Testing the Noncommutative Standard Model in W-Pair-Production at the LHC

T. Ohl R. Rückl C. Speckner

Lehrstuhl für Theoretische Physik II, Universität Würzburg

LHC-D Workshop — SUSY and BSM II
Bonn, 22.02.2007

Outline

noncommutative field theory: realized on Minkowski spacetime by replacing the product between fields with the

Moyal-Weyl $\star$-product

$$\hat{\phi}(x) \star \hat{\eta}(x) = \hat{\phi}(x) \exp\left(\frac{i}{2} \frac{1}{\Lambda_{\mathrm{NC}}^2} \overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu \theta^{\mu\nu} \right) \hat{\eta}(x)$$

$\longrightarrow$ non-vanishig commutator

$$[\hat{x}^\mu \overset{\star}{,} \hat{x}^\nu] = \frac{i}{\Lambda_{\mathrm{NC}}^2} \theta^{\mu\nu} = i\lambda\theta^{\mu\nu}$$

parametrization

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

problem: local Lie Algebra not closed w.r.t. Moyal-Weyl-commutator

$$\left[\hat{\tau}_i(x) T_i \stackrel{\star}{,} \hat{\tau}_j(x) T_j\right] = \frac{1}{2}\left\{\hat{\tau}_i(x) \stackrel{\star}{,} \hat{\tau}_j(x)\right\}\left[T_i, T_j\right] + \frac{1}{2}\left[\hat{\tau}_i(x) \stackrel{\star}{,} \hat{\tau}_j(x)\right]\left\{T_i, T_j\right\}$$

solution: choose gauge generators and gauge field from the enveloping associative algebra

caveat: in general: additional degrees of freedom in the gauge field!

problem: local Lie Algebra not closed w.r.t. Moyal-Weyl-commutator

$$\left[\hat{\tau}_i(x) T_i \overset{\star}{,} \hat{\tau}_j(x) T_j\right] = \frac{1}{2}\left\{\hat{\tau}_i(x) \overset{\star}{,} \hat{\tau}_j(x)\right\}\left[T_i, T_j\right] + \frac{1}{2}\left[\hat{\tau}_i(x) \overset{\star}{,} \hat{\tau}_j(x)\right]\left\{T_i, T_j\right\}$$

solution: choose gauge generators and gauge field from the enveloping associative algebra

caveat: in general: additional degrees of freedom in the gauge field!

Seiberg-Witten-Map

Replace gauge parameter and gauge field with maps

$$\tau \rightarrow \hat{\tau}(A, \tau) \quad , \quad A^\mu \rightarrow \hat{A}^\mu(A)$$

from Lie Algebra to enveloping algebra such that

$$\begin{array}{ccc} A^\mu & \xrightarrow{\text{SWM}} & \hat{A}^\mu(A) \\ \downarrow \tau & & \downarrow \hat{\tau}(\tau, A) \\ A^\mu_\tau & \xrightarrow{\text{SWM}} & \hat{A}^\mu_{\hat{\tau}} \end{array}$$

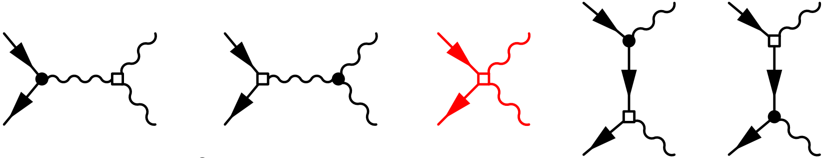
Expand $\star$-product, SWM and a similar map $\hat{\Psi}(\Psi, A)$ for the matter fields in orders of λ and insert them into the Standard Model lagrangian $\longrightarrow$ Noncommutative Standard Model

- invariance under “ordinary” gauge transformations order by order in λ
- corrections to Standard Model vertices
- new vertices not allowed in the Standard Model
- dependence of the gauge sector on the choice of representation; in the scenario under discussion parametrized by a new parameter

$$-\frac{1}{4g^2} \leq \kappa_2 \leq \frac{1}{4g^2}$$

- additional ambiguities due to the SWM being not unique

NCSM Feynman diagrams contributing to $q\bar{q} \longrightarrow W^+W^-$ at first order in λ:

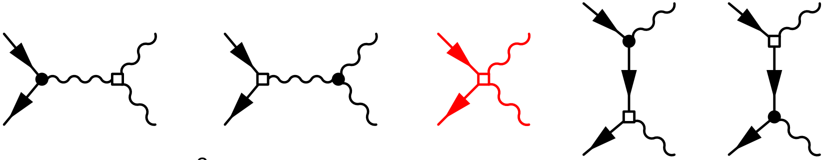


Expansion of $|\mathