(B) EWSB dynamics

B1. Weakly-coupled EWSB dynamics : the Higgs model

In the weakly-interacting limit, as discussed, it is useful (and justified) to think in terms of a scalar condensate breaking the EW symmetry spontaneously. The longitudinal W, Z thus correspond to field quanta along the valley of degenerate points of the orbit which minimizes the scalar potential (massless excitations). The Higgs boson, instead is a quantum excitation in the orthogonal direction (massive mode).

The study of the properties of the Higgs boson can give information on the dynamics at the origin of the condensate.

As we anticipated, in the SM the scalar field responsible for the condensate is an elementary field, i.e. one appearing in the microscopic Lagrangian. The model gives the most minimal realization of EWSB dynamics and leads to specific, testable predictions :

(i) existance of a custodial symmetry, ensuring $|\rho - 1| \lesssim O(1‰)$

(ii) Higgs couplings predicted in terms of the particles' masses

$$y_\psi = \frac{m_\psi}{v} \qquad g_{hVV} = \frac{2 m_V^2}{v} \qquad g_{hhVV} = \frac{m_V^2}{v^2} \quad (V = W, Z)$$

(iii) Higgs cubic and quartic self interactions predicted in terms of the Higgs mass

$$\lambda_3 = \frac{m_h^2}{2v} \qquad \lambda_4 = \frac{m_h^2}{8v^2}$$

Problem of the SM is the instability of the $(H^\dagger H)$ coefficient under UV radiative corrections (Hierarchy Problem)

B.2 Strongly-coupled EWSB dynamics

Assuming that the EW interactions remain perturbative, one can use the standard language of EWSB in terms of a scalar condensate. The latter could be a composite operator of some additional dynamics in the strongly-coupled regime.

In order to solve the hierarchy problem and generates all scales naturally, the new theory should have no relevant singlet operator.
A gauge theory with fermion matter fields and no (elementary) scalar is the prototype of strongly-interacting EWSB dynamics.

The condensate must be singlet under the new "color" group (otherwise it could not have a vev → see Elitzur's theorem) and must transform non-trivially (in fact, as a doublet) under the EW symmetry. Since the strenght of the EW interaction is much weaker (by assumption) than the new color force, we can consider effectively the EW symmetry as a global one in first approximation. Clearly $SU(2)_L \times U(1)_Y$ must be a subgroup of the global symmetry group of the strong dynamics.

The simplest example of scalar composite operator getting a vev is that of a fermion bilinear

$$H \sim \bar{\Psi}\Psi \qquad \langle \bar{\Psi}\Psi \rangle \neq 0$$

Although one cannot prove in full generality that a non-vanishing condensate exists which breaks spontaneously the global symmetry of the strong dynamics, there are interesting exceptions.

In particular the Vafa-Witten theorem (Vafa, Witten, NPB 234 (1984) 173) states that vector global symmetries in a vector-like gauge theory can<u>not</u> be spontaneously broken.

<u>Notice</u>: vector symmetry is defined to be the global invariance of a theory where all fermions have a mass

Furthermore, 't Hooft anomaly matching can be used to identify those theories where the axial part of the global symmetry <u>must</u> be spontaneously broken (ex: SU(N) vector-like gauge theories with N even, where baryons are bosons). <u>See</u>: 't Hooft in "Recent Developments in gauge theories" Cargese Summer school, 1980

In the following let us assume that a non-vanishing fermion bilinear condensate exists which breaks spontaneously part of the global symmetry.
The question is thus under which assumption the condensate will break the EW subgroup of the global symmetry, hence leading to the BEH mechanism.
As we will see, in some cases such breaking is inevitable, in others instead it may occur depending on the relative orientation of the EW gauged subgroup inside the linearly-realized global group.

Let us start by stating two useful theorems.
Consider a gauge theory defined in terms of fermions transforming as the direct sum of irreducible, finite-dimensional, unitary representations:

$$\psi = \bigoplus_{k} \psi^{(r_k)}$$

Then it follows:

Theorem 1 (mass term)

A mass term $\psi^{(r_i)}\psi^{(r_j)}$ is allowed in the Lagrangian if:

(A) $r_j \sim \bar{r}_i$

and (only for $r_i = r_j$)

(B) $\psi^{(r_i)}\psi^{(r_j)}$ is overall symmetric in gauge and flavor space

Here $r_j \sim \bar{r}_i$ means that r_j is (unitary) equivalent to $\bar{r}_i$, the conjugate representation of r_i; that is, there exist S unitary such that

$$S^{-1} U(r_j) S = U^*(r_i) \qquad S^\dagger S = \mathbb{1}$$

where $U(r_i), U(r_j)$ are the representations of the gauge group (confining part and weak gauging) on the fermions

$$\psi^{(r_i)} \to U(r_i)\,\psi^{(r_i)}$$

For the above equivalence to hold, of course, r_i and r_j must have the same dimension.

