### The Toric SO(10) F-Theory Landscape

Wilfried Buchmuller

in collaboration with Markus Dierigl, Paul Oehlmann & Fabian Ruehle

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#### UV Completion of the Standard Model

- Structure of Standard Model points towards "grand unification" of strong and electroweak interactions (quark and lepton content, gauge group, "unification" of gauge couplings, small neutrino masses ...)
- Strong theoretical arguments for supersymmetry at "high" energy scales (gravity, extra dimensions, string theory)
- Energy scale of grand unification:  $\Lambda_{GUT} \simeq 10^{15} \dots 10^{16} \text{ GeV}$ energy scale of supersymmetry breaking:  $\Lambda_{SB} \simeq ??$
- This talk: 6d supergravity GUTs,  $\Lambda_{\rm SB} \sim R_{\rm c}^{-1} \sim \Lambda_{\rm GUT}$

# Split symmetries

WB, Dierigl, Ruehle, Schweizer '15, '16

Consider SO(10) GUT group in 6d, broken at orbifold fixed points to standard SU(5)xU(1), Pati-Salam SU(4)xSU(2)xSU(2) and flipped SU(5)xU(1), with SM group as intersection; bulk fields 45, 16, 16\*, 10's [Asaka,WB, Covi '02; Hall, Nomura et al '02; ...]; full 6d gauge symmetry:



N 16's from charged bulk 16-plet and N flux quanta:

16  $[SO(10)] \sim 5^* + 10 + 1 [SU(5)] \sim q, l, u^c, e^c, d^c, \nu^c [G_{SM}]$ 

Higgs fields from uncharged bulk 10-plets, form split multiplets:

$$H_1 \supset H_u$$
,  $H_2 \supset H_d$ ,  $\Psi \supset D^c$ ,  $N^c$ ,  $\Psi^c \supset D$ ,  $N$ 

Flux **breaks supersymmetry** [Bachas '95], soft SUSY breaking only for quark-lepton families:

$$M^{2} = m_{\tilde{q}}^{2} = m_{\tilde{l}}^{2} = \frac{4\pi N}{V_{2}} \sim (10^{15} \text{ GeV})^{2}$$
$$m_{3/2} \sim 10^{14} \text{ GeV}, \quad m_{\tilde{q}}^{2} = m_{\tilde{l}}^{2} > m_{3/2} \sim m_{1/2} \gg m_{\tilde{h}}$$

Emerging picture of **Split Symmetries** (reminiscent of "split/spread SUSY" [Arkani-Hamed, Dimopoulos; Giudice, Romanino '04; Hall, Nomura '11]):

- supersymmetry breaking is large for scalar quarks and leptons because they form complete GUT multiplets
- supersymmetry breaking can be small for gauge and Higgs fields because they form incomplete GUT multiplets (THDM)

## 6d SO(10) F-theory vacua

- Can full 6d SO(10)xU(1) SUGRA model (including singlets) be embedded into string theory?
- Considerable work on 6d F-theory vacua: Morisson, Taylor, Park, Cvetic, Schafer-Nameki, Weigand, Grimm, Palti, Klevers, Ruehle, Oehlmann, ... (2012 - 2016)
- Systematic way to construct elliptically fibered CY threefolds using toric geometry
- SO(10)xU(1) symmetry can be realized using Kodaira classification and Mordell-Weil U(1) factor

6d F-theory vacua: elliptically fibered CY threefolds; start from 2d (reflexive) polygon  $F_3$ :

Batyrev '94



2d toric ambient space with 4 homogeneous coordinates  $[u:v:w:e_1]$ and 2  $\mathbb{C}^*$  scale transformations modded out, yields  $dP_1$ ; define four divisors

$$D_{u}, D_{v}, D_{w}, D_{e_{1}}, \quad D_{x_{i}} = \{x_{i} = 0\}$$
$$\sum_{i} Q_{ai} D_{x_{i}} \sim 0, \ a = 1, 2; \quad x_{i} \to \lambda^{Q_{ai}} x_{i}, \ \lambda \in \mathbb{C}^{*}$$

coordinates	vertices	divisor classes	[v]	$[e_1]$
u	(1, -1)	[v]	0	1
v	(-1, 0)	[v]	0	1
w	(0,1)	$[ve_1]$	1	0
$e_1$	(0, -1)	$[e_1]$	1	-1

