Exploring EFTs of Self-Interacting Dark Matter

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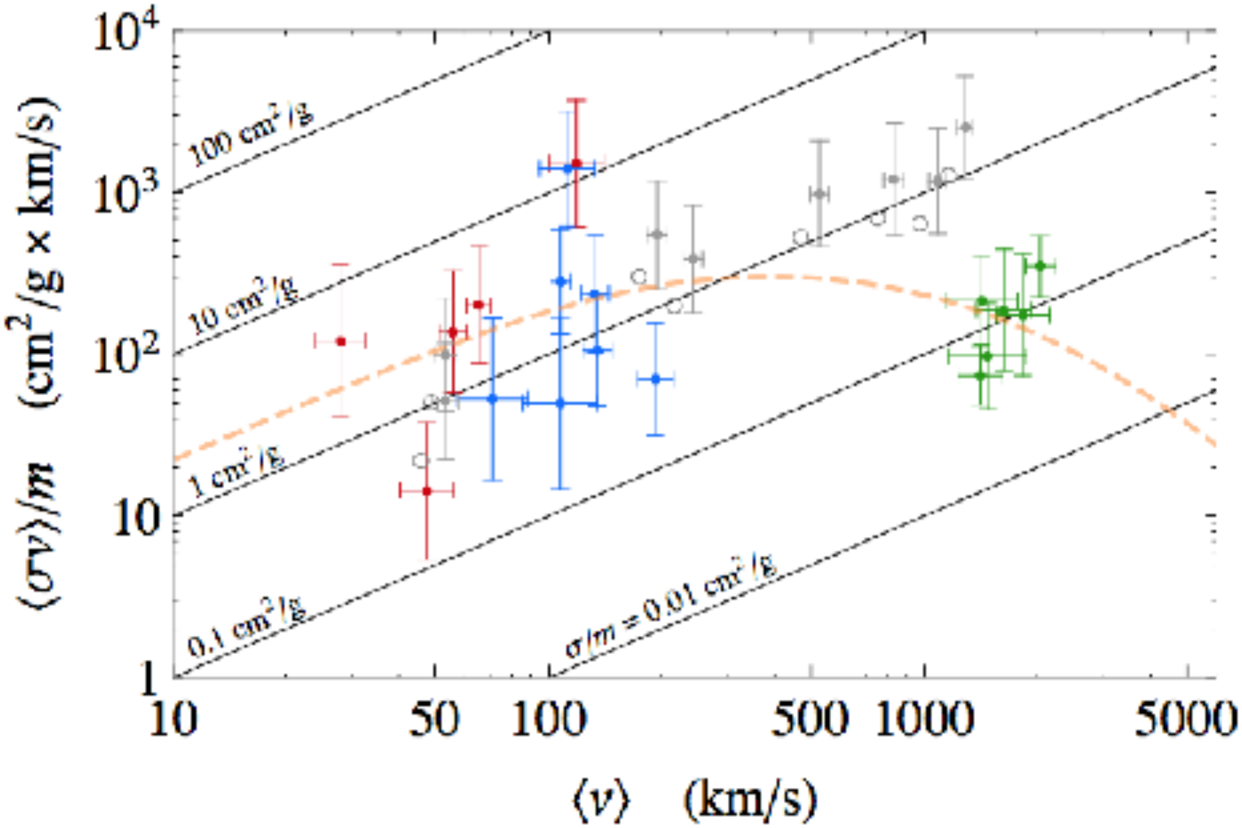
Why do we go beyond WIMPs?

- Core vs. Cusp Problem
  - Simulations show cuspy profiles whereas rotation curve observations show cored profiles
- Missing Satellites Problem
  - CDM simulations show an overprediction of subhalos and associated dwarf galaxies as compared to observations
- Too Big to Fail Problem
  - Most luminous galaxies predicted to inhabit the most massive subhalos.
  - Massive subhalos are expected to form stars and should host observable galaxies.
  - Low mass galaxies have observed velocities too small to be consistent with the mass of the subhalos they are expected to inhabit.
- Diversity Problem

Baryonic Feedback vs. Self-Interacting Dark Matter

- Supernova driven outflows can help:
  - Flatten the dark matter cusp into a core
  - Deplete baryons and render low mass halos incapable of forming satellites
- SIDM is an interesting alternative
  - Alleviate core vs cusp problem and too big to fail problem by scattering
  - Can give rather interesting signals in experiments depending on how it interacts with the Standard Model