It is easy to show that S is unique up to a phase if $r_{i,j}$ are irreducible representations

proof: suppose R unitary exists such that

$$R^{-1} V(r_j) R = U^*(r_i)$$

<u>Useful references</u> :

- Georgi "Lie Algebra in Particle Physics" (Frontiers in Physics)
- Peskin, Lectures at Les Houches 1982

then

$$S^{-1} U(r_j) S = R^{-1} U(r_j) R$$

$$U(r_j) SR^{-1} = SR^{-1} U(r_j)$$

$$\left[U(r_j) , SR^{-1} \right] = 0$$

By Schur's lemma it follows that SR^{-1} is a multiple of the identity. By unitarity of S and R the constant of proportionality can only be a phase

$$\begin{aligned} R &= \lambda S \\ R^{+} &= \lambda^{*} S^{+} \end{aligned} \Rightarrow \quad RR^{+} = |\lambda|^2 SS^{+} \quad \Rightarrow \quad |\lambda|^2 = 1$$

It is a simple result of group theory that the product $r_i \otimes r_j$ contains a singlet, as required to write a mass term, if and only if $r_i \sim \bar{r}_j$, i.e. if condition (A) is fullfilled:

$$r_i \otimes r_j \supset \text{singlet} \quad \Leftrightarrow \quad r_i \sim \bar{r}_j$$

If $r_i = r_j$ then condition (A) states that r_i is equivalent to its adjoint. In this case it follows that

$$S^{*} S = S S^{*} = \pm 1 \qquad (\text{for } r_i = r_j)$$

hence (since S is unitary) one has

$$\boxed{S = \pm S^{T}}$$

proof:

$$S^{-1} U(r_1) S = U^*(r_1)$$

$$(S^{-1})^* U^*(r_1) S^* = U(r_1) \quad \text{(complex conjugate)}$$

Substituting in the first equation:

$$S^{-1}\left[(S^{-1})^* U^*(r_1) S^*\right] S = U^*(r_1)$$

$$U^*(r_1) S^* S = S^* S\, U^*(r_1)$$

$$[U^*(r_1), S^* S] = 0 \Rightarrow S^* S = c \cdot \mathbb{1}$$

(by Schur's lemma)

Then:

$$SS^* = (S^T S^*) SS^* = S^T (c \cdot \mathbb{1}) S^* = c\, \mathbb{1}$$

$$c^* \mathbb{1} = (SS^*)^* = S^* S = c\, \mathbb{1} \Rightarrow \boxed{c = c^*}$$

Also:

$$c^2 \mathbb{1} = SS^* (S^* S)^T = S(S^* S^T) S^\dagger = SS^\dagger = \mathbb{1}$$

$$\text{hence } c^2 = 1 \Rightarrow \boxed{c = \pm 1}$$

It is then possible to distinguish two cases

(i) $S = S^T$ $\quad r_i$ is called real-positive, or real.

In this case there exists R unitary such that $\hat{U} = R^{-1} U(r_1) R$ has generators purely imaginary and antisymmetric, i.e. $\hat{U} = \exp(T)$ with $-T = T^T$ and real.
It can be shown that $RR^T = S$.

proof : For S unitary and symmetric, it is always possible to decompose

$$S = RR^T$$

with R unitary.

Then $S^{-1} U(r_1) S = U^*(r_1)$

$$(R^T)^{-1} R^{-1} U(r_1) RR^T = U^*(r_1)$$

$$R^{-1} U(r_1) R = R^T U^*(r_1)(R^T)^{-1}$$

Let us define $\hat{U} \equiv R^{-1} U R$

It follows

$$(\hat{U}^T)^{-1} = (R^T U^T (R^{-1})^T)^{-1}$$
$$= R^T U^* (R^T)^{-1}$$

Hence, by the identity derived above, it follows

$$\hat{U} = (\hat{U}^T)^{-1} \Rightarrow \hat{U}^T = \hat{U}^{-1} \Rightarrow \hat{U} = e^T$$

with T real and antisymmetric.

