One Lecture on the Hierarchy Problem

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Abstract

This is the summary of a lecture given during the 2017 Spring School on Superstring Theory at the ICTP. It contains a pedagogical introduction to effective field theory and a description of fine-tuning problems in quantum field theory. I also briefly mention the main ideas behind known solutions to the electroweak hierarchy problem.

1 Introduction

Often, accidental cancellations between unrelated parameters signal that our description of Nature is incomplete. A well-known example is the rest energy of the electron in classical electrodynamics. In natural units ($\hbar = c = 1$) we have

$$m_e = m_{e,0} + \frac{e^2}{4\pi r_e}\,. \tag{1.1}$$

The first term on the right-hand side is the bare electron mass in the Lagrangian. The second accounts for the energy stored in the electric field generated by the electron.

Experimentally we know that $m_e \approx 0.511$ MeV. Cheating a little for illustrative purposes we can use our modern knowledge of the electron radius $r_e \lesssim \text{TeV}^{-1}$ to cut-off the divergence of the Coulomb self-energy. This corresponds to not having observed deviations from a point-like behavior at LEP []. Putting together these two measurements we conclude that only an accidental cancellation between the two terms on the right-hand side of (1.1) can explain the observed value of the electron mass.

This apparent fine-tuning is hiding something deep. At the length scales in our calculation classical electrodynamics breaks down and we need to include quantum effects to obtain the correct result. Restoring units, we can not ignore quantum mechanics below

$$c\Delta t \lesssim \frac{\hbar c}{\Delta E} \approx \frac{\hbar}{m_e c}\,, \tag{1.2}$$

or in natural units for $r_e \lesssim 1/m_e$. So the result of our classical calculation is not correct. If we include the contribution of photons and positrons from vacuum fluctuations [1], the term that diverges as $1/r_e$ is cancelled by virtue of a new symmetry. The chiral symmetry that emerges in quantum electrodynamics as m_e goes to zero. Only a term logarithmic in $1/r_e$ and proportional to $m_{e,0}$ survives, as dictated by the selection rules of this new symmetry,

$$m_e = m_{e,0}\left[1 + \frac{3\alpha}{4\pi}\log\frac{1}{m_e r_e}\right]. \tag{1.3}$$

Now we have a correction of less than 10% even for an electron that stays point-like up to the Planck scale. Incidentally, pushing classical electrodynamics beyond its limits of validity has other surprising consequences, including the emergence of an acausal behavior for the electron on time scales of $\mathcal{O}(e^2/m_e)$ [2].

Setting violations of causality aside, we have just seen that what appeared as an accidental cancellation was pointing to a more fundamental description of our physical system in terms of quantum mechanics.

This is not the only case in which apparent coincidences is signaling the emergence of a new paradigm. A second classic example that has a completely different resolution is that of planetary orbits in the solar system. In 1596 Johannes Kepler published the *Mysterium Cosmographicum*, where he showed that each of the five Platonic solids can be uniquely inscribed into and circumscribed by a sphere. If ordered in a specific pattern (octahedron, icosahedron, dodecahedron, tetrahedron, cube) the spheres reproduced, within the experimental accuracy of the time, the orbits of the six known planets, from Mercury to Saturn. This seems a striking coincidence that requires finely tuned values of unrelated parameters. Alternatively, as Kepler did, one could see it as an example of God's refined aesthetic sense.

Today we know that the explanation is different, but still paradigm-shifting. Not only we are not unique in any way, but we are just a tiny speck of dust in an unimaginably vast universe. This kind of approximate accidents become likely if we think about the staggering number of other stars, planets and solar systems over which we have to integrate small probabilities.

I hope that these two examples convinced you that fine-tuning problems in physics are worthy of attention, as they often lead to the emergence of a new understanding of the Universe. Today we are facing two problems of this kind and they might have answers that are just as deep as the historical examples given above.

The first and most dramatic of the two puzzles concerns the size of the cosmological constant: $\Lambda \approx (10^{-3}\ \mathrm{eV})^4$ that is much smaller than all the particle physics scales that we know and should naively contribute to it (except neutrino masses). I am not going to discuss this problem here. I refer the interested reader to the reviews [3, 4, 5, 6] and their references.

In the following I describe another fine-tuning problem in modern theoretical physics, the one related to the Higgs boson mass, also known as the *hierarchy problem* [7, 8, 9, 10, 11]. To state it precisely we first have to make sense of the illusory divergences of quantum field theory. We have already encountered one example in the Coulomb self-energy of the electron as $r_e \to 0$.

To this end, in the next section I introduce Effective Field Theory (EFT). In Section 3 I use EFT to state the problem first in a toy model and then in the Standard Model of particle physics. In Section 4 I briefly summarize the main ideas behind the solutions that have been proposed.

