LPT-Orsay

Neutrino Physics: theory and phenomenology

Cargese 2018 International Summer School

Asmaa Abada

inVisiblesPlus elusiVes

two lectures

- Basics: history and basic concepts
- Oscillation phenomena and searches from many fronts
- Properties and Nature
- Theoretical frameworks and (Minimal) New Physics Models
- Effective approach and some applications

Some references

- C. Giunti, C.W. Kim, “Fundamentals of Neutrino Physics and Astrophysics, Oxford University Press.
- R. N. Mohapatra and P. Pal, “Massive Neutrinos in Physics and Astrophysics, World Scientific
- M. Fukugita, T. Yanagida, “Physics of Neutrinos: and Application to Astrophysics (Theoretical and Mathematical Physics) ", Springer

Part 1

- Neutrino Problem: brief chronology
- Oscillation phenomena
- Searches from many fronts: present situation
- Theoretical frameworks and Minimal New Physics Models

Neutrinos are the most elusive particles of the Standard Model

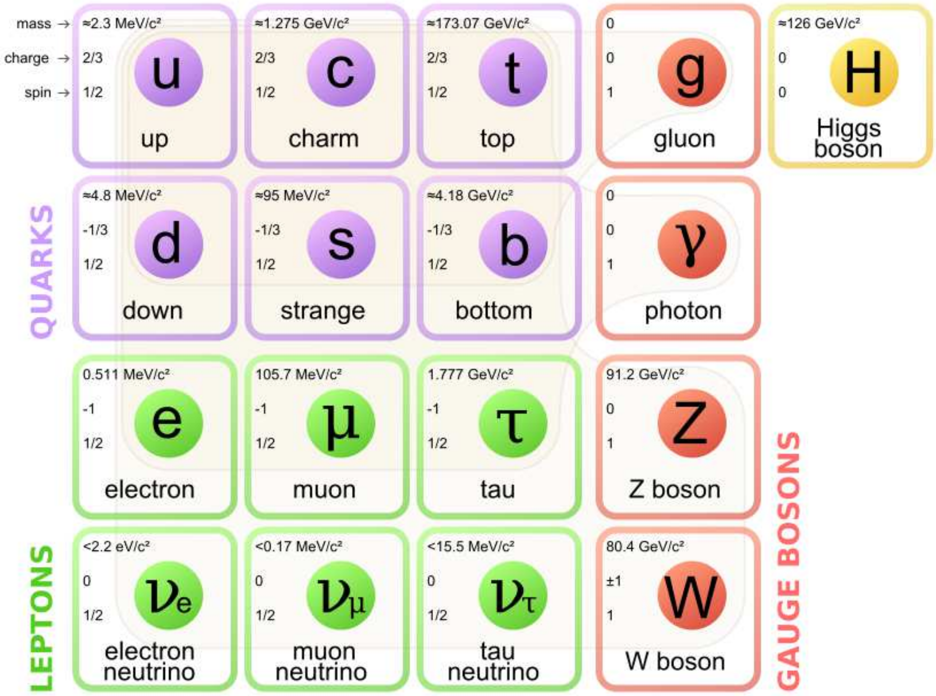
$$Q_{em} = 0, \quad Q_{color} = 0.$$

- **Provide informations on the essential features of the SM:**
  "left" nature of the weak interaction and family structure
- **more importantly, they call for physics beyond the Standard Model**

The Standard Model of particle physics

- **Standard Model:** renormalisable QFT formulation based on $SU(3)_c \times SU(2)_L \times U(1)_Y$

$\Rightarrow$ **successful description** of (most) elementary particles and their interactions



- Gauge bosons $\leftrightsquigarrow$

  strong, weak, electromagnetic interactions

- Quarks (strong, weak, electric);

  charged leptons (weak, electric);

  neutral leptons (weak)

- Higgs boson: EW symmetry breaking;

  elementary particle masses

☞ Despite its *remarkable success*, is the **SM the ultimate description of Nature?**

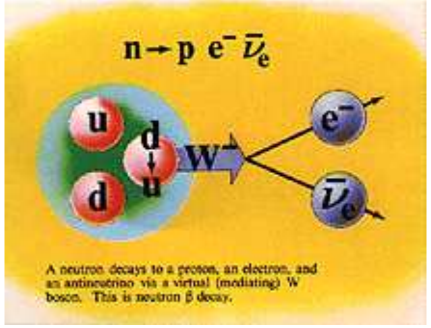
Theoretical caveats (hierarchy problem, choice of gauge group, family/flavour puzzle, ...)

Observational problems: dark matter candidate, baryon asymmetry of the Universe,

massive neutrinos!

Brief history of the neutrinos ν

- ν **birth:** **“Rescue” conservation of energy** in nucleus beta decay $n \to p + e^- + \overline{\nu}_e$



> *“Dear Radioactive Ladies and Gentlemen,*
>
> *... because of the wrong statistics of the N and Li^6 nuclei...and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. ... electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle ...*
>
> *...The continuous beta spectrum would then become understandable...”*
>
> *Pauli, 1930*

- **Fermi Theory for β decay** (Named **“neutrino”** 1934), following)
  the “neutron” discovery by **Chadwick** (1933), assumed **massless**
- **(Anti)Electron neutrino:** detected in **1956** by **Cowan** and **Reines**; so they come **in flavours!**
- **Muon neutrino:** $\pi^- \to \mu^- \overline{\nu}_\mu$ discovery in **1962** by **Lederman, Schwartz** and **Steinberger**
- **3 neutrino families:** Z boson **decay width, CERN 1989**
- **Tau neutrino:** $\tau^- \to \pi^- \pi^- \pi^+ \pi^- \pi^+ \nu_\tau$ direct evidence in **2000** by **DONUT at Fermilab**

☞ **Neutrinos in the SM:** 3 **massless** states! ν_e, ν_μ and ν_τ

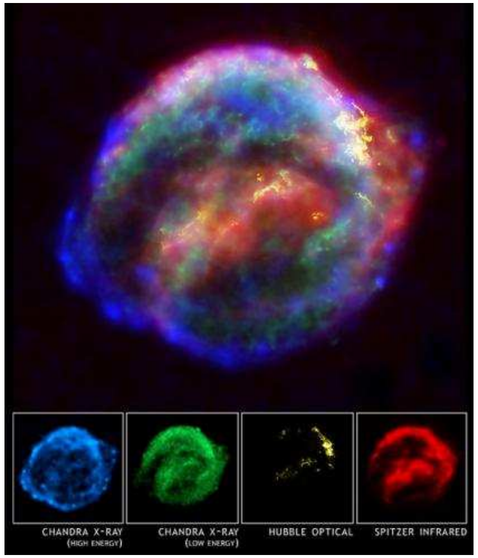
Studying neutrinos: rich sources in Nature & hand-made

- About $\sim 10^{11}$ **neutrinos** cross a cm^2 of our skin!

FACT: about 65 million neutrinos pass through your thumbnail every second.

- About $\sim 10^{8}$ **neutrinos** from natural radioactivity,

- even $\sim 10^{11}$ **relic neutrinos** from the Big-Bang



- Supernova, e.g., SN1987A $\sim 10^{11}$ **neutrinos**
  similar as solar flux during few seconds

→ **Neutrino Astronomy era!**

Studying neutrinos: sources & detectors

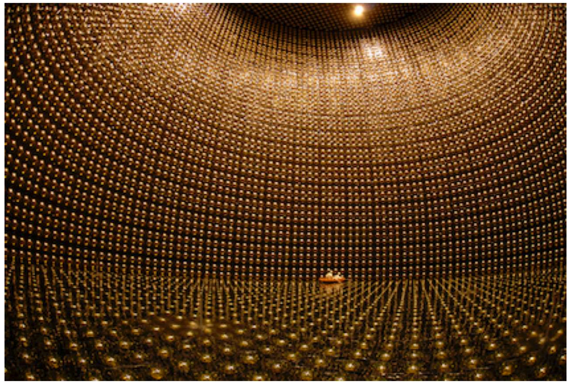
- **Neutrino sources** have been experimentally and observationally explored,
  huge impact for particle & astroparticle physics and astronomy!

- A **world-wide effort** to detect and study ν's
  from different sources, using distinct methods...