Red: Dwarf galaxy data

Blue: Low Surface Brightness galaxy data

Green: Cluster data

Gray: SIDM N-body simulation halos

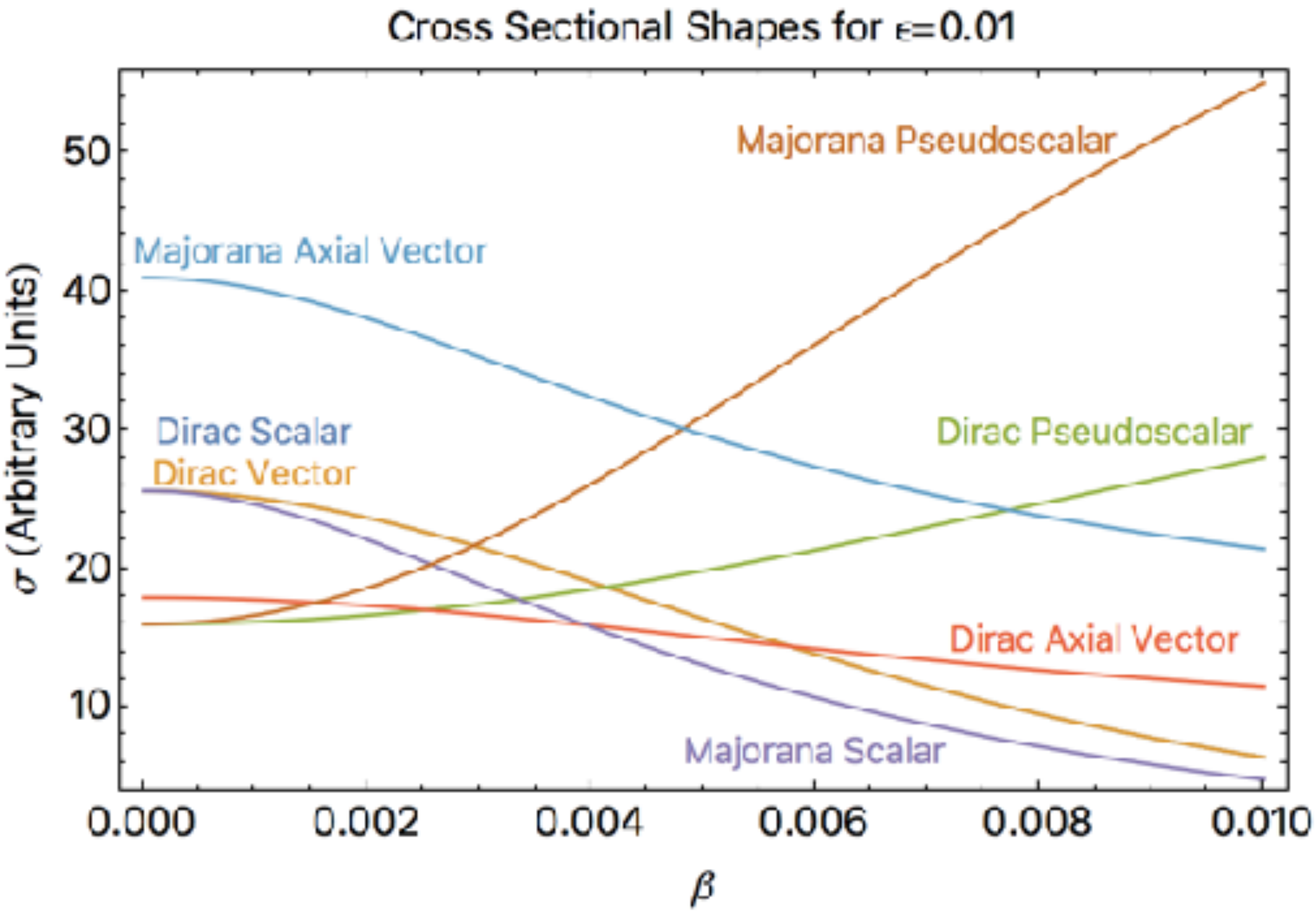
Best fit dark photon model curve shown

Kaplinghat, Tulin, Yu [1508.03339]

EFT Approach

- Process we consider is DM scattering
- Classify all EFTs with a light mediator and fermionic dark matter
  - Study scalar, vector, pseudoscalar and axial vector interactions
  - **Dirac** and Majorana fermions and **Symmetric** vs. Asymmetric

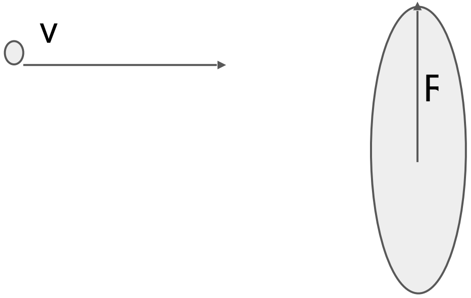
Type	Process	Channels
Majorana	$\chi\chi \to \chi\chi$	s, t, u
Dirac, Asymmetric	$\chi\chi \to \chi\chi$	t, u
Dirac, Symmetric	$\chi\bar{\chi} \to \chi\bar{\chi}$	s, t



$$\epsilon = \frac{m_\phi}{m_\chi}$$

Sommerfeld Enhancement

- A Classical Analogy



w/o gravity $\sigma_0 = \pi R^2$

w/ gravity $\sigma = \pi b_{max}^2 = \sigma_0\left(1 + \frac{v_{esc}^2}{v^2}\right)$

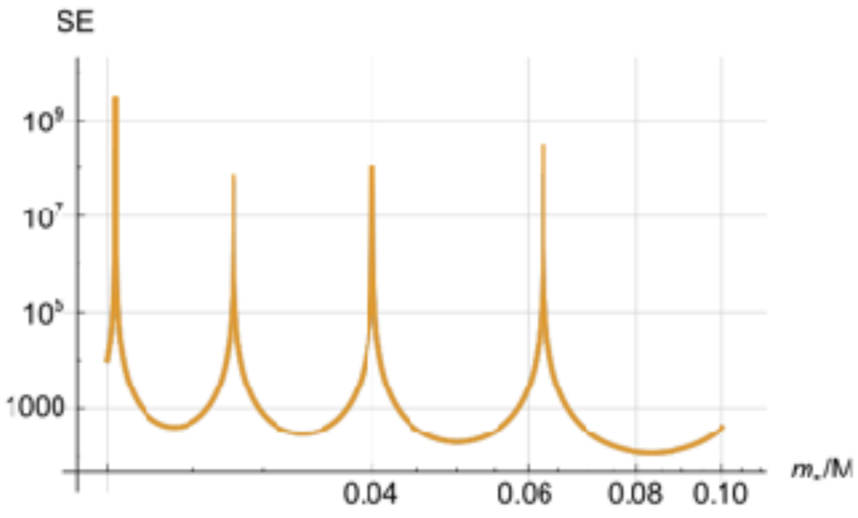
- Non-perturbative effect that can be treated quantum mechanically
  - Match a field theory calculation onto a quantum mechanical potential
  - Solve the Schrodinger Equation

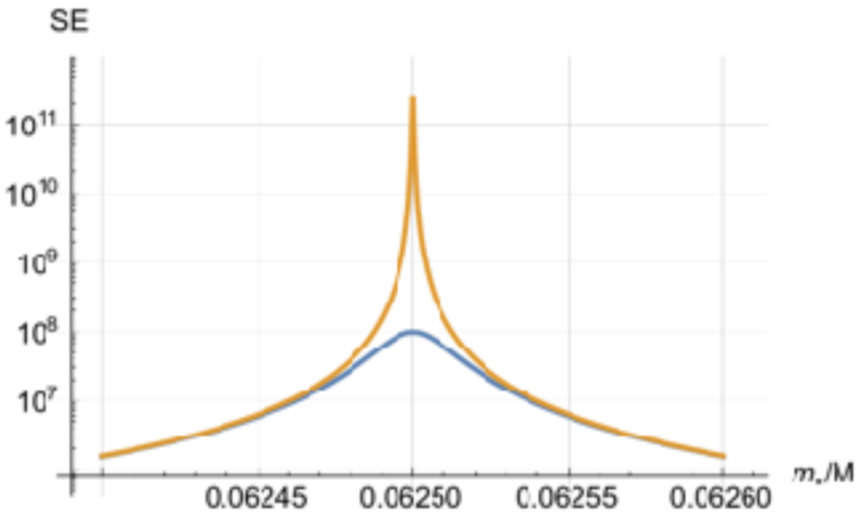
$$S = \frac{|\Psi(0)|^2}{|\Psi^0(0)|^2}$$

Arkani-Hamed, Finkbeiner, Slatyer, Weiner [0810.0713]; Lepage [9706029]

Hulthen Potential

$$V(r) = -\frac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}}.$$





Blum, Sato, Slatyer [1603.01383]

Various Non-Relativistic Potentials

Mediator	Interaction	$\frac{1}{r}$	$\frac{s_1 \cdot s_2}{r}$	$\frac{3(s_1 \cdot \hat{r})(s_2 \cdot \hat{r}) - s_1 \cdot s_2}{r^3}$
Scalar	$\lambda_s \bar{\chi} \chi \phi$	$-\lambda_s^2$	0	0
Vector	$\lambda_v \bar{\chi} \gamma^\mu \chi A_\mu$	$\pm\left(1 + \frac{m_A^2}{4m_\chi^2}\right)$	$\pm\frac{2\lambda_v^2 m_A^2}{3m_\chi^2}$	$\mp\frac{\lambda_v^2}{m_\chi^2}\left(1 + m_A r + \frac{m_A^2 r^2}{3}\right)$
Pseudoscalar	$i\lambda_p \bar{\chi} \gamma^5 \chi \phi$	0	$\frac{\lambda_p^2 m_\phi^2}{3m_\chi^2}$	$\frac{\lambda_p^2}{m_\chi^2}\left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)$
Axial Vector	$\lambda_a \bar{\chi} \gamma^5 \gamma^\mu \chi A_\mu$	0	$\frac{-8\lambda_a^2}{3}\left(1 - \frac{m_A^2}{8m_\chi^2}\right)$	$\lambda_a^2\left(\frac{1}{m_\chi^2} + \frac{4}{m_A^2}\right)\left(1 + m_A r + \frac{m_A^2 r^2}{3}\right)$

Bellazzini, Cliche, Tanedo [1307.1129]

Thank You!

Backup

Scattering Cross Sections

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\left\{\left(\frac{s-4m_\chi^2}{s-m_\phi^2}\right)^2 + \left(\frac{t-4m_\chi^2}{t-m_\phi^2}\right)^2 - \frac{16m_\chi^4 - 4m_\chi^2 s - 4m_\chi^2 t - st}{(s-m_\phi^2)(t-m_\phi^2)}\right\}$$

Scalar

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{32\pi^2 s}\left\{\frac{s^2+2st+2t^2-8m_\chi^2 t+8m_\chi^4}{(s-m_\phi^2)^2} + \frac{t^2+2st+2s^2-8m_\chi^2 s+8m_\chi^4}{(t-m_\phi^2)^2} - \frac{8m_\chi^4-2(s+t)^2}{(s-m_\phi^2)(t-m_\phi^2)}\right\}$$

Vector

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\left\{\left(\frac{s}{s-m_\phi^2}\right)^2 + \left(\frac{t}{t-m_\phi^2}\right)^2 + \frac{st}{(s-m_\phi^2)(t-m_\phi^2)}\right\}$$

Pseudoscalar

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\frac{2}{m_\phi^4(m_\phi^2-s)^2(m_\phi^2-t)^2}\Big\{&(64m_\chi^8(m_\phi^2-s)(m_\phi^2-t)\\ &-32m_\chi^6(5m_\phi^6+m_\phi^4(-6s-6t+u)+m_\phi^2(2s^2+2st-su+2t^2-tu)+stu)\\ &+4m_\chi^4(22m_\phi^8-4m_\phi^6(4s+4t-3u)+m_\phi^4(-3s^2+6s(t-2u)-3t^2-12tu+u^2)\\ &+m_\phi^2(5s^3-s^2(t-4u)-s(t^2-4tu+u^2)+t(5t^2+4tu-u^2))-st(s^2-4st+t^2-u^2))\\ &-4m_\chi^2m_\phi^2(m_\phi^6(3s+3t+5u)-m_\phi^4(2s^2+6st+5su+2t^2+5tu)+m_\phi^2(s^2(t+u)+st(t+3u)+t^2u)+st(s^2+t^2))\\ &+m_\phi^4(m_\phi^4(s^2+t^2+4u^2)-2m_\phi^2(s^3+2su^2+t^3+2tu^2)+s^4+s^2u^2+2stu^2+t^4+t^2u^2))\Big\}\end{aligned}$$

Axial Vector

$$\epsilon = \frac{m_\phi}{m_\chi}$$

Non-Relativistic Cross Sections

$$v = \beta$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM,scalar} = \frac{\lambda^4}{4096\pi^2 m_\chi^2} \frac{256}{(\epsilon^2 - 2v^2(cos\theta - 1))^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM,vector} = \frac{\lambda^4}{4096\pi^2 m_\chi^2} \frac{256}{(\epsilon^2 - 2v^2(cos\theta - 1))^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM,pseudoscalar} = \frac{\lambda^4}{4096\pi^2 m_\chi^2} \frac{16\epsilon^4 - 96v^2\epsilon^2(cos\theta - 1) + 192v^4(cos\theta - 1)^2}{(\epsilon^2 - 2v^2(cos\theta - 1))^2}$$

