \qquad \Rightarrow \qquad SO(4) \to SO(3)$$

[Sikivie, Susskind, Voloshin, Zakharov NPB 173 (1980) 189]

The three W's are triplets of the custodial $SO(3) \sim SU(2)$ and for vanishing hypercharge gauge coupling (i.e. vanishing Weinberg angle) one would obtain $m_W = m_Z$.
The $U(1)_Y$ gauging breaks explicitly the custodial $SO(3)$ (as well as the quark Yukawas) and implies

$$\rho \equiv \frac{m_W}{m_Z \cos\theta_W} = 1 + \Delta\rho_{SM}$$

↑
O(a few %.)
From top and hypercharge corrections

Experimentally $|\rho - 1 - \Delta\rho_{SM}| <$ a few ‰.

Let us come back to the spontaneous breaking. For the spontaneous breaking of a global symmetry, the Goldstone theorem implies the existence of massless Nambu-Goldstone bosons (NGBs).
We know experimentally that the three NGB associated to $SO(4) \to SO(3)$ (or $SU(2)_L \times U(1)_Y \to U(1)_{em}$) are not in the spectrum. How is this possible?

This question was asked and then addressed in the '60 by a number of physicists.

Anderson [Phys. Rev. 130 (1963) 439] was the first to suggest that the theorem could not apply in the case of local symmetries and that NGBs were "eaten-up" to form the longitudinal vector boson degrees of freedom.

Within a year a number of people independently derived what is known now as the Brout-Englert-Higgs (BEH) mechanism

F. Englert, R. Brout	PRL 13 (1964) 321	(26 June)
P. Higgs	Phys. Lett. 12 (1964) 132	(27 July)
P. Higgs	PRL 13 (1964) 508	(31 August)
Guralnik, Hagen, Kibble	PRL 13 (1964) 585	(12 October)

The failure of Goldstone's theorem was identified and depends on the choice of the gauge:

(1) if one quantize the theory with a Lorentz-violating gauge fixing, like the Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$, the assumption of Lorentz invariance fails along with the crucial property that the commutator $[Q_V, O]$ of the regularized charge $Q_V \equiv \int_V d^3x\, J^0(\vec{x},t)$ with an observable O is time invariant in the limit $V \to \infty$

(2) if one quantize the theory in a Lorentz-invariant way (e.g. with a Lorentz gauge fixing $\partial_\mu A^\mu = 0$), then the NGBs do exist but are not part of the physical states.

The gauge-fixing condition

$$\langle \Psi | \partial_\mu A^\mu | \Psi \rangle = 0$$

on a generic physical state $|\psi\rangle$ projects out the NGB in the sense that their contribution to the description of asymptotic states has no physical consequences.

In other words, the NGB can be removed from the asymptotic states but their virtual contributions should be retained.
In particular, it follows that the S-matrix is unitary once restricted to the subspace of physical states, as it happens for the ghosts or the unphysical polarizations (scalar and longitudinal) of the photon in the Gupta-Bleuler quantization.

So the Goldstone theorem fails and there are no NGBs in the spectrum. The physical spectrum is thus read from perturbation theory in a suitable gauge (the unitary gauge). It is said that the scalar degrees of freedom corresponding to the NGBs in the global case are "eaten" to form the longitudinal polarizations of massive spin-1 field.
Hence, one of the predictions of spontaneous breaking of a local symmetry seems to be the existence of massive spin-1 field, as opposed to theories with massless gauge fields.
Notice that this procedure constructs the asymptotic states by interpolating them from the vacuum with the (gauge-dependent) fields of the Lagrangian.

A2. Beyond Naive Perturbation Theory: gauge-invariant description

The standard procedure summarized above is successful in that it correctly reproduces the observed features of the EW sector.

There are however a few points of concern that arise from a more careful analysis.

- First, the standard picture relies on a gauge-fixing (required to define the theory in the continuum), but physical properties must be gauge invariant.