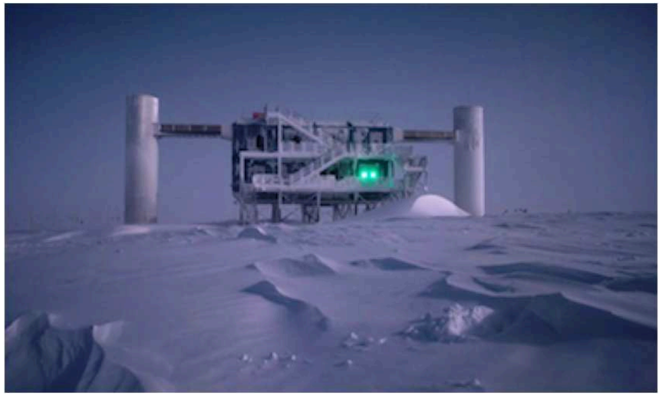
Laboratory: reactors, accelerators

Cosmic rays: atmospheric neutrinos (ν_{atm}), ultra-high energy neutrinos

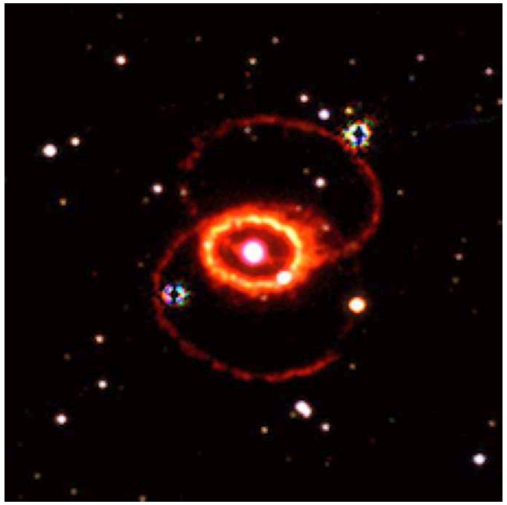
Astrophysical: solar neutrinos (ν_{sol}), supernovae



Super-Kamokande



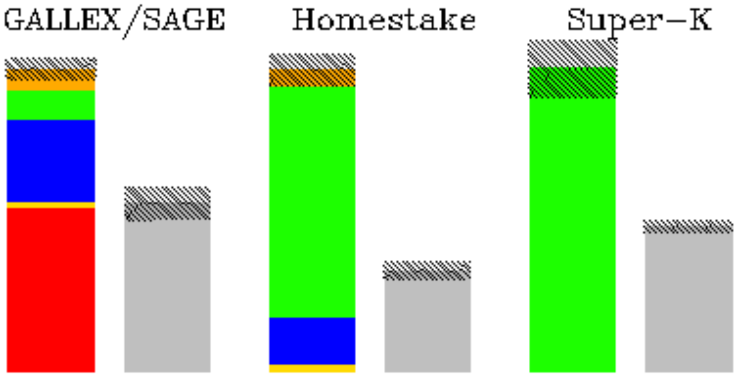
IceCube Neutrino Observatory

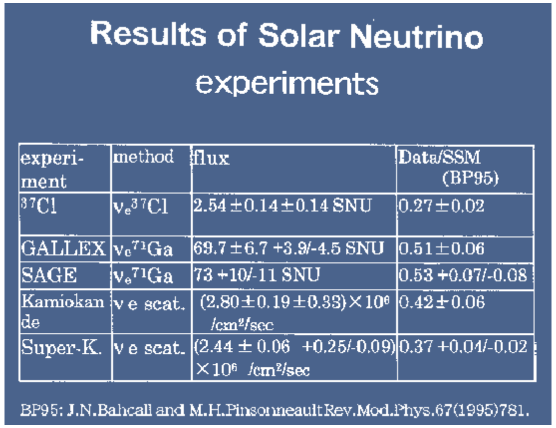


SN1987A

Studying neutrinos: unexpected news

A puzzling and surprising discovery: **the solar ν_e and atmospheric ν_μ fluxes...**





Results of Solar Neutrino experiments

experiment	method	flux	Data/SSM (BP95)
^{37}Cl	$\nu_e{}^{37}$Cl	2.54 ±0.14 ±0.14 SNU	0.27 ±0.02
GALLEX	$\nu_e{}^{71}$Ga	69.7 ±6.7 +3.9/-4.5 SNU	0.51 ±0.06
SAGE	$\nu_e{}^{71}$Ga	73 +10/-11 SNU	0.53 +0.07/-0.08
Kamiokande	ν e scat.	(2.80 ±0.19 ±0.33) ×10^6 /cm^2/sec	0.42 ±0.06
Super-K.	ν e scat.	(2.44 ± 0.06 +0.25/-0.09) ×10^6 /cm^2/sec	0.37 +0.04/-0.02

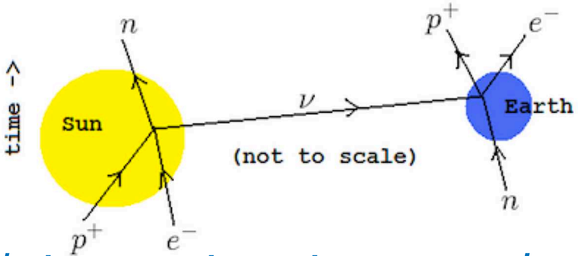
BP95: J.N.Bahcall and M.H.Pinsonneault Rev.Mod.Phys.67(1995)781.

☞**Solar neutrino problem:** detection of **only 1/3 of expected flux of solar ν_e's**

☞**Atmospheric neutrino problem:** detection of $\#\nu_e \sim \#\nu_\mu$, expected $\#\nu_\mu \sim 2\#\nu_e$

$$\begin{aligned} \pi^\pm\ ,\ K^\pm \to \quad & \mu^\pm \quad +\nu_\mu(\overline{\nu}_\mu) \\ & \mu^\pm \quad \to e^\pm\ +\ \nu_e(\overline{\nu}_e)\ +\ \overline{\nu}_\mu(\nu_\mu) \end{aligned}$$

- ▶**"Unexpected" production of ν_α:** *do charged currents violate lepton flavours?*
- ▶**"Disappearance" of propagating ν_α:** *do neutrinos oscillate?*
- ▶**Standard Solar model** predictions: *to be challenged?*



➙ **Massive implications:** *Propagating neutrinos are not weak interaction eigenstates!*

Neutrino oscillations → massive states, leptonic mixing!

- Illustrative 2-family example

2 **massive states** ($\nu_2, \nu_3 :\ \Delta m_\nu = \sqrt{|m_2^2 - m_3^2|}$) related to 2 flavour states $\nu_\alpha = U^*_{\alpha i}\,\nu_i$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

- Consider **a relativistic neutrino**, produced (e.g.) in muon decay??

(i) Production of **weak eigenstate**: $|\nu(t=0)\rangle = |\nu_\mu\rangle = \cos\theta\,|\nu_2\rangle + \sin\theta\,|\nu_3\rangle$

(ii) Travel distance L to the **detector**, during which it *oscillates*

$$|\nu(t)\rangle = \cos\theta e^{-iE_2 t}|\nu_2\rangle + \sin\theta e^{-iE_3 t}|\nu_3\rangle, \quad \text{with } t \simeq L \text{ and } E \sim \sqrt{p^2+m^2}$$

(using wave packet formalism we arrive to the same expressions)

(iii) At the detector, it produces μ in **charged current** scattering, with probability

$$\mathcal{P}^{2\nu}_{\mu\to\mu}(L,t) = |\langle\nu_\mu|\nu(t)\rangle|^2 = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_\nu^2\, L}{4E}\right) \quad \neq 1$$

☞ **Indisputable:** Oscillations hold if and only if νs are massive and mix

➙ The minimal SM is incomplete!

Charged current interaction not diagonal in flavour space

$$\mathcal{L}_{int} = -\frac{g}{\sqrt{2}} \bar{\ell}^i_L \gamma^\mu \nu^j_L \boldsymbol{U}_{ij} W^+_\mu + h.c. \ ,$$

For $n = 3$ ➡ Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix

$$U = \begin{pmatrix} c_{12}\ c_{13} & s_{12}\ c_{13} & s_{13}e^{-i\delta} \\ -s_{12}\ c_{23} - c_{12}\ s_{23}\ s_{13}e^{i\delta} & c_{12}\ c_{23} - s_{12}\ s_{23}\ s_{13}e^{i\delta} & s_{23}\ c_{13} \\ s_{12}\ s_{23} - c_{12}\ c_{23}\ s_{13}e^{i\delta} & -c_{12}\ s_{23} - s_{12}\ c_{23}\ s_{13}e^{i\delta} & c_{23}\ c_{13} \end{pmatrix} \mathrm{Diag}\left\{e^{i\alpha_1}, e^{i\alpha_2}, 1\right\}$$

[Chau-Keung parametrisation]

δ Dirac phase, $\alpha_{1,2}$ Majorana[a], $\theta_{12}, \theta_{23}, \theta_{13}$

m_1, m_2, m_3 mass eingenvalues, if $m_3 > 0$, $m_{1,2} = |m_{1,2}|e^{i\alpha_{1,2}}$

---

[a] We will discuss the Nature Majorana or Dirac later

Transition Probabilities

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta; L) \quad &= \quad \delta_{\alpha\beta} - 4 \sum_{j<k} \mathrm{Re}\left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}\right) \sin^2\left(\frac{\Delta m^2_{jk} L}{4E}\right) \\ &\pm \quad 2 \sum_{j<k} \mathrm{Im}\left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}\right) \sin\left(\frac{\Delta m^2_{jk} L}{2E}\right) \;, \quad \Delta m^2_{jk} = m^2_j - m^2_k \end{aligned}$$

☞ ± ➞ − for neutrinos, + for antineutrinos

☞ Appearance (Desappearance) oscillation probability: $\alpha \neq \beta$ ($\alpha = \beta$)

☞ Oscillation experiments do not give the nature : Dirac or Majorana : $\overline{\nu} \equiv \nu$!

