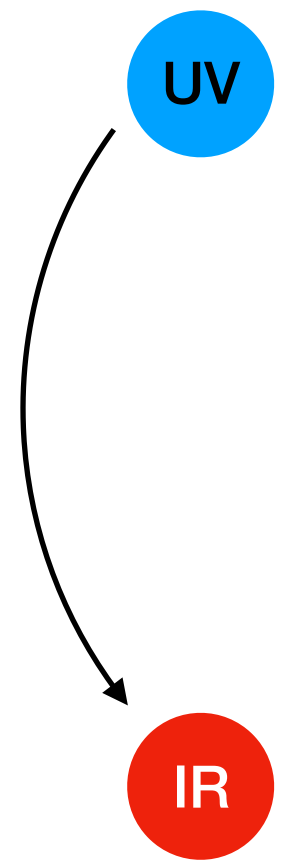
Beyond Positivity Bounds of Scattering Amplitudes

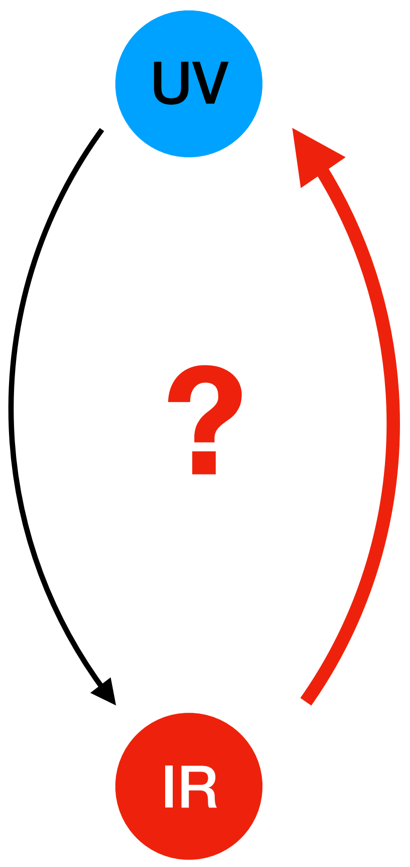
Francesco Sgarlata
SISSA/ISAS & INFN Trieste

Based on PRL 120 (2018) no.16, 161101 (B.Bellazzini, F.Riva, J. Serra, FS)
and working in progress

ma per seguir virtute e canoscenza

SISSA


UV
IR



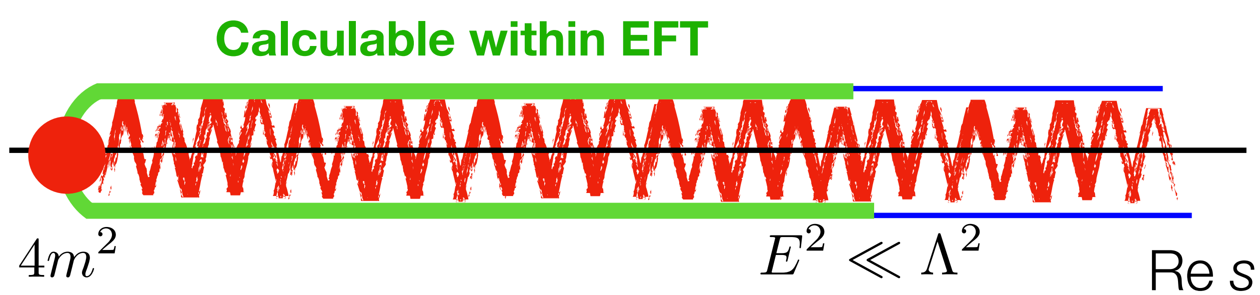
IR question: Does any EFT admit consistent UV completion?

Beyond Positivity Bounds

$$\frac{\partial^2}{\partial s^2}\mathcal{M}^{2\to 2}(s,t=0) = \sum_X \int_{4m^2}^{\infty} \frac{ds}{\pi s^2}\sigma_{12\to X}(s) > 0$$

IR — UV

Fully non-perturbative



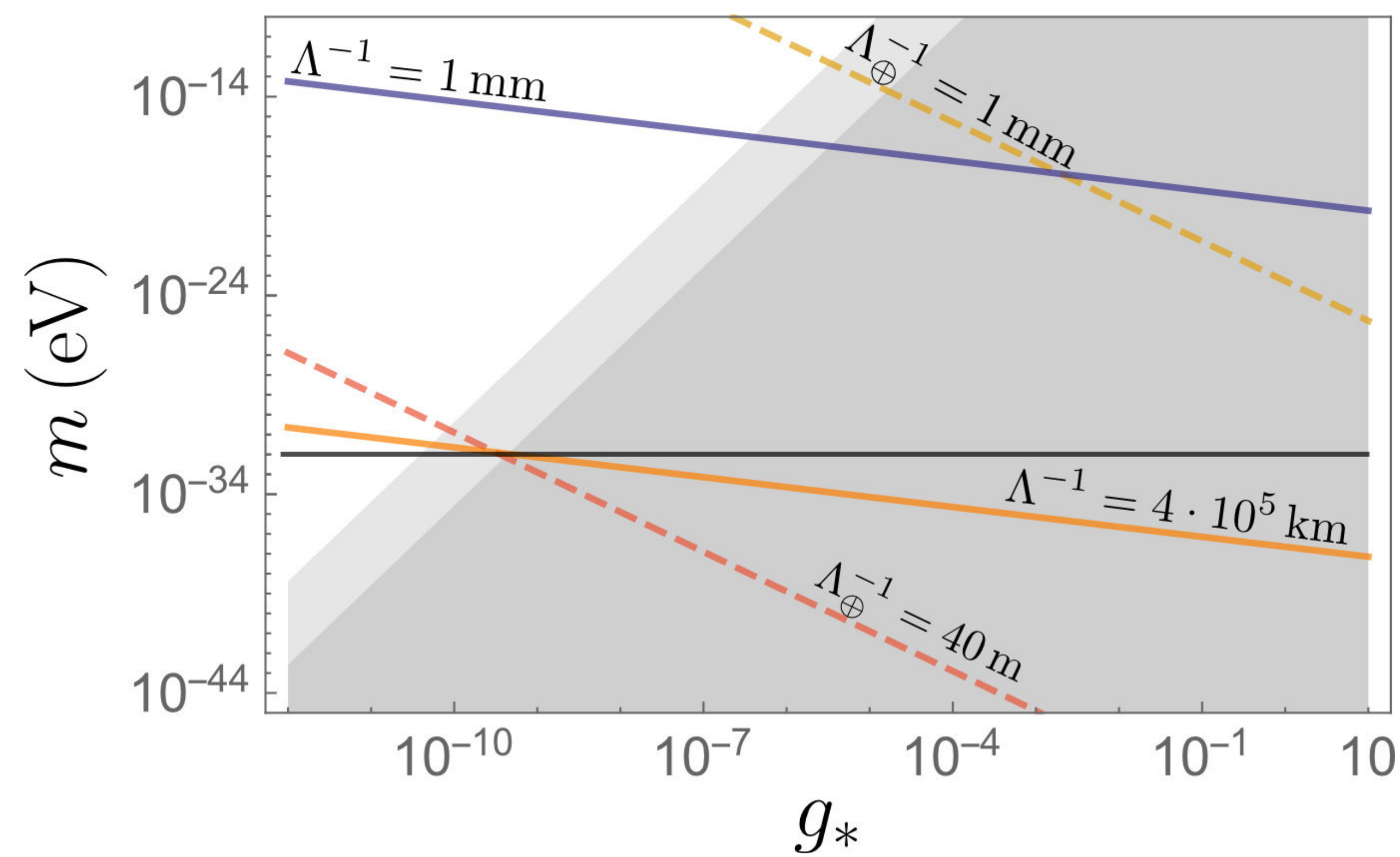
IR-residue > loop-factor x $\int_0^{E^2\ll\Lambda^2} ds\,[...]$

Very useful when LHS suppressed > RHS unsuppressed

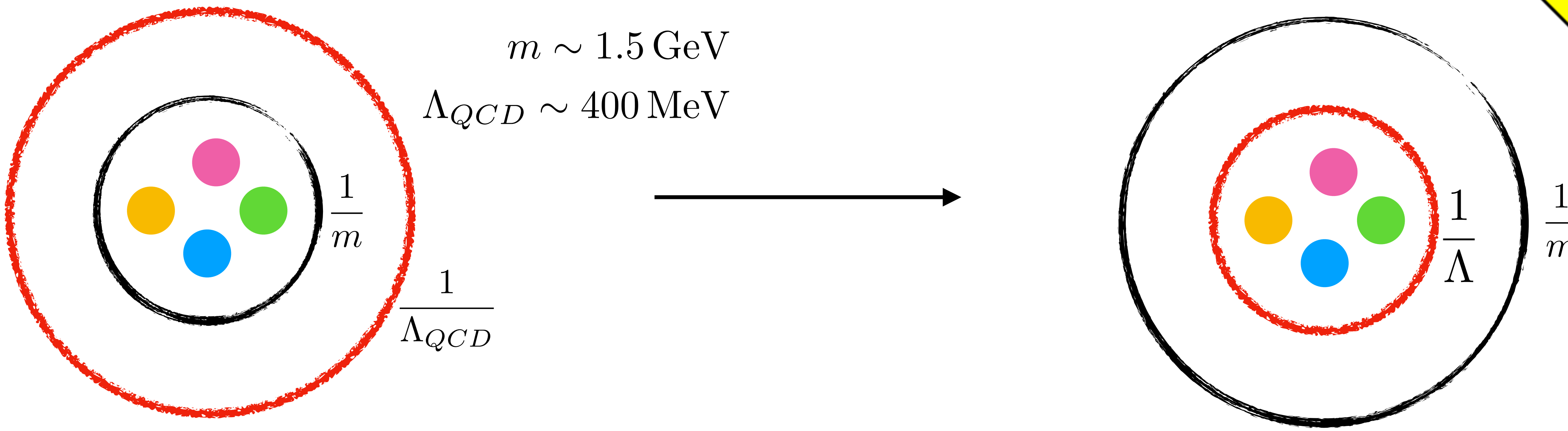
dRGT Massive Gravity

$$\frac{1}{\Lambda} \simeq r_{\rm moon} \left(\frac{g_*}{4.5 \cdot 10^{-10}}\right)^{-1/3} \left(\frac{m}{10^{-32}\,\mathrm{eV}}\right)^{-2/3}$$

PRL 120 (2018) no.16, 161101
B.Bellazzini, F.Riva, J.Serra, FS



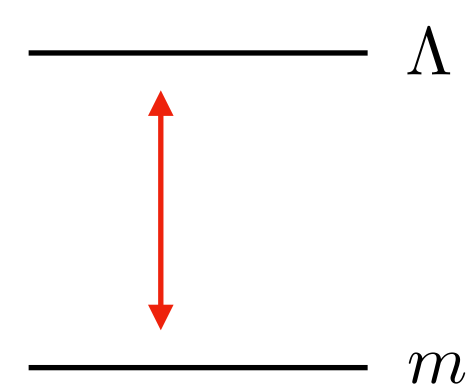
Massive higher spin EFTs



- Natural question : can we find the EFT for these particles?

<u>**Preliminary results for massive spin-3**</u>

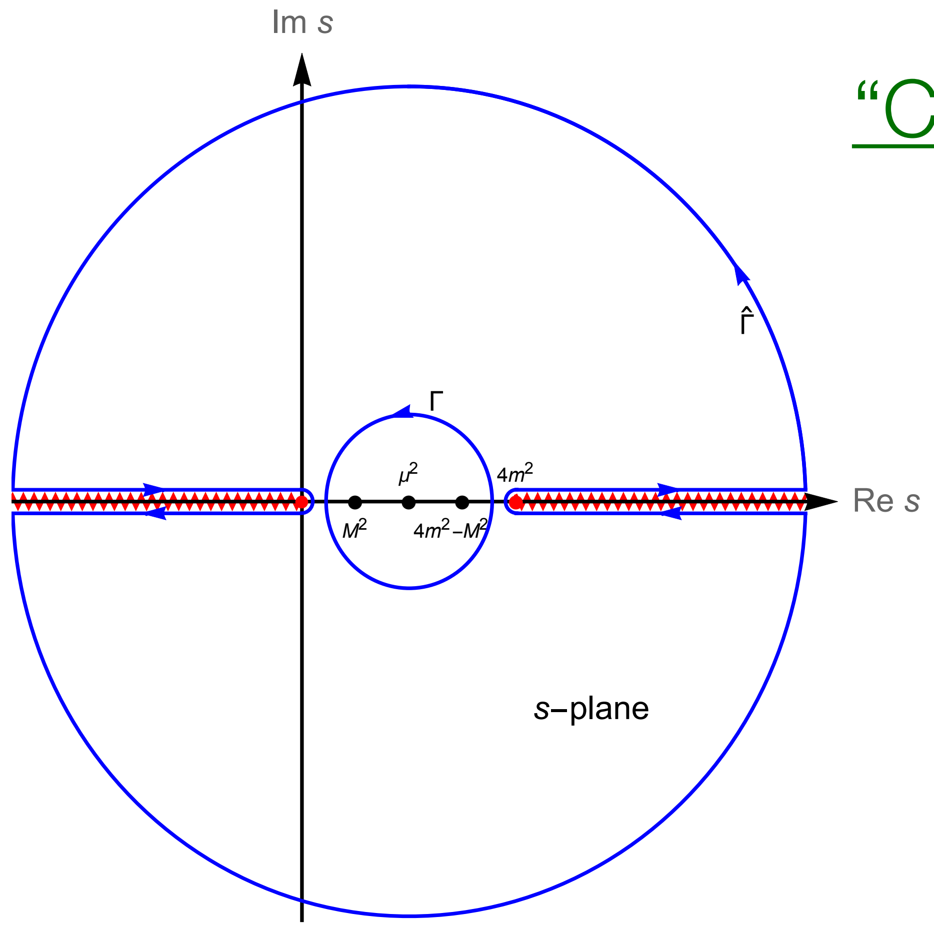
$$\Lambda < \frac{m}{(g_*/4\pi)^{1/8}}$$



As we send $m \to 0$
the interactions die → **We recover no-go theorems**

BACKUP SLIDES

Beyond Positivity Bounds



"Can any EFT be UV completed?"

$$\frac{\partial^2}{\partial s^2}\mathcal{M}^{2\to 2}(s,t=0) = \sum_X \int_{4m^2}^{\infty} \frac{ds}{\pi s^2}\sigma_{12\to X}(s) > 0$$