determine dual polygon ( $F_3^* = F_{14}$ ) :

$$\begin{split} \langle m_i, v_j \rangle &\geq -1 \ \forall \ i, j \ , \\ p_{F_3} &= \sum_{m_i \in F_3^*} s_i \prod_{v_j \in F_3} x_j^{\langle m_i, v_j \rangle + 1} \\ &= s_1 u^3 e_1^2 + s_2 u^2 v e_1^2 + s_3 u v^2 e_1^2 + s_4 v^3 e_1^2 + s_5 u^2 w e_1 \\ &+ s_6 u v w e_1 + s_7 v^2 w e_1 + s_8 u w^2 + s_9 v w^2 \end{split}$$

Classification of reflexive polyhedra: in 2d 16; in 4d 473.800.776 (Kreuzer, Skarke), yields CY threefolds

Torus as hypersurface in 2d ambient space:

$$\mathcal{E} = \{p_{F_3} = 0\}$$

with two points, yielding Mordell-Weil U(I) group after fibration:

$$\hat{s}_0 = D_{e_1} \cap \mathcal{E}, \quad \hat{s}_1 = \{t_P = 0\} \cap \mathcal{E}$$
  
 $\sigma_1 \equiv [\hat{s}_1] - [\hat{s}_0] = [v] - [e_1]$ 

Fibration of torus over  $\mathbb{P}^1$  yields K3; coefficients of polynomial (# 28) now dependent on base coordinates, can be tuned to obtain wanted singularity structure

Consider polynomial for torus (F3) with base-dependent coefficients, rewrite equation for torus in Weierstrass form:

$$F = -y^2 + x^3 + fx + g = 0$$

with dependence on base coordinates  $z_0, d_i(z_0, z_1)$ ,

$$f = z_0^2 \left( -\frac{1}{3} d_5^2 d_7^2 + z_0 R_1 + \mathcal{O}(z_0^2) \right),$$
$$g = z_0^3 \left( -\frac{2}{27} d_5^3 d_7^3 + z_0 R_2 + \mathcal{O}(z_0^2) \right)$$

torus is singular at point (x, y) if discriminant vanishes,

$$F = \frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = 0,$$
$$\Delta = 4f^3 + 27g^2 = 0$$

Kodaira classification: order of singularity determines non-Abelian gauge group,  $Ord(f, g, \Delta) = (2, 3, 7)$  yields SO(10):

$$\Delta = z_0^7 (P + z_0 R + \mathcal{O}(z_0^2)),$$
  
$$P = -d_5^3 d_7^3 (d_3 d_5 - d_1 d_7)^2 d_9^2$$

Vanishing of P at some points of basis leads to stronger singularities, and therefore larger symmetries, at these points:

$Ord(f, g, \Delta)$	fiber singularity
(2, 3, 7)	SO(10)
(2,3,8)	SO(12)
(3,4,8)	$E_6$
(3,4,8)	$E_6$
(2,3,8)	SO(12)
	$\begin{array}{ c c } \mathrm{Ord}(f,g,\Delta) \\ \hline (2,3,7) \\ (2,3,8) \\ (3,4,8) \\ (3,4,8) \\ (2,3,8) \end{array}$

Finally, resolve singularities by adding a `top' and calculate intersection numbers



Polytope formed from polygon F3 with an SO(10) top with two levels; vertices listed in table, base not shown; CY 3-fold cut out by hypersurface equation (divisors not all independent); classification: torus polygons [Grassi, Perduca '12], SO(10) tops [Bouchard, Skarke '03]



From fibration of torus over sphere to K3 with resolved SO(10) singularity

Intersection pattern at resolved SO(10) singularity:



Global GUT model building, starting from toric geometry [Morrison, Taylor, Cvetic, Schafer-Nameki, Weigand, Grimm, Palti, Klevers, Ruehle, Oehlmann, ... '12 ... ]; at enhanced symmetry points `coset matter' is generated, i.e. 16's and 10's:





top: Yukawa coupling  $16_{3/4}16_{-1/4}10_{-1/2}$  at locus  $z_0 = d_5 = d_7 = 0$ bottom: Yukawa coupling  $16_{3/4}16_{3/4}10_{-3/2}$  at locus  $z_0 = d_7 = d_9 = 0$ 



standard F-theory picture: 3 different matter curves intersecting at locus of Yukawa coupling in 4-fold (property of base)