☞ oscillations arise when $L \sim L_{\mathrm{osc}} \Rightarrow \frac{\Delta m^2 L}{4\pi E} \sim 1 \;\Leftrightarrow\; \Delta m^2(\mathrm{eV}^2) \sim \frac{E(\mathrm{GeV})}{L(\mathrm{km})}$

☞ e.g., $n = 2$: $P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2\left(\frac{L}{L_{\mathrm{osc}}}\pi\right)$, $L_{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2} \simeq 2.48\mathrm{km}\left(\frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)}\right)$

Accessible Δm^2

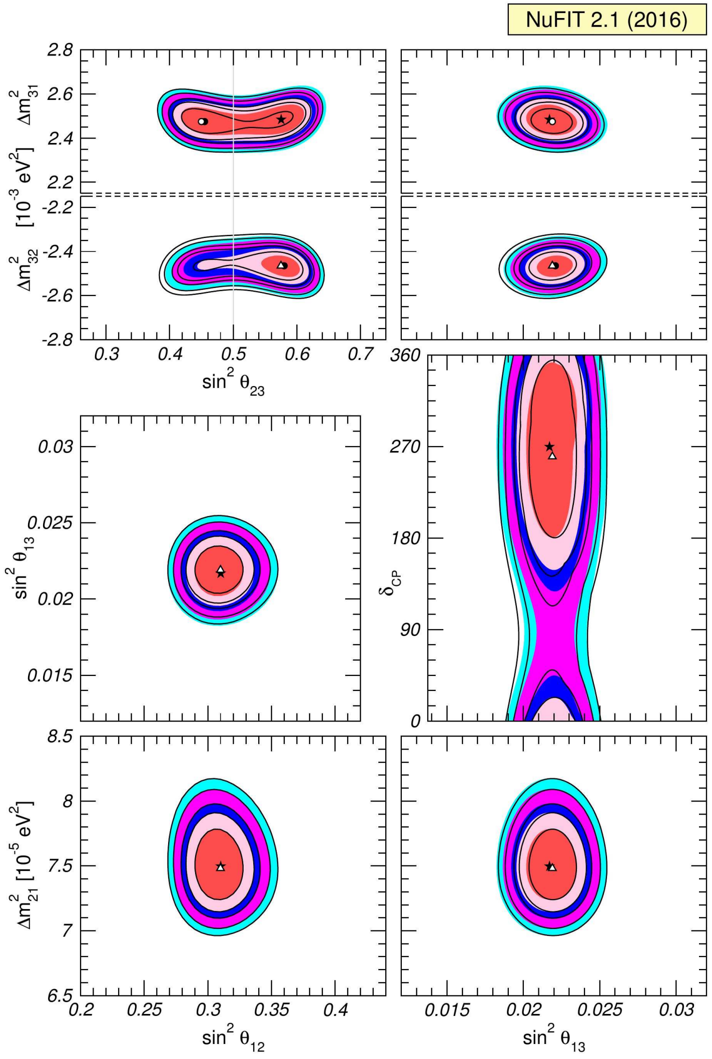
Depending on ν sources (E) and positions L of the detector :

L(km)	E(GeV)	Δm^2(eV^2)	Source
10^8	10^{-3}	10^{-11}	solar ν
10^4	1	10^{-4}	atmospheric ν
10^3	10	10^{-2}	ν from accelerators (long distance)
0.1	1	10	ν from accelerators (short distance)
0.1	10^{-3}	10^{-2}	ν from reactors

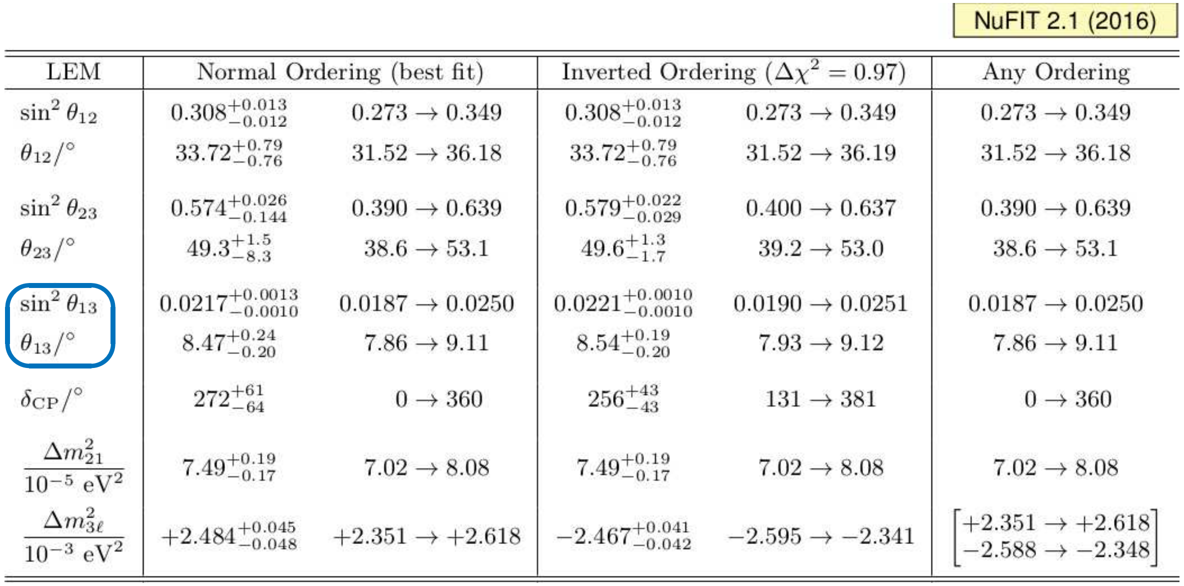
☞ **Facts:** ν change flavours after propagating a finite distance

Solar $\nu_e \to \nu_{\mu,\tau}$	$\Delta m^2_{\text{sol}} \simeq 7.6 \times 10^{-5}\ \text{eV}^2$ $\sin^2\theta_{\text{sol}} \simeq 0.30$	SNO, BOREXino, Super-Kamiokande, GALLEX/GNO, SAGE, Homestake, Kamiokande
Atmospheric $\nu_\mu \to \nu_\tau$	$\Delta m^2_{\text{atm}} \simeq 2.4 \times 10^{-3}\ \text{eV}^2$ $\sin^2\theta_{\text{atm}} \simeq 0.50$	IMB, MAcro, Soudan-2, Kamiokande, Super-Kamiokande
LBL Accelerator ν_μ disappearance		K2K, T2K, MINOS
LBL Accelerator $\nu_\mu \to \nu_\tau$		Opera
LBL Accelerator $\nu_\mu \to \nu_e$	Δm^2_{atm} $\sin^2\theta_{\text{Chooz}} \simeq 0.023$	T2K, MINOS
LBL Reactor $\overline{\nu}_e$ disappearance		Daya Bay, RENO Double Chooz
SBL Accelerator $\nu_\mu(\overline{\nu}_\mu) \to \nu_e(\overline{\nu}_e)$	$\Delta m^2 \simeq 1\text{eV}^2$ (?) $\sin^2\theta \simeq 0.1$ (?)	LSND, MiniBooNE ++ Solar: GALLEX, SAGE++
SBL Reactor $\overline{\nu}_e$ disappearance		Bugey, ILL, Rovno,...

Lepton mixing & neutrino data: current status



$\Rightarrow$



NuFIT 2.1 (2016)

LEM	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 0.97$)		Any Ordering
$\sin^2\theta_{12}$	$0.308^{+0.013}_{-0.012}$	$0.273 \to 0.349$	$0.308^{+0.013}_{-0.012}$	$0.273 \to 0.349$	$0.273 \to 0.349$
$\theta_{12}/^\circ$	$33.72^{+0.79}_{-0.76}$	$31.52 \to 36.18$	$33.72^{+0.79}_{-0.76}$	$31.52 \to 36.19$	$31.52 \to 36.18$
$\sin^2\theta_{23}$	$0.574^{+0.026}_{-0.144}$	$0.390 \to 0.639$	$0.579^{+0.022}_{-0.029}$	$0.400 \to 0.637$	$0.390 \to 0.639$
$\theta_{23}/^\circ$	$49.3^{+1.5}_{-8.3}$	$38.6 \to 53.1$	$49.6^{+1.3}_{-1.7}$	$39.2 \to 53.0$	$38.6 \to 53.1$
$\sin^2\theta_{13}$	$0.0217^{+0.0013}_{-0.0010}$	$0.0187 \to 0.0250$	$0.0221^{+0.0010}_{-0.0010}$	$0.0190 \to 0.0251$	$0.0187 \to 0.0250$
$\theta_{13}/^\circ$	$8.47^{+0.24}_{-0.20}$	$7.86 \to 9.11$	$8.54^{+0.19}_{-0.20}$	$7.93 \to 9.12$	$7.86 \to 9.11$
$\delta_{\mathrm{CP}}/^\circ$	272^{+61}_{-64}	$0 \to 360$	256^{+43}_{-43}	$131 \to 381$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.17}$	$7.02 \to 8.08$	$7.49^{+0.19}_{-0.17}$	$7.02 \to 8.08$	$7.02 \to 8.08$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.484^{+0.045}_{-0.048}$	$+2.351 \to +2.618$	$-2.467^{+0.041}_{-0.042}$	$-2.595 \to -2.341$	$\begin{bmatrix} +2.351 \to +2.618 \\ -2.588 \to -2.348 \end{bmatrix}$

- ▶ **"Precision era"** for **neutrino physics**
- ▶ Only **three** oscillation **parameters** **unknown...**

  $\boldsymbol{\theta_{23}}$ octant; $\boldsymbol{\delta_{\mathrm{CP}}}$; $\boldsymbol{\nu}$-mass ordering

- ▶ Exciting **experimental roadmap** ahead!

Lepton mixing & neutrino data: Leptonic CP Asymmetry

$$\begin{aligned}\Delta_{CP}(\alpha\beta) &\equiv P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \\ &= 4\sum_{j>k} \mathrm{Im}\left(U_{\alpha j}U^*_{\beta j}U^*_{\alpha j}U_{\beta k}\right)^* \sin\left(\Delta m^2_{jk}\frac{L}{2E}\right)\end{aligned}$$

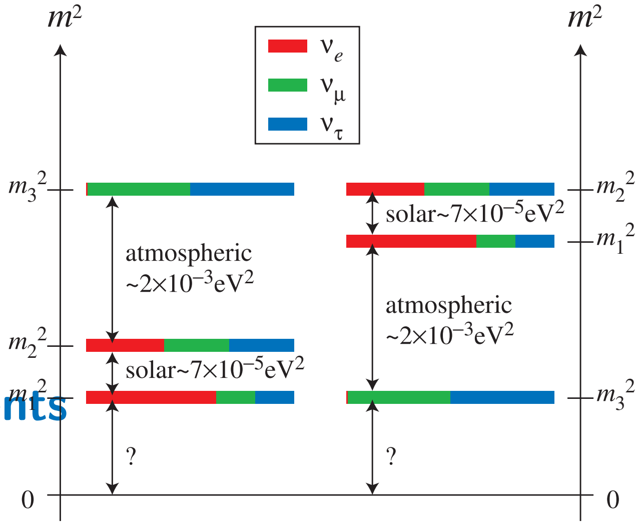
- Cannot be observed in **appearance experiments**

$$CPT \to \Delta_{CP}(e\mu) = \Delta_{CP}(\mu\tau) = \Delta_{CP}(\tau e) \equiv 16\mathcal{J}\ell_{12}\ell_{23}\ell_{31}$$

- $\mathcal{J} \equiv \mathrm{Im}\left(U_{e3}U^*_{e1}U^*_{\mu 3}U_{\mu 1}\right) \simeq \sin 2\theta_{23}\sin 2\theta_{12}\sin\theta_{13}\sin\delta$
  (Jarlskog Invariant)
- $\ell_{ij} \equiv \sin\left(1.27\Delta m^2_{ij}(\mathrm{eV})^2\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\right)$
- θ_{23} large (OK) and also Δm^2_{13}.
- θ_{12} large (OK) and also Δm^2_{12}.
- θ_{13} conditions the measurement of CPV phase: $\theta_{13} \sim 8.5^0$

▶ **What about (absolute) neutrino masses?**

Lepton mixing & neutrino data: Absolute ν masses



- ▶ **Oscillation data:** only **two squared-mass differences**

  Undetermined mass ordering:

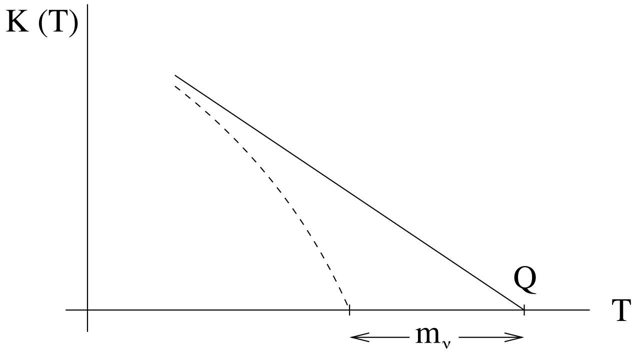
  normal $[m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}]$

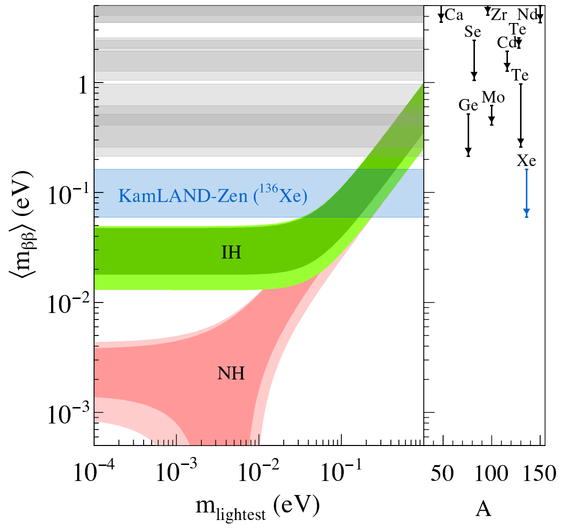
  inverted $[m_{\nu_3} \ll m_{\nu_1} \lesssim m_{\nu_2}]$

  Unknown absolute mass scale: need direct experiments

- ▶ **Resolving** the absolute mass scale
  - **Tritium decays** $({}^3\text{H} \rightarrow {}^3\text{He} + \overline{\nu}_e + e^-)$: $\boldsymbol{m_{\nu_e} \lesssim 2.1}$ **eV** [Troitsk]

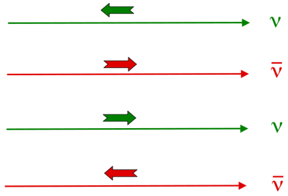
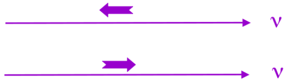
    ☝ June 11 2018, the KATRIN experiment has been inaugurated!
  - **$0\nu2\beta$ decays** (if Majorana ν): $\boldsymbol{|m_{ee}| \lesssim 0.3}$ **eV** [GERDA, KamLAND-Zen]
  - **Cosmology** (CMB, LSS, Lyα): $\boldsymbol{\sum_i m_{\nu_i} \lesssim 0.23 \rightarrow 0.12}$ **eV**





[KamLAND-Zen Coll., '15]

What about the nature: Dirac or Majorana?

Dirac spinor	Majorana spinor
	
Collection of 4 states	2 states
$\psi = \psi_L + \psi_R$	$\psi = \psi_L + (\psi_L)^c$, $(\psi_L)^c = (\psi)^c_R$
Conservation of L	Non-conservation of L

- ▶ both descriptions are possible for neutrinos

$$-\mathcal{L}_m^{\text{Dirac}} = m\overline{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L) \longleftarrow \text{all fermions}$$

$$-\mathcal{L}_m^{\text{Majorana}} = \frac{m}{2}\overline{\psi^c}\psi + \frac{m}{2}\overline{\psi}\psi^c = \frac{m}{2}\psi^T C\psi + \frac{m}{2}\overline{\psi}C\overline{\psi}^T,\ \psi^c = C\gamma_0\psi^*,\ C = i\gamma_2\gamma_0 \longleftarrow \text{only for } \nu$$

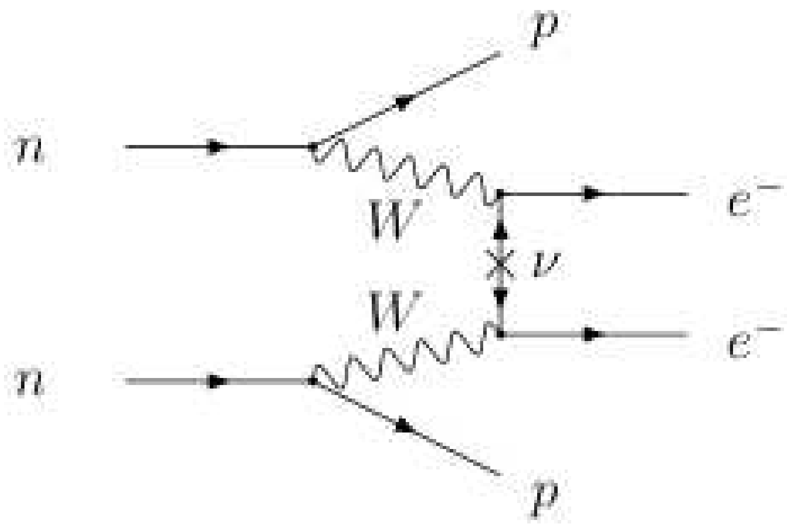
- ☞ Both break gauge invariance (need SSB)
- ☞ $\mathcal{L}_m^{\text{Majorana}}$ violates Lepton number symmetry
- ☞ Lepton number symmetry is accidental in the SM
- ☞ To distinguish them: mass observables and not kinematical observables

