Lecture 2: From Diagrams to Cross Sections

Last time, we saw master formula for collider physics.

$$AB \to 12 \cdots n$$

$$\sigma_M = \frac{1}{2E_{cm}^2} \sum_{n=2}^{\infty} \int d\Phi_n \left| M_{AB \to 12 \cdots n} \right|^2 f_M(\Phi_n)$$

Things to remember:

- In principle, $12 \cdots n$ consists only of collider-stable (or quasi-stable) particles

  yes p, e^-, γ, μ^-, K_L, $\pi^\pm$, $K^\pm$, etc.

  not $\pi^0 \to \gamma\gamma$ (though in practice collinear $\gamma\gamma$ are tagged as π^0)

  not d, u, s, c, b, $W/Z/h$ t

  If we want to write e.g. $pp \to t\bar{t}$, we need to massage master formula.

- Must define a specific measurement function f_M (unless you want total cross section)
- Must include all configurations that could contribute to $f_M(\Phi_n)$ (very important for jets)

This is very daunting.

$$\underbrace{p\ \ p}_{\text{these are my beams at fixed } E_{cm}} \to \underbrace{37\,\pi^+ + 34\,\pi^- + 42\,\underset{\hookrightarrow\gamma\gamma}{\pi^0} + \text{etc.}}_{\text{this is what I have to detect}}$$

I really need to do integral over 113$^+$-body phase space?!

I really need to calculate $2 \to 113^+$ scattering amplitude?!

Luckily, there are simplifications, if you choose the appropriate measurement function.

If you are sufficiently agnostic as to what happens to beam remnant, and if you use a suitable jet algorithm, then you can replace.

$$\begin{array}{ll} p_A p_B \to \text{hadron}_1 + \text{hadron}_2 + \cdots \text{hadron}_n & f_M(\text{hadrons}) \\ \Downarrow & \Downarrow \\ \text{Parton}_A\ \text{Parton}_B \to \text{Parton}_1 + \text{Parton}_2 + \cdots \text{Parton}_m & \tilde{f}_M(\text{partons}) \end{array}$$

with $m \ll n$

Depends on choice of f_M, only proved in a small subset of cases. Crucial for making predictions at the LHC.

More formally, we call this <u>factorization</u>, if.

$$|M|^2 f^{\mathcal{O}} \simeq |M_A|^2 |M_B|^2 |M_C|^2 \cdots \tilde{f}^{\mathcal{O}} + \text{small, controlled corrections}$$

This depends crucially on the observable $\mathcal{O}$.

A remarkable statement, when true. Says that quantum mechanical interference in $|M|^2$ can be neglected and replaced by a series of semi-classical probabilities. (Usually only true up to corrections.)

For proton-proton collisions, the most important factorization is for parton distribution functions.

$$pp \to X \implies \sum_{ij} A_i B_j \to X$$

where i, j could be quarks, antiquarks or gluons.

Not true in general, only for observables that don't care about "beam remnant"

Instead of fixed p_{CM} have variable $\hat{p}_{CM}$

$$P_{CM} = \underset{\uparrow}{x_A}\underset{\substack{\uparrow\\ \text{proton}}}{\left(E_{beam}, 0, 0, E_{beam}\right)} + \underset{\uparrow}{x_B}\underset{\substack{\uparrow\\ \text{proton}}}{\left(E_{beam}, 0, 0, -E_{beam}\right)}$$

momentum fraction of parton

We then have probability distribution over x_A and x_B

$$f_i(x_A, \underset{\substack{\uparrow\\ \text{"factorization scale"}}}{\mu}) \qquad f_j(x_B, \mu)$$

(scale where you aren't able to resolve proton structure)

These lectures will not discuss DGLAP evolution

Most convenient to write in terms of parton luminosity function

$$\sigma_M = \int d\hat{s}\; \mathcal{L}_{ij}(\hat{s})\; \underbrace{\frac{1}{2\hat{s}} \sum_{n=2}^{\infty} \int d\Phi_n(\hat{s}) \left| M_{ij\to 123\ldots n} \right|^2 f_\mu(\Phi_n)}_{\text{cross section for } ij \to 123\ldots \text{ at } E_{cm}^2 = \hat{s}}$$