The second case is :

(ii) $S = -S^T$ r_i is called real negative, or pseudo-real

Notice that if $r_i = r_j$ then condition (A) is not sufficient to guarantee that a mass term can be written. This is indeed the case if overall in gauge and flavor indices the bilinear $\psi^{(r_i)}\psi^{(r_j)}$ is symmetric. This is ensured by condition (B).

One can show that a color singlet fermion bilinear is symmetric (antisymmetric) in flavor indices if the fermion field transforms as a real (pseudoreal) representation of the color group (see Peskin).

Examples :

r^i = adjoint of $SU(N_c)$

$$\psi^{i,a}_{\alpha} \psi^{j,b}_{\beta} \underbrace{\delta^{ij}}_{\text{color}} \underset{\substack{\uparrow \\ \text{flavor} \\ \text{(symmetric)}}}{F^{ab}} \underbrace{\varepsilon_{\alpha\beta}}_{\text{Lorentz}}$$

r^i = fundamental of $SU(2)_c$

$$\psi^{i,a}_{\alpha} \psi^{j,b}_{\beta} \underbrace{\varepsilon^{ij}}_{\text{color}} \underset{\substack{\uparrow \\ \text{flavor} \\ \text{(antisymmetric)}}}{F^{ab}} \underbrace{\varepsilon^{\alpha\beta}}_{\text{Lorentz}}$$

Notice that a theory with only 1 fundamental of $SU(2)_c$ is not consistent since it has global anomalies.
A theory with one 4 of $SU(2)_c$ (spin 3/2) has instead no global anomalies. Since the 4 is pseudoreal, the mass term is not allowed by symmetry (F^{ab} would be the identity, hence symmetric)

On the pattern of global symmetry breaking

1) vectorlike theories ($SU(n)_L \times SU(n)_R$)

i) diagonal $SU(n)_V$ vectorial group unbroken (Vafa - Witten)

ii) $SU(n)_L \times SU(n)_R \to SU(n)_V$ implied for $n > 2$ by 't Hooft anomaly matching

2) theories with real representations ($SU(n)$)

i) vectorial $SO(n)$ unbroken for <u>n even</u> (Kosower)

ii) $SU(n) \to SO(n)$ if n is <u>even</u> and fermions do not transform trivially under the center of the gauge group, by anomaly matching (Kosower)

3) theories with pseudo-real representations ($SU(n)$)

i) vectorial $Sp(n)$ unbroken for <u>n even</u> (Kosower)

ii) $SU(n) \to Sp(n)$ for <u>n even</u> by 't Hooft anomaly matching (Kosower)

Kosower Phys. Lett. 144B (1984) 215

The other useful result is

Theorem 2 (condensate)

A scalar condensate $\langle \psi(R_i)\, \psi(R_j) \rangle$ can be a singlet of the gauge group only if conditions (A) and (B) are satisfied

The condensate will necessarily be a singlet of the new strong color force, while it can "break spontaneously" the weakly-coupled part of the gauge symmetry in the perturbative sense explained before.

Then, consider a product gauge group $G_{strong} \times G_{weak}$ when G_{strong} gets strong while $G_{weak} \supseteq G_{EW}$ remains weakly-coupled ($G_{EW} = SU(2)_L \times U(1)_Y$). Theorems 1 and 2 can thus be formulated by focusing on the local subgroup $G \equiv G_{strong} \times G_{EW}$. One has the following two possibilities:

1. conditions (A), (B) are not satisfied, the condensate breaks necessarily G_{EW} (Technicolor Theories)

2. conditions (A), (B) are satisfied and the condensate preserves or breaks G_{EW} depending on vacuum alignment

An example of theories of the first kind comes in fact from the SM itself.

Consider the limit $\Lambda_{SU(2)} \ll v \ll \Lambda_{QCD}$, i.e. one in which the Higgs vev v is below the dynamical scale of QCD but much above that of $SU(2)_L$ itself.

At the scale Λ_{QCD} a quark bilinear forms which is color singlet. Let us consider only two flavors of quarks for simplicity

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\square$	$\square$	$\frac{1}{6}$
u_R	$\square$	1	$+\frac{2}{3}$
d_R	$\square$	1	$-\frac{1}{3}$

For $SU(3)_c$ singlet condensates one then possible:

$$\langle \bar{u}_L u_R \rangle , \ \langle \bar{u}_L d_R \rangle , \ \langle \bar{d}_L u_R \rangle , \ \langle \bar{d}_L d_R \rangle$$

plus their hermitean conjugates.
The Vafa-Witten theorem ensures that the $SU(3)_c$ dynamics leaves an $SU(2)_V$ vector-like subgroup unbroken

Vector-like subgroup = maximal subgroup which is left unbroken when all quarks are massive

Vafa-Witten theorem tells about the orientation of the condensate with respect to a given orientation of the mass term.