  For example, the very fact that the Higgs field gets a vev is a gauge-dependent statement

  Also, it is clear that asymptotic states can only be excited by gauge-invariant (local) operators, and not by the gauge-dependent Lagrangian fields.

  Gauge invariance is a redundancy in the description of the theory which arises as a consequence of the request of locality

(Ex: SUSY theories with different gauge groups and same global symmetry are dual and describe the same physics)

What is then the physical significance of the gauge-dependent vev of the Higgs field? Can we really talk about spontaneous breaking of a local symmetry, given that the latter is just a redundancy?

A well known result by Elitzur helps us to substantiate the above arguments:

Elitzur's theorem

S. Elitzur, Phys. Rev. D12 (1975) 3978

see proof in Itzykson and Drouffe "Statistical Field Theory", vol I, sec. 6.1.3

" In a gauge theory with a compact gauge group there cannot be any local order parameter "

Corollary: " There cannot be spontaneous breaking of a local (gauge) symmetry "

proof:

Let define the theory on an Euclidean lattice and be $\{\varphi\}$ a collection of lattice fields with values on a compact group manifold. This includes the lattice fields $U_\mu(x)$ (link fields). Since the group is compact, there is no need to make a gauge fixing, as first noticed by Wilson.

Notice: a gauge theory on the lattice is defined in terms of link fields $U_\mu(x)$ which have values on the group manifold rather than its algebra and transform as

$$U_\mu(x) \to \Omega(x)\, U_\mu(x)\, \Omega^\dagger(x+\hat{\mu})$$

under gauge transformations. The link variable is the discretized version of the Wilson line

$$U(x,y) = P \exp\Big(\int_{C:\,x\to y} d\xi^\mu A_\mu \Big)$$

Let $f(\varphi)$ be a <u>local</u> functional over the fields φ which is not gauge invariant, that is:

$$\int dg \; f(g\varphi) = 0$$

where dg is the Haar measure over the Lie group manifold and $g\varphi$ is obtained by acting on φ with a gauge transformation. <u>Notice</u> that the symmetry is local, that is the group measure should be interpreted as

$$dg = \prod_{\{x\}} dg(x)$$

Then we need to proove that for any such f one has

$$\langle f \rangle \equiv \lim_{J\to 0} \lim_{N\to\infty} \langle f \rangle_{N,J}$$

with $$\langle f \rangle_{N,J} = Z^{-1}_{N,J} \int d\varphi \; e^{-S(\varphi) + J\cdot\varphi} \; f(\varphi)$$

$$Z_{N,J} = \int d\varphi \; e^{-S(\varphi) + J\cdot\varphi}$$

Here $d\varphi = \prod_{x,\mu} dU_\mu(x) \cdot d\chi(x)$ where $d\chi$ is the measure over matter fields

$$J\cdot\varphi \equiv \int d^4x \; J(x)\, \varphi(x)$$

and N is the number of lattice points. Notice the importance of first taking the thermodynamical limit ($N\to\infty$) and then the limit of vanishing external source ($J\to 0$).

The functional f is local, i.e. it is function of a (finite) set of fields $\{\varphi(x)\}_{x\in V}$ where V is some finite volume. Let us indicate such set with φ' and its complement with φ'':

$$\{\varphi'\} = \{\varphi(x)\}_{x\in V}$$

$$\{\varphi''\} = \{\varphi(x)\}_{x\notin V}$$

We can compute $\langle f\rangle_{N,J}$ by performing a change of variable $\varphi \to g\varphi$ in the integral and use the fact that the Haar measure is invariant (e.g. $dU = d(\Omega_1 U \Omega_2)$ $\forall\, \Omega_{1,2}$) as well as is the action:

$$\langle f\rangle_{N,J} = Z_{N,J}^{-1} \int d\varphi \; e^{-S(\varphi) + J'\cdot g\varphi' + J''\cdot\varphi''} \; f(g\varphi')$$

Here we used the fact that it is possible to restrict g to the subgroup of transformations which act on the set φ' only, i.e. those which leave φ'' invariant ($g\varphi'' = \varphi''$).