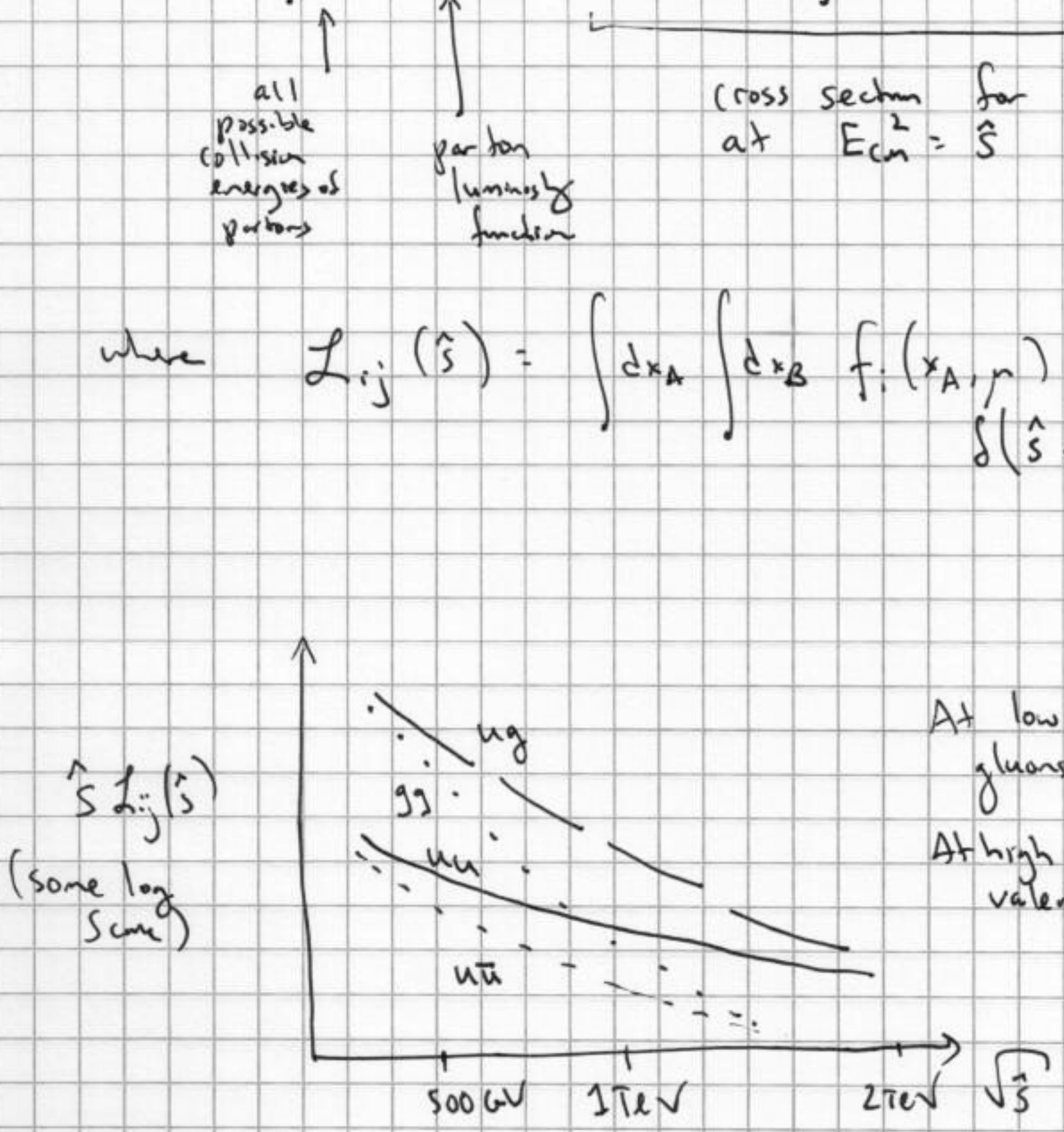
↑ all possible collision energies of partons

↑ parton luminosity function

where

$$\mathcal{L}_{ij}(\hat{s}) = \int dx_A \int dx_B\; f_i(x_A,\mu)\, f_j(x_B,\mu)\; \delta(\hat{s} - x_A x_B S)$$

↑ E_{cm}^2 for protons.