The maximal vector-like subgroup in $SU(2)_L \times SU(2)_R$ is $SU(2)$ and the quark mass term can always be diagonalized so that $m_u = m_d \neq 0$

$$\mathcal{L}_m = -(\bar{u}_L \; \bar{d}_L) \begin{pmatrix} m & \\ & m \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix} + h.c.$$

In such basis, the VW theorem states that

$$\langle \bar{u}_L u_R \rangle = \langle \bar{d}_L d_R \rangle \quad (\text{possibly} \neq 0)$$

and $\langle \bar{u}_L d_R \rangle = 0 = \langle \bar{d}_L u_R \rangle$

The condensate thus breaks $SU(2)_L \times U(1)_Y$. This is in fact a simple consequence of theorem 2, since the quarks transform as overall complex representations of $SU(3)_c \times SU(2)_L \times U(1)_Y$. Since the condensate is necessarily an $SU(3)_c$ singlet, then it must break the EW group. In this case the strong dynamics which breaks $SU(2)_L \times U(1)_Y$ is $SU(3)_c$ color itself!

Notice that the electromagnetic invariance $U(1)_{em}$ is not broken by the condensate, as in fact the unbroken $SU(2)_V$ contains $U(1)_{em}$. Thus, the pattern of spontaneous symmetry breaking is

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$$

which is the same breaking pattern one has in the real case $v \gg \Lambda_{QCD}$.

The W and Z get mass through their coupling to the $SU(2)_L \times SU(2)_R$ currents made of quarks.
Consider for example the resummed propagator for vector bosons:

where  $= \langle J_\mu^a(q)\, J_\nu^b(-q) \rangle \equiv \int d^4x\, e^{-iq\cdot x} \langle 0 | T\left(J_\mu^a(x)\, J_\nu^b(0) \right) | 0 \rangle$

Given the tree-level propagator in a ξ gauge:

$$-\frac{i}{q^2}\left(\eta^{\mu\nu} - (1-\xi)\,\frac{q^\mu q^\nu}{q^2} \right) \equiv -\frac{i}{q^2}\left((\mathbb{P}_T)_{\mu\nu} + \xi\,(\mathbb{P}_L)_{\mu\nu} \right)$$

one obtains the resummed propagator ($A, B = L, R$)

$$= \frac{-i}{q^2 - g_A g_B \, \Pi_{AB}(q^2)} (P_T)_{\mu\nu} - \frac{i}{q^2} \xi \, (P_L)_{\mu\nu}$$

where current conservation implies

$$\langle J_\mu^a J_\nu^b \rangle = (P_T)_{\mu\nu} \, \delta^{ab} \, \Pi(q^2)$$

By invariance under $SU(2)_V$ and parity one has

$$\langle J_V^a J_V^b \rangle = (P_T)_{\mu\nu} \, \delta^{ab} \, \Pi_{VV}(q^2)$$

$$\langle J_A^a J_A^b \rangle = (P_T)_{\mu\nu} \, \delta^{ab} \, \Pi_{AA}(q^2)$$

$$\langle J_V^a J_A^b \rangle = 0$$

$$\begin{cases} \Pi_{LL}(q^2) = \Pi_{RR}(q^2) = \frac{1}{2} \left(\Pi_{VV}(q^2) + \Pi_{AA} \right) \\ \Pi_{LR}(q^2) = \Pi_{VV}(q^2) - \Pi_{AA}(q^2) \end{cases}$$

The W, Z mass matrix arises from the poles of the transverse part of the propagator :

$$\Pi_{LL}(0) = \Pi_{RR}(0) = f_\pi^2$$

$$\Pi_{LR}(0) = - f_\pi^2$$

For $\Pi(0) \neq 0$ the correlator

$$\langle J_\mu(q)\, J_\nu(-q) \rangle = \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Pi(q^2)$$

has a pole for $q^2 \to 0$

The pole indeed corresponds to the exchange of the NGBs of $SU(2)_L \times SU(2)_R \to SU(2)_V$:

$$\otimes - - \overset{\pi}{- -} - \otimes \qquad \langle 0 |\, J_A^\mu \,| \pi(p) \rangle = -i f_\pi p^\mu$$

$$\frac{q_\mu q_\nu}{q^2} f_\pi^2$$

One obtains the mass matrix

$$(m^2)_{ij} = \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix}$$

which gives the physical mass eigenvalues

$$m_\gamma = 0$$

$$m_W = g f_\pi \simeq 55 \text{ MeV}$$

$$m_Z = \sqrt{g^2 + g'^2}\, f_\pi = \frac{m_W}{\cos\theta_W} \simeq 57 \text{ MeV}$$