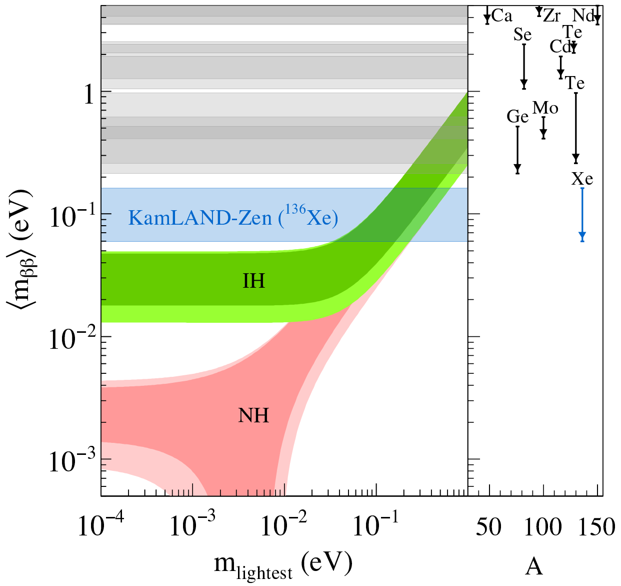
Experimental test for Lepton Number violation $(0\nu\beta\beta)$

$$(A,Z) \longrightarrow (A,Z+2) + e^- + e^- + \overline{\nu_e} + \overline{\nu_e}$$

Furry ➡ $(A,Z) \longrightarrow (A,Z+2) + e^- + e^-$

$$\overline{\nu_e} \equiv \nu_e$$





- $\Delta L = 2$ ➡ Majorana
- $(0\nu\beta\beta)$ Amplitude proportional to effective Majorana mass :$|m_{ee}| = |\sum_i U_{ei}^2 m_i|$
- m_{ee} depend on m_i, Dirac and Majorana CP phases and mixings angles
- A signal $(0\nu\beta\beta)$ ➡absolute mass, prove LNV, thus the Majorana nature for ν

☞ **Indisputable:** νs are massive and mix

↓

The minimal SM is incomplete!

↓

An observational Caveat that is also theoretical one!

- ν mixings "add fuel to the fire": add to the fermion flavour puzzle!

$$U_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \lambda \sim 0.2, A \simeq 0.8, \rho \simeq 0.1, \eta \simeq 0.4$$

→ Quarks: small mixing angles, 1 Dirac CPV phase

$$U_{PMNS} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & -s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \mathrm{diag}\left(e^{i\alpha_1}, e^{i\alpha_2}, 1\right)$$

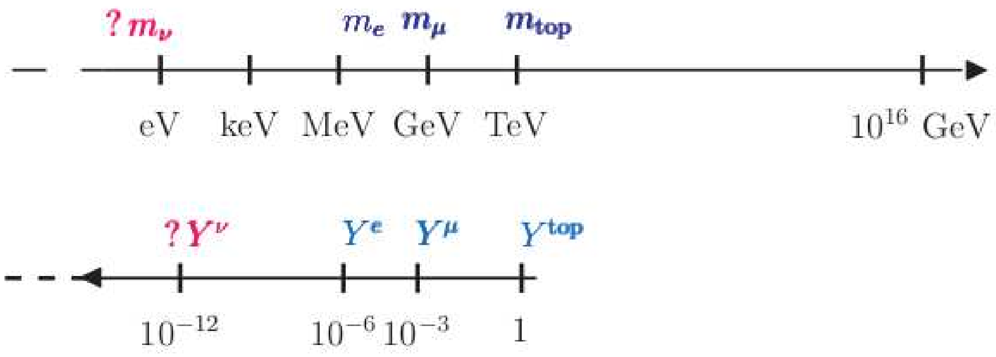
Leptons: 2 large mixing angles, 1 Dirac + 2 Majorana CPV phases

⇒ Very different mixing pattern for Leptons and Quarks

➞ Is this related to different mass generation mechanisms?

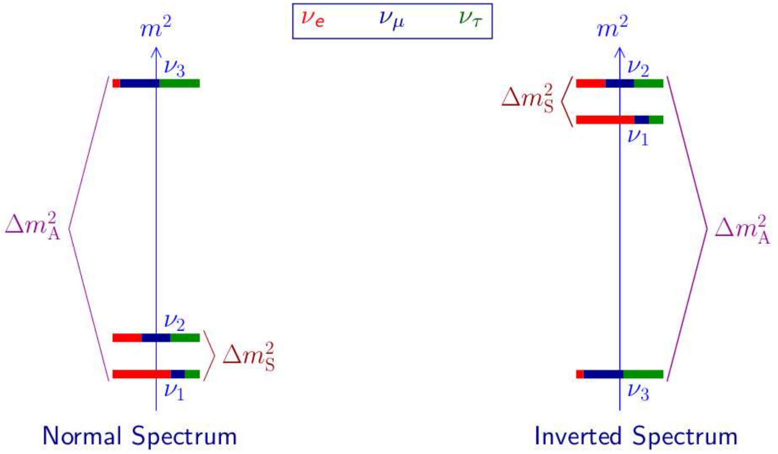
- ν data worsens fermion hierarchy problem!

→ **Why are ν so light?**

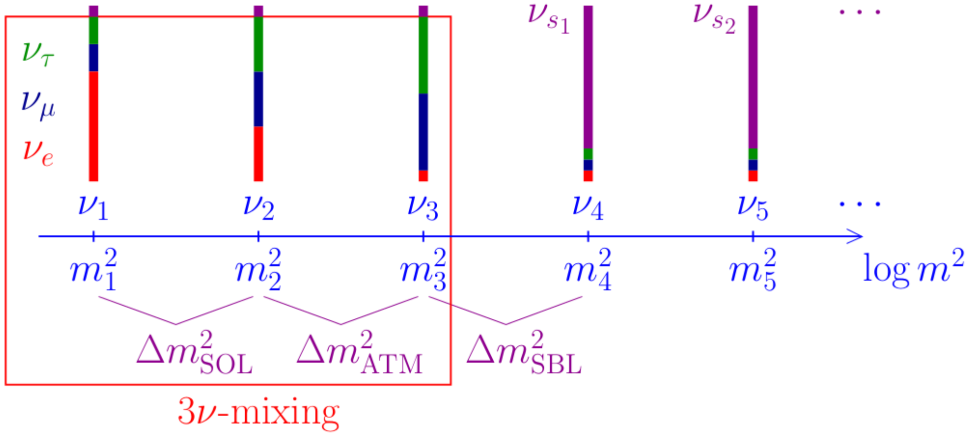


→ **What is the absolute neutrino mass scale?**

▶ **Are there some extra fermionic gauge singlets (steriles)?**



3-ν mixing scheme



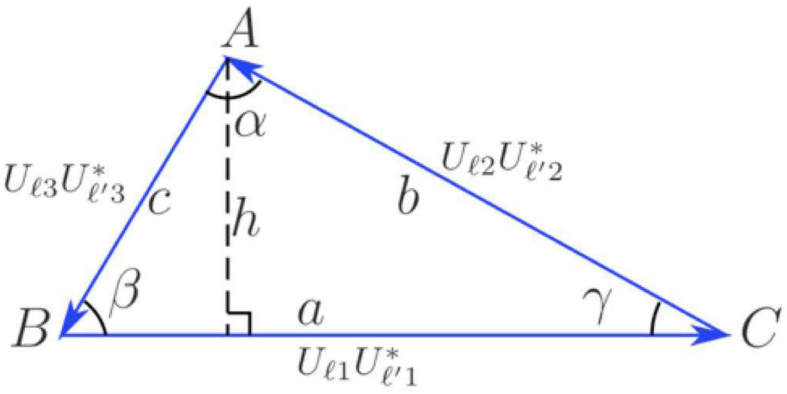
3+?-ν mixing schemes

➞ Does this mean that U_{PMNS} is incomplete? Non-Unitary ?