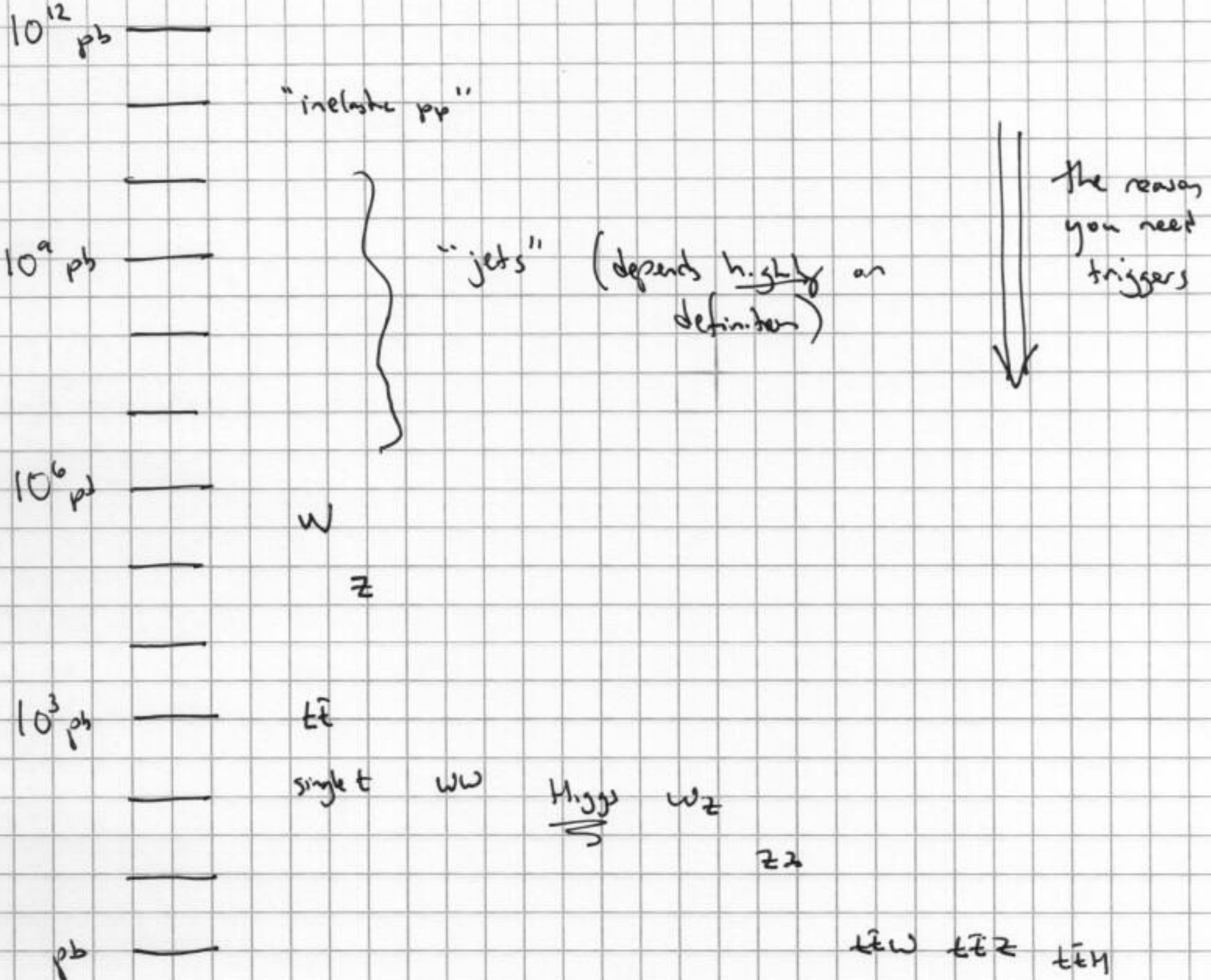


At low energies, gluons dominate

At high energies, valence quarks dominate.

Because parton luminosities (and cross sections) fall with $\hat{s}$, scattering dominated by low $\hat{s}$ processes.