2 Effective Field Theory

Imagine to know that your theory is valid up to some energy scale M_*. If you only need to make a prediction for measured quantities at $E \ll M_*$, it is not necessary to include in your calculation all the details of the dynamics at the high scale. For example, you can describe the energy levels of the Hydrogen atom with excellent accuracy, without knowing anything about the mass of the top quark. The error that you are making is of order $\alpha m_e/m_t$ and if your experimental precision is inferior, this is perfectly acceptable. Some of the low-energy parameters that you need for the calculation are more sensitive to m_t, for example the proton mass m_p and the fine structure constant α. However these are all quantities that you can measure at low energy, forgetting about their ultraviolet (UV) origin.

If we could not describe the low-energy dynamics only in terms of low-energy degrees of freedom, at least to some finite precision, we would not have been able to make predictions for any physical system. So the fact that UV sensitive quantities can all be fixed through low-energy measurements must be independent of our specific example.

However this does not mean that every trace of the UV dynamics disappears in the low-energy theory. There are very non-trivial consequences of UV physics that survive at low energy. One classic example is the spin-statistics theorem. In non-relativistic quantum mechanics it is just a (measured) fact of life, but in quantum field theory it emerges from causality. Other than symmetry constraints, the UV dynamics also leaves behind small corrections to low-energy observables (the $\alpha m_e/m_t$ error in the case of the Hydrogen atom). Therefore if we had a systematic way of building a low-energy theory from a more complete theory we would have accomplished two remarkable tasks. We would have considerably simplified our low-energy calculations and at the same time we would have a way to reconstruct, at least to some extent, the UV dynamics from low-energy measurements. Effective Field Theory is precisely the systematic construction that we are looking for. In the rest of this section I often follow [12].

To see EFT at work, take a scale $M \lesssim M_*$ and split the degrees of freedom in the path integral into two parts, the high-frequency and the low-frequency modes,

$$\int \mathcal{D}\phi e^{iS(\phi)} = \int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)} , \tag{2.1}$$

$$\begin{aligned} \omega_{\phi_H} &> M , \\ \omega_{\phi_L} &< M . \end{aligned} \tag{2.2}$$

If we know how to do the path integral over the high-frequency modes we obtain a description of the system in terms of the low-energy degrees of freedom

$$\int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)} = \int \mathcal{D}\phi_L e^{iS_M(\phi_L)} . \tag{2.3}$$

This is all we need and we have not even restricted the validity of the theory. In principle we can use $S_M(\phi_L)$ to make predictions up to M_*. In practice this suggest that we have not really gained anything and in fact most of the time the path integral can not be solved exactly. However we can at least consider the previous equation as a definition of the low-energy action

$$e^{iS_M(\phi_L)} \equiv \int \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)} \,. \tag{2.4}$$

In some cases this gives us a way to compute $S_M(\phi_L)$ in a perturbative expansion. Even when this is not possible, we can always write $S_M(\phi_L)$ as an infinite sum of operators built from the low-frequency fields and consistent with all the low-energy symmetries of the problem,

$$S_M(\phi_L) = \int d^d x \sum_i c_i \mathcal{O}_i(\phi_L) \,. \tag{2.5}$$

Note that some of these operators are non-local by a $1/M$ amount, since we have integrated out fields with $\omega_{\phi_H} > M$. So this is also an expansion in derivatives. It might seem that this infinite sum requires the full knowledge of $S(\phi_L, \phi_H)$ to be useful. However here the power of *broken* symmetries comes to our rescue.

This is familiar from quantum and classical mechanics. Even in systems that are not rotationally invariant, for example, selection rules of the rotational symmetry are extremely useful to predict relations between matrix elements. If you prefer a quantum field theory equivalent you can think about Isospin in QCD and its breaking by the quark masses or flavor symmetries in the Standard Model (SM) and their breaking by the Yukawa matrices.

In our case we need an even simpler symmetry. We can just use dimensional analysis, which should more appropriately be called the selection rules of the dilatation operator [13].

If we set $\hbar = c = 1$, our operators have some dimension δ_i in units of energy $[\mathcal{O}_i] = E^{\delta_i}$. Since the action is dimensionless ($\hbar = 1$) we must have $[c_i] = E^{d-\delta_i}$.