Since at this level g is arbitrary, we can average over all possible values of g, obtaining

$$\langle f\rangle_{N,J} = Z_{N,J}^{-1} \int d\varphi \, dg \; e^{-S(\varphi) + J'\cdot g\varphi' + J''\cdot\varphi''} \; f(g\varphi')$$

We can rewrite

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$$\langle f \rangle_{N,J} = Z_{N,J}^{-1} \int dq\, dg\, e^{-S(q) + J''\cdot q''} f(gq')$$
$$+ Z_{N,J}^{-1} \int dq\, dg\, e^{-S(q) + J''\cdot q''} \left(e^{J'\cdot gq'} - 1 \right) f(gq')$$

The first term vanishes trivially because by assumption $\int dg\, f(gq') = 0$.
The factor in parenthesis in the second term can be boundes as

$$\left| e^{J'\cdot gq'} - 1 \right| \leq \eta(\varepsilon)$$

with $\eta(\varepsilon)$ vanishing uniformingly with ε in the case the source J is bounded by ε.
The crucial point is that the above inequality holds for any field configuration q' and for any number of sites N.
This is true for gauge systems thanks to the locality of gauge transformations, while is is not true for theories with a symmetry because in that case $J\cdot q$ is an extensive quantity proportional to N.

Since the above inequality does not depend on N we can first take the thermodynamical limit $(N\to\infty)$ and obtain

$$\lim_{N\to\infty} \langle f \rangle_{N,J} = \lim_{N\to\infty} \frac{\int dg \int dq\, e^{-S(q) + J''\cdot q''} \left(e^{J'\cdot gq'} - 1 \right) f(gq')}{\int dg \int dq\, e^{-S(q) + J''\cdot q''} \left[\left(e^{J'\cdot gq'} - 1 \right) + 1 \right]}$$

$$\leq \eta(\varepsilon)\, \sup(f)$$

Finally, taking $J\to 0$ one obtains $\langle f \rangle = 0$.

Let us stress again that the result obtained above relies on the fact that on the lattice the theory does not need a gauge fixing. Had we instead introduced a gauge fixing, the expectation value of local functionals f could have been non-vanishing.
This in fact depends on the kind of gauge fixing which is chosen:
for gauge fixings where the field's fluctuation in a gauge orbit are small compared to the orbit's lenght, a perturbative calculation will reproduce accurately the value of the Green functions.
More precisely, the gauge-invariant Green functions of dressed operators are given in terms of the Green functions of gauge-dependent fields in perturbation theory.
This is the result of the paper by Frohlich, Morchio, Strocchi NPB 190 (1981) 553.

How this non-perturbative, rigorous result can be reconciled with the naive, perturbative approach ?

The answer to this question was given in the early '80s by several physicists independently :

- Fröhlich, Morchio, Strocchi   Phys. Lett. 97B (1980) 249
  NPB 190 (1981) 553

- 't Hooft   "Which topological features of a gauge theory can be responsible for permanent confinement ?"
  in "Recent Developments in Gauge Theories"
  NATO Advanced Study Institutes Series, vol. 59 pag 117
  Springer

- Banks, Rabinovici   NPB 160 (1979) 349

In particular Fröhlich, Morchio and Strocchi (FMS) introduced a perturbative expansion in terms of gauge-invariant composite operators showing that it matches (hence justifies it) the standard perturbative approach based on gauge-dependent Green functions and fields in suitable gauges.

The composite fields are in one-to-one correspondence with the elementary fields of the standard procedure and are obtained by dressing them with Higgs fields.