Notice that the leading-order relation between the W and Z masses

$$m_W = m_Z \cos\theta_W \qquad (\leftrightarrow \rho = 1)$$

is exactly the same as for the SM with $v \simeq 246 \gg \Lambda_{QCD}$.
This is because the QCD dynamics itself possesses (similarly to the Higgs scalar sector) a custodial invariance.

A few comments are in order:

1. notice that the values of the W and Z are not quite correct since have been obtained using $g \simeq 0.6$ which is the value at $\mu \simeq 100$ GeV. One should run g and g' down to the Λ_{QCD} scale

2. for $v \ll f_\pi$ but finite the degrees of freedom eaten to form the longitudinal W and Z are not exactly the NGBs from QCD chiral symmetry breaking, but rather have a small component along the NGBs from the Higgs breaking. The physical "pion" (i.e. the light scalars remaining in the spectrum) are the ortogonal combination

$$\begin{cases} |W_L, Z_L\rangle = \dfrac{f_\pi}{\sqrt{f_\pi^2+v^2}} |\pi_{QCD}\rangle + \dfrac{v}{\sqrt{f_\pi^2+v^2}} |\chi_{Higgs}\rangle \\[2ex] |\pi\rangle_{phys} = -\dfrac{v}{\sqrt{f_\pi^2+v^2}} |\pi_{QCD}\rangle + \dfrac{f_\pi}{\sqrt{f_\pi^2+v^2}} |\chi_{Higgs}\rangle \end{cases}$$

3. Quarks get a dynamical mass from the condensate. Depending on the value of v and of the Yukawas, this may dominate over the contribution to the quark mass from the Higgs condensate. Leptons get mass only from the Higgs Yukawas.

Theories of type 1 have been used as prototypes for Technicolor. An example is Minimal Technicolor:

$$G^{\text{strong}} = SU(N_{TC}) \quad \text{with} \quad N_{TC} \geq 3$$

$$G^{\text{weak}} = SU(2)_L \times U(1)_Y$$

$$\Psi = \{Q, U^c, D^c\}$$ (all left-handed Weyl fermions)

$$Q = (\square, 2)_0$$

$$U^c = (\bar{\square}, 1)_{-\frac{1}{2}}$$

$$D^c = (\bar{\square}, 1)_{+\frac{1}{2}}$$

$\Big\} \leftarrow$ with this assignment of quantum numbers all anomalies vanish (no need of extra "leptons")

Vafa-Witten theorem implies the following pattern of global symmetry breaking:

$$SU(2)_L \times SU(2)_R \times U(1)_V \rightarrow SU(2)_V \times U(1)_V$$

(we assume that chiral symmetry is indeed broken, as for example it must happen if N_{TC} is even)

Since the gauged subgroup of $SU(2)_L \times SU(2)_R$ is chiral ($Y = T_{3R}$), then $SU(2)_L \times U(1)_Y$ is necessarily broken by the condensate

$$\langle Q U^c \rangle = \langle Q D^c \rangle \neq 0$$

It is worth mentioning that while in the above examples it is condition (A) of Theorem 2 which is violated (i.e. the fermion representations are complex), it is possible to construct theories where the theorem fails because (B) does not hold.

Consider for example a theory with gauge group $SU(N_{TC}) \times SU(2)_L \equiv G$ with $N_{TC} \geq 3$, and a single Weyl fermion ψ transforming as $(\text{adj}, 4)$ of G. In this case the global symmetry with respect to the TC dynamics is $SU(4)$ of which $SU(2)_L$ is a subgroup.