The largest scale in our theory is M and we can always write

$$\begin{aligned} c_i &= \gamma_0 M^{d-\delta_i} + \gamma_1 M_1^{d-\delta_i} + \gamma_2 M_2^{d-\delta_i} + \ldots = g_i M^{d-\delta_i} \,, \\ g_i &\equiv \gamma_0 + \gamma_1 \left(\frac{M_1}{M}\right)^{d-\delta_i} + \ldots \end{aligned} \tag{2.6}$$

where $M_1, M_2, \ldots < M$. This just means that even if the c_i receive contributions from multiple scales we can always parametrize them in terms of the largest scale in the theory times some dimensionless coefficient. From simple dimensional analysis we expect $g_i = \mathcal{O}(1)$ unless some extra symmetry is at work. The selection rules of the dilatation operator are what determined the form of c_i, i.e. all contributions must have dimensions $E^{d-\delta_i}$ and the largest one can be at most $\sim M^{d-\delta_i}$.

Now we are in a position to estimate the contribution of each term in the sum (2.5) to low-energy observables. Using again dimensional analysis we have

$$\int d^d x \mathcal{O}_i \approx E^{\delta_i - d} \,, \tag{2.7}$$

so each term in the sum contributes to a low-energy measurement an amount

$$c_i \int d^d x \mathcal{O}_i \approx g_i \left(\frac{E}{M}\right)^{\delta_i - d} \,. \tag{2.8}$$

We see immediately that operators with $\delta_i > d$ are suppressed when $E \ll M$, so if we are interested in a finite level of precision we need only a finite number of operators for our calculation! Not surprisingly operators with $\delta_i > d$ are called *irrelevant*, those with $\delta_i < d$ *relevant* and the ones with $\delta_i = d$ *marginal*.

Here resides the power of Effective Field Theory. We have just seen that ignoring completely the high energy dynamics, we can write a finite set of operators based on the fields and symmetries that we observe at low energy and make predictions to an arbitrary level of accuracy. If our experimental precision is sufficient we can even probe operators suppressed by powers of $1/M$ and obtain information on the scale at which new phenomena should appear.

This is not all. The very simple construction that we have just seen can do something else for us. Given low-energy observations it can tell us if they arise from a "reasonable" high-energy theory. In other words it tells us if we should be surprised or not. For example we can imagine that at low energy we measure the theory of a free massless scalar[1]

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} \tag{2.9}$$

and we know from our experimental observations that it is valid at least up to $E \approx M$. Is this surprising from an EFT perspective? The answer is no. We can easily imagine that the UV theory possesses a shift symmetry, $\phi \to \phi + c$, that prevents interactions from being generated when we integrate out high-frequency modes. Of course we expect higher order terms consistent with the symmetry, as for example $(\partial^2\phi)^2/M^2$, but measuring them might be beyond our experimental capabilities.

What about a free massive scalar with $m \ll M$?

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} - \frac{m^2\phi^2}{2}\,. \tag{2.10}$$

The answer is still no. There is nothing surprising in this Lagrangian and this can be seen in at least two ways. I will discuss the most unusual one that I have learned from [13]. In momentum space the Lagrangian

$$\phi(-k)\left(k^2 - m^2\right)\phi(k) \tag{2.11}$$

has an infinite number of symmetries $\phi(k) \to e^{i\alpha(k)}\phi(k)$ with $\alpha(-k) = -\alpha(k)$. To better understand it, we can expand α in odd powers of k,

$$\alpha(k) = a_\mu k^\mu + a_{\mu\nu\rho}k^\mu k^\nu k^\rho + \ldots \tag{2.12}$$

and notice that the linear term corresponds to translations. Its generator in position space is just $i\partial_\mu$ and the corresponding conserved current is the stress-energy tensor $T^{\mu\nu}$. The higher order terms are generated by higher powers of derivatives and are associated with higher-spin currents. The algebra is trivial (for example $[\partial, \partial^3] = 0$) and obviously does not contain dilatations or special conformal transformations. This symmetry is broken by higher-point interactions and preserves the form of the free Lagrangian.

[1] Here and in the following when Lorentz contractions are obvious I suppress the corresponding indices.

Finally it is time to consider a surprising example:

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} - \frac{\lambda\phi^4}{4}\,. \tag{2.13}$$

In this theory both the shift symmetry and the momentum-space symmetry of the free action are broken by λ. So we expect a mass term of order $m^2 \sim \frac{\lambda M^2}{16\pi^2}$. The linear dependence on λ can be deduced using the selection rules of the broken symmetries of the free action[2], while that on M^2 is as usual dictated by the dilatation symmetry. We can also explicitly obtain m^2 by integrating out high-momentum modes of ϕ at one-loop from the diagram obtained by contracting two of the ϕ legs in the ϕ^4 vertex.

Not observing this mass term is indeed surprising and as we will see in the following it is a simple example of a fine-tuning problem. You might object that (2.13) is scale invariant and a mass term should not be generated. However the symmetry is broken by the scale M that limits the validity of (2.13). For example it could be (or be proportional to) the physical mass of a new particle that interacts with ϕ or be the scale at which λ hits a Landau pole. Even if the theory at M was transitioning smoothly to a UV fixed point we would still expect contributions to m^2 of $\mathcal{O}(\lambda M^2)$ [14].