Consider for example the simplified case in which only $SU(2)_L$ is gauged, while hypercharge acts as a global invariance and let focus on the bosonic and leptonic sector.
The $SU(2)_L$-invariant composite operators then classify as follows (up to an appropriate normalization factor $N(H^\dagger H)$ function of $(H^\dagger H)$):

operator	$U(1)_Y$	perturbative limit	
$(H^\dagger H)$	0	$h + \dots$	
$H^\dagger W^i_{\mu\nu} \sigma^i H$	0	$W^3_{\mu\nu} + \dots$	custodial triplet
$H^{\dagger c} W^i_{\mu\nu} \sigma^i H$	+1	$W^+_{\mu\nu} + \dots$	$\mathrm{Tr}(\mathcal{H}^\dagger W^i_{\mu\nu} \sigma^i \mathcal{H} \underbrace{\sigma^a}_{SU(2)_R})$
$H^\dagger l_L$	-1	$e_L + \dots$	
$H^{\dagger c} l_L$	0	$\nu_L + \dots$	

together with the elementary fields $B_{\mu\nu}$ $(Y=0)$ and e_R $(Y=-1)$.
The fields interpolating the Z and the photon are obtained through appropriate linear combinations of $H^\dagger W^i_{\mu\nu} \sigma^i H$ and $B_{\mu\nu}$.

Each operator is fully gauge invariant, local and has a definite global $U(1)_Y$ charge (one can reconstruct custodial $SU(2)_R$ multiplets in the bosonic sector).

By means of these composite operators one can construct asymptotic states and extract physical properties through perturbation theory. For example, the two-point function of $H^\dagger(x) H(x)$ has a pole corresponding to the physical Higgs mass, etc.

The existence of a gauge-invariant description is one of the main results of the FMS analysis. The construction of the composite operators does not depend on the specific representation of the Higgs field but relies only on the existence of a non-trivial gauge orbit, $H^\dagger H = v^2 \neq 0$, which minimizes the potential.

The more realistic case in which there is a non trivial residual gauge invariance of the orbit $H^\dagger H = v^2$ (i.e. the "unbroken" gauge group $U(1)em$) requires a more careful discussion.

For an unbroken gauge group, invariant composite operators are constructed through non-local strings

$$\overline{\psi}(x)\ P\ e^{i\int_x^y d\xi_\mu A^\mu(\xi)}\ \psi(y)$$

An asymptotic state with a given charge is obtained by sending one of the extrema of the string to infinity (ex: $x \to \infty$), i.e. by "removing" one charge by sending it to infinity.

The theorem by FMS then proves that there always exists a set of gauge-covariant local fields $\Phi^i(x)$, linear in the elementary fermion and vector fields and obtained by dressing them with Higgs fields, with which one can construct the gauge-invariant strings.
The operators $\Phi^i(x)$ transform under the same representations of the gauge $SU(2)_L \times U(1)_Y$ group as the elementary fields.

Notice: this assumes that they decompose into the same $U(1)_{em}$ representations (with the same charge) as the elementary fields. In this one removing one end of the string one obtains all possible $U(1)_{em}$ charges.

In the case of $SU(2)_L \times U(1)_Y$ the covariant local fields $\Psi^i(x)$ are the following

i) the same gauge-invariant fields listed before corresponding to neutral states ($H^\dagger H \sim h$, $H^{c\dagger} \ell_L \sim \nu_L$, $H^\dagger W^i_{\mu\nu} \sigma^i H \sim W^3_{\mu\nu}$)

ii) the following covariant fields (up to normalizing factor):

	$SU(2)_L \times U(1)_Y$	perturbative limit
$H^\alpha (H^\dagger \ell_L)$	$2_{+1/2}$	e_L
$(H^\dagger \sigma^\pm H^c)(H^{c\dagger} W^i_{\mu\nu} \sigma^i H)$	3_0	$W^\pm_{\mu\nu}$
e_R	1_{-1}	e_R

In conclusion, the analysis of FMS shows that the crucial feature of the Higgs phenomenon is not the existence of a symmetry breaking local order parameter, but rather the existence of a minimizing gauge orbit $\{v\}$, $v \neq 0$.