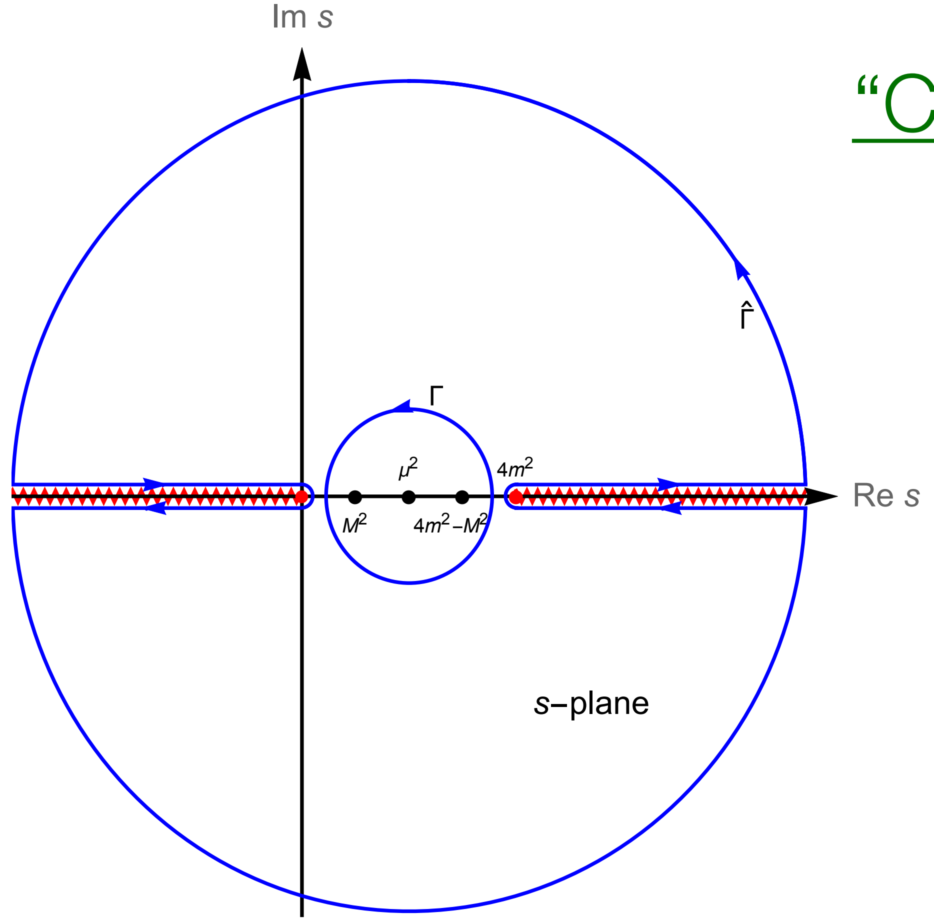
IR — UV

Fully non-perturbative

Assumptions: analyticity, locality, crossing symmetry, unitarity

Conclusion : s^2 coefficient is strictly positive

Beyond Positivity Bounds



"Can any EFT be UV completed?"

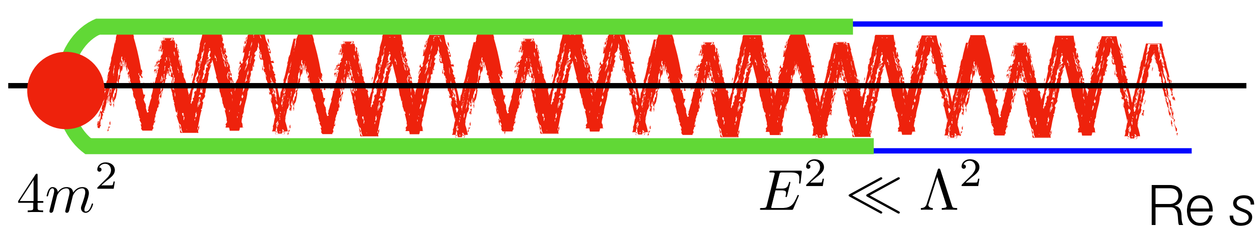
$$\frac{\partial^2}{\partial s^2}\mathcal{M}^{2\to 2}(s,t=0) = \sum_X \int_{4m^2}^{\infty} \frac{ds}{\pi s^2}\sigma_{12\to X}(s) > 0$$

IR — UV

Fully non-perturbative

Assumptions: analyticity, locality, crossing symmetry, unitarity

Conclusion : s^2 coefficient is strictly positive



IR-residue > loop-factor x $\int_0^{E^2\ll\Lambda^2} ds\,[...]$

PRL 120 (2018) no.16, 161101
B.Bellazzini, F.Riva, J.Serra, FS

Very useful when LHS suppressed > RHS unsuppressed

Galileon

$$\pi \to \pi + c_\mu x^\mu + d \qquad -\frac{1}{2}(\partial\pi)^2\left[1 + \frac{c_3}{2\Lambda^3}\Box\pi + \frac{c_4}{2\Lambda^6}\left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right) + \ldots\right]$$

$$\mathcal{M}(\pi\pi \to \pi\pi) = -\frac{3}{4}(c_3^2 - 2c_4)\frac{stu}{\Lambda^6} \to 0$$

The theory is sick. We can add a tiny mass deformation $\mathcal{M}(s, t=0) \sim \frac{c_3^2 {m_\pi}^2 s^2}{\Lambda^6}$

Usual positives give no new informations IR-residue $\sim m^2 > 0$

Can the mass deformation be arbitrarily small?

IR-residue > loop-factor x $\int_0^{E^2 \ll \Lambda^2} ds\,[\ldots]$

suppressed

$$m^2 > \Lambda^2\left(\frac{3}{320}\right)\frac{(c_3 - 2c_4/c_3)^2}{16\pi^2}\left(\frac{E}{\Lambda}\right)^8$$

The massless limit is not smooth. As $m \to 0$ the interactions switch off.

