# HOW LIGHT CAN GAUGE BOSONS OF FLAVOR BE?

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# IN THE STANDARD MODEL…

There is no explanation for:

- family replication and inter-family **mass hierarchy** (in fact the **Yukawa hierarchy** since all masses are proportional to the Higgs VEV)
- the weak **mixing pattern**: small angles for quarks, large angles for neutrinos
- **neutrino masses:** very small (seesaw?), mass hierarchy is yet unknown

$$m_e : m_\mu : m_\tau \sim 1/k\,\epsilon^2 : k\epsilon : 1 \qquad \sin\theta^q_{12} \sim \sqrt{\epsilon} \sim 4\epsilon \qquad \sin\theta^q_{23} \sim \epsilon \qquad k \sim 3$$

$$m_d : m_s : m_b \sim \epsilon^2 : \epsilon : 1 \qquad \sin\theta^q_{13} \sim \epsilon^2$$

$$m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1$$

$$\epsilon \simeq \frac{1}{20} \div \frac{1}{30}$$

# STANDARD MODEL $\otimes$ FAMILY SYMMETRY

- A gauge **family symmetry** can be introduced and **mass hierarchy** between families can be related to the **hierarchy of the symmetry breaking**.
- Family symmetry should **not allow fermions to have mass** until this symmetry is spontaneously broken. So it should not be **vector-like**: L and R fermions should transform differently under family symmetry.

Maximal family symmetry could be a chiral symmetry:

$$\boxed{U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e}$$

$$q_{Li} \sim 3_q \qquad u_R^j \sim \bar{3}_u \qquad d_R^\alpha \sim \bar{3}_d \qquad l_{L\beta} \sim 3_l \qquad e_R^k \sim \bar{3}_e$$

In grand unification SU(5) is reduced to

$$U(3)^2 = U(3)_l \times U(3)_e$$

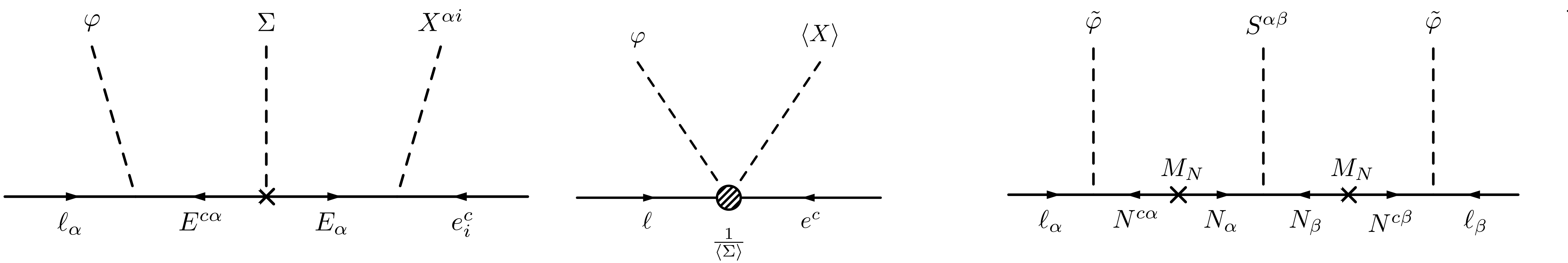
$$(q,\, \bar{u},\, \bar{e})_i = (10, 3_e)$$

$$(l,\, \bar{d})_\alpha = (\bar{5}, 3_l)$$

$SU(3)_f$ gauged, U(1) global

# EFFECTIVE OPERATORS FOR FERMION MASSES

$$\mathcal{L} = \sum_n \left( \frac{\xi_n^i \xi_n^j}{M^2} \bar{\phi} \bar{u}_j q_i + \frac{\eta_n^\alpha \xi_n^i}{M^2} \phi \bar{d}_\alpha q_i + \frac{\xi_n^k \eta_n^\alpha}{M^2} \phi \bar{e}_k l_\alpha + \frac{\eta_n^\alpha \eta_n^\beta}{M^3} \bar{\phi} \bar{\phi} l_\alpha l_\beta \right)$$