▶ Strong Potential for CP violation

☞ Unitarity triangle surface $\propto J_{\rm CP}^{\rm lepton}$: $J_{\rm CP,max}^{\rm lepton} \simeq 1000 \times J_{\rm CP}^{\rm quark}$

$$J_{\rm CP}^{\rm quark} = 2.96 \times 10^{-5}, \quad J_{\rm CP,max} \simeq 3.29 \times 10^{-2}$$



$$\mathcal{J} \equiv \mathrm{Im}\left(U_{e3}U^*_{e1}U^*_{\mu 3}U_{\mu 1}\right)$$

$$\mathcal{J} = \sin 2\theta_{23} \sin 2\theta_{12} \sin\theta_{13} \sin\delta$$

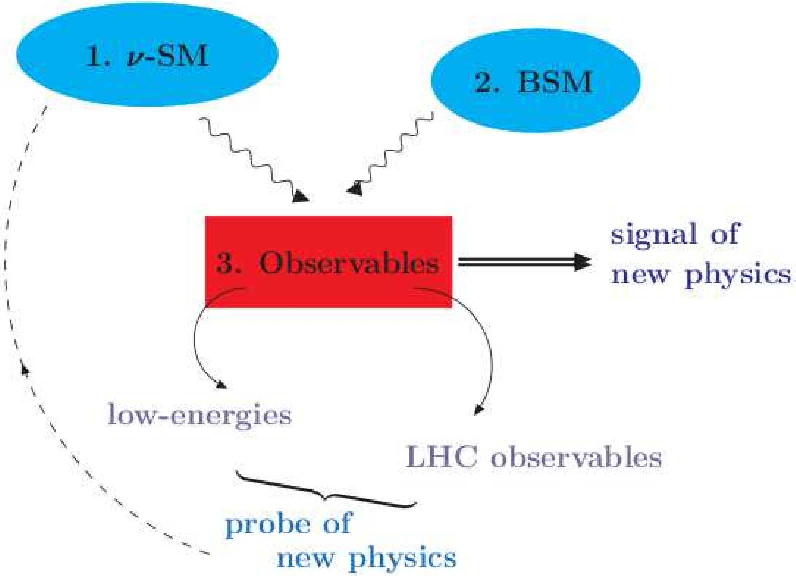
Unitarity Triangle (in e, μ)

Jarlskog Invariant

➞ New possibility for having Baryogenesis from Leptogenesis ?

Lepton mixing & massive neutrinos: a gateway to NP

- ☞ ν-SM = New Physics just to explain ν masses and mixings
- ☞ ν-SM will allow for many new phenomena
- ☞ SM has other issues that call for BSM



➡ **Determination of ν-SM/BSM model requires combinations of $\neq$ observables**

$m_\nu \neq 0 \Rightarrow$ New Physics Scale

Standard Model

- ν_L and no ν_R $\Longrightarrow$ No Dirac mass term: $\mathcal{L}_{m_D} = m_D\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right)$

- No Higgs triplet $\Longrightarrow$ No Majorana mass term: $\mathcal{L}_{m_M} = \frac{1}{2}M\overline{\nu_L^c}\nu_L + h.c.$

Majorana field: $\Psi_\nu = \nu_L + \nu_L^c$ → $\Psi_\nu = \Psi_\nu^c$ → $\overline{\nu_L^c}\nu_L = \nu_L^T C \nu_L, \quad C = i\gamma^2\gamma^0$

- Lepton number symmetry is accidental $\Longrightarrow$ Non-renormalisable operators dim 5, 6 ..

$$\boxed{\text{SM} \equiv \text{Effective theory of a larger one valid at a scale } \Lambda}$$

↓

$$\delta\mathcal{L}^{d=5} = c^{d=5}\mathcal{O}^{d=5}, \quad \mathcal{O}^{d=5} = \frac{1}{\Lambda}\left\{\left(\phi\ell\right)^{\mathrm{T}}\left(\phi\ell\right) + h.c.\right\} \xrightarrow{\langle\phi\rangle = v} m_\nu \sim v^2/\Lambda$$

$m_\nu \sim \sqrt{\Delta m^2_{\mathrm{atm}}} \sim \sqrt{2\times 10^{-3}\mathrm{eV}^2} \Rightarrow \Lambda \sim 10^{15}$ GeV (Remarkably near Λ_{GUT} !)

Beyond the Standard Model

Typically 3 possible ways to generate $m_\nu \neq 0$:

- Seesaw mechanism can be achieved via
  1. type I with RH neutrino exchange
  2. type II with scalar triplet exchange
  3. type III with fermionic triplet exchange
- Radiative corrections → MSSM extended $+\not{R}_p$, Zee model, $\cdots$
- Extra dimensions → alternative to the seesaw

Neutrino masses at Tree-Level

Example: SM $+\nu_R$

ν_R (massive and Majorana) : gauge singlet ➜ does not break gauge invariance!!

$$\mathcal{L}_{\mathrm{SM}} \to \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\text{type-I}}$$

$$\mathcal{L}_{\text{type-I}} = -\sum_{ij}^{a} \left(Y_\nu^{ij} \overline{L_i} \tilde{\phi} \nu_{Rj} + \frac{1}{2} M_R^{ij} \nu_{Rj} \nu_R + h.c. \right)$$

General mass term Dirac+Majorana[a]

($+\nu_L$ with or without Majorana mass[b])

$$\mathcal{L}_M^{(\nu)} = -\frac{1}{2} \left(\overline{\nu}_L \ \overline{\nu^c}_R \right) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

---

[a] Minkowski, Ramond,Yanagida, Mohapatra, Senjanovic,Gell Mann, ...

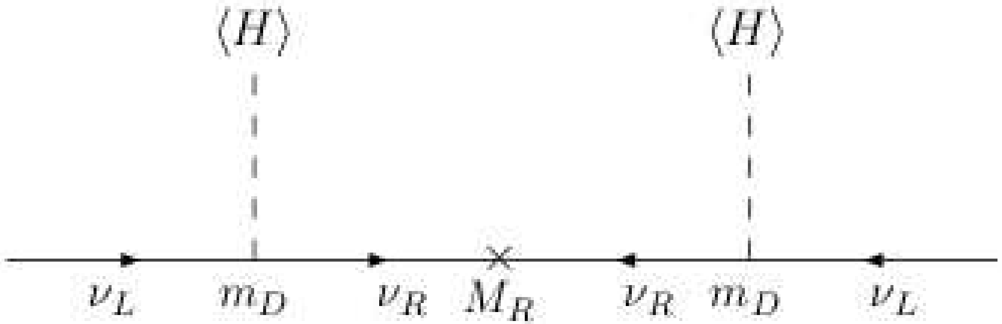
[b] such a term $\overline{\nu_L^c} m_L \nu_L$ ➜ Isospin Triplet

Diagonalisation : $\mathcal{L}_M^{(\nu)} = \frac{1}{2}\left(\overline{L}\ \overline{R^c}\right)\begin{pmatrix} m_L & 0 \\ 0 & M_R \end{pmatrix}\begin{pmatrix} L^c \\ R \end{pmatrix} + h.c.$

mass eigenstates: [in the limit $m_D \ll M_R$]

$$\nu \simeq L + L^c = \nu^c \ , \quad \rightarrow \quad \tilde{m}_L \sim -m_D \frac{1}{M_R} m_D^T$$

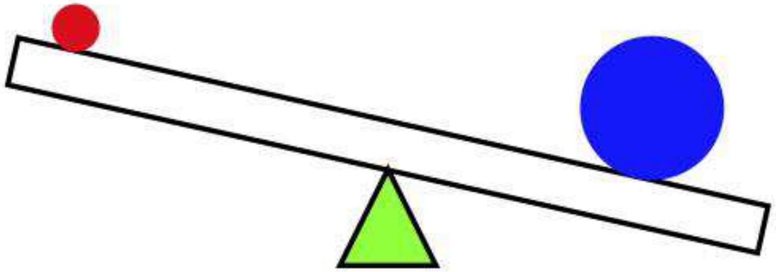
$$N \simeq R + R^c = N^c \ , \quad \rightarrow \quad \tilde{M}_R \sim M_R$$



$$M_R \sim \Lambda$$

One generation case:

$$\begin{aligned} \tilde{m}_L &\sim \frac{m_D^2}{M_R} \ll M_R \\ \tilde{M}_R &\sim M_R \end{aligned}$$



$$\begin{cases} m_D \sim 200\text{GeV} & \rightarrow \geq \text{heaviest fermion} \\ M_R \sim 10^{15}\text{GeV} \rightarrow \text{close to GUT} \end{cases}$$

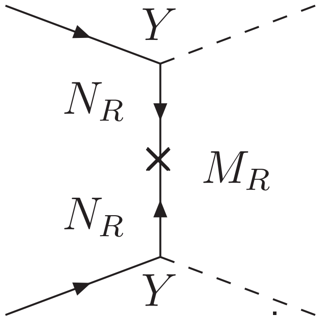
$$\Downarrow$$

$$m_\nu \propto \sqrt{\Delta m^2_{\text{atm}}} \sim (10^{-2} \;-\; 10^{-1})\text{eV}$$

☞ **Neutrino mass generation mechanism at tree-level, other options?**

→ Other Seesaw Mechanisms

Seesaw I, II, III



type I (fermionic singlet)