The gauge-invariant approach tends to the standard perturbative approach in the weakly-coupled limit and in all gauges where the size of the field's fluctuations are much smaller than the radius of the gauge orbit.

What happens in the strongly-coupled limit? To answer this question let's imagine to reduce the value of the EW vev in the SM (e.g. by letting $\mu^2 \to 0^+$).
For $v \downarrow$ the mass of the gauge fields (as well as that of the Higgs boson) become lighter and lighter: $m_W \sim g(v) \cdot v$.
The coupling $g(v)$ however increases (logarithmically) since $SU(2)_L$ is asymptotically free in the SM.
For v sufficiently low ($v \sim$) the coupling $g(v)$ becomes non perturbative and m_W is of the order of the dynamical $SU(2)_L$ scale. In this limit the theory is non-perturbative and the spectrum cannot be read from perturbation theory. Rather, it will be qualitatively similar to a confining gauge theory with additional composite states of mass $m \approx \Lambda_{SU(2)} \approx m_W$. One can thus think of the W, Z and Higgs boson as composite particles.

One might ask if passing from the Higgs phase to the confining one (as obtained for example for $v \ll \Lambda_{SU(2)}$) is characterized by a (thermodynamical) phase transition.
A well known result of

Osterwalder, Seiler   Ann. of Phys. 110 (1978) 440
Fradkin, Shenker   Phys. Rev. D19 (1979) 3682

shows that in theories with scalar (i.e. Higgs) fields in the fundamental there is no such phase transition.
That is, the Higgs and the confinement phases are actually one and the same phase.
However for more general Higgs field representations the two are genuinely distict and separated by a phase transition.

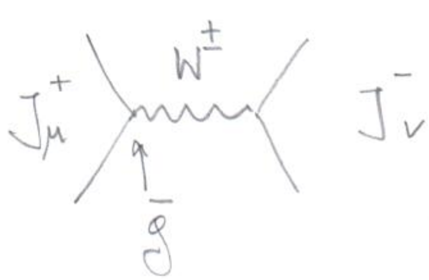
Before the discovery of the W, Z bosons the possible realization of the SM theory in the confining region ($v \ll \Lambda_{SU(2)}$) was seriously considered by

Abbott and Farhi   Phys. Lett. 101B (1981) 69

The $SU(2)_L$ interaction was strong and confining, while the additional $U(1)$ factor (i.e. hypercharge) was identified with EM charge
(Hence the gauge group is $SU(2)_L \times U(1)_{em}$ with $g_1 \to e$).

The $SU(2)_L$ dynamics forms $W^\pm$ and W^0 as bound states ($W^\pm \sim H^{tc} W^i_{\mu\nu} \sigma^i H$, $W^0 \sim H^\dagger W^i_{\mu\nu} \sigma^i H$), and also the left-handed fermions are equally composite.

Thanks to the $SU(2)_c$ global invariance of the strong dynamics the neutral and charged current structure of the theory was reproducing the known experimental facts
For example charged currents arise through the coupling between the composite $W^\pm$ and the composite left-handed fermions



$$\sim \frac{\bar{g}^2}{8 M_W^2} = \frac{G_F}{\sqrt{2}}$$

$\bar{g}$ is a coupling among composites analog to the ρNN coupling (experimentally $g_{\rho NN} \approx 2.5 - 5.5$) and is expected to be O(a few). Given the experimental value of G_F, the W mass was thus expected to be somewhat heavier than the value predicted in the weakly-coupled SM.

Also, the model predicted that at around $M_W \sim O(100\ \text{GeV})$ the composite structure of W should appear, i.e. new resonances should appear from the strong sector in addition to the intermediate vector bosons.

Notice that composite W's could arise from a different microscopic dynamics, one which does not involve any Higgs scalar.

