Particle DM without Prejudice

Lecture Plan

1. Cosmo warmup + WIMPs
2. Beyond WIMPs
3. Dark Pheno

reviews

- Feng 1003.0904
- Lisanti 1603.03797
- Slatyer 1710.05137

DM Basics

- evidence:
  - rotation curves
  - cluster mergers
  - $\boxed{\text{CMB}}$

- energy budget:

$\Omega_\Lambda = 0.68$

$\boxed{\Omega_{DM} = 0.27}$

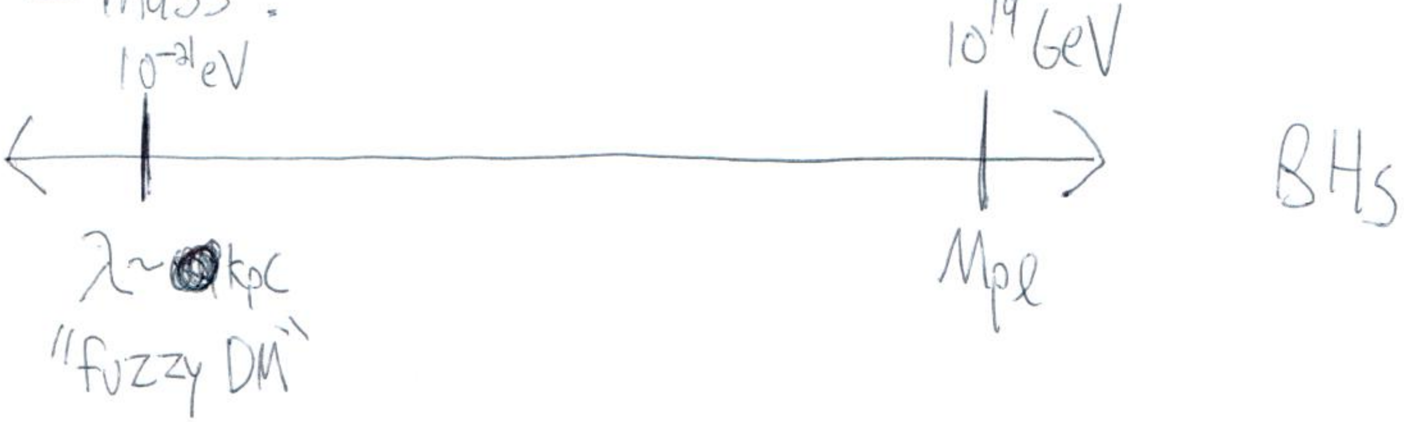
$\Omega_b = 0.05$

$\Omega_\nu \lesssim 0.003$ (predict $\gtrsim .001$)

$\Omega_\gamma = 5 \times 10^{-5}$

DM Properties

- dark: $|Q| \ll 1$
- cold: $v \ll 1$
- cosmologically stable: $\tau_{DM} \gtrsim 10^{11}$ y
- spin? $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$
- mass?

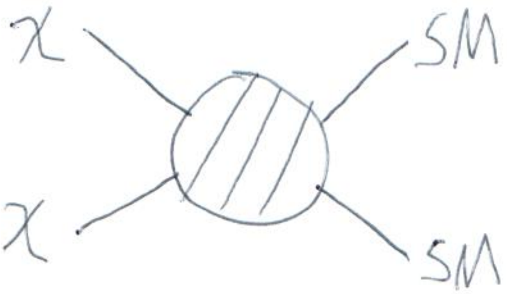


- one or multi-component?