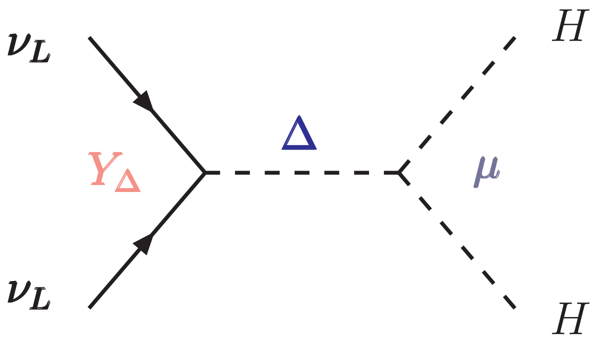
$$m_\nu = -\frac{1}{2}v^2\, Y_N^T \frac{1}{M_N}\, Y_N$$

Minkowski, Gell-Man,

Ramond, Slansky

Yanagida, Glashow

Mohapatra, Senjanovic



type II (scalar triplet)

$$m_\nu = -2v^2 Y_\Delta \frac{\mu_\Delta}{M_\Delta^2}$$

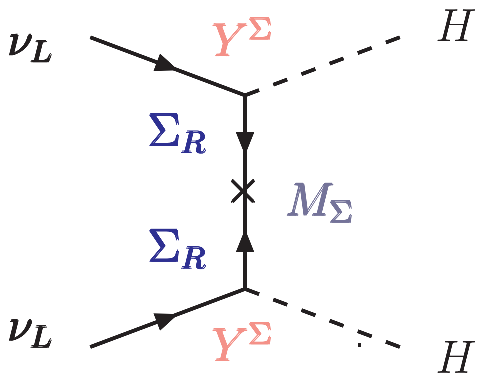
Magg, Wetterich,

Nussinov

Mohapatra, Senjanovic

Schechter, Valle

Ma, Sarkar



type III (fermionic triplet)

$$m_\nu = -\frac{v^2}{2}\, Y_\Sigma^T\, \frac{1}{M_\Sigma}\, Y_\Sigma$$

Ma, Hambye et al.

Bajc, Senjanovic, Lin

A.A., Biggio, Bonnet, Gavela,

Notari, Strumia, Papucci, Dorsner

Fileviez-Perez, Foot, Lew...

☞ **How to disentangle among the different possibilities (BSM)?**

→ $\boxed{\text{Use first the effective approach}}$

☞ Neutrino masses require the addition of new fields

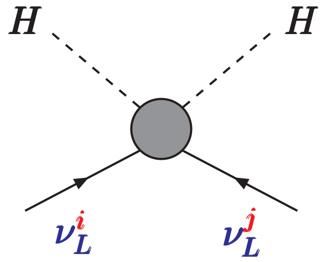
- ▶ Effects at low energy: effective theorie approach

☞ heavy fermion: $\frac{1}{\not{D}-M} \sim -\frac{1}{M} - \frac{1}{M}\not{D}\frac{1}{M} + \ldots$

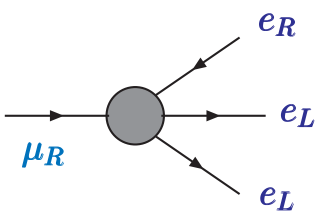
☞ heavy scalar : $\frac{1}{D^2-M^2} \sim -\frac{1}{M^2} - \frac{D^2}{M^4} + \ldots$

➞ $$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{1}{M} c^{d=5}\mathcal{O}^{d=5} + \frac{1}{M^2} c^{d=6}\mathcal{O}^{d=6} + \cdots$$

$$\Delta\mathcal{L}^{d\geq 5} = \frac{c^{d=5}}{M} \times$$



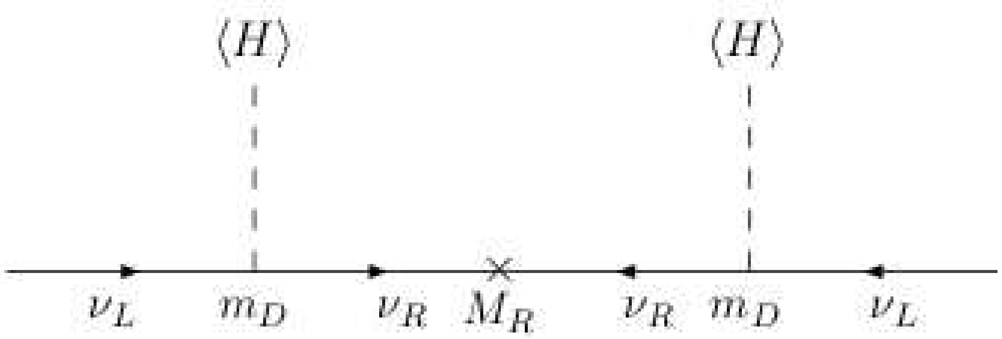
$$+ \frac{c^{d=6}_{\mu eee}}{M^2} \times$$

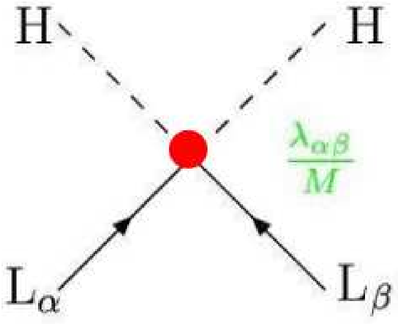


$$+ \frac{c^{d=6}_{\ell_i\ell_j\gamma}}{M^2} \ldots$$

Dimension 5

$$\delta\mathcal{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta} \left(\overline{\ell^c_{L\alpha}}\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger\, \ell_{L\beta}\right) + \text{h.c.}\,,$$





$$m_\nu = \frac{v^2 Y^\dagger Y}{M_R}$$

$$m_\nu = c^{d=5} v^2, \quad c^{d=5} = \frac{Y^\dagger Y}{M_R}$$

☞ $Y \sim 1$ → $M_R \sim M_{\rm GUT}$

☞ $Y \sim 10^{-6}$ → $M_R \sim \text{TeV}$

$\mathcal{O}^{d=5}$ Operator violates lepton number L → Majorana neutrinos

- $\mathcal{O}^{d=5}$ **is common to all models of Majorana neutrinos**

Higher order operators

☞ $\mathcal{O}^{d=5}$ operator: same for all SM extensions incorporating massive MAJORANA

☞ $\mathcal{O}^{d=6}$: 3 "types" of Dimension 6 operators relevant for **cLFV** **(dipole and 3-body)**

2 lepton-Higgs-photon: $\mathcal{O}^6_{\ell_i\ell_j\gamma} \sim L_i\sigma^{\mu\nu}e_j H F_{\mu\nu}$

$\mathcal{O}^6_{\ell_i\ell_i\gamma}$ ➞ anomalous magnetic or electric moments $(\propto$ Re or Im $\mathcal{C}^6_{\ell_i\ell_i\gamma}/\Lambda^2)$

$\mathcal{O}^6_{\ell_i\ell_j\gamma}$ ➞ **radiative decays** $\ell_i \to \ell_j\gamma$ $(\propto \mathcal{C}^6_{\ell_i\ell_j\gamma}/\Lambda^2)$

4 lepton: $\mathcal{O}^6_{\ell_i\ell_j\ell_k\ell_l} \sim (\ell_i\gamma_\mu P_{L,R}\ell_j)(\ell_k\gamma^\mu P_{L,R}\ell_l) \rightsquigarrow$ **3-body decays** $\ell_i \to \ell_j\ell_k\ell_l$, ...

2 lepton-2 quarks: $\mathcal{O}^6_{\ell_i\ell_j q_k q_l} \sim (\ell_i\gamma_\mu P_{L,R}\ell_j)(q_k\gamma^\mu P_{L,R}q_l)$ $\mu - e$ **in Nuclei, meson decays,**

(Higher order $\mathcal{O}^{d=7,8,..}$: ν (transitional) magnetic moments, NSI, unitarity violation, ...)