$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing}$$

- Three scalar triplets of U(3)e $\xi^i_{(n)} \sim \bar{3}_e$ with VEV hierarchy

$$v_3 : v_2 : v_1 \sim \epsilon^2 : \epsilon : 1$$

- Three scale triplets of U(3)l $\eta^\alpha_{(n)} \sim \bar{3}_l$ VEV with almost no hierarchy

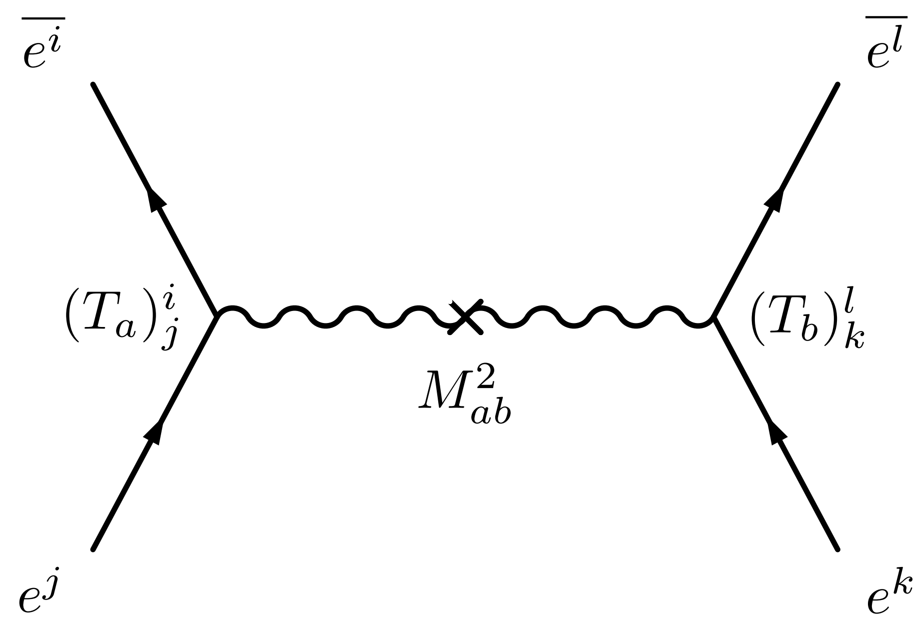
$$u_3 : u_2 : u_1 \sim \epsilon_L^2 : \epsilon_L : 1 \quad \epsilon_L \sim O(1)$$

$$M \sim \begin{pmatrix} O(\epsilon^2) & 0 & 0 \\ O(\epsilon^2) & O(\epsilon) & 0 \\ O(\epsilon^2) & O(\epsilon) & 1 \end{pmatrix} \cdot \frac{v_3 u_3}{M^2} v_{\rm EW}$$

$$u_{1,2,3} \sim v_3 \qquad v_2 \gtrsim ?$$

# HOW LIGHT CAN GAUGE BOSONS OF FLAVOR BE

$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2}$ nothing




**$SU(2)_e$** gauge symmetry limit ($v_3 >> v_2$):

- SU(2) gauge bosons have equal masses **$M^2=g^2v_2^2/2$**
- there are no FCNC thanks to custodial symmetry

Then constraints on masses are proportional to violation of **custodial symmetry** (corrections of order $\epsilon = v_2/v_3$ )

$$J^{(e)\mu}_{\pm/3} = \frac{1}{2}\begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}\gamma^\mu V^{(e)\dagger}\left(\begin{array}{c|c} \sqrt{2}\sigma_\pm/\sigma_3 & 0 \\ \hline 0 & 0 \end{array}\right) V^{(e)}\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

# HOW LIGHT CAN GAUGE BOSONS OF FLAVOR BE

$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2}$ nothing $\qquad SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2}$ nothing

Most stringent limits:

Lepton Flavour Violating modes with right handed leptons $\Rightarrow$ $v_2 \gtrsim \mathrm{TeV}$

Lepton Flavour Violating modes with left handed leptons $\Rightarrow$ $u_2 \gtrsim 50\,\mathrm{TeV}$, $u_3 \gtrsim 100\,\mathrm{TeV}$

Compositeness limits $\Rightarrow$ $\boxed{v_2 \gtrsim 2\text{-}3\,\mathrm{TeV}}$ $\Rightarrow$ $v_3 \gtrsim 100\,\mathrm{TeV}$

- This is a limit on the scale. The mass of the horizontal gauge bosons ($g^2 v_2^2/2$) can be even smaller.

# CONCLUSION

- **All** breaking scales can be **50-100 TeV** respecting experimental constraints.
- $SU(2)_e$ subgroup is broken at scale

$$v_2 \simeq v_3/\epsilon \gtrsim 2\text{-}3\,\mathrm{TeV}$$

- Horizontal gauge bosons can be **as light as few TeV**, without contradiction with the experimental limits (flavour changing processes with leptons, compositeness limits K-mesons system, B-mesons-system, etc.) because of custodial symmetry.

# IN THE STANDARD MODEL…

- Weak eigenstates are not mass eigenstates. Fermion mixing in charged currents is

$$V_{CKM} = V_L^u V_L^{d\dagger} \qquad V_{\text{PMNS}} = V_{\nu L}^\dagger V_{eL}$$

- CP is broken in the Yukawa sector. The mixing matrix is the source of CP violation
- Yukawa couplings (and also the photon/Z couplings) are diagonal in mass basis:

$$\mathbf{M}_{ij}^{(f)} = -\frac{v}{\sqrt{2}}\mathbf{Y}_{ij}^{(f)} \qquad (V^\dagger V = 1)$$

  **no flavour changing neutral currents at tree level**

- all flavour changing and CP-violation is originated from loop diagrams
- there is no mixing in the right particles sector (unless right W bosons exist)

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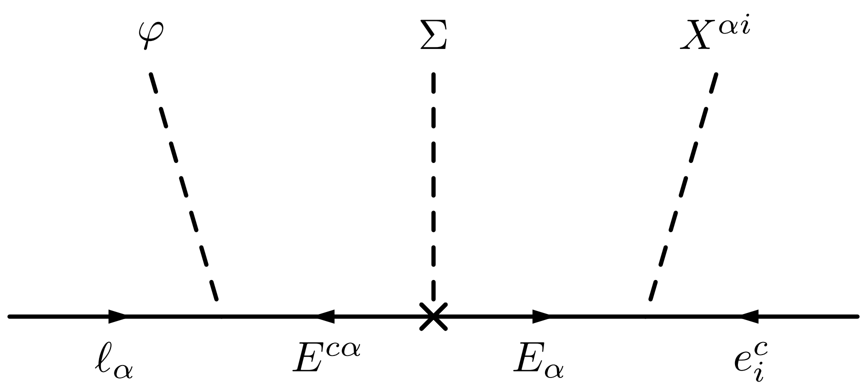
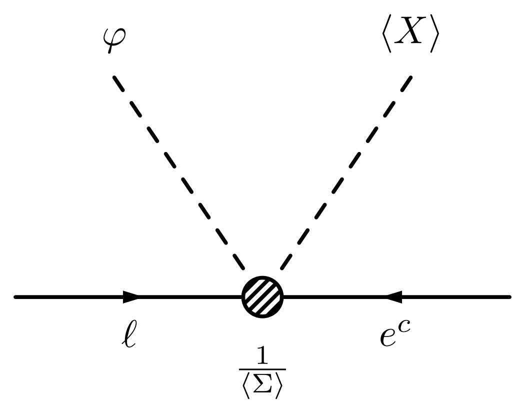
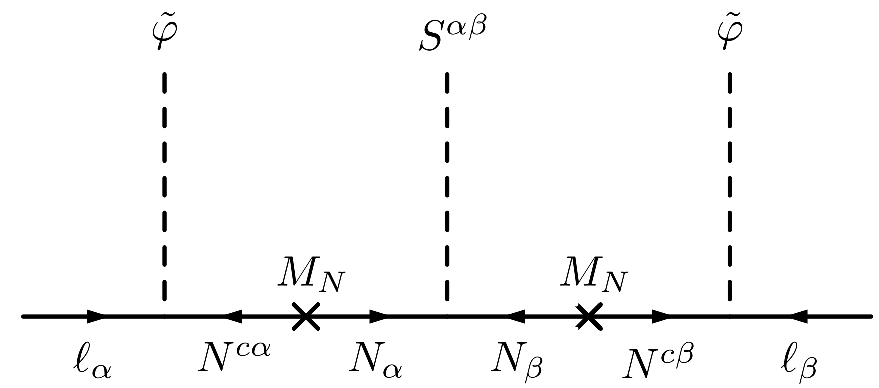
# EFFECTIVE OPERATORS FOR FERMION MASSES

$$\mathcal{L} = \frac{\lambda_u}{M_U} X_u^{ij} \bar{\phi} \bar{u}_j q_i + \frac{\lambda_d}{M_D} X_d^{i\alpha} \phi \bar{d}_\alpha q_i + \frac{\lambda_e}{M_E} X_e^{\beta k} \phi \bar{e}_k l_\beta + \lambda_\nu \frac{S^{\alpha\beta}}{M_N^2} \bar{\phi}\bar{\phi} l_\alpha l_\beta$$




Fermion masses can be parametrized:

$$m_e : m_\mu : m_\tau \sim 1/k\,\epsilon^2 : k\epsilon : 1$$

$$m_d : m_s : m_b \sim \epsilon^2 : \epsilon : 1$$

$$m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1$$

$$\sin\theta_{12}^q \sim \sqrt{\epsilon} \sim 4\epsilon \quad \sin\theta_{23}^q \sim \epsilon$$

$$\sin\theta_{13}^q \sim \epsilon^2$$

$$\epsilon \simeq \frac{1}{20} \div \frac{1}{30}$$

$$k \sim 3$$

- Three scalar triplets of U(3)$_e$

  $\xi_{(n)}^i \sim \bar{3}_e$

  with VEV hierarchy

  $$v_3 : v_2 : v_1 \sim \epsilon^2 : \epsilon : 1$$

  $SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2}$ nothing

- Three scale triplets of U(3)$_l$

  $\eta_{(n)}^\alpha \sim \bar{3}_l$

  VEV with almost no hierarchy

  $$u_3 : u_2 : u_1 \sim \epsilon_L^2 : \epsilon_L : 1$$

  $$\epsilon_L \sim O(1)$$

  $$u_{1,2,3} \sim v_3$$

- inducing VEV to X$_e$ and S via couplings

  $$\mu_X^* \xi_{(l)} X^\dagger \eta_{(n)} + \text{h.c.}$$

  $$\langle X_e^{\alpha i} \rangle \sim \begin{pmatrix} O(\epsilon^2) & 0 & 0 \\ O(\epsilon^2) & O(\epsilon) & 0 \\ O(\epsilon^2) & O(\epsilon) & 1 \end{pmatrix} \cdot \frac{\mu_X v_3 u_3}{M_X^2}$$

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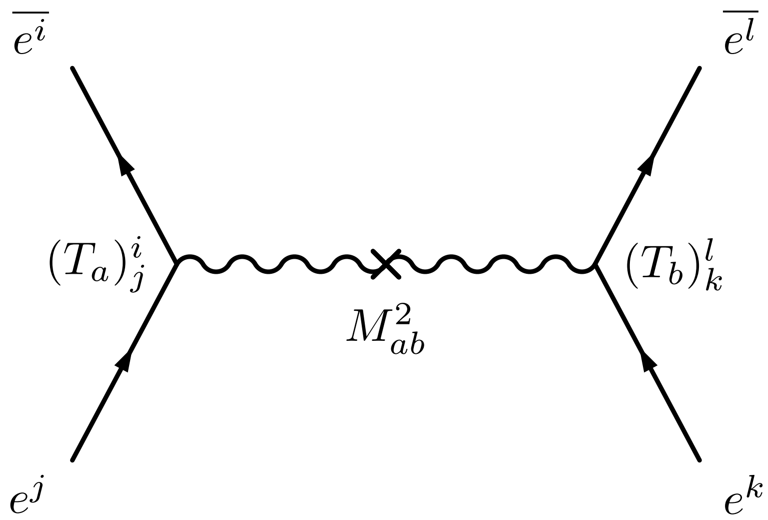