For example in the model by

Fritzsch, Mandelbaum PLB 102 (1981) 319

the W is a bound state of a new confining $SU(N)_H$ gauge group made of constituents fermions α, β called haplons (from Greek "haplos" = simple)

	$SU(3)_c$	$U(1)_{em}$	$SU(N)_H$
α_L	3	$-\frac{1}{2}$	$\square$
β_L	3	$+\frac{1}{2}$	$\square$
α_R	3	$-\frac{1}{2}$	$\square$
β_R	3	$+\frac{1}{2}$	$\square$

The strong dynamics has thus an $SU(2)_L \times SU(2)_R$ chiral global symmetry (after color is turned on) with $U(1)_{em}$ gauging a subgroup of the vectorial $SU(2)_V$.

The custodial triplet of W's has constituents

$$\bar{\alpha}\beta \sim W^-$$
$$\bar{\beta}\alpha \sim W^+$$
$$(\bar{\alpha}\alpha - \bar{\beta}\beta) \sim W^3$$

plus a singlet $W^0 \sim (\bar{\alpha}\alpha + \bar{\beta}\beta)$ which is expected to be heavier.

The model was also proposed before the discovery of the W, Z and to be viable the authors needed to assume that the $SU(2)_L \times SU(2)_R$ chiral symmetry is unbroken (to avoid the appearance of light NGBs and to ensure the lightness of the SM fermions).

Notice that this is compatible with 't Hooft anomaly matching for $n_f = 2$.

A modern reformulation of the composite W scenario was given by

Cui, Gherghetta, Wells JHEP 0911 (2009) 080

in terms of a Randall-Sundrum setup.

A3. Physical significance of EWSB

The spontaneous breaking of a global symmetry has specific physical consequences dictated by

1. the Noether current → existence of massless NGBs
2. conservation of the classical current

For example, in the case of QCD the (partial) conservation of the axial current implies the Goldberger - Treiman relation

$$g_{\pi NN} = \frac{2 m_N \, g_A(0)}{f_\pi} \simeq 12.7 \qquad (\text{exp. } g_{\pi NN} \simeq 13.5)$$

It is thus natural to ask: what are the physical consequences of an underlying microscopic theory in the Higgs phase / region? What about, in particular, the case of the EW symmetry?

We saw that in the strongly-coupled region of parameter space the EW sector of the SM is not distinguishable qualitatively from a confining theory.

In the weakly-coupled region (Higgs region) of the parameter space on the other hand (v large, g weak), the spectrum is characterized by

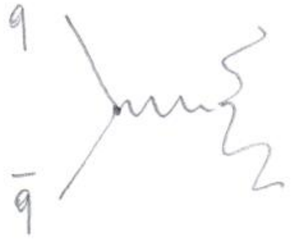
light and weakly-interacting vector bosons (W, Z)

In this sense the evidence for EWSB is the existence of spin-1 W, Z bosons which are light (compared to the scale $4\pi v$ – remember for example that $m_W \sim \bar{g} v \approx 4\pi v$ in the model by Abbott and Fahri) and weakly coupled.

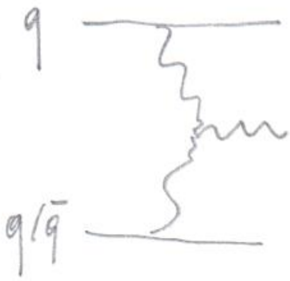
By weak coupling we (also) mean that the W_T and Z_T should look elementary and show no sign of inner structure.

For example, a direct test of the inner structure of the transverse W_T, Z_T come from Triple Gauge Coupling (TGC) measurements.