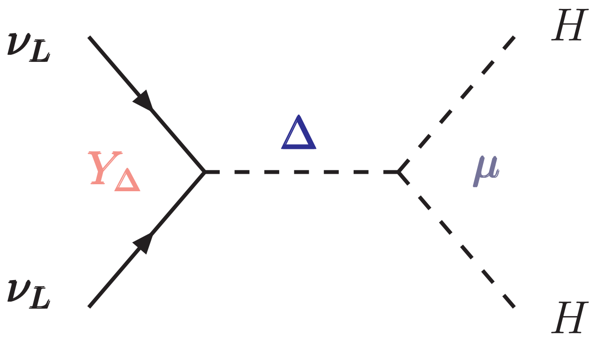
☞ **A specific example: Seesaw models**

Dimension 6 operators

Model	Effective Lagrangian $\mathcal{L}_{eff} = c_i \mathcal{O}_i$		
	$c^{d=5}$	$c_i^{d=6}$	$\mathcal{O}_i^{d=6}$
Fermionic Singlet	$Y_N^T \frac{1}{M_N} Y_N$	$\left(Y_N^\dagger \frac{1}{M_N^\dagger} \frac{1}{M_N} Y_N \right)_{\alpha\beta}$	LFV $\left(\overline{\ell_{L\alpha}} \widetilde{\phi} \right) i \not{\partial} \left(\widetilde{\phi}^\dagger \ell_{L\beta} \right)$
Scalar Triplet	$4 Y_\Delta \frac{\mu_\Delta}{M_\Delta^2}$	$\frac{1}{M_\Delta^2} Y_{\Delta\alpha\beta} Y^\dagger_{\Delta\gamma\delta}$	LFV $\left(\overline{\widetilde{\ell_{L\alpha}}} \vec{\tau} \ell_{L\beta} \right) \left(\overline{\ell_{L\gamma}} \vec{\tau} \widetilde{\ell_{L\delta}} \right)$
		$\frac{\lvert\mu_\Delta\rvert^2}{M_\Delta^4}$	Higgs-Gauge $\left(\phi^\dagger \vec{\tau} \widetilde{\phi} \right) \left(\overleftarrow{D_\mu} \overrightarrow{D^\mu} \right) \left(\widetilde{\phi}^\dagger \vec{\tau} \phi \right)$
		$-2 \left(\lambda_3 + \lambda_5 \right) \frac{\lvert\mu_\Delta\rvert^2}{M_\Delta^4}$	$\left(\phi^\dagger \phi \right)^3$ Higgs
Fermionic Triplet	$Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$	$\left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma \right)_{\alpha\beta}$	LFV $\left(\overline{\ell_{L\alpha}} \vec{\tau} \widetilde{\phi} \right) i \not{D} \left(\widetilde{\phi}^\dagger \vec{\tau} \ell_{L\beta} \right)$

Fermions: if $Y \sim \mathcal{O}(1)$, $c^{d=6} \sim (c^{d=5})^2$ and the smallness m_ν would preclude observable effects from $\mathcal{O}_i^{d=6}$. Not the case for scalars!

Case of scalar triplet (type II)



$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix} \sim (1,3,2) \qquad L_\Delta = -2$$

Yukawa couplings:

$$Y_{\Delta ij} \overline{(l_L)^c}_{ia} \, (l_L)_{jb} \, (i\tau_2\tau_\alpha)_{ab} \, \Delta^\alpha + h.c.$$

Scalar coupling:

$$\mu \; \phi^t_a \phi_b \, (i\tau_2\tau_\alpha) \left(\Delta^\dagger\right)^\alpha + h.c.$$

$$-M_\Delta{}^2 \Delta^\dagger\Delta - \tfrac{1}{2}\lambda_2 \left(\Delta^\dagger\Delta\right)^2$$

$$-\lambda_3 \left(\phi^\dagger\phi\right) \left(\Delta^\dagger\Delta\right) + \ldots$$

d=5 Operator (Mass) $m_\nu = v^2 \, Y_\Delta \, \dfrac{\mu}{M_\Delta{}^2}$ ➞ 2 different scales μ, M_Δ

possible to have $Y_\Delta \sim \mathcal{O}(1)$ $M_\Delta \sim 1\,\text{TeV}$ ($\mu \sim 100$ eV)

Low energy effects of dimension 6 operators:

$\frac{1}{2M_\Delta^2} Y_{\Delta ij} Y^\dagger_{\Delta kl} \left(\overline{l_{Li}}\gamma^\mu l_{Lk}\right) \left(\overline{l_{Lj}}\gamma_\mu l_{Ll}\right) \rightarrow$ **LFV**, $g-2$, EDMs

constraints not suppressed by μ

$$\left.\begin{aligned} &-2\frac{\mu^2}{M_\Delta^4}\partial_\mu\left(\phi^\dagger\phi\right)\partial^\mu\left(\phi^\dagger\phi\right)\\ &2\lambda_3\frac{\mu^2}{M_\Delta^4}\left(\phi^\dagger\phi\right)^3\\ &4\frac{\mu^2}{M_\Delta^4}\left[\phi^\dagger D_\mu\phi\right]^\dagger\left[\phi^\dagger D_\mu\phi\right]\end{aligned}\right\}\rightarrow$$

EW precision data,
couplings to gauge bosons

$-2\frac{\mu^2}{M_\Delta^4}\left(\phi^\dagger\phi\right)\left\{Y_e\bar{l}e_R\phi + Y_d\bar{q}d\phi - Y_u\bar{q}i\tau_2 u\phi + h.c.\right\} \rightarrow$ top physics...

Constraining the type II seesaw

★ Scalar triplet: bounds from low energy constraints

☞ $Y_\Delta \lesssim 10^{-1} \times \left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)$ or stronger

☞ If observation of $\mu \to e\gamma$ at MEG (sensitivity of 10^{-13})

- for $Y_\Delta \sim \mathcal{O}(1)$ ➙ $15\ \mathrm{TeV} < M_\Delta < 50\ \mathrm{TeV}$
- for $Y_\Delta \sim \mathcal{O}(10^{-2})$ ➙ $0.15\ \mathrm{TeV} < M_\Delta < 0.50\ \mathrm{TeV}$

★ Scalar triplet: bounds from LHC

☞ If M_Δ turns out to be as low as $\mathcal{O}(\mathrm{TeV})$ ➙ possibility of clean signals in colliders (LHC)

LHC constraints on scalar triplet

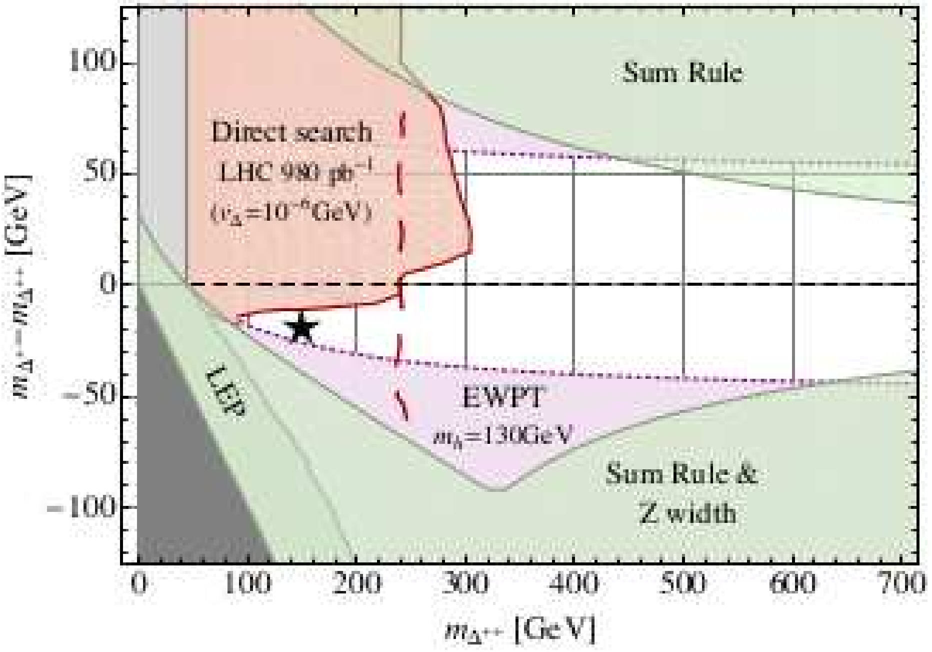
★ Production of Δ^{++} and Δ^{--}, decaying into pairs of same-sign leptons

➞ striking signals, free from SM backgrounds

★ Drell-Yann Production $\begin{cases} M_{\Delta^{++}} \sim 200\ \text{GeV} \Rightarrow \sigma(\gamma^*, Z^* \to \Delta^{++}\Delta^{--}) \sim 100\ \text{fb} \\ M_{\Delta^{++}} \sim 900\ \text{GeV} \Rightarrow \sigma(\gamma^*, Z^* \to \Delta^{++}\Delta^{--}) \sim 0.1\ \text{fb} \end{cases}$

★ Decay product $\begin{cases} \Gamma(\Delta^{\pm\pm} \to W^\pm W^\pm) \sim \mu^2 M_\Delta^3 \\ \Gamma(\Delta^{\pm\pm} \to \ell_i^\pm\ \ell_j^\pm) \sim Y_{\Delta ij} M_\Delta \end{cases}$

➞ **LHC:** so far, only **negative search** results ⇒ **constraints on parameter space** $(M_\Delta, \mu, Y_\Delta)$



Melfo et.al., arXiv:1108.4416

Summary

Up to now, we have seen

- ☞ **Indisputable:** νs are massive and mix
- ☞ **Majorana and Dirac nature :** both are possible
- ☞ **SM$_{m_\nu}$:** strong potential for CP violation
- ☞ **The SM must be extended:** extended Higgs sector, New particles, ...
- ☞ **The Seesaw mechanism:** fermionic and/or scalar new fields
  ➜ Seesaw type I, II, III
- ☞ **The effective approach:** Dim 5 operator common to all NP extensions
- ☞ **The effective approach:** Dimension 6 operatorw may help in disentangling among NP scenarios

☝ **Where is the scale of NP?**

Part 2

- Focus on extension with Fermionic singlets
- BAU from leptogenesis
- Lowering the scale, different models
- Sterile fermions