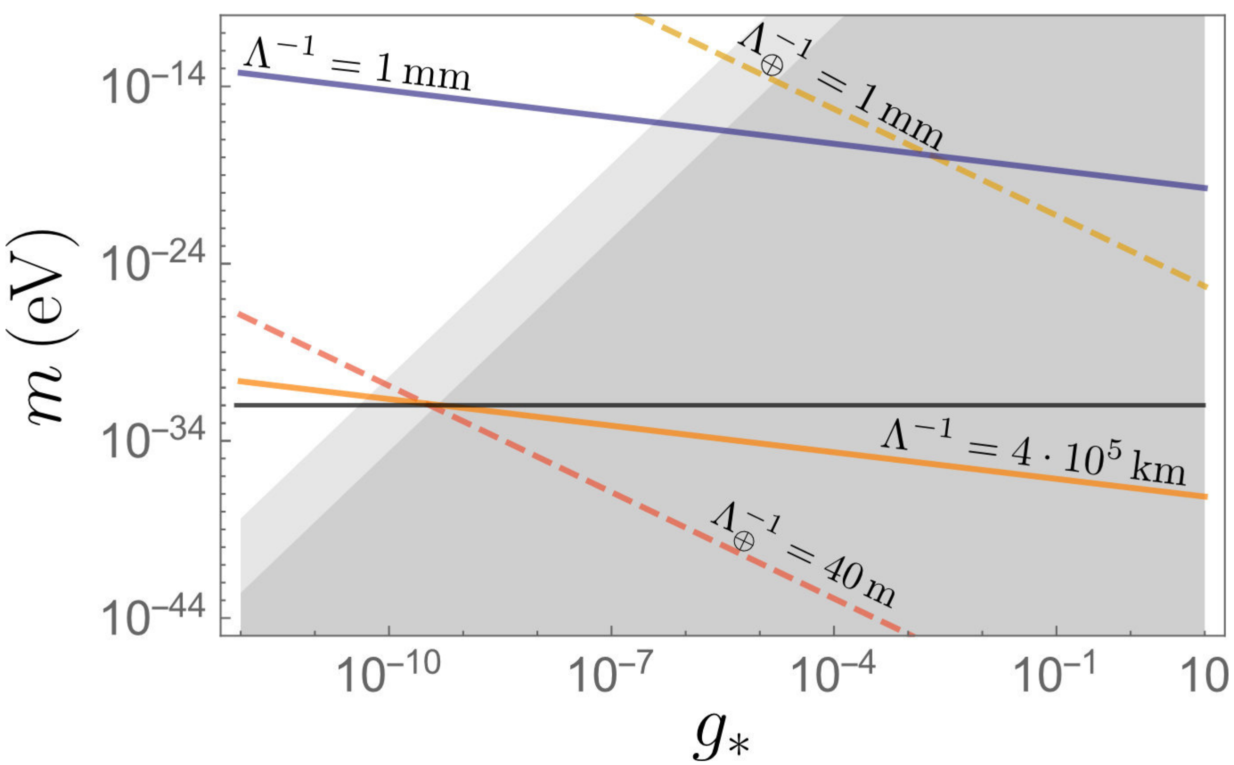
Beyond Positivity Bounds

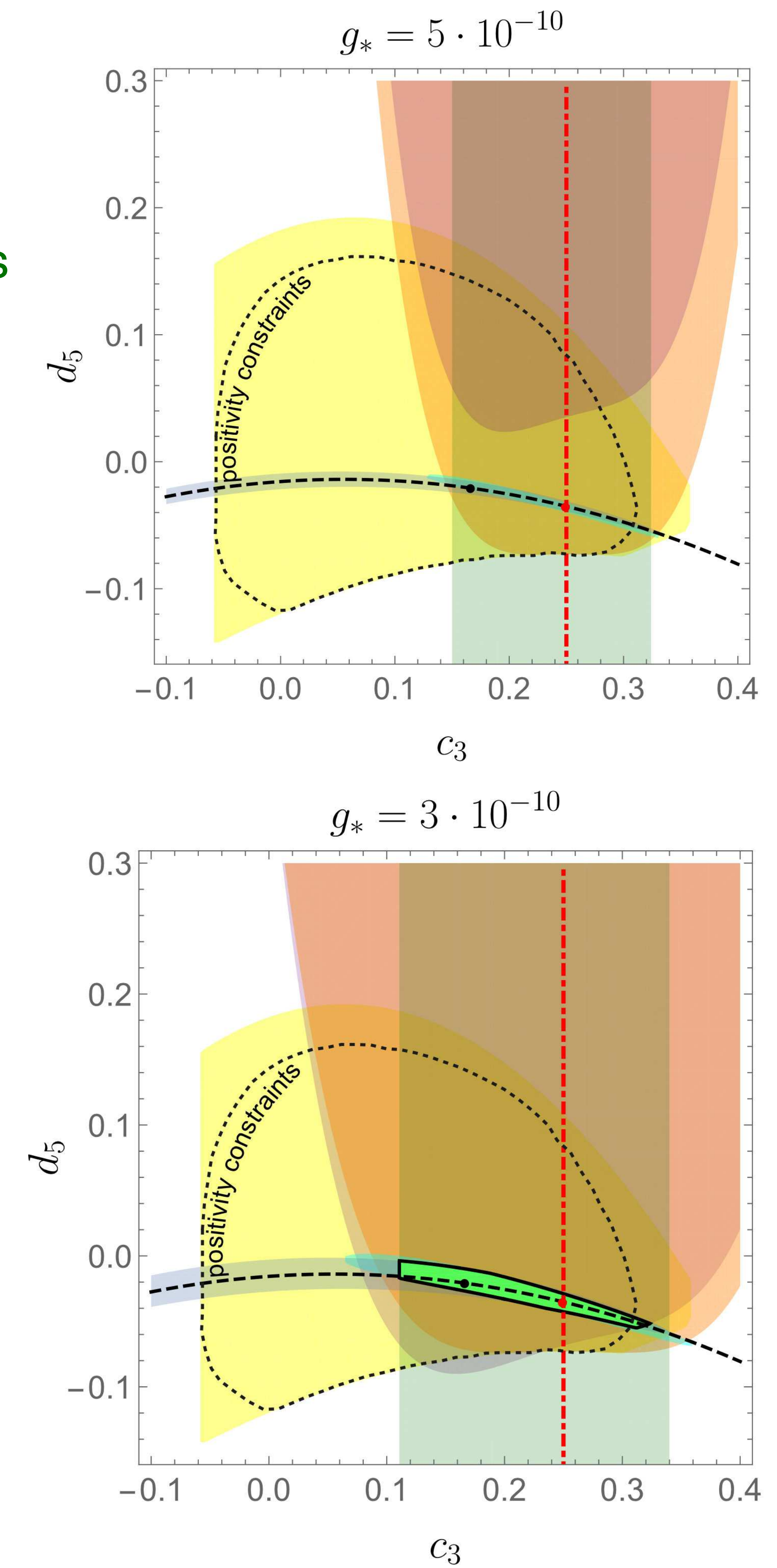
- Galileon theories, dRGT massive gravity

PRL 120 (2018) no.16, 161101
B.Bellazzini, F.Riva, J.Serra, FS

$$\frac{1}{\Lambda} \simeq r_{\rm moon} \left(\frac{g_*}{4.5\cdot 10^{-10}}\right)^{-1/3} \left(\frac{m}{10^{-32}\,\rm eV}\right)^{-2/3}$$

Strong bound!

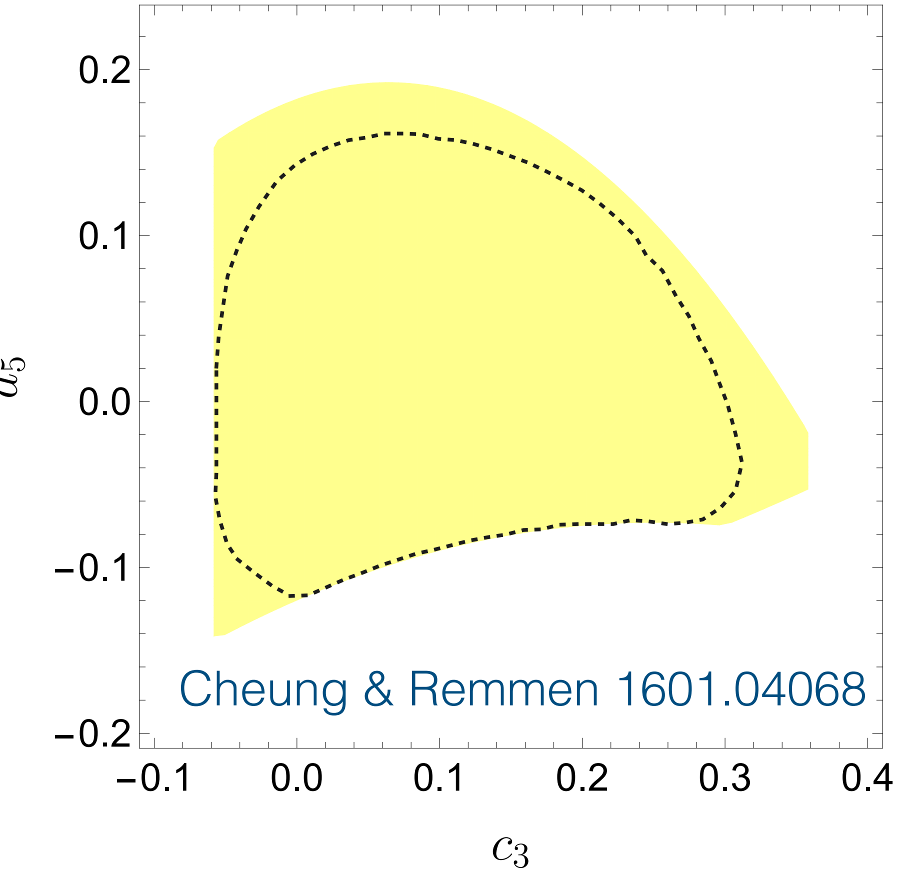




- Effective theory of massive higher spins? Work in progress

dRGT massive gravity

We explicitly break diff-invariance by adding a mass term to the Einstein Hilbert action



$$S_{\rm dRGT} = \int d^4\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{M_{Pl}^2 m^2}{8}V(g,h)\right]$$