#### C.3 Polygon F<sub>3</sub>

vertices:  $w: (0,1,0), u: (1,-1,0), e_1: (0,-1,0), v: (-1,0,0)$ gauge group: SO(10) × U(1)

Top 1

vertices: 
$$z_0: (0,0,1), f_2: (1,0,1), f_3: (0,1,1), f_4: (1,1,1), g_1: (1,1,2), g_2: (1,2,2)$$
  
factorization:  $s_1 = d_1 z_0, s_2 = d_2 z_0^2, s_3 = d_3 z_0^2, s_4 = d_4 z_0^3, s_5 = d_5, s_6 = d_6 z_0, s_7 = d_7 z_0, s_8 = d_8, s_9 = d_9$ 

			1
locus	representation	multiplicity	
$z_0 = d_5 = 0$	16 <sub>3/4</sub>	$(2K_B^{-1}-\mathcal{S}_7)\mathcal{Z}$	
$z_0 = d_9 = 0$	10 <sub>3/2</sub>	$S_9Z$	
$z_0 = d_3 d_5 - d_1 d_7 = 0$	$10_{-1/2}$	$(3K_B^{-1}-\mathcal{S}_9-2\mathcal{Z})\mathcal{Z}$	
$z_0 = d_7 = 0$	$16_{-1/4}$	$(\mathcal{S}_7 - \mathcal{Z})\mathcal{Z}$	
	<b>1</b> <sub>3</sub>	$(K_B^{-1}-\mathcal{S}_7+\mathcal{S}_9)\mathcal{S}_9$	(C.7)
	<b>1</b> <sub>2</sub>	$6(K_B^{-1})^2 + S_7^2 + K_B^{-1}(-5S_7 + 4S_9 - 2Z)$	(0.1)
		$+\mathcal{S}_7(2\mathcal{S}_9+\mathcal{Z})-\mathcal{S}_9(2\mathcal{S}_9+5\mathcal{Z})$	
	<b>1</b> <sub>1</sub>	$12(K_B^{-1})^2 - 4\mathcal{S}_7^2 - \mathcal{S}_9^2 + 6\mathcal{Z}^2$	
		$+K_B^{-1}(8\mathcal{S}_7-\mathcal{S}_9-25\mathcal{Z})+\mathcal{S}_7(\mathcal{S}_9+4\mathcal{Z})$	
	10	$18 + 11(K_B^{-1})^2 + 3S_7^2 + 2S_9^2 + 10Z^2 + 5S_9Z$	
		$-K_B^{-1}(3S_7 + 4S_9 + 15Z) - 5S_7Z - 2S_7S_9$	

Euler number: 
$$\chi = -24(K_B^{-1})^2 + 8K_B^{-1}S_9 - 4S_9^2 + 6K_B^{-1}S_7 + 4S_7S_9 - 6S_7^2 + 30K_B^{-1}Z - 10S_9Z + 10S_7Z - 20Z^2$$
  
anomaly coefficients:  $a \sim K_B^{-1}$ ,  $b \sim -Z$ ,  $b_{11} \sim -(6K_B^{-1} - 2S_7 + 4S_9 - \frac{5}{4}Z)$ 

complete information of single model; all properties geometrically determined, multiplicities via intersection numbers of base divisors; in total 36 models

#### Multiplicities for Hirzebruch surface $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$ as base:

representation	locus	multiplicity
$10_{3/2}$	$z = d_9 = 0$	2
$16_{3/4}$	$z = d_5 = 0$	0
$16_{-1/4}$	$z = d_7 = 0$	4
$10_{-1/2}$	$z = d_3 d_5 - d_1 d_7 = 0$	4
45	z = 0	0
$1_3$	$d_8 = d_9 = 0$	2
$1_2$	V(2)	36
$1_1$	V(3)	76
$1_{0}$	moduli	51 + 1
T	tensor	1

### Conclusions

- Supersymmetric extensions of Standard Model strongly motivated, but what is the scale of SUSY breaking?
- Higher-dimensional GUT models with flux lead to GUT scale for SUSY breaking; emerging low energy spectrum reminiscent of `spread' SUSY (THDM + higgsino + ...)
- Embedding of 6d SUGRA models with SO(10)xU(1) gauge symmetry into F-theory possible
- Open questions: F-theory compactifications on orbifolds, incorporation of Wilson line breaking, high-scale supersymmetry breaking ...