• thermal relic: DM was in thermal equilibrium

T_d

ex) WIMP



1. Cosmo Warmup + WIMPs

PLAN

I expanding Universe

II cosmic thermodynamics

III WIMP Miracle

I Expanding Universe

- homogeneous, isotropic $\Rightarrow$ FRW metric

$$ds^2 = dt^2 - \underset{\substack{\uparrow \\ \text{scale factor}}}{a^2}(t)\left[\frac{dr}{1-\underset{\substack{\uparrow \\ \text{curvature, } k\approx 0}}{kr^2}} + r^2 d\Omega^2\right] \qquad a(t_0)=1$$

- Hubble: $H \equiv \frac{\dot{a}}{a}$ $\qquad H_0 = 100\, h \,\frac{\text{km}}{\text{s-Mpc}}$ $\qquad h \approx 0.67$

- Friedmann eq: $H^2 = \frac{8\pi G}{3}\rho$

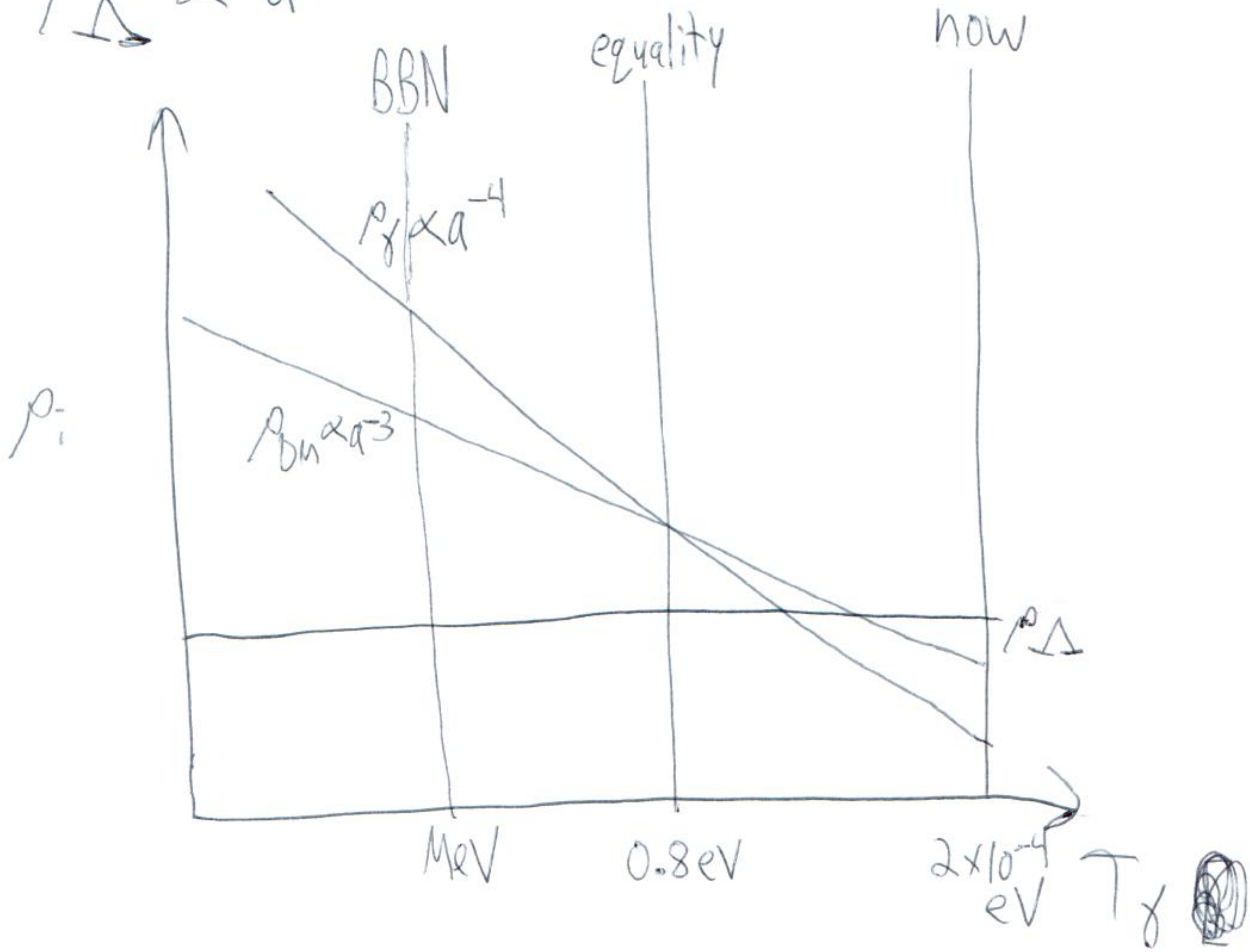
$$\Omega_i \equiv \frac{\rho_i}{\rho_c} \qquad \rho_c h^{-2} = 1.1\times 10^{-5}\ \text{GeV}/\text{cm}^3$$