A very standard plot to see is "staircase to heaven".
Useful for visualizing hierarchies of cross sections



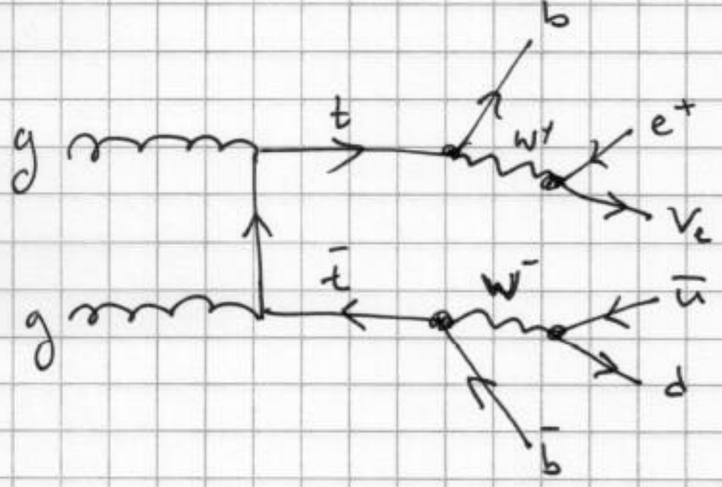
Huge range of processes! These are backgrounds to each other, and backgrounds to new physics.

But what do we even mean by $t\bar{t}$ cross section?

We don't see top quark. Using gg parton channel:

We can calculate this, but we can't measure this (at least not directly)

Something closer ...



If we don't ask detailed questions about beams, this is okay, as long as we use suitable jet algorithm to reconstruct b, $\bar{b}$, $\bar{u}$, d.

But naively, order of these diagrams is very different. (i.e. number of gauge couplings.)

$$\underset{\underset{\text{whatever that is}}{\uparrow}}{\sigma_{t\bar{t}}} = \sum_{ij} \int d\hat{s}\, \mathcal{L}_{ij}(\hat{s})\, \frac{1}{2\hat{s}} \int d\Phi_2 \left| M_{ij \to tt} \right|^2 \underset{\underset{\text{accepting fract}}{\uparrow}}{f_{tt}}(\Phi_2)$$

or . . .

$$\sum_{ij} \int d\hat{s}\, \mathcal{L}_{ij}(\hat{s})\, \frac{1}{2\hat{s}} \int d\Phi_4 \left| M_{ij \to bW^+\bar{b}W^-} \right|^2 f_{bW\bar{b}W}(\Phi_4)$$

or . . .

$$\sum_{ij} \int d\hat{s}\, \mathcal{L}_{ij}(\hat{s})\, \frac{1}{2\hat{s}} \sum_{klmn} \int d\Phi_6 \left| M_{ij \to b\bar{b}klmn} \right|^2 f(\Phi_6)$$

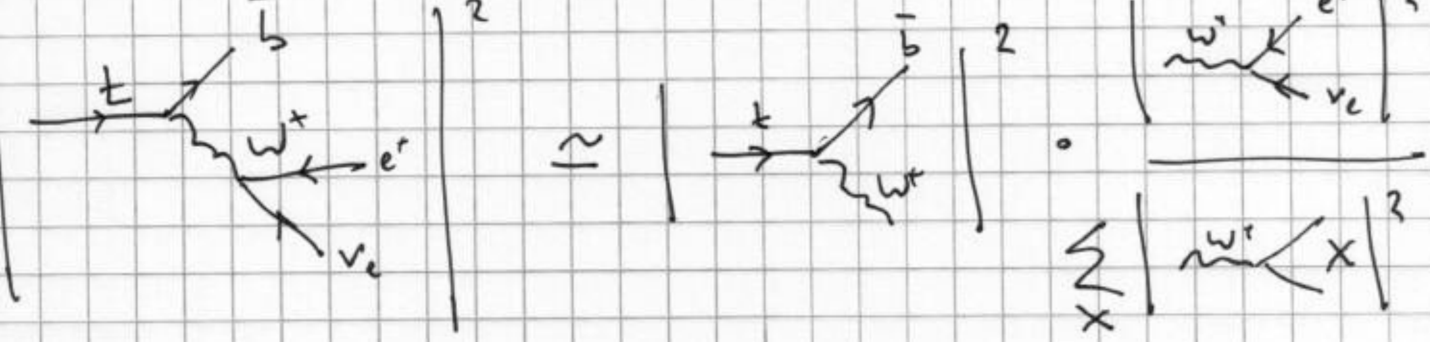
Which is it? For appropriate choice of f, all of these are reasonable approximations!