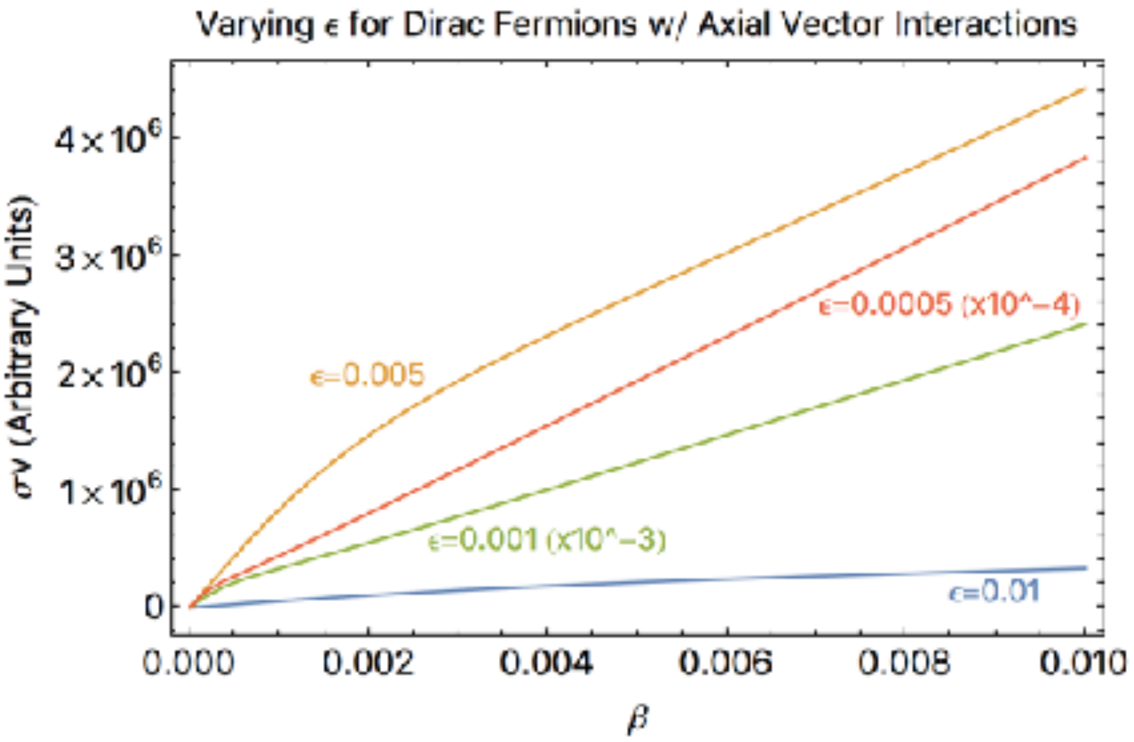
$$\left(\frac{d\sigma}{d\Omega}\right)_{CM,axialvector} = \frac{16\lambda^4}{4096\pi^2 m_\chi^2} \frac{112\epsilon^4 - 256v^2\epsilon^2(cos\theta - 1) + 192v^4(cos\theta - 1)^2}{\epsilon^4(\epsilon^2 - 2v^2(cos\theta - 1))^2}$$

Things can get a little complicated….