The experimental processes which are considered are

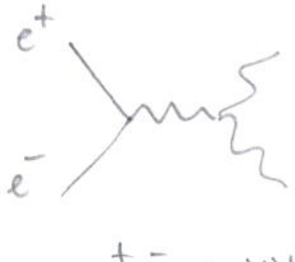


$q\bar{q} \to VV$



$qq/q\bar{q} \to V + JJ$

at the LHC and Tevatron



$e^+e^- \to VV$

at LEP2

The effects of compositeness in the cubic vertex among vector bosons can be parametrized in terms of dim-6 effective operators (corresponding to an expansion in momenta of the form factors)

For example consider the operator

$$g \frac{c_{3W}}{m_W^2} \, \varepsilon^{abc} \, W^a_{\mu\nu} \, W^b_{\nu\rho} \, W^c_{\rho\mu}$$

which gives a correction to the triple vertex growing with energy^2

 $\sim g \left(1 + c_{3W} \frac{E^2}{m_W^2} \right)$

While the leading term remains weak, the correction grows strong for $E \sim \Lambda_s$ with the strong scale defined by

$$c_{3W} \frac{\Lambda_s^2}{m_W^2} = \frac{4\pi}{g}$$

Hence $\quad \Lambda_s = \sqrt{4\pi} \, \frac{1}{\sqrt{\frac{g \, c_{3W}}{m_W^2}}}$

Currently the strongest constraints on Λ_s come from the LHC. A CMS analysis of $Zjj \to \ell\ell jj$ events give

$$\left| \frac{g \, c_{3W}}{m_W^2} \right| < 2.2 \text{ TeV}^{-2}$$

[CMS collaboration arXiv: 1712.09814]
(13 TeV, 36 fb^{-1})

which implies

$$\boxed{\Lambda_S > 2.4 \text{ TeV}}$$

Comparable bounds come from LEP and Tevatron.
The lenght $l_S = 1/\Lambda_S$ can be interpreted as the smallest distance at which the elementary nature of the W and Z has been tested.

Hence the experimental evidence is that the W, Z are light (in agreement with the SM perturbative prediction) and elementary (hence weakly coupled) up to a scale $\Lambda_S \approx 2\text{-}3$ TeV.
This is evidence that the underlying microscopic theory is one in which the EW local symmetry is "spontaneously broken".

This situation should be contrasted, for example, with that of the ρ resonance in QCD:

* the ρ is an isospin triplet spin-1 state with mass $m_\rho = 775$ MeV and width $\Gamma_\rho = 149$ MeV (relatively narrow)

* several people considered the possibility that the ρ is a gauge field of spontaneously-broken symmetry

$$\underset{\substack{\wr\\ \rho}}{SU(2)_L \times SU(2)_H} \times SU(2)_R \longrightarrow SU(2)_V$$

The symmetry breaking pattern implies 6 NGBs of which:

3 NGBs eaten to give mass to the ρ

3 NGBs remain in the spectrum = pions

The gauge $SU(2)_H$ group was called <u>Hidden Local Symmetry</u>

Sakurai Currents and Mesons 1969

Schwinger PRL 24B (1967) 473

Wess, Zumino Phys. Rev. 163 (1967) 1727

Weinberg Phys. Rev. 166 (1968) 1568

Bando et al. PRL 54 (1985) 1215

However, although HLS is a theoretically useful tool under some circumstances, the idea of the ρ as a gauge field does not quite work because:

1. the ρ is not weakly coupled <u>Ex</u>: $g_{\rho\pi\pi} = 6.04 \sim \frac{4\pi}{2}$

2. at the scale $\Lambda_S \approx 1\,\text{GeV}$ one starts resolving the composite structure of the ρ in terms of quarks and gluons, and other resonances appear.

In the case of the ρ <u>both</u> the transverse and the longitudinal polarizations are composites. But what about the W and Z ?

While the evidence for elementary transverse W_T, Z_T comes from the spectrum (their mass), their interactions with fermions and TGC measurements, the evidence for elementary W_L, Z_L is more elusive and still not conclusive.

A well-known probe of the W_L, Z_L compositeness is Vector Boson Fusion (VBF) scattering) $V_L V_L \to V_L V_L$ $(V = W, Z)$.
If longitudinal polarizations are composite, they are expected to become strongly interacting at high energies.
This is in full analogies with the scattering of NGBs from the SB of a global symmetry.

Since, as we discussed, EW interactions are weak and perturbative, the longitudinal W and Z can be thought of NGBs eaten in the BEH mechanism. In the SM these degrees of freedom are part of the Higgs field, together with the Higgs boson.

For example, pions in QCD are (pseudo) NGBs from the spontaneous breaking of chiral symmetry $SU(2)_L \times SU(2)_R \to SU(2)_V$ and can be described by the chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma \right] + O(p^4)$$

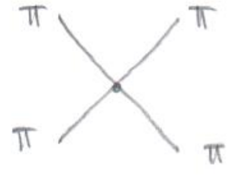
$$\Sigma = \exp\left(i\, \sigma^a \pi^a(x) / f_\pi \right) \qquad f_\pi = 92\ \mathrm{MeV} \quad \text{pion decay constant}$$

Expanding in powers of pion fields

$$\mathcal{L} = \frac{1}{6 f_\pi^2} \left[(\pi^a \partial_\mu \pi^a)^2 - \pi^a \pi^a (\partial_\mu \pi^b \partial^\mu \pi^b) \right] + O(p^4)$$

Hence the scattering amplitude for pion scattering grows with the energy

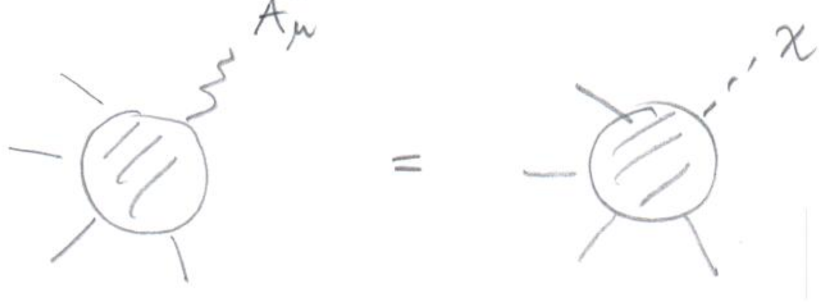
$$A(\pi\pi \to \pi\pi) \sim \frac{E^2}{f_\pi^2}$$



and gets strong at $E \approx 4\pi f_\pi$.

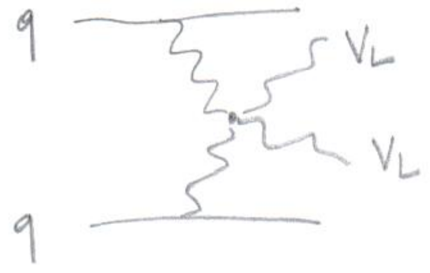
The so-called Equivalence Theorem relates the scattering amplitudes of longitudinal vector bosons to those of would-be NGBs (i.e. the unphysical modes) in a Lorentz covariant gauge.

Cornwall, Levin, Tiktopoulos Phys. Rev. D10 (1974) 1145
Vayonakis Lett. Nuov. Cim. 17 (1976) 383
Lee, Quigg, Thacker Phys. Rev. D16 (1977) 1519



$$\left(1 + O\left(\frac{M_A^2}{E^2} \right) \right)$$

Hence WW scattering probes the strength of the longitudinal W_L, Z_L similarly to the scattering of pions in QCD



In practice extracting the information on the scattering of longitudinal vector bosons from the process $pp \to VV\,jj$ is challenging due to an accidental enhancement of the contribution of transverse modes.

see for example: Contino et al. JHEP 1005 (2010) 089

Current experimental data on VBF from the LHC are not yet testing the nature of the longitudinal W and Z but are rather sensitive to the scattering of the transverse polarizations.

However, in theories which reduce to the SM in a decoupling limit (the exception being TC models), the longitudinal W and Z are part of the Higgs field together with the Higgs boson.

Tests on their nature thus come from testing Higgs compositeness.

Whether the Higgs field is elementary or composite in fact depends on the dynamics responsible for the EWSB.