We assume that the TC group confines at a scale $\Lambda_{TC} \gg v = 246\,\text{GeV}$ and ask whether the SM $SU(2)_L$ is spontaneously broken by the condensate $\langle \psi\psi \rangle$. In this case ψ is a pseudo-real representation r of G, since it is possible to find a unitary transformation $S = -S^T$ so that $r \sim \bar{r}$. Consequently, $\psi\psi$ can be a singlet of G but this turns out to be antisymmetric under the exchange of the two fermion fields. Hence condition (B) is violated and the TC-preserving condensate breaks $SU(2)_L$. Notice that the theory is free from global anomalies

(see: Witten Phys. Lett. 117 B (1982) 324
Bar NPB 650 (2003) 522)

Technicolor theories can thus be constructed with pseudo-real representations provided there is no global symmetry group.

In theories of type 2, instead, whether or not the condensate effectively breaks the weak group G^{EW} depends of vacuum alignment.

The conditions (A) and (B) ensure that G^{EW} can be embedded into the linearly-realized global subgroup, at least classically. Whether this alignment is realized at the quantum level is a dynamical issue.

An important class of theories which realize this situation are, in fact, theories vectorlike under the confining color group. An example is given by QCD with two flavors, in the chiral limit $m_q = 0$, when the gauging of $U(1)_{em}$ is turned on (hence the analogy is between G_strong x G_EW and SU(3)_c x U(1)_em)

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad SU(2)_L \times SU(2)_R \times U(1)_V \to SU(2)_V \times U(1)_V$$

$$q_R = \begin{pmatrix} u \\ d \end{pmatrix} \qquad Q = T_{3L} + T_{3R} + B/2$$

The vacuum alignment can be established by expanding the theory around a gauge-preserving vacuum and computing the scalar potential for the NGB modes. A vacuum misalignment would manifest itself in a non-zero vev for the NGBs.

In the case of QCD we have seen that the 1-loop potential for the pions has the form:

$$V(\pi) \simeq \frac{3}{8\pi^2} \alpha_{em} \sin^2\left(\frac{|\pi|}{f_\pi}\right) \int_0^{+\infty} dQ^2 \, \Pi_{LR}(Q^2)$$

where $|\pi| \equiv \sqrt{(\pi^1)^2 + (\pi^2)^2} = \sqrt{\pi^+ \pi^-}$

and the form factor Π_{LR} is defined by

$$\Pi_{LR}(q^2) \equiv \Pi_{AA}(q^2) - \Pi_{VV}(q^2)$$

$$\langle J_V^{\mu,a} \, J_V^{\nu,b} \rangle = \Pi_{VV}(q^2) \, (P_T)^{\mu\nu} \, \delta^{ab}$$

$$\langle J_A^{\mu,a} \, J_A^{\nu,b} \rangle = \Pi_{AA}(q^2) \, (P_T)^{\mu\nu} \, \delta^{ab}$$

Hence the minimum of the potential is at $\langle |\pi| \rangle = 0$ if the integral $\int_0^{+\infty} dQ^2 \, \Pi_{LR}(Q^2)$ is positive.
This is indeed true due to a theorem due to Witten:

(Witten, PRL 51 (1983) 2351)

In a vectorlike, confining gauge theory at $\theta = 0$ it follows that $\langle J_V^\mu(k) J_V^\mu(-k) \rangle - \langle J_A^\mu(k) J_A^\mu(-k) \rangle \geq 0$

for any Euclidean k.

Notice that in the Euclidean $(P_T)^{\mu\mu} \geq 0$ hence the theorem implies that $\Pi_{LR}(k^2) \geq 0$.

There is an important class of theories of type 2, namely COMPOSITE HIGGS THEORIES.
These are those that fullfill the additional two requirements:

1) In the limit in which the vacuum is aligned in the $SU(2)_L \times U(1)_Y$ preserving direction, the spectrum of NGBs should include an EW doublet with hypercharge $+1/2$ (Composite Higgs field)

2) The theory should have a (tunable) mechanism to misalign the vacuum in an $SU(2)_L \times U(1)_Y$ -breaking direction

As an example of theory which fullfills at least condition (1), consider again Minimal Technicolor

	$SU(N_{TC})$	$SU(2)_L$	$U(1)_Y$
Q	$\square$	$\square$	0
U^c	$\overline{\square}$	1	$-\frac{1}{2}$
D^c	$\overline{\square}$	1	$+\frac{1}{2}$

In the special case $N_{TC} = 2$, because the fundamental representation of $SU(2)_{TC}$ is pseudoreal, the global symmetry is $SU(4)$ rather than $SU(2)_L \times SU(2)_R$. Indeed Q can be mixed with U^c, D^c.
Furthermore, the representations are overall pseudoreal and conditions (A), (B) of theorem 2 are verified.