1011.1232 de Rham, Gabadadze, Tolley

$h^2, h^3, h^4 \ldots$

- Coefficients fixed to propagate only 5 d.o.f.'s
- Only two independent coefficients c_3, d_5

IR-residue > loop-factor x $\int_0^{E^2 \ll \Lambda^2} ds\,[\ldots]$

$$\Sigma_{\rm IR}^{SS} = \frac{2m^2}{9\Lambda_3^6}\left(7 - 6c_3(1+3c_3) + 48d_5\right) > 0$$

$$\Sigma_{\rm IR}^{VV} = \frac{m^2}{16\Lambda_3^6}\left(5 + 72c_3 - 240c_3^2\right) > 0$$

$$\Sigma_{\rm IR}^{VS} = \frac{m^2}{48\Lambda_3^6}\left(91 - 312c_3 + 432c_3^2 + 384d_5\right) > 0$$

...

mass-suppressed

Hard scattering contribution

$$\mathcal{M}^{SS} = \frac{st(s+t)}{6\Lambda_3^6}\left(1 - 4c_3(1-9c_3) + 64d_5\right)$$

$$\mathcal{M}^{VV} = \frac{9st(s+t)}{32\Lambda_3^6}(1-4c_3)^2$$

$$\mathcal{M}^{VS} = \frac{3t}{4\Lambda_3^6}\left(c_3(1-2c_3)(s^2+st-t^2) - \frac{5s^2+5st-9t^2}{72}\right)$$

...

mass unsuppressed

dRGT massive gravity

We can derive a lower theoretical bound on the graviton mass

$$\left(\frac{m}{4\pi M_{\rm Pl}}\right) > \frac{1}{F_i(c_3,d_5)}\left(\frac{g_*}{4\pi}\right)^4 \cdot \delta^6 \cdot [1\pm\delta] \qquad g_* \equiv \left(\frac{\Lambda}{\Lambda_3}\right)^3$$

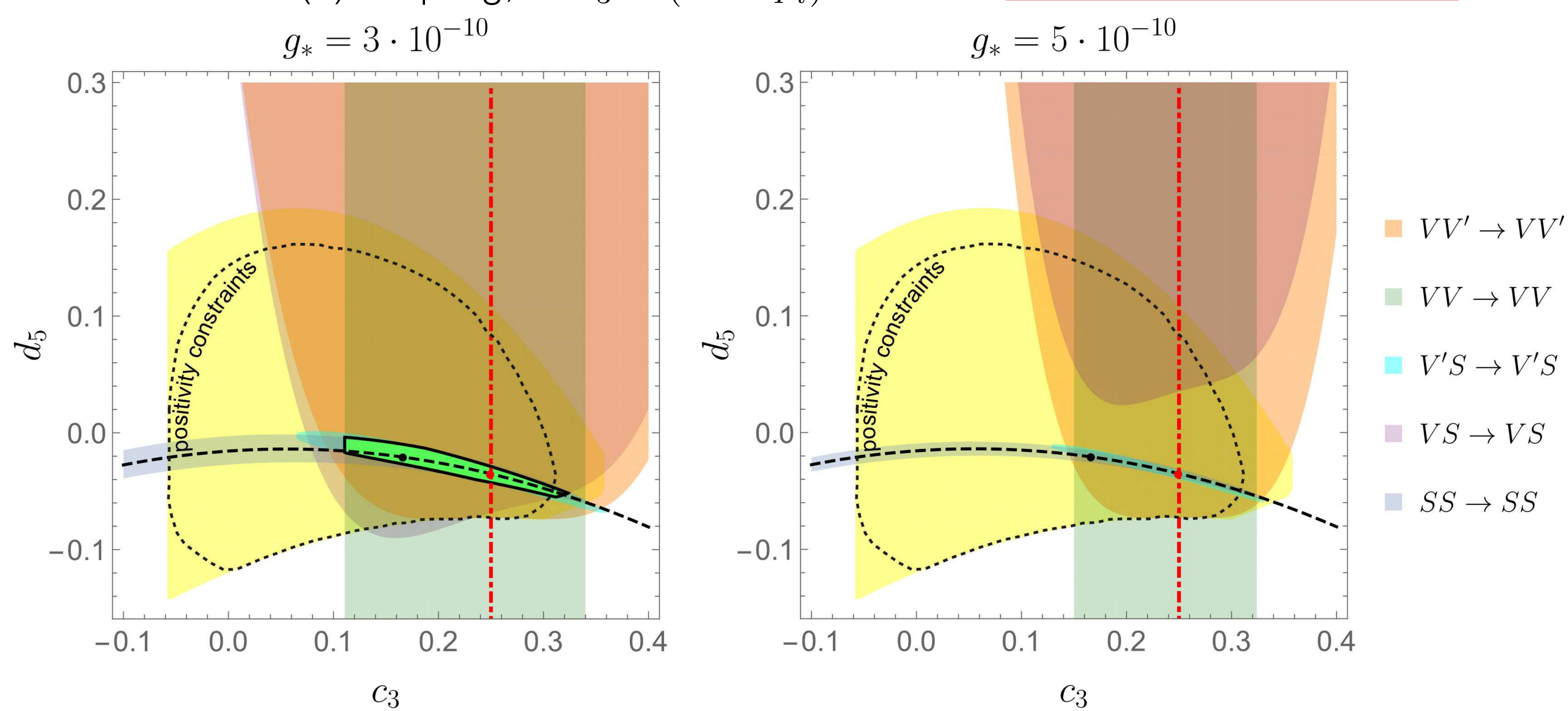
The most conservative bound is obtained by picking the maximum of minimums of $F_i(c_3,d_5)$

$$m > 10^{-32}\text{eV}\left(\frac{g_*}{4.5\cdot 10^{-10}}\right)^4\left(\frac{\delta}{1\%}\right)^6$$

PRL 120 (2018) no.16, 161101
B.Bellazzini, F.Riva, J.Serra, FS

The experimental bound on the graviton mass is $m < 10^{-32}\text{eV} \longrightarrow g_* < 4.5\cdot 10^{-10}$

In the literature it is assumed O(1) coupling, or $\Lambda_3 = (m^2 M_{Pl})^{1/3} = \Lambda$ *Such scenario is ruled out!*