Before concluding, it is worth to mention that the way I presented this simple EFT construction rests on completely solid ground. Integrating out one small momentum shell at a time ($M - dM < \omega < M$, then $M - 2dM < \omega < M - dM$, ...) we generate a flow in the space of possible actions

$$\frac{\partial S_M}{\partial M} = \mathcal{F}(S_M)\,. \tag{2.14}$$

In this picture $\mathcal{F}$ is a smooth function of the couplings and there are no divergences anywhere (we are always integrating between an IR and a UV cutoff). If we expand this differential equation around a solution, irrelevant operators correspond to negative eigenvalues, meaning that the flow is erasing information while going to low energy and converging towards zero. I refer to [12] for more details.

Finally, you might wonder how to assign operator dimensions. For small deviations from a free action (i.e. small couplings) we can assign operator dimensions starting from kinetic terms. For example

$$S = \int d^d x \frac{(\partial\phi)^2}{2}\,, \tag{2.15}$$

implies that $[\phi] = E^{(d-2)/2}$. Then for operators built out of ϕ and its derivatives we can deduce the eigenvalues of the flow $(d - \delta_i)$ by our simple dimensional analysis arguments. At strong coupling we have to take into account also the running of operator dimensions, but this does not invalidate our categorization of operators, it only moves some from one category to another (typically from marginal to relevant or irrelevant).

This suggests the modern interpretation of quantum field theory that is still absent from many textbooks. We can think of any quantum field theory as an EFT valid up to some scale

[2] As an exercise you can check how the higher-spin symmetry of the free action enforces $m^2 \propto \lambda$.

M. Renormalization is just the flow of the action from M to the energy at which we make our measurements. The flow is generated by integrating out high-momentum modes. There are no divergences that need to be cancelled by counterterms. There are only matching calculations between different effective theories to be performed at physical scales (as we will see in the next section). We can always consider these scales one by one, first we have M than maybe new physics appears again at $10M$ and so on.

From a pragmatic point of view this is just the most efficient way of describing our finite experimental knowledge. However this also hints to the more radical possibility that there isn't any quantum field theory valid to arbitrarily high energies.

This concludes our brief introduction to EFT. Through some of the examples in this section we have already seen the essence of the hierarchy problem. It is the absence of a term in the action predicted by symmetry. However it is worth to see it emerge directly in a toy version of the Standard Model from a more pedestrian computation. This is going to make the usual statements about fine-tuning and accidental cancellations more concrete.

3 Solutions

This section is not a comprehensive summary of the research work that has been conducted on the hierarchy problem. Many variations over known solutions are not even going to be mentioned. It has to be considered just as a broad-brush account of the main ideas behind this ongoing theoretical effort.

Most of the solutions to the hierarchy problem are reminiscent of one of the two examples presented in the introduction. Either new dynamical degrees of freedom appear around the Higgs mass to enlarge the symmetries of the SM or our Higgs is not alone, but part of a multiverse where many different values of m_h are populated.

In the first category we have pseudo-Goldstone Higgs models [18, 19, 20, 21, 22, 23, 24, 25], models with extra dimensions [26, 27, 28, 29, 30], supersymmetric models [31, 32, 33, 34, 35, 36, 37, 38, 39] and attempts to construct beyond the SM theories without new scales [40]. In the second, models that incorporate an anthropic solution [41, 42, 43, 44, 45] and Nnaturalness [46]. A third possibility that was proposed recently ties the size of the Higgs mass to a modified cosmological history of our universe [47].

The simplest possible solution, which is now almost unanimously considered excluded by experiment, does not quite fit in any of the previous classes. The basic picture is the following: it is possible that the flow of our UV theory down to low energy is logarithmically slow. This happens if up at the UV scale M_* we have only marginal operators and the theory is sufficiently weakly coupled. An example of this behavior can be found in asymptotically free gauge theories as QCD. Nobody finds surprising that $\Lambda_{QCD} \ll M_{\rm Pl}$, since this hierarchy can be explained in terms of a small coupling in the UV and its logarithmic running,

$$\Lambda_{QCD} \sim M_* e^{-1/g_s^2(M_*)} \,. \tag{3.1}$$

The same could be true for the Higgs boson. Maybe this particle is a composite object of a confining gauge group that at scales much above m_h is better described in terms of its fundamental fermionic constituents. This idea, known under the name of Technicolor, requires new degrees of freedom around m_h, but it is not protecting the Higgs mass via any symmetry. In this sense it is not a representative of the first class of solutions.

If a gauge group were to confine around 100 GeV we would have already observed a plethora of new particles at colliders, not to mention the deviations from the SM that we would expect in precision measurements of the flavor-changing and electroweak parts of the SM Lagrangian and have not been observed.