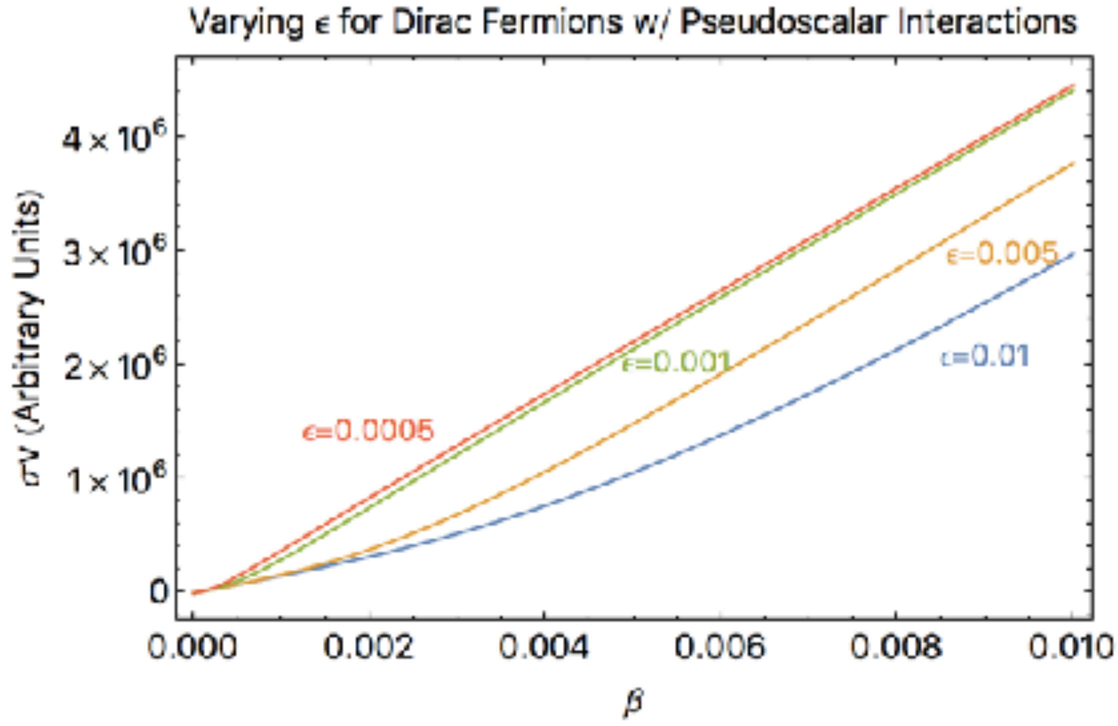
$$
\begin{aligned}
&\sigma_{CM,\,\text{axial vector}} \qquad (115)\\
&= \lambda^4 \Bigg\{ \frac{[-6(8((\epsilon^2+6)\epsilon^2\delta^{10} - (\epsilon^6+16\epsilon^4+76\epsilon^2+16)\delta^8 + 8(\epsilon^6+11\epsilon^4+42\epsilon^2+24)\delta^6 - 8(3\epsilon^6+32\epsilon^4+88\epsilon^2+96)\delta^4}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&+ \frac{32(\epsilon^6+12\epsilon^4+24\epsilon^2-32)\delta^2 - 512\epsilon^2(\epsilon^2+4))v^6 + 2((\epsilon^4-16)\epsilon^2\delta^{12} + (-12\epsilon^6+8\epsilon^4+208\epsilon^2+64)\delta^8}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&+ \frac{8(7\epsilon^6-8\epsilon^4-120\epsilon^2-96)\delta^6 - 32(5\epsilon^6-8\epsilon^4-88\epsilon^2-128)\delta^4 + 256(\epsilon^6-2\epsilon^4-28\epsilon^2-50)\delta^2 - 512(\epsilon^6+4\epsilon^4-32))v^4}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&- \frac{2((\epsilon^4+2\epsilon^2-8)\epsilon^2\delta^{10} - 2(7\epsilon^6+12\epsilon^4-56\epsilon^2-32)\delta^8 + 8(9\epsilon^6+16\epsilon^4-56\epsilon^2-160)\delta^6 - 32(7\epsilon^6+20\epsilon^4-160)\delta^4}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&+ \frac{128(3\epsilon^6+12\epsilon^4+40\epsilon^2-32)\delta^2 - 4096\epsilon^2)v^2 + ((\epsilon^2+4)\epsilon^2\delta^8 - 4(3\epsilon^4+12\epsilon^2+8)\delta^6 + 32(\epsilon^4+4\epsilon^2+16)\delta^2}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&- \frac{512)(\delta^2-4)\delta^2\epsilon^2)(\log\delta)\epsilon^2\delta^4 + 3(8((\epsilon^2+6)\epsilon^2\delta^{10} - (\epsilon^6+16\epsilon^4+76\epsilon^2+16)\delta^8 + 8(\epsilon^6+11\epsilon^4+42\epsilon^2+24)\delta^6}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&- \frac{8(3\epsilon^6+32\epsilon^4+88\epsilon^2+96)\delta^4 + 32(\epsilon^6+12\epsilon^4+24\epsilon^2-32)\delta^2 - 512\epsilon^2(\epsilon^2+4))v^6 + 2((\epsilon^4-16)\epsilon^2\delta^{10}}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&+ \frac{(-12\epsilon^6+8\epsilon^4+208\epsilon^2+64)\delta^8 + 8(7\epsilon^6-8\epsilon^4-120\epsilon^2-96)\delta^6 - 32(5\epsilon^6-8\epsilon^4-88\epsilon^2-128)\delta^4 + 256(\epsilon^6-2\epsilon^4-28\epsilon^2-50)\delta^2}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&- \frac{512(\epsilon^6+4\epsilon^4-32))v^4 - 2((\epsilon^4+2\epsilon^2-8)\epsilon^2\delta^{10} - 2(7\epsilon^6+12\epsilon^4-56\epsilon^2-32)\delta^8 + 8(9\epsilon^6+16\epsilon^4-56\epsilon^2-160)\delta^6}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&- \frac{32(7\epsilon^6+20\epsilon^4-160)\delta^4 + 128(3\epsilon^6+12\epsilon^4+40\epsilon^2-32)\delta^2 - 4096\epsilon^2)v^2 + ((\epsilon^2+4)\epsilon^2\delta^8 - 4(3\epsilon^4+12\epsilon^2+8)\delta^4}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&+ \frac{32(\epsilon^4+4\epsilon^2+16)\delta^2 - 512)(\delta^2-4)\delta^2\epsilon^2)(\log\delta^2)\epsilon^2\delta^4 + 8v^2(((\epsilon^8-12\epsilon^6+192\epsilon^4-576\epsilon^2-256)\delta^{10}}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&+ \frac{32(4\epsilon^6-57\epsilon^4+76\epsilon^2+64)\delta^8 - 64(\epsilon^8+8\epsilon^6-2\epsilon^4+152\epsilon^2+64)\delta^6 + 256(\epsilon^6+4\epsilon^4+152\epsilon^2+72)\epsilon^2\delta^4}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&- \frac{256\epsilon^2(\epsilon^6+4\epsilon^4-40\epsilon^2-288)\delta^2 - 8192\epsilon^4)v^4 - 48\delta^2(\delta^2-4)\epsilon^2((\epsilon^2-16)\delta^6 + (22\epsilon^2+80)\delta^4 + 32(3\epsilon^2-4)\delta^2 + 32(3\epsilon^2+8))v^2}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)}\\
&+ \frac{48(\delta^2-4)^2(7\delta^4-4\delta^2+16)\delta^2\epsilon^4))}{1536 m^2 \pi v^2 \delta^4(-4v^2+\delta^2-4)^2(4v^2+\delta^2)\epsilon^6(4v^2+\epsilon^2)} \Bigg\}
\end{aligned}
$$