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GIUDICE, GROJEAN, POMAROL, RATTAZZI JHEP 0706 (2007) 045

A generalization of Vafa-Witten Theorem plus 't Hooft anomaly matching imply that the pattern of global symmetry breaking is

$$SU(4) \to Sp(4)$$

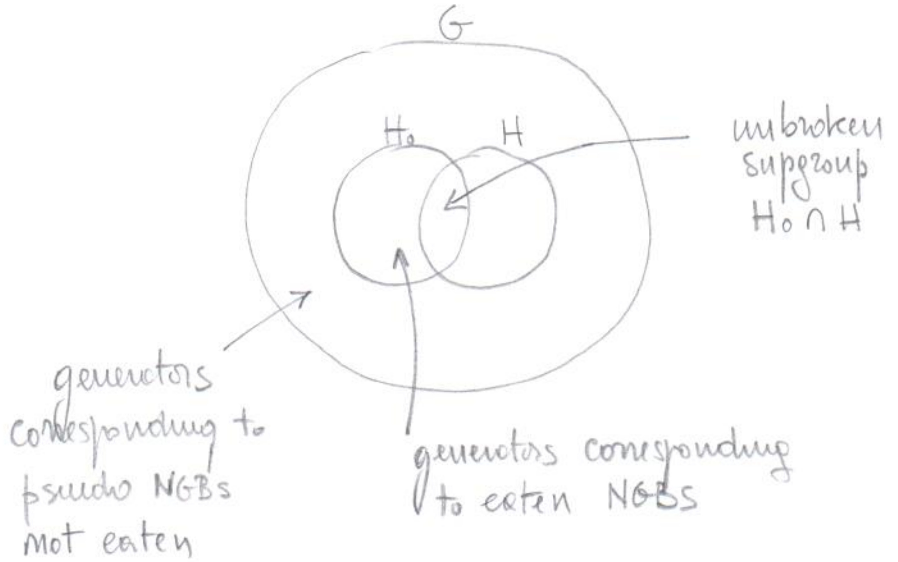
Notice that at the algebra level $SU(4) \sim SO(6)$ and $Sp(4) \sim SO(5)$

There are thus $15-10=5$ NGBs which transform as a

$2_{+\frac{1}{2}} \oplus 1_0$ under $SU(2)_L \times U(1)_Y$.

In order to be realistic, a CH theory should have a mechanism to misalign the vacuum in an $SU(2)_L \times U(1)_Y$ breaking direction.

At the level of the algebra of generators, the cartoon which describes vacuum misalignment is the following



G = global symmetry group

H = subgroup unbroken by strong dynamics

H_0 = weakly gauged subgroup.

$$\# \text{ eaten NGBs} = \dim(H_0) - \dim(H \cap H_0)$$

$$\# \text{ pseudo NGBs} = \dim(G) - \dim(H) - \# \text{ eaten NGBs}$$

Each (uneaten) pseudo NGBs parametrizes one misalignment angle.

The simplest coset realizing the above picture (with custodial symmetry) is $G = SO(5) \to SO(4) = H$ (minimal CH)

[Agashe, Contino, Pomarol NPB 719 (2005) 165]

In this case there are $10 - 6 = 4$ NGBs which transform as a composite Higgs doublet under the EW subgroup $SU(2)_L \times U(1)_Y$ of $SO(5)$.

Generically, the unbroken subgroup $H \cap H_0$ can be at most a $U(1)$:

$$H = SO(4)$$

$$H_0 = SU(2)_L \times U(1)_Y$$

$$H \cap H_0 = U(1)_{em}$$

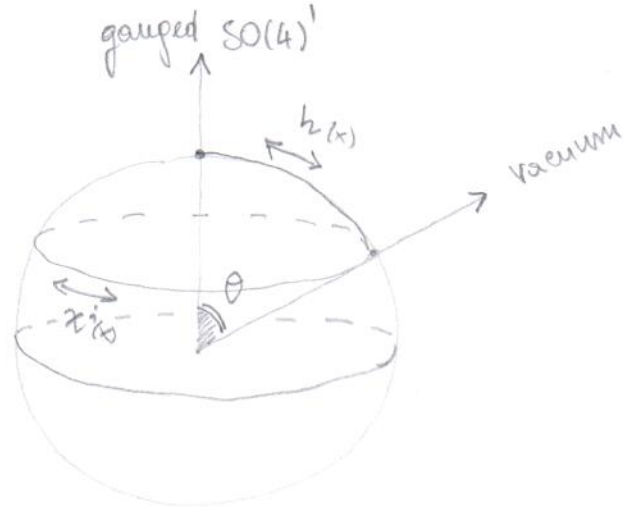
Hence, there are $\dim(H_0) - \dim(H \cap H_0) = 3$ eaten NGBs and 1 pseudo NGB (the Higgs boson), corresponding to 1 misalignment angle.