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Most stringent limits:

| LFV modes | Operators | Experimental constraints |
|---|---|---|
| $\mu \to eee$ | $-\frac{2G_{\mu eee}}{\sqrt{2}}\overline{e_R}\gamma^\mu \mu_R \overline{e_R}\gamma^\mu e_R$ | $\frac{2\lvert G_{\mu eee}\rvert}{\sqrt{2}} \sim \frac{\epsilon^3}{v_2^2} < 10^{-6}\frac{4G_F}{\sqrt{2}}$ |
| $\tau^- \to \mu^- e^+ e^-$ | $-\frac{2G_{\tau\mu ee}}{\sqrt{2}}\overline{\mu_R}\gamma^\mu \tau_R \overline{e_R}\gamma^\mu e_R$ | $\frac{2\lvert G_{\tau\mu ee}\rvert}{\sqrt{2}} \sim \frac{\epsilon}{v_2^2} < 10^{-4}\frac{4G_F}{\sqrt{2}}$ |
| $\tau \to \mu\mu\mu$ | $-\frac{2G_{\tau\mu\mu\mu}}{\sqrt{2}}\overline{\mu_R}\gamma^\mu \tau_R \overline{\mu_R}\gamma^\mu \mu_R$ | $\frac{2\lvert G_{\tau\mu\mu\mu}\rvert}{\sqrt{2}} \sim \frac{\epsilon}{v_2^2} < 10^{-4}\frac{4G_F}{\sqrt{2}}$ |
| $\mu \to eee$ | $-\frac{2G_{\mu eee}}{\sqrt{2}}\overline{e_L}\gamma^\mu \mu_L \overline{e_L}\gamma^\mu e_L$ | $\frac{2\lvert G_{\mu eee}\rvert}{\sqrt{2}} \sim \frac{u_2^3}{u_3^3}\frac{1}{u_2^2} < 10^{-6}\frac{4G_F}{\sqrt{2}}$ |

→ $v_2 \gtrsim$ TeV

→ $u_2 \gtrsim 50$ TeV, $u_3 \gtrsim 100$ TeV

Compositeness limits → $\boxed{v_2 \gtrsim \text{2-3 TeV}}$ → $v_3 \gtrsim 100$ TeV

$$\Gamma(K^+ \to \pi^+ \mu^- e^+)/\Gamma_{\text{total}} < 5.2 \cdot 10^{-10}$$

$$v_2 > 30\,\text{TeV}$$

$$\mathcal{L}_C = \pm \frac{g^2}{2\Lambda_{RR}^2} \bar{\psi}_R \gamma_\mu \psi_R \bar{\psi}_R \gamma^\mu \psi_R \qquad \frac{g^2}{4\pi} = 1$$

$$\begin{aligned} M_{12} =& \frac{G_F^2 m_W^2}{4\pi^2} \tilde{F}^* \langle K^0 | (\overline{d_L} \gamma^\mu s_L)(\overline{d_L} \gamma_\mu s_L) | \bar{K}^0 \rangle = \\ &= -\frac{G_F^2 m_W^2}{12\pi^2} \tilde{F}^* f_K^2 m_K B_K e^{i(\xi_s - \xi_d - \xi)} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{K\bar{K}\text{SM}} =& \frac{-G_F^2 M_W^2}{4\pi^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) \tilde{F} + h.c. = \\ &= -\frac{1 - i\alpha_{SM}}{\Lambda_{SM}^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) + h.c. \\ &\sim -\frac{1 - i10^{-2}}{(1,6 \cdot 10^6 \text{GeV})^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) + h.c. \end{aligned}$$

$$\begin{aligned} \left|\frac{1 - i\alpha_{ds}^{(d)}}{\Lambda_{ds}^{(d)2}}\right| &= \left|\frac{(V_{3d}^{(d)} V_{3s}^{(d)*})^2}{2v_2^2}\right| = \\ &= X_d \left|\frac{(V_{Ltd} V_{Lts}^*)^2}{2v_2^2}\right| \\ &\approx \frac{(\epsilon^3)^2}{2v_2^2} \end{aligned}$$