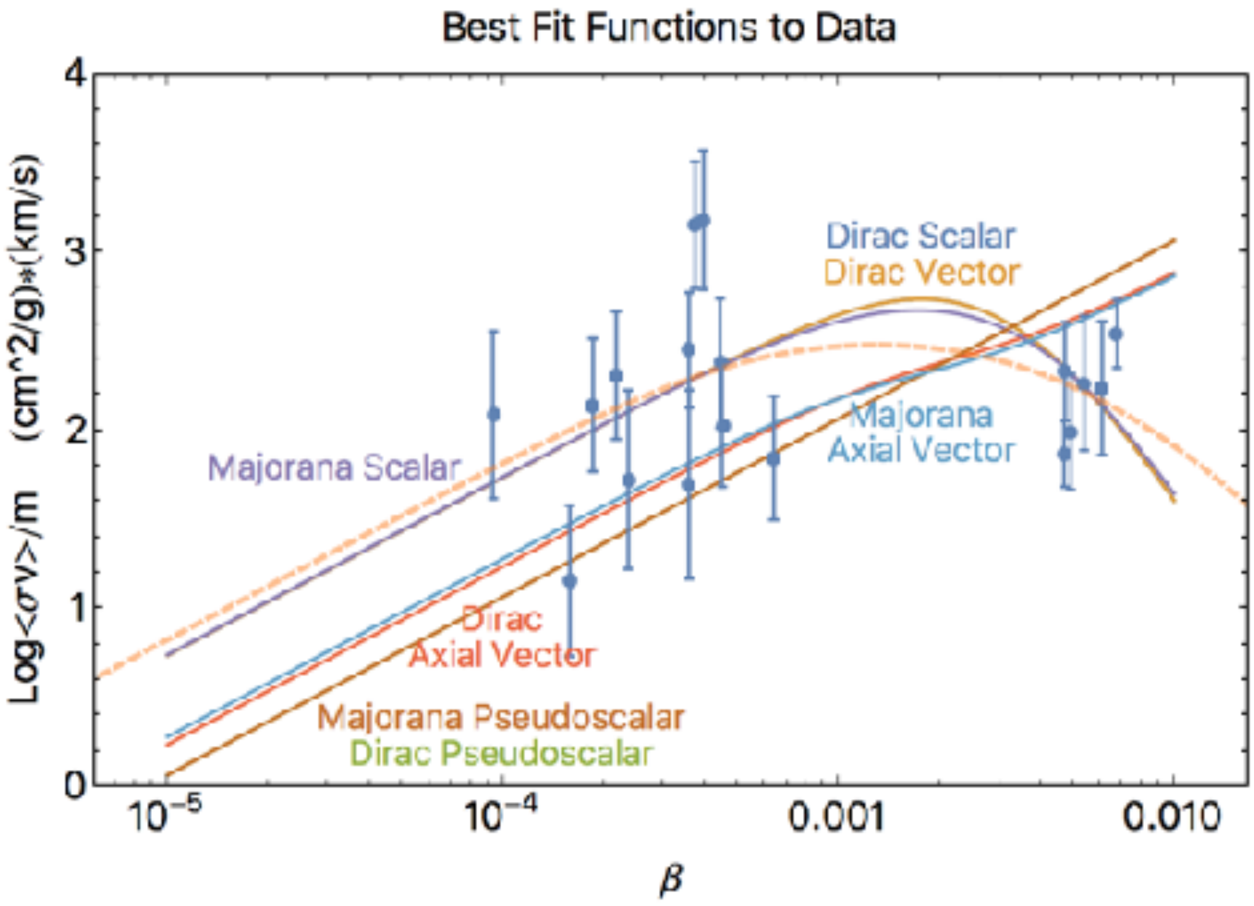
$$\epsilon = \frac{m_\phi}{m_\chi} \qquad \delta = \frac{m_{Higgs}}{m_\chi}$$

Mediator Mass Effects - Axial Vector Case



Mediator Mass Effects - Pseudoscalar Case





Best Fit Functions to Data
Log<σv>/m (cm^2/g)*(km/s)
β
0
1
2
3
4
10^-5
10^-4
0.001
0.010
Dirac Scalar
Dirac Vector
Majorana Scalar
Majorana
Axial Vector
Dirac
Axial Vector
Majorana Pseudoscalar
Dirac Pseudoscalar

Coulomb Potential

$$V(r) = \frac{-\alpha}{2r}$$

This potential admits an analytic solution for the Sommerfeld enhancement factor

$$S = \left| \frac{\frac{\pi}{\epsilon_v}}{1 - exp[-\frac{\pi}{\epsilon_v}]} \right|$$

As v becomes large, S starts to approach 1.

As v approaches 0, S behaves like 1/v and starts to diverge.

Important in the nonrelativistic limit!

Asymmetric Dark Matter Cross Sections

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\frac{1}{2}\left\{\frac{2(t-4m_\chi^2)^2}{(m_\phi^2-t)^2} + \frac{2(u-4m_\chi^2)^2}{(m_\phi^2-u)^2} - \frac{16m_\chi^4 - 8m_\chi^2(-s+t+u) - s^2 + t^2 + u^2}{(m_\phi^2-t)(m_\phi^2-u)}\right\}$$

Scalar

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\left\{\frac{8(\frac{s}{2}-m_\chi^2)^2}{(m_\phi^2-t)^2} + \frac{8(\frac{s}{2}-m_\chi^2)^2}{(m_\phi^2-u)^2} + \frac{8t(m_\chi^2-\frac{t}{2})}{(m_\phi^2-t)^2} + \frac{8(m_\chi^2-\frac{u}{2})^2}{(m_\phi^2-t)^2} + \frac{8(m_\chi^2-\frac{t}{2})^2}{(m_\phi^2-u)^2}\right.$$
$$\left. + \frac{8u(m_\chi^2-\frac{u}{2})}{(m_\phi^2-u)^2} + \frac{4(4m_\chi^4 - 6m_\chi^2 s + 2m_\chi^2 t + 2m_\chi^2 u + s^2)}{(m_\phi^2-t)(m_\phi^2-u)} + \frac{4t^2}{(m_\phi^2-t)^2} + \frac{4u^2}{(m_\phi^2-u)^2}\right\}$$

Vector

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\frac{1}{2}\left\{-\frac{-16m_\chi^4 + 8m_\chi^2 s - s^2 + t^2 + u^2}{(m_\phi^2-t)(m_\phi^2-u)} + \frac{2t^2}{(m_\phi^2-t)^2} + \frac{2u^2}{(m_\phi^2-u)^2}\right\}$$