- comoving volume: $V \propto a^3$
- redshift: $T_\gamma \propto a^{-1}$

$$\rho_\gamma \propto \frac{T_\gamma}{V} \propto a^{-4}$$

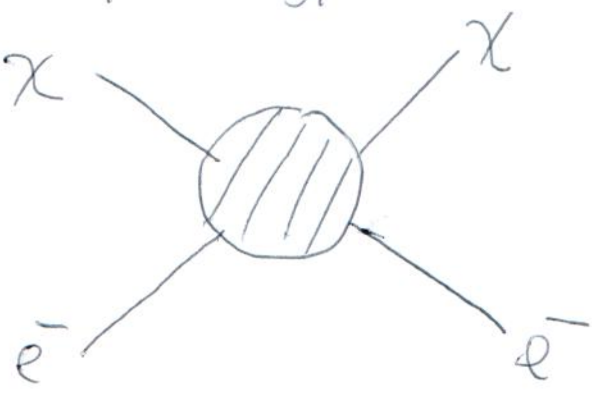
$$\rho_{DM} \propto \frac{m_{DM}}{V} \propto a^{-3}$$

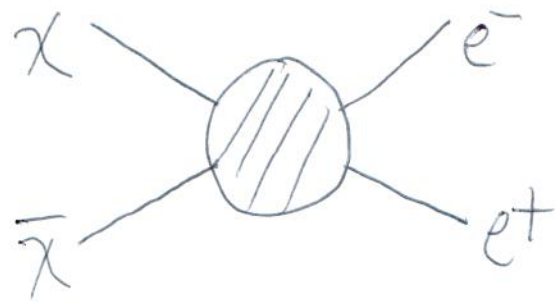
$$\rho_\Lambda \propto a^0$$



II Cosmic Thermodynamics

kinetic equilibrium	chemical equilibrium
rapid energy transfer	rapid number changing





- number/energy density:

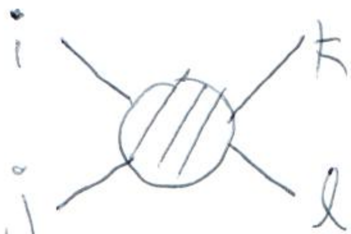
$$n = \frac{g}{(2\pi)^3} \int d^3p \, f(\vec{p}) \qquad \rho = \frac{g}{(2\pi)^3} \int d^3p \, E(\vec{p}) \, f(\vec{p})$$

phase space distribution

$$E = \sqrt{|\vec{p}|^2 + m^2}$$

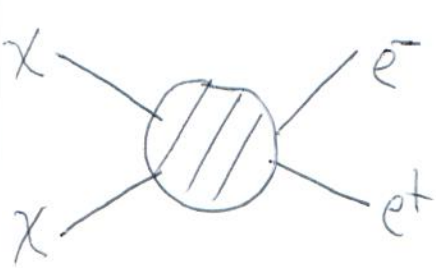
- kinetic equil. $\Rightarrow$ $f(\vec{p}) = \dfrac{1}{e^{\frac{E-\mu}{T}} \pm 1}$ + fermion, − boson

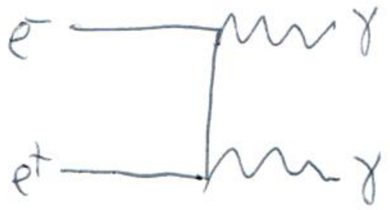
- chemical equil. $\Rightarrow$



$$\mu_i + \mu_j = \mu_k + \mu_l$$

ex) What is μ_χ if the following are in equil.?