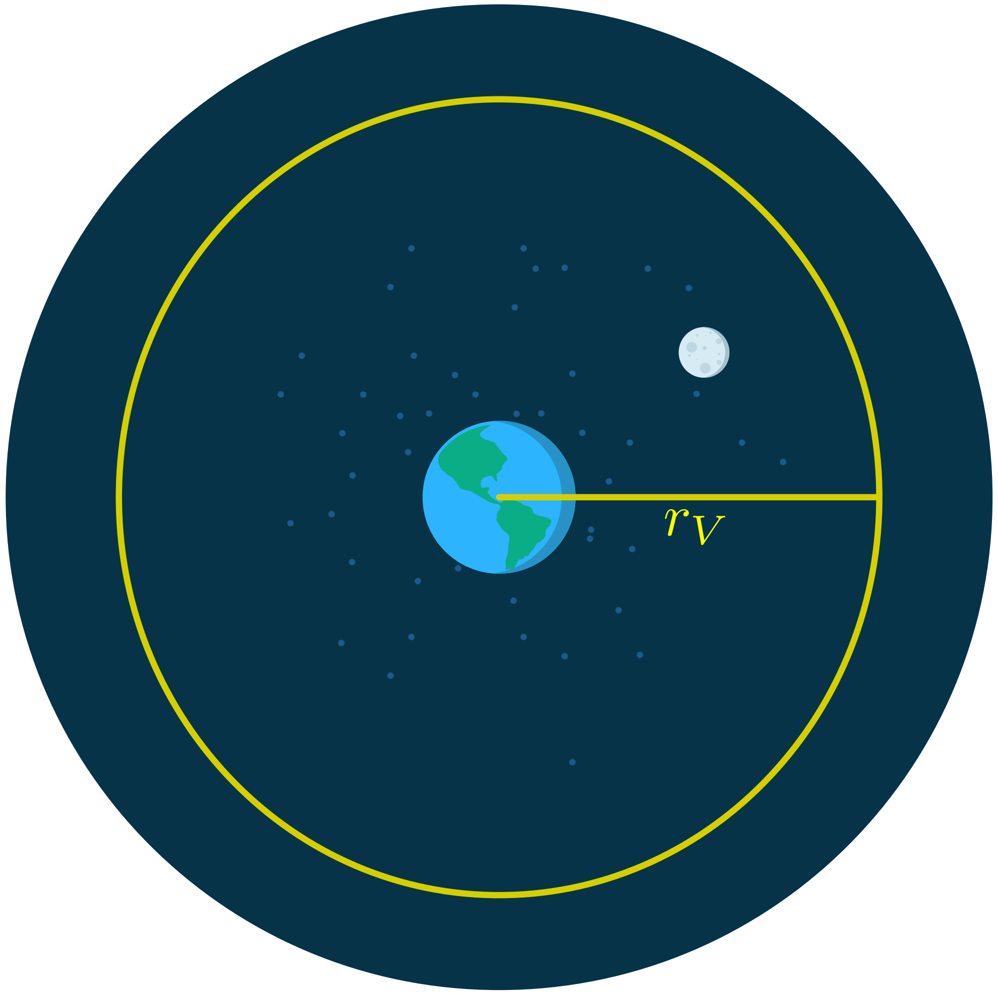


The fate of massive gravity

$$\frac{1}{\Lambda} \simeq r_{\text{moon}} \left(\frac{g_*}{4.5 \cdot 10^{-10}} \right)^{-1/3} \left(\frac{m}{10^{-32}\,\text{eV}} \right)^{-2/3}$$

The computation shown so far has been performed in flat space-time.

What about physics around massive bodies?



Non linearities $r_V = \frac{1}{\Lambda_3} \left(\frac{M_\oplus}{M_{Pl}} \right)^{1/3}$

Gravitational potential for a test massive body

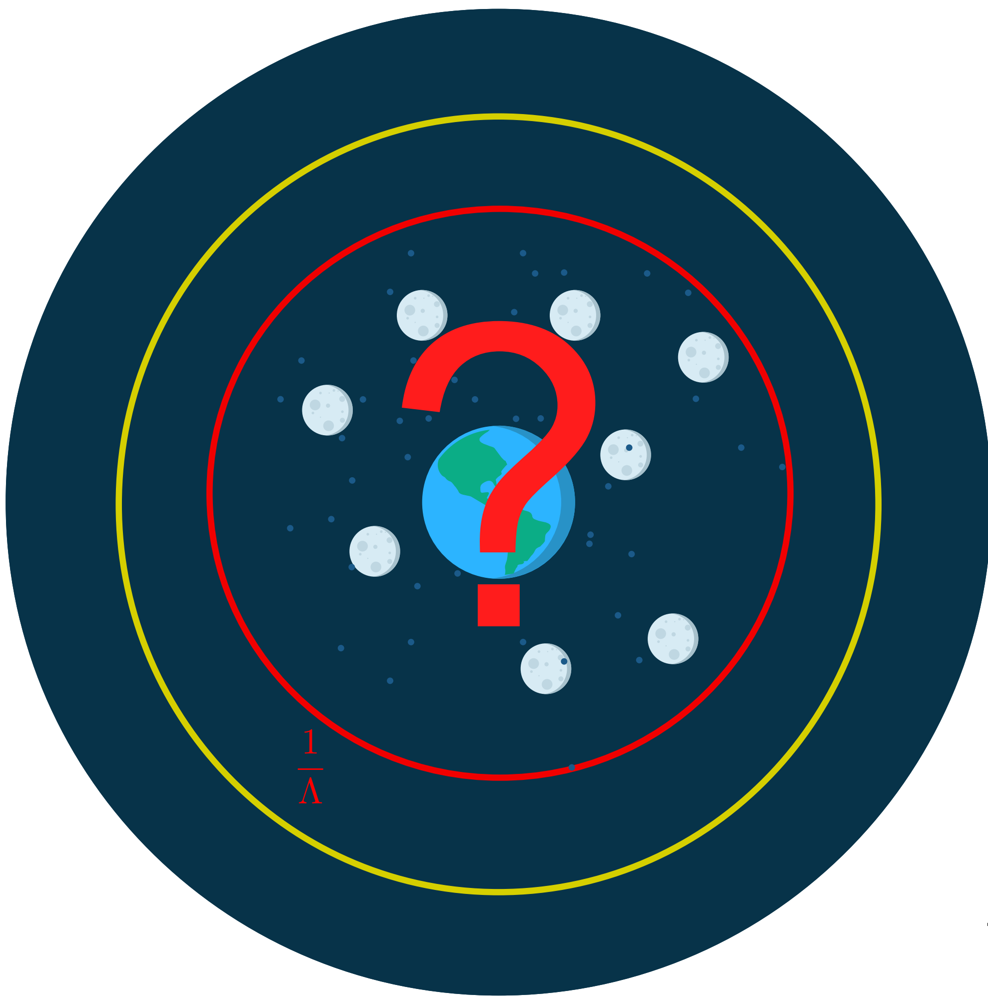
$$\left(\frac{M_\oplus\, m_{\text{test}}}{M_{\text{Pl}}^2} \right) \frac{1}{r} \left[1 + \left(\frac{r_V}{r} \right)^3 + \dots \right]$$

The fate of massive gravity

$$\frac{1}{\Lambda} \simeq r_{\text{moon}} \left(\frac{g_*}{4.5 \cdot 10^{-10}} \right)^{-1/3} \left(\frac{m}{10^{-32}\,\text{eV}} \right)^{-2/3}$$

The computation shown so far has been performed in flat space-time.

What about physics around massive bodies?



Non linearities $r_V = \frac{1}{\Lambda_3} \left(\frac{M_\oplus}{M_{Pl}} \right)^{1/3}$

Gravitational potential for a test massive body

$$\left(\frac{M_\oplus\, m_{\text{test}}}{M_{\text{Pl}}^2} \right) \frac{1}{r} \left[1 + \left(\frac{r_V}{r} \right)^3 + \ldots \right] \times \left(1 + \frac{1}{r\Lambda} + \ldots\right)$$

quantum corrections

$(\partial/\Lambda)^{2n}$

Vainshtein screening breaks at $r \sim \frac{1}{\Lambda} \approx 10^{3\div4} \frac{1}{\Lambda_3} \approx (1 \div 10)\, r_{\text{moon}}$