How can that be?!

The Narrow Width Approximation.

Work horse of cascade decay analyses. (essential for BSM searches)

Easiest to understand for top decays.



A very important example of factorization

$$\text{Key}: \underset{\text{propagator}}{\left|\frac{1}{q^2-\underset{\substack{\uparrow\\ \text{mass}}}{M^2}+iM\underset{\substack{\uparrow\\ \text{total width}}}{\Gamma}}\right|^2} \simeq \delta(\underset{\substack{\uparrow\\ \text{on-shell condition}}}{q^2-M^2})\cdot\frac{\pi}{M\Gamma}$$

<u>If</u> you can ignore finite width effects, including quantum mechanical interference, then this is a reasonable approximation.

Recall:

$$d\Phi_3\left({}^{p}t \to {}^{p}b\,{}^{p}e\,{}^{p}\nu\right) = d\Phi_2\left({}^{p}t \to {}^{p}b\,{}^{p}W\right) \times \frac{dm_W^2}{2\pi}\, d\Phi_2\left({}^{p}W \to {}^{p}e\,{}^{p}\nu\right)$$

This is just phase space, no approximations.

$$\left|M_{t\to be\nu}\right|^2 = \left|\frac{\sum_{\epsilon\in\text{polarization}} M^{\epsilon}_{t\to bW}\, M^{\epsilon}_{W\to e\nu}}{q^2 - M_W^2 - iM_W\Gamma_W}\right|^2$$

Again, no approximations. Now the approximation:

$$\simeq \sum_{\epsilon\in\text{polarizations}} \left|M^{\epsilon}_{t\to bW}\right|^2 \left|M^{\epsilon}_{W\to e\nu}\right|^2 \underbrace{\delta(q^2 - m_W^2)\frac{\pi}{\Gamma_W M}}_{\substack{\text{NWA on}\\ \text{propagator}}}$$

Putting the pieces together:

$$\Gamma_{t\to be\nu} = \frac{1}{2m_t}\int d\Phi_3 \left|M_{t\to be\nu}\right|^2$$

$$\simeq \frac{1}{2m_t} \sum_{\epsilon\in\text{polarization}} \int d\Phi_2 \left|M^{\epsilon}_{t\to bW}\right|^2 \int \frac{dm_W^2}{2\pi}\cdot\frac{\pi}{\Gamma_W M}\,\delta(q^2 - m_W^2) \cdot \int d\Phi_2 \left|M^{\epsilon}_{W\to e\nu}\right|^2$$

After the dust settles, and ignoring polarization issue. (i.e. summing / averaging as appropriate)

$$\Gamma_{t\to be\nu} = \Gamma_{t\to bW} \cdot \underbrace{\frac{\Gamma_{W\to e\nu}}{\Gamma_W^{\text{total}}}}_{Br(W\to e\nu)} \quad \xleftarrow{\text{did you see how I did that?}}$$

So full 3-body top decay factorizes into a 2-body decay times a branching fraction.

This is why $gg \to t\bar{t} \simeq gg \to b\bar{b}e\nu\bar{u}d$ up to branching ratio effects.

Only true to the extent to which narrow width approximation holds. In general, cannot make this simplification, though it is ubiquitous.

To summarize this lecture, master formula says we have to calculate

$$pp \to \text{(quasi-) stable particles.}$$

With PDFS, you can replace

pp with $\displaystyle\sum_{ij} \underset{\uparrow\ \ \uparrow\ \text{partons.}}{i\ \ j}$

With NWA, we can often group final state particles into "bundles" where cross section is dominated by on-shell intermediate resonances.

Have to think very carefully if that is true!

Next time: extend this logic to QCD radiation.