$$2\mu_\chi = \mu_{e^-} + \mu_{e^+} \qquad \mu_{e^-} + \mu_{e^+} = 2\mu_\gamma \qquad \mu_\gamma + \mu_{e^-} = 2\mu_\gamma + \mu_{e^-}$$

$$\mu_\chi = 0 \Leftarrow \mu_{e^-} + \mu_{e^+} = 0 \Leftarrow \mu_\gamma = 0$$

- relativistic densities ($\mu \ll T$):

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{(boson)} \\ \frac{3}{4} n_{boson} & \text{(fermion)} \end{cases} \qquad \rho = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{(boson)} \\ \frac{7}{8} \rho_{boson} & \text{fermion} \end{cases}$$

- non-relativistic boson/fermion ($\mu \ll m$):

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \qquad \rho = m n$$

- total radiation: $\rho_r = \frac{\pi^2}{30} g_* T^4$

$$g_* \approx \sum_{\text{rel. bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{rel. fermions}} g_j \left(\frac{T_j}{T}\right)^4$$

SM:
$g_* = 106.75 \quad T > m_t$
$ 3.36 \quad T < m_e$

- Hubble (radiation domination):

$$H = \sqrt{\frac{8\pi G}{3}\rho_r} = \underbrace{\sqrt{\frac{8\pi}{90}}\,\pi}_{1.66} \sqrt{g_*}\,\frac{T^2}{M_{pl}}$$

entropy

- entropy S in a comoving volume $V \propto a^3$ is conserved (in equilibrium)

- entropy density: $$s = \frac{S}{V} \overset{2^{nd}\text{ Law}}{=} \frac{\rho + P}{T} \approx \frac{4}{3}\frac{\rho}{T}$$

- relativistic pressure: $P \approx \frac{\rho}{3}$

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

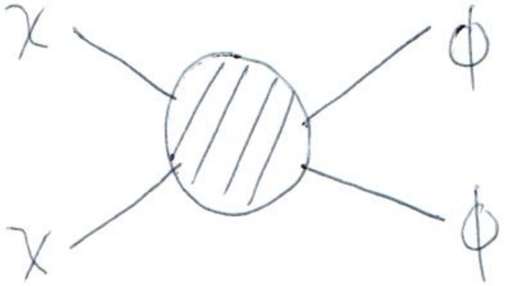
$$g_{*s} = \sum_{\text{rel. bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{rel. fermions}} g_j \left(\frac{T_j}{T}\right)^3$$

- comoving density:

$$Y_i \equiv \frac{n_i}{s} \propto n_i \times V$$

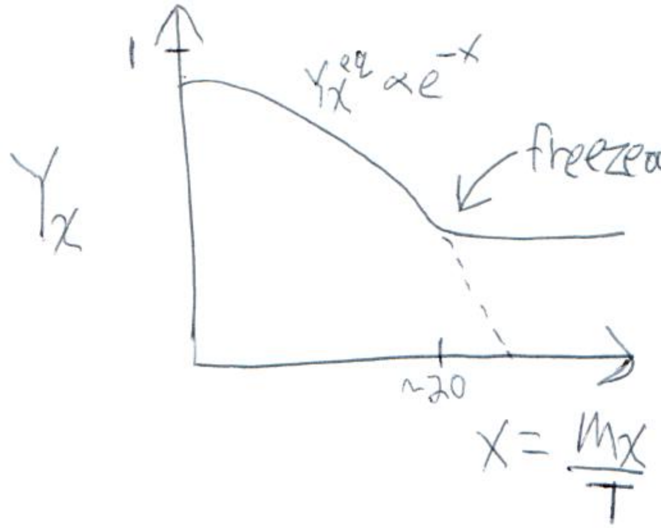
conserved for an inert particle

III WIMP "Miracle"



$\phi \in$ SM, BSM

- assume: $T_d = T_\gamma$



(A) sudden freezeout

(B) Boltzmann Eq.