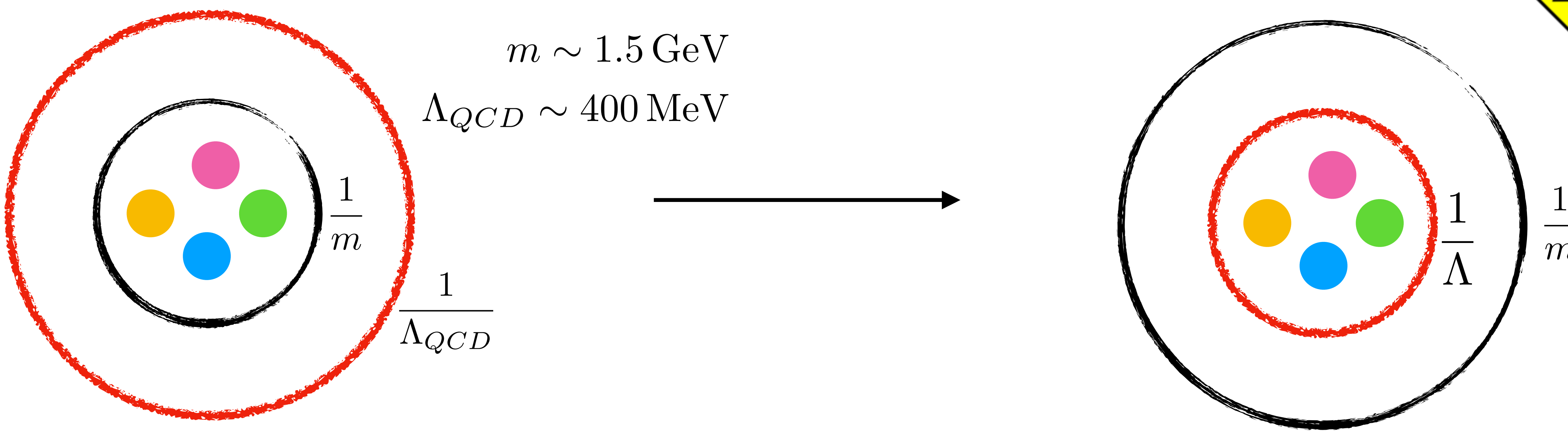
The angular precession of the perihelion of the Moon gets modified

$$\left(\delta\phi^{\pi}\big|_{r=1/\Lambda} \sim \right) \pi \left(\frac{r}{r_V} \right)^{3/2} \sim 10^{-11} \div 10^{-10}$$

$$\delta\phi^{\text{exp}}\big|_{moon} \sim 10^{-11}$$

EFT for massive higher spins

- In flat space, massless spin s>=1 (s >= 2) cannot couple minimally to EM (gravity) (Weinberg-Witten theorem)
- No obstruction for (charged) massive higher spin particles: they do exists!
- Natural question : can we find the EFT for these particles?



For massive higher-spin particles minimally coupled to EM

$$\Lambda_s < \frac{m}{e^{1/(2s-1)}}$$

M. Porrati, R. Rahman (2009)

As $m \to 0$ the cutoff must go to zero ⟶ no-go theorems

Bound without coupling the higher spin to external fields?

Attempts for massive spin 3

- We consider a sector of a massless interacting spin-3 particle ⟶ irrelevant interactions (super soft)

$$\delta\phi^{\alpha\beta\gamma} = \partial^{(\alpha}\xi^{\beta\gamma)} \qquad \xi^{\mu}_{\mu} = 0$$

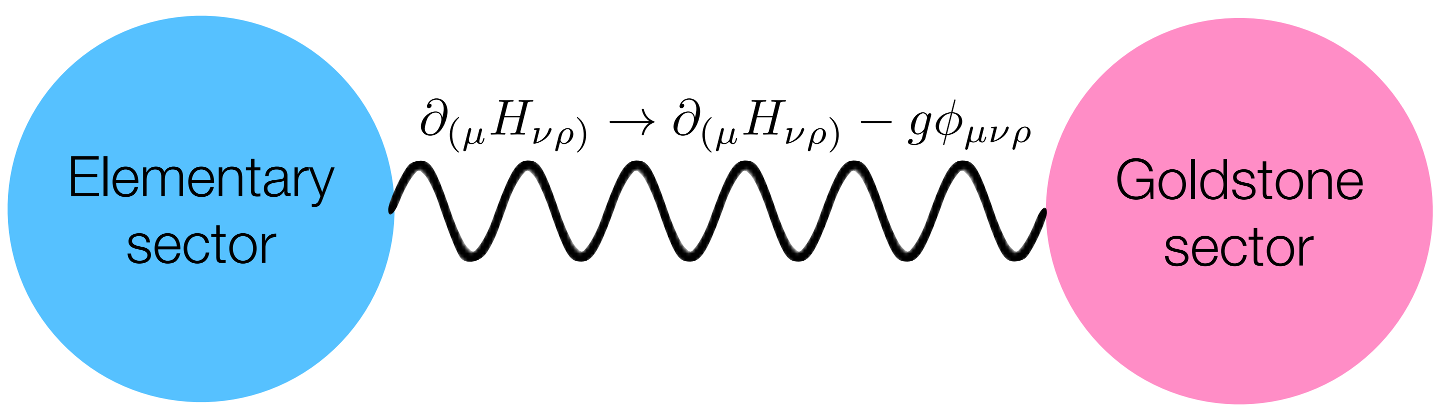
$$R^{\alpha\beta\gamma}_{\mu\nu\rho} = \partial_{[\mu\nu\rho}\phi^{\alpha\beta\gamma]}$$

$$\mathcal{L}_{El} = \mathcal{L}^{\text{kin}}_{\phi} + \frac{g^2}{\Lambda^{12}} R^4 + \ldots$$

$$\mathcal{A}^{TT}(s, t=0) \sim \frac{s^6}{\Lambda^{12}}$$

- Goldstone sector $H_{\mu\nu}(x) \to H_{\mu\nu}(x) + \lambda_{\mu\nu}$ with $\lambda^{\mu}_{\mu} = 0$

$$\mathcal{L}_{\text{Gold}} = \mathcal{L}^{\text{kin}}_{H} + c_1 H^2 + \mathcal{L}_{int}$$



$c_1 = \frac{3}{4}m^2$ Tuned to propagate the 7 d.o.f.

Attempts for massive spin 3

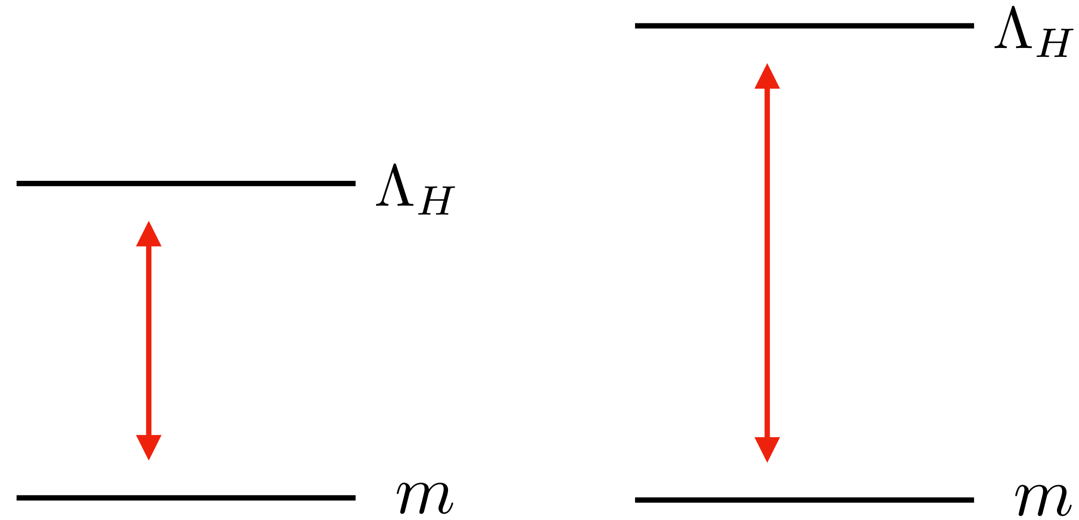
- The goldstone sector induces gauge-symmetry breaking interactions $\mathcal{L}_{int} = \epsilon\,\phi^4 + \frac{g^2}{\Lambda^{12}} R^4 + \ldots$

$$\epsilon = g_*^2 \frac{m^4}{\Lambda_H^4}$$

- Scalar modes scattering $\mathcal{A}^{SSSS}(s) \sim \epsilon \frac{s^6}{m^{12}} = g_*^2 \frac{s^6}{(\Lambda_H m^2)^4}$