An analogous counting can be done in a slightly more convenient way if one thinks of gauging an $SO(4)'$ subgroup instead of $SU(2)_L \times U(1)_Y$. Since $SO(4)' \sim SU(2) \times SU(2)$, in this way the invariance under custodial symmetry remains manifest. One has

$$H = SO(4) \qquad \# \text{ eaten NGB} = \dim(SO(4)') - \dim(SO(3))$$
$$= 3$$
$$H_0 = SO(4)'$$

$$H \cap H_0 = SO(3) \qquad \# \text{ pseudo NGBs} = 1$$

There is 1 misalignment angle which characterize the relative direction between the true vacuum and the gauged $SO(4)'$.
The NGBs live on the coset space $G/H = SO(5)/SO(4) = S^4$; the cartoon is thus the following



$\theta =$ misalignment angle

$h =$ Higgs boson (pseudo NGB)

$\chi^i =$ eaten NGBs

EWSB thus occurs through vacuum misalignment. There is however an alternative and equivalent description in terms of a 2-step symmetry breaking as follows:

1. at leading order in perturbation theory (tree level), define the theory by expanding around the $\theta = 0$ vacuum.
   That is described by the spontaneous breaking

$$SO(5) \xrightarrow[f]{} SO(4)$$

   occurring at the dynamical scale f.
   The NGBs form an $SU(2)_L$ doublet.

2. The composite Higgs gets a potential at the radiative level and acquires a vev $\langle H \rangle = v$ breaking

$$SO(4) \xrightarrow[v]{} SO(3)$$

One has

$$\boxed{\sin^2\theta = \left(\frac{v}{f}\right)^2 \equiv \xi}$$

The limit

$$\xi \to 0 \qquad \text{with} \quad v \text{ fixed}$$
$$f \to \infty$$

is a decoupling limit in which all the composite states become heavy (and thus decouple) except for the Higgs doublet.
One thus recovers the SM. Notice that:

1. the degree of tuning is $\quad FT \approx O(\xi)$

2. all corrections to the SM predictions scale with ξ.
   For example

$$\frac{\delta c_{Higgs}}{c_{Higgs}} \sim O(\xi) \qquad\qquad \frac{\delta O_{EWPT}}{O_{EWPT}} \sim O(\xi)$$

See for example:

Giudice, Grojean, Pomarol, Rattazzi JHEP 0706 (2007) 045

Witten's theorem imply that the EW contribution to the Higgs potential will tend to align the vacuum in the $SU(2)_L \times U(1)_Y$ - preserving direction. Therefore, additional dynamics is needed to obtain the required EWSB through misalignment.

Several mechanism have been proposed to get vacuum misalignment. In particular there are the following two options:

1. vacuum misalignment from new gauge interactions

The idea is to enlarge the weakly gauge group G_{weak} (so that $G_{weak} \supset G_{EW}$) and choose representations for the fermions which are

- overall complex under $G_{weak} \times G_{strong}$
- real under $G_{EW} \times G_{strong}$

Then theorem 2 implies that the condensate can preserve G_{EW} but must break G_{weak}. Vacuum misalignment is thus controlled by the relative strenght of the EW group and of the additional weak group.

This idea was proposed first by

Banks NPB 243 (1984) 125

and then adopted by Georgi and Kaplan in subsequent models.
See:

GEORGI, KAPLAN, GALISON Phys. Lett. 143B (1984) 152
GEORGI, KAPLAN PLB 145 (1984) 216
DUGAN, GEORGI, KAPLAN NPB 254 (1985) 299

2. vacuum misalignment from top interactions

The strength of the interaction between the strong sector (comprising the Higgs) and the top quark is constrained by the requirement of generating a large enough top quark mass. It thus cannot be too weak and can dominate over the EW contribution.

An example where top correction to the Higgs potential can successfully misalign the vacuum is given by theories with top quark partial compositeness.

See :
- Agashe, Contino, Pomarol NPB 719 (2005) 165
- Contino, Da Rold, Pomarol PRD 75 (2007) 055014
- Giudice, Grojean, Pomarol, Rattazzi JHEP 0706 (2007) 045