However a small deformation of this picture inspired by QCD might still work. If the Higgs boson is the equivalent of a pion, its mass can be much smaller than the confining scale. It is set by the explicit breaking of a new global symmetry that is spontaneously broken by the strong dynamics. This set of ideas, that I have mentioned above under the name of pseudo-Goldstone Higgs models, is still viable and is currently being probed by the LHC. For a more comprehensive overview I refer to [25]. Models with extra dimensions that are still experimentally allowed can always be described by a four dimensional theory of this type.

All these models have two concrete difficulties in reproducing experimental observations that require some amount of model building gymnastics to be kept under control. The first is that we have not observed any flavor changing processes beyond the SM and these measurements are sensitive to new physics in the multi-TeV range, especially if strongly coupled [48, 49]. The second is that Higgs boson couplings to SM particles are known to the $20-30\%$ level [50, 51]. This requires some separation between the measured Higgs vacuum expectation value v and the scale at which the new global symmetry is spontaneously broken f. Otherwise operators generated by the strong dynamics such as

$$\frac{(\partial |H|^2)^2}{f^2}, \frac{(H^\dagger \overleftrightarrow{D} H)^2}{f^2}, \ldots \tag{3.2}$$

would give unacceptably large v^2/f^2 corrections to the measured Higgs couplings. This separation requires either some amount of tuning or new structure in the theory [52].

This is a paradigmatic example of the fate of this class of solutions to the hierarchy problem in the last few decades. They start as a simple and entirely plausible idea, but the absence of positive experimental evidence progressively makes them more convoluted.

A similar destiny has befallen supersymmetric extensions of the SM. Supersymmetry protects the Higgs mass by tying it to the mass of its fermionic partner, the Higgsino. The latter is protected by chiral symmetry, which can be described as follows. The two Weyl components of a Dirac fermion ψ_L and ψ_R,

$$\overline{\Psi} i\partial\!\!\!/ \Psi - M_\Psi \overline{\Psi}\Psi = \overline{\psi}_L i\partial\!\!\!/ \psi_L + \overline{\psi}_R i\partial\!\!\!/ \psi_R - M_\Psi \left(\bar{\psi}_L \psi_R + h.c.\right) , \tag{3.3}$$

in absence of a mass term are decoupled. Their phase can be changed independently without affecting the dynamics. The selection rules of this symmetry insure that all contributions to the fermion mass are proportional to M_Ψ. This can be seen by promoting M_Ψ to a field and by assigning it transformation properties that preserve the chiral symmetry even when the mass term in (4.3) is present in the Lagrangian. This is a useful technique that allows us to keep track of the powers of M_Ψ (or any other parameter breaking a symmetry) entering our observables.

After this short description of chiral symmetry we can go back to the status of supersymmetric solutions. Adding a fermionic partner for the Higgs is not enough. For supersymmetry to be an honest symmetry we have to double the SM particle content promoting every particle to a supermultiplet with the same mass. If we failed to do it the $\mathcal{O}(1)$ couplings of the Higgs to other SM particles (in particular the top quark and the gauge bosons) would break supersymmetry, restoring the problem. To see this we consider the effect on the Higgs mass of the supersymmetric partners of the top quark and we let their masses $m_{\tilde{t}_1,\tilde{t}_2}$ be free parameters. Then if $m_{\tilde{t}_1,\tilde{t}_2} \gg m_t$ we are going to introduce tuning from terms of the form $\delta m_H^2 \propto (m_{\tilde{t}_1}^2 - m_t^2)$.

In the Minimal Supersymmetric Standard Model (MSSM) we have two complex scalars (stops) with mass matrix[3]

$$\begin{pmatrix} m_{Q_3}^2 + m_t^2 + m_Z^2\left(\frac{1}{2} - \frac{2}{3}s_W^2\right)\cos 2\beta & v\left(y_t A_t \sin\beta - \mu y_t \cos\beta\right) \\ v\left(y_t A_t \sin\beta - \mu y_t \cos\beta\right) & m_{u_3}^2 + m_t^2 + m_Z^2 \frac{2}{3} s_W^2 \cos 2\beta \end{pmatrix}, \tag{3.4}$$

where m_{Q_3}, m_{u_3}, A_t are parameters that softly break supersymmetry and allow the stop masses to be different from the top mass. s_W is the usual sine of the Weinberg angle, while μ and $\tan\beta$ characterize the Higgs sector of the theory. In the MSSM we need two Higgs doublets, H_u and H_d, in order to write Yukawa couplings in the superpotential. Their supersymmetric interactions are given by