- There is an interaction which increase the strong coupling scale

$$\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}\phi_{\mu_1\nu_1\rho}\phi_{\mu_2\nu_2}{}^{\rho}\phi_{\mu_3\nu_3\sigma}\phi_{\mu_4\nu_4}{}^{\sigma} \qquad \mathcal{A}^{SSSS}(s) \sim \epsilon \frac{s^5}{m^{10}} = g_*^2 \frac{s^5}{(\Lambda_H m^{3/2})^4}$$



As we send $m \to 0$ the interactions die

Beyond positivities

$$\Lambda_H < \frac{m}{(g_*/4\pi)^{1/8}} \qquad \Lambda_H < \frac{m}{(g_*/4\pi)^{1/6}}$$

Scaling of amplitudes

$$\mathcal{L}_{\rm int} = c_1 \frac{g_*^2}{\Lambda^{12}} \left(R^{\rho_1\rho_2\rho_3}{}_{\mu\nu\sigma} R_{\rho_1\rho_2\rho_3}{}^{\mu\nu\sigma}\right)^2 + c_2\epsilon^2 \frac{g_*^2}{\Lambda^6} \left(R^{\rho_1\rho_2\rho_3}{}_{\mu\nu\sigma} R_{\rho_1\rho_2\rho_3}{}^{\mu\nu\sigma}\right) \phi_{\mu\nu\sigma}\phi^{\mu\nu\sigma} + c_3\epsilon^4 g_*^2 (\phi_{\mu\nu\rho})^4$$

Table 1: Scalings of amplitudes

	c_1/Λ^{12}	$c_2\epsilon^2/\Lambda^6$	$c_3\epsilon^4$	h^4 tuned		c_1/Λ^{12}	$c_2\epsilon^2/\Lambda^6$	$c_3\epsilon^4$	h^4 tuned
$TTTT$	s^6	s^3	s^0	0	$TTSS$	s^5m^2	s^6/m^6	s^3/m^6	s^3/m^6
$SSSS$	s^4m^4	s^5/m^4	s^6/m^{12}	s^5/m^{10}	$VVSS$	s^5m^2	s^6/m^6	s^5/m^{10}	s^5/m^{10}
$VVVV$	s^6	s^5/m^4	s^4/m^8	s^3/m^6	$TTVV$	s^6	s^5/m^4	s^2/m^4	s^2/m^4
$H'H'H'H'$	s^4m^4	s^3	s^2/m^4	s^2/m^4	$HHHH$	s^4m^4	s^3	s^2/m^4	0

Gauging of the goldstone sector

$$\mathcal{L}_{\rm Gold} = \frac{\Lambda_*^4}{g_*^2}\hat{\mathcal{L}}\left[\frac{\partial_\mu \bar{H}_{\nu\rho}}{\Lambda_*}, \bar{H}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda_*^2}, \frac{\partial}{\Lambda_*}\right]$$
$$= \frac{\Lambda_*^2}{g_*^2}\left[\alpha_1\,\partial_\mu \bar{H}_{\nu\rho}\partial^\mu \bar{H}^{\nu\rho} + \alpha_2\,\partial_\mu \bar{H}^{\mu\rho}\partial^\nu \bar{H}_{\nu\rho} + \alpha_3\,\partial_\nu \bar{H}\partial_\mu \bar{H}^{\mu\nu} + \alpha_4 \partial_\mu \bar{H}\partial^\mu \bar{H} + \alpha_5 R\bar{H}\right]$$
$$+ c_1\frac{\Lambda_*^4}{g_*^2}\bar{H}^2 + \mathcal{L}_{\rm Gint}$$

$$\alpha_1 = -\frac{1}{2}\,, \qquad \alpha_2 = 1\,, \qquad \alpha_3 - 2\alpha_5 = -1\,, \qquad \alpha_4 + 2\alpha_5 = \frac{1}{2}$$

Conserved current of the shift symmetry $\delta\mathcal{L}_{\rm Gold} = \Lambda_*^2/g_*^2\,\xi_{\nu\rho}\partial_\mu \mathcal{J}^{\mu\nu\rho}$

$$\mathcal{J}^{\mu\nu\rho} = \left(-2\alpha_1\partial^\mu \bar{H}^{\nu\rho} - 2\alpha_2\partial^\nu \bar{H}^{\mu\rho} - \alpha_3\eta^{\mu\nu}\partial^\rho \bar{H}\right)$$
$$= \left(\partial^\mu \bar{H}^{\nu\rho} - 2\partial^\nu \bar{H}^{\mu\rho} + \eta^{\mu\nu}\partial^\rho \bar{H}\right)$$

Gauging of the goldstone sector

Covariant derivatives

$$D_{(\mu}\tilde{H}_{\nu\rho)} \equiv \partial_{(\mu}\bar{H}_{\nu\rho)} - g\phi_{\mu\nu\rho}$$

$$\bar{R}_{\mu\nu} \equiv R_{\mu\nu} + \Box\bar{H}_{\mu\nu} - g\partial^\sigma\phi_{\sigma\mu\nu}$$

$$\mathcal{L}^{\tilde{H}}_{kin} = \frac{\Lambda_*^2}{g_*^2}\left[-\frac{1}{6}D_{(\mu}\tilde{H}_{\nu\rho)}D^{(\mu}\tilde{H}^{\nu\rho)} + \frac{1}{2}D_{(\mu}\tilde{H}^{\mu}_{\rho)}D_{(\nu}\tilde{H}^{\nu\rho)} - \frac{3}{2}\left(D_{(\mu}\tilde{H}^{\mu}_{\rho)} - \partial_\rho\tilde{H}\right)\partial^\rho\tilde{H}\right]$$

After gauging, the mass term is generated

$$\mathcal{L}^{\phi}_{\text{mass}} = -\frac{g^2}{6g_*^2}\Lambda_*^2\left[\phi^2_{\mu\nu\rho} - 3\phi^2_{\mu}\right]$$

$$g = g_*\frac{\sqrt{3}m}{\Lambda_*}$$

Power counting of gauge-breaking interactions

$$\frac{1}{g_*^2}\left(D_{(\mu}\tilde{H}_{\nu\rho)}\right)^4 \supset \frac{g^4}{g_*^2}\phi^4_{\mu\nu\rho} = \#g^2\frac{m^2}{\Lambda_*^2}\phi^4_{\mu\nu\rho} = 3\#g_*^2\frac{m^4}{\Lambda_*^4}\phi^4_{\mu\nu\rho}$$

Gauging of the goldstone sector

Equation of the motion in the gauge $H_{\mu\nu} = \frac{\eta_{\mu\nu}}{4} H$

$$\mathcal{F}_{\mu\nu\rho} - \frac{1}{2}\eta_{(\mu\nu}\mathcal{F}_{\rho)} - m^2\phi_{\mu\nu\rho} + m^2\eta_{(\mu\nu}\phi_{\rho)} + \frac{1}{4}g\Lambda_*^2 J_{\mu\nu\rho} = 0$$

$$8c_1\Lambda_*^4 H - g\Lambda_*^2\partial_\mu\phi^\mu - \frac{3}{2}\Lambda_*^2\Box H = 0$$

$$J^{\mu\nu\rho} = \frac{1}{3}\eta^{(\mu\nu}\partial^{\rho)}H$$

$$\mathcal{F}_{\mu\nu\rho} = \Box\phi_{\mu\nu\rho} + \partial_{(\mu}\partial_\nu\phi_{\rho)} - \partial_\alpha\partial_{(\mu}\phi^\alpha_{\nu\rho)}$$