(A) Sudden Freeze

units: $g_* = 1$

$\hbar = 1$

$2 = 1$

thermal avg.

- freezeout: $\Gamma = n_\chi \langle \sigma v \rangle = H$

$$\Omega_\chi \propto m_\chi Y_\chi$$

- equality: $\rho_\chi^{eq} = \rho_\gamma^{eq} \sim T_{eq} s_{eq}$

$$m_\chi Y_\chi^{FO} = m_\chi Y_\chi^{eq} = \frac{\rho_\chi^{eq}}{s_{eq}} \sim T_{eq}$$

$$\boxed{\Omega_\chi \sim \frac{m_\chi Y_\chi}{T_{eq}}}$$

$$\Omega_\chi \sim \frac{m_\chi Y_\chi^{FO}}{T_{eq}} \sim \frac{m_\chi n_\chi^{FO}}{s_{FO} T_{eq}} \sim \frac{m_\chi H_{FO}}{s_{FO} T_{eq} \langle \sigma v \rangle}$$

use: $H_{FO} \sim \dfrac{T_{FO}^2}{M_{pl}}$ $\quad s_{FO} \sim T_{FO}^3$ $\quad T_{FO} \sim m_\chi$

$$\Omega_\chi \sim \frac{1}{T_{eq} \cdot M_{pl} \langle \sigma v \rangle} \Rightarrow \boxed{\langle \sigma v \rangle \sim \frac{1}{T_{eq} \cdot M_{pl}}}$$

- WIMP miracle: $\sqrt{T_{eq} \cdot M_{pl}} \sim 100 \text{ TeV}$

$$\langle \sigma v \rangle \sim \frac{\alpha_w^2}{m_\chi^2} \Rightarrow \boxed{m_\chi \sim \alpha_w \sqrt{T_{eq} \cdot M_{pl}}} \sim \text{TeV}$$

(B) Boltzmann Eq.

Gondolo + Gelmini 1991

- inert particle: $\frac{1}{a^3}\frac{d(na^3)}{dt}=0$

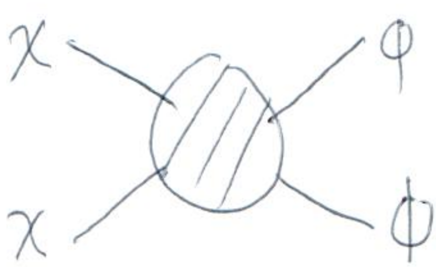
$$\dot{n}+3Hn=0$$

- Boltzmann Eq:

$$\dot{n}+3Hn=\frac{g}{(2\pi)^3}\int\frac{d^3p}{E}\,C[F]$$

collision operator

ex) DM freezeout:



$$\dot{n}_\chi+3Hn_\chi=-n_\chi^2\langle\sigma v\rangle_{\chi\chi}+n_\phi^2\langle\sigma v\rangle_{\phi\phi}$$

- detailed balance:

$$(n_\chi^{eq})^2\langle\sigma v\rangle_{\chi\chi}=(n_\phi^{eq})^2\langle\sigma v\rangle_{\phi\phi}$$

- assume ϕ is in equil. $n_\phi = n_\phi^{eq}$

$$\boxed{\dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle_{\chi\chi}\left(n_\chi^2 - (n_\chi^{eq})^2\right)}$$

$$n_\chi^{eq}\langle\sigma v\rangle > H \Rightarrow n_\chi \simeq n_\chi^{eq}$$

$$n_\chi^{eq}\langle\sigma v\rangle < H \Rightarrow \frac{n_\chi}{s} \to \text{constant}$$

$$\Omega_\chi h^2 = \left(\frac{s_0}{\rho_c h^{-2}}\right) m_\chi Y_\chi \qquad s_0 = 2900/\text{cm}^3$$

$$\Omega_\chi h^2 \approx 0.12 \Rightarrow \langle\sigma v\rangle \simeq 3\times 10^{-26}\ \text{cm}^3/\text{s}$$

$$= \frac{1}{(20\,\text{TeV})^2}$$