$$\mathcal{W}_{\text{MSSM}} = \mu H_u H_d + y_u Q H_u u^c + y_d Q H_d d^c + y_e Q H_d e^c\,, \tag{3.5}$$

this defines μ. $\tan\beta = v_u/v_d$ is given by the ratio of the vacuum expectation values of the two doublets. All these definitions are just instrumental to get to the tuning of the Higgs mass. The interactions

$$\mathcal{L}_{\text{MSSM}} \supset -|y_t|^2|H_u|^2\left(|\tilde{Q}_t|^2 + |\tilde{t}^c|^2\right) - \left(y_t A_t \tilde{Q}_t H_u^0 \tilde{t}^c + \mu^* y_t \tilde{Q}_t H_d^{0*}\tilde{t}^c + h.c.\right)\,, \tag{3.6}$$

at one-loop contribute to the supersymmetry breaking H_u mass parameter in the Lagrangian

$$\mathcal{L}_{\text{MSSM}} \supset -m_{H_u}^2 |H_u|^2\,. \tag{3.7}$$

For $m_{Q_3}, m_{u_3}, A_t \gg m_t$ we have

$$\delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2}\left(|m_{Q_3}|^2 + |m_{u_3}|^2 + |A_t|^2\right)\log\frac{\Lambda}{\text{TeV}}\,, \tag{3.8}$$

where Λ is the scale at which supersymmetry breaking effects are mediated to the MSSM. If m_{Q_3}, m_{u_3} or A_t are larger than m_h we have reintroduced a fine-tuning problem.

From this discussion the simplest phenomenological problems of this idea are already clear. We expect new particles charged under SM gauge groups near the weak scale and we do not observe them neither directly nor indirectly. Compared to the composite Higgs case, the issue is mitigated by the perturbative couplings of the new particles to the SM, but it is not completely absent.

[3] For simplicity we assume all parameters to be real, their phases are in any case strongly constrained by EDM measurements [53].

Nonetheless having only weak couplings introduces another problem. At tree-level in the MSSM

$$m_h < m_Z |\cos 2\beta| \,. \tag{3.9}$$

This means that we need one-loop corrections to raise m_h to its observed value. The leading ones come from the correction to the Higgs quartic coupling given by stop loops[4]. Including the leading two-loop effects we have [54]

$$\delta m_h^2 \approx \frac{3 G_F}{\sqrt{2}\pi^2}\left[m_t^4(Q_1)\log\frac{M_s^2}{m_t^2} + m_t^4(Q_2)\frac{X_t^2}{M_s^2}\left(1 - \frac{X_t^2}{12 M_s^2}\right)\right] . \tag{3.10}$$

Here, $M_s^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$, $Q_1 = \sqrt{m_t M_s}$, $Q_2 = M_s$, $X_t = A_t - \mu\cot\beta$ and $m_t(Q)$ is the running top mass. In the limit $m_{Q_3}, m_{u_3}, A_t \gg m_t, m_Z, \mu$, the physical stop masses in terms of the parameters in (4.4) read

$$m_{\tilde{t}_{1,2}}^2 \approx \frac{1}{2}\left(m_{Q_3}^2 + m_{u_3}^2 \mp \sqrt{\left(m_{Q_3}^2 - m_{u_3}^2\right)^2 + 2|A_t|^2 v^2 (1 - \cos 2\beta)}\right) . \tag{3.11}$$

As we expected from dimensional analysis the contributions of the stops to the Higgs quartic grow logarithmically with their mass. Raising this contribution is in direct tension with the desire of minimizing fine tuning from $\delta m_{H_u}^2 \sim m_{\tilde{t}_{1,2}}^2$. The same is true for the term proportional to A_t and μ. The latter is the Higgsino mass and would introduce tuning already at tree-level. Of course, as you might have guessed, there are ways around this problem, but require adding more structure to the theory, for example changing the Higgs potential by the addition of a gauge-singlet scalar [55].

In summary lack of positive experimental evidence is forcing us to add extra layers to the simplest supersymmetric models and/or to accept some amount of fine-tuning. This of course does not make them experimentally excluded and the community looks forward to new LHC studies for more information. For additional details on supersymmetry phenomenology and current collider bounds see [56, 57, 58].

Obviously the above discussion is quite general. We can keep pushing up the scale of new physics and still consider dynamical solutions to the hierarchy problem acceptable, if we are willing to tolerate growing amounts of fine-tuning ($\sim E^2/m_h^2$ where E is the energy scale that we can probe without finding new physics). The question of how much tuning is reasonable to expect in a physical theory can not be answered quantitatively. However borrowing Riccardo Barbieri's words, a honest physicist should set in his heart a tuning threshold past which he/she stops working on this kind of model building. The important implicit part is that this threshold should not vary with time (something that very few physicists had the moral strength to accomplish).