Pseudoscalar

$$
\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s} 4 \Bigg(& \frac{4t^2 m_\chi^4}{m_\phi^4 \left(m_\phi^2 - t\right)^2} + \frac{4u^2 m_\chi^4}{m_\phi^4 \left(m_\phi^2 - u\right)^2} + \frac{2\left(15m_\chi^4 - 8tm_\chi^2 - s^2 + t^2 + u^2\right) m_\chi^4}{m_\phi^4 \left(m_h^2 - t\right)\left(m_\phi^2 - u\right)} \\
& + \frac{2\left(16m_\chi^4 - 8um_\chi^2 - s^2 + t^2 + u^2\right) m_\chi^4}{m_\phi^4 \left(m_\phi^2 - t\right)\left(m_h^2 - u\right)} + \frac{16m_\chi^4}{\left(m_\phi^2 - t\right)^2} + \frac{4\left(t - 4m_\chi^2\right)^2 m_\chi^4}{m_\phi^4 \left(m_h^2 - t\right)^2} + \frac{8\left(s - 2m_\chi^2\right) m_\chi^4}{\left(m_\phi^3 - m_\phi t\right)^2} + \frac{8\left(u - 2m_\chi^2\right) m_\chi^4}{\left(m_\phi^3 - m_\phi t\right)^2} \\
& - \frac{2\left(-16m_\chi^4 + 8sm_\chi^2 - s^2 + t^2 + u^2\right) m_\chi^4}{m_\phi^4 \left(m_\phi^2 - t\right)\left(m_\phi^2 - u\right)} + \frac{16m_\chi^4}{\left(m_\phi^2 - u\right)^2} - \frac{2\left(15m_\chi^4 + 8(s - t - u)m_\chi^2 - s^2 + t^2 + u^2\right) m_\chi^4}{m_\phi^4 \left(m_h^2 - t\right)\left(m_h^2 - u\right)} \\
& + \frac{4\left(u - 4m_\chi^2\right)^2 m_\chi^4}{m_\phi^4 \left(m_h^2 - u\right)^2} + \frac{8\left(s - 2m_\chi^2\right) m_\chi^4}{\left(m_\phi^3 - m_\phi u\right)^2} + \frac{8\left(t - 2m_\chi^2\right) m_\chi^4}{\left(m_\phi^3 - m_\phi u\right)^2} - \frac{8tm_\chi^2}{\left(m_\phi^2 - t\right)^2} - \frac{2\left(8m_\chi^4 - 2(s + u)m_\chi^2 + t^2\right) m_\chi^2}{m_\phi^2 \left(m_\phi^2 - t\right)\left(m_\phi^2 - u\right)} \\
& - \frac{2\left(8m_\chi^4 - 2(s + t)m_\chi^2 + u^2\right) m_\chi^2}{m_\phi^2 \left(m_\phi^2 - t\right)\left(m_\phi^2 - u\right)} - \frac{2\left(15m_\chi^4 - 2(s - 4t - u)m_\chi^2 + t^2\right) m_\chi^2}{m_\phi^2 \left(m_h^2 - t\right)\left(m_\phi^2 - u\right)} - \frac{8um_\chi^2}{\left(m_\phi^2 - u\right)^2} \\
& - \frac{2\left(16m_\chi^4 + 2(s - t - 4u)m_\chi^2 + u^2\right) m_\chi^2}{m_\phi^2 \left(m_\phi^2 - t\right)\left(m_h^2 - u\right)} + \frac{t^2}{\left(m_\phi^2 - t\right)^2} + \frac{u^2}{\left(m_\phi^2 - u\right)^2} + \frac{2\left(m_\chi^2 - \frac{u}{2}\right)^2}{\left(m_\phi^2 - t\right)^2} - \frac{t\left(t - 2m_\chi^2\right)}{\left(m_\phi^2 - t\right)^2} \\
& + \frac{4\left(m_\chi^2 t - 2m_\chi^4\right)}{\left(m_\phi^2 - t\right)^2} + \frac{\left(s - 2m_\chi^2\right)^2}{2\left(m_\phi^2 - t\right)^2} + \frac{20m_\chi^4 - 2(3s + t + u)m_\chi^2 + s^2}{\left(m_\phi^2 - t\right)\left(m_\phi^2 - u\right)} + \frac{2\left(m_\chi^2 - \frac{t}{2}\right)^2}{\left(m_\phi^2 - u\right)^2} - \frac{u\left(u - 2m_\chi^2\right)}{\left(m_\phi^2 - u\right)^2} \\
& + \frac{4\left(m_\chi^2 u - 2m_\chi^4\right)}{\left(m_\phi^2 - u\right)^2} + \frac{\left(s - 2m_\chi^2\right)^2}{2\left(m_\phi^2 - u\right)^2} \Bigg)
\end{aligned}
$$

Axial Vector

Majorana Fermions

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\left\{\left(\frac{s-4m_\chi^2}{s-m_\phi^2}\right)^2 + \left(\frac{t-4m_\chi^2}{t-m_\phi^2}\right)^2 + \left(\frac{u-4m_\chi^2}{u-m_\phi^2}\right)^2 + \frac{st-4m_\chi^2 u}{(s-m_\phi^2)(t-m_\phi^2)} + \frac{tu-4m_\chi^2 s}{(t-m_\phi^2)(u-m_\phi^2)} + \frac{su-4m_\chi^2 t}{(s-m_\phi^2)(u-m_\phi^2)}\right\}$$

Scalar

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\lambda^4}{64\pi^2 s}\left\{\left(\frac{s}{s-m_\phi^2}\right)^2 + \left(\frac{t}{t-m_\phi^2}\right)^2 + \left(\frac{u}{u-m_\phi^2}\right)^2 + \frac{st}{(s-m_\phi^2)(t-m_\phi^2)} + \frac{tu}{(t-m_\phi^2)(u-m_\phi^2)} + \frac{su}{(s-m_\phi^2)(u-m_\phi^2)}\right\}$$