I will not discuss here other dynamical solutions to the hierarchy problem. They can be found in the list of references at the beginning of this section. We can instead turn to two solutions that have a central cosmological component, the relaxion and Nnaturalness. The original relaxion solution can be summarized by this potential valid up to a cut-off M

$$V = \left(-M^2 + g\phi\right)|H|^2 + V_\phi(g\phi) + \frac{\phi}{f}\widetilde{G}^a_{\mu\nu}G^{\mu\nu a} \,, \tag{3.12}$$

$$V_\phi(g\phi) = g^2\phi^2 + gM^2\phi + \ldots \,, \tag{3.13}$$

[4]Recall that the physical Higgs mass is $\propto \sqrt{\lambda}$.

accompanied by an exponentially large number of e-folds of low scale inflation ($H_I \sim \Lambda_{QCD}$, where H_I is the Hubble parameter during inflation). If we imagine that the relaxion field ϕ starts from $\phi \gtrsim M^2/g$, during inflation it is going to slowly roll down its potential until it arrives at a field value where the Higgs mass crosses zero. If we are at $T \sim H_I \lesssim \Lambda_{QCD}$ this point is special from the relaxion point of view. It is where the barriers of size $f_\pi^2 m_\pi^2$ generated by $\frac{\phi}{f}\widetilde{G}^a_{\mu\nu}G^{\mu\nu a}$, start to appear, since they are proportional to the Higgs vev, $m_\pi \propto m_u + m_d \propto v$.

If inflation is still ongoing (i.e. the relaxion kinetic energy is negligible), the rolling of ϕ is going to stop when the slope of

$$\frac{\phi}{f}\widetilde{G}^a_{\mu\nu}G^{\mu\nu a} \sim f_\pi^2 m_\pi^2 \cos\frac{\phi}{f} \tag{3.14}$$

equals the slope of the other part of the potential $gM^2\phi$. This happens at

$$g \approx \frac{f_\pi^2 m_\pi^2}{fM^2} \approx 10^{-21}\ \text{GeV}\left(\frac{10^9\ \text{GeV}}{f}\right)\left(\frac{10\ \text{TeV}}{M}\right)^2 . \tag{3.15}$$

The value of f is chosen to respect current bounds on axion interactions and we have taken a low value of the cut-off M. Following our EFT discussion, it is technically natural to take g so small, since it is breaking the shift symmetry of ϕ. However the value of g implies trans-Planckian field excursions

$$\Delta\phi \gtrsim M^2/g \gg M_{\text{Pl}} \tag{3.16}$$

that in our EFT formulation are allowed, but are usually problematic when gravity is taken into account [59]. As mentioned above the solution also requires an exponentially large number of e-folds

$$N = \int dt H_I = \int d\phi \frac{H_I}{\dot{\phi}} \approx \Delta\phi \frac{H_I}{\dot{\phi}} \approx \Delta\phi \frac{H_I^2}{V'} \approx \frac{H_I^2}{g^2} . \tag{3.17}$$

Note that we have not solved the strong CP problem. In this model $\theta \sim \mathcal{O}(1)$. If we want to solve it g becomes smaller by a factor of $\theta \sim 10^{-10}$ [47]. There are developments over this basic picture that can avoid trans-Planckian field excursions and raise the maximum cut-off of the theory [60, 61, 62, 63, 64, 65, 66]. However by now you should know that they come at a price. For example introducing a coincidence between the weak scale and the vector-like masses of new fermions.

It is now time to discuss the class of ideas closest to the resolution of Kepler's observation. We start with Nnaturalness and then briefly comment on the multiverse and the anthropic principle. In the case of Nnaturalness we imagine that multiple copies of the SM exist and that they have different values of the Higgs mass. The point $m_H^2 = 0$ is not special in any way, so we have both sectors with $m_H^2 > 0$ and sectors with $m_H^2 < 0$. We take a uniform distribution for m_H^2, so if the theory has N sectors and a cut-off M, the lightest Higgs is at $m_H \approx M/\sqrt{N}$. We identify this sector with the SM that we observe and imagine that all the other sectors are coupled to us only through gravity. Obviously in this setup it is expected to have sectors with a Higgs mass that appears unnaturally small and arises from a cancellation. We just need to have enough sectors, given a cut-off M. However even a relatively low cut-off $M \approx 10$ TeV, requires a large number of new sectors $N \approx 10^4$ to get at least one with the observed Higgs mass.

It seems that we have already explained the size of the Higgs mass with this "brute force" approach, however there is still one experimental fact that we have not taken into account. Why is most of the energy density contained in the sector with the smallest negative m_H^2? The observed value of $\Delta N_{\rm eff}$ (all the energy density gravitationally coupled to us normalized to that contained in one SM neutrino) has an upper bound of approximately 0.5 at the epoch of recombination [67].

We can not simply give it special couplings to the inflaton or to whatever reheats the Universe, otherwise we would not have really solved the problem. We would still need to explain why the smallest negative m_H^2 sector is also the one that couples to the inflaton. Nnaturalness explains the smallness of the observed Higgs mass only if all the sectors are treated democratically.

To obtain the observed value of $\Delta N_{\rm eff}$ we have to imagine that at some point the energy density was dominated by a gauge-singlet field, the *reheaton*. For illustrative purposes I take it to be a scalar ϕ. Then we can couple ϕ to all the Higgs bosons with the most relevant coupling that we can write down

$$a \sum_i \phi |H_i|^2 \tag{3.18}$$

and let ϕ decays reheat the SM and all other sectors. If $m_\phi \lesssim m_{H_i}, \forall i$ we can compute the decays in the EFT where we have integrated out all the Higgs bosons. The leading operators that we need to consider are[5]

$$\frac{a}{m_{h_i}} y_\psi \phi \bar{\psi}\psi\,, \quad \text{if } m_{H_i}^2 < 0 \tag{3.19}$$

$$\frac{a}{m_{H_i}^2} \phi F^2\,, \quad \text{if } m_{H_i}^2 > 0\,. \tag{3.20}$$

Here F is the field strength of any $SU(2)_L \times U(1)_Y$ gauge boson and this operator is allowed because only QCD is breaking the electroweak symmetry in sectors with $m_{H_i}^2 > 0$, where the Higgs boson does not have a vev. So $m_W, m_Z \sim \Lambda_{QCD} \ll m_{H_i}$. As we did in the previous section, we distinguish between m_{h_i} the physical Higgs mass and the coefficient of $|H_i|^2$ in the Lagrangian, m_{H_i}. They coincide only for sectors with $m_{H_i}^2 > 0$.

From the operators above it is clear that even with equal couplings to all sectors the reaheaton decays preferentially to the lightest one with $m_{H_i}^2 < 0$ since

$$\Gamma_{m_{H_i}^2 < 0} \sim \frac{1}{m_{h_i}^2} \tag{3.21}$$

$$\Gamma_{m_{H_i}^2 > 0} \sim \frac{1}{m_{H_i}^4}\,. \tag{3.22}$$

This is not quite enough to meet experimental constraints, but it is the parametric argument underlying the experimental feasibility of Nnaturalness. For more details I refer to the original paper [46].

[5] As an exercise check this explicitly. What other operators that can lead to ϕ decays are present in the $m_{H_i}^2 > 0$ sectors up to dimension five?

To conclude this section I mention the possibility that our universe is only one of many causally disconnected universes, each of which has different values of the fundamental parameters. In the presence of a landscape of vacua as provided by string theory [68, 69, 70, 71] there is a mechanism to generate an exponentially large number of them through inflation [72, 73, 74]. Then it is natural to ask why we live in one with such a small value of the Higgs mass. The answer is that observers exist only in universes with a Higgs mass close to its observed value. If m_h is larger than its observed value by a factor of a few, nuclei heavier than Hydrogen become unstable and decay, while if it is smaller the proton decays [75, 76]. Universes with $m_H^2 > 0$ are even less hospitable [75, 77]. However this arguments rely on varying only the Higgs vev, keeping the other SM parameters fixed. They are not valid if also Yukawa couplings vary from universe to universe.

This idea has the advantage of being able to accommodate in the same framework also a solution to the cosmological constant problem through Weinberg's argument [78] and to look simple if compared to the status of dynamical solutions. Nonetheless at the moment it is just an appealing qualitative picture. An actual model going from inflation to a distribution of SM parameters that justifies anthropic arguments has not been written and might be a prohibitively difficult task. It is also possible that it will lead to the same (or a larger) amount of convoluted model building needed to make the other solutions experimentally viable. In summary I find unfair to compare the apparent simple elegance of this idea with what is left of other simple and beautiful ideas after detailed experimental scrutiny.

This concludes my account of the main ideas behind known solutions to the hierarchy problem. Reflecting the time limitations of the actual lecture it was remarkably short, incomplete and dotted with personal idiosyncrasies. So I encourage every interested student to go carefully through the references.

4 Conclusion

The conclusion of this lecture is that there is no conclusion. The electroweak hierarchy problem is more confusing and fascinating than ever. The null results from LEP, LHC, searches for flavor-changing neutral currents, searches for WIMP dark matter and many other beyond the SM explorations, are shaking our belief in what used to be considered the most plausible solutions. It is too early to discard them, but not too early to look for alternatives. Whatever the final answer will be, it will not be the resolution of a technical problem, but a choice between different views of the Universe.

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