Pseudoscalar

$$
\begin{aligned}
&\left(\frac{d\sigma}{d\Omega}\right)_{CM} \\
&= \frac{e^4}{256\pi^2 s}\left\{ \frac{64(4m_\chi^2 - s)^2 m_\chi^4}{m_\phi^4 (s - m_\phi^2)^2} + \frac{64(4m_\chi^2 - t)^2 m_\chi^4}{m_\phi^4 (t - m_\phi^2)^2} + \frac{64(4m_\chi^2 - u)^2 m_\chi^4}{m_\phi^4 (u - m_\phi^2)^2} \right. \\
&- \frac{32(16m_\chi^4 - 8(s + t - u)m_\chi^2 + s^2 + t^2 - u^2)m_\chi^4}{m_\phi^4 (s - m_\phi^2)(t - m_\phi^2)} - \frac{32(16m_\chi^4 - 8(s - t + u)m_\chi^2 + s^2 - t^2 + u^2)m_\chi^4}{m_\phi^4 (s - m_\phi^2)(u - m_\phi^2)} \\
&- \frac{32(16m_\chi^4 + 8(s - t - u)m_\chi^2 - s^2 + t^2 + u^2)m_\chi^4}{m_\phi^4 (t - m_\phi^2)(u - m_\phi^2)} + \frac{\frac{32m_\chi^4(16m_\chi^4 - 8sm_\chi^2 + s^2 + t^2 - u^2)}{m_\phi^4} - \frac{32m_\chi^2(16m_\chi^4 - 2(4s + t - u)m_\chi^2 + s^2)}{m_\phi^2}}{(s - m_\phi^2)(t - m_\phi^2)} \\
&+ \frac{\frac{32m_\chi^4(16m_\chi^4 - 8sm_\chi^2 + s^2 - t^2 + u^2)}{m_\phi^4} - \frac{32m_\chi^2(16m_\chi^4 - 2(4s - t + u)m_\chi^2 + s^2)}{m_\phi^2}}{(s - m_\phi^2)(u - m_\phi^2)} + \frac{\frac{32m_\chi^4(16m_\chi^4 - 8tm_\chi^2 - s^2 + t^2 + u^2)}{m_\phi^4} - \frac{32m_\chi^2(16m_\chi^4 + 2(s - 4t - u)m_\chi^2 + t^2)}{m_\phi^2}}{(t - m_\phi^2)(u - m_\phi^2)} \\
&- \frac{\frac{32(-16m_\chi^4 + 8sm_\chi^2 - s^2 + t^2 + u^2)m_\chi^4}{m_\phi^4} + \frac{32(8m_\chi^4 - 2(s + u)m_\chi^2 + t^2)m_\chi^2}{m_\phi^2} + \frac{32(8m_\chi^4 - 2(s + t)m_\chi^2 + u^2)m_\chi^2}{m_\phi^2} - 16(20m_\chi^4 - 2(3s + t + u)m_\chi^2 + s^2)}{(t - m_\phi^2)(u - m_\phi^2)} \\
&- \frac{\frac{32(-16m_\chi^4 + 8tm_\chi^2 + s^2 - t^2 - u^2)m_\chi^4}{m_\phi^4} + \frac{32(8m_\chi^4 - 2(t + u)m_\chi^2 + s^2)m_\chi^2}{m_\phi^2} + \frac{32(8m_\chi^4 - 2(s + t)m_\chi^2 + u^2)m_\chi^2}{m_\phi^2} - 16(20m_\chi^4 - 2(s + 3t + u)m_\chi^2 + t^2)}{(s - m_\phi^2)(u - m_\phi^2)} \\
&+ \frac{\frac{64u^2 m_\chi^4}{m_\phi^4} + \frac{128(-4m_\chi^2 - s + t)m_\chi^4}{m_\phi^2} + 8(24m_\chi^4 - 4(s + t + u)m_\chi^2 + s^2 + t^2)}{(u - m_\phi^2)^2} \\
&- \frac{\frac{32(-16m_\chi^4 + 8um_\chi^2 - s^2 - t^2 - u^2)m_\chi^4}{m_\phi^4} + \frac{32(8m_\chi^4 - 2(t + u)m_\chi^2 + s^2)m_\chi^2}{m_\phi^2} + \frac{32(8m_\chi^4 - 2(s + u)m_\chi^2 + t^2)m_\chi^2}{m_\phi^2} - 16(20m_\chi^4 - 2(s + t + 3u)m_\chi^2 + u^2)}{(s - m_\phi^2)(t - m_\phi^2)} \\
&+ \frac{\frac{64t^2 m_\chi^4}{m_\phi^4} + \frac{128(-4m_\chi^2 + s + u)m_\chi^4}{m_\phi^2} + 8(24m_\chi^4 - 4(s + t + u)m_\chi^2 + s^2 + u^2)}{(t - m_\phi^2)^2} \\
&+ \frac{\frac{64s^2 m_\chi^4}{m_\phi^4} - \frac{128(4m_\chi^2 - t - u)m_\chi^4}{m_\phi^2} + 8(24m_\chi^4 - 4(s + t + u)m_\chi^2 + t^2 + u^2)}{(s - m_\phi^2)^2} \\
&+ \frac{\frac{32m_\chi^4(16m_\chi^4 - 8tm_\chi^2 - s^2 + t^2 - u^2)}{m_\phi^4} - \frac{32m_\chi^2(16m_\chi^4 - 2(s + 4t - u)m_\chi^2 + t^2)}{m_\phi^2}}{(s - m_\phi^2)(t - m_\phi^2)} + \frac{\frac{32m_\chi^4(16m_\chi^4 - 8um_\chi^2 + s^2 - t^2 + u^2)}{m_\phi^4} - \frac{32m_\chi^2(16m_\chi^4 - 2(s - t + 4u)m_\chi^2 - u^2)}{m_\phi^2}}{(s - m_\phi^2)(u - m_\phi^2)} \\
&\left. + \frac{\frac{32m_\chi^4(16m_\chi^4 - 8um_\chi^2 - s^2 + t^2 + u^2)}{m_\phi^4} - \frac{32m_\chi^2(16m_\chi^4 + 2(s - t - 4u)m_\chi^2 + u^2)}{m_\phi^2}}{(t - m_\phi^2)(u - m_\phi^2)} \right\}
\end{aligned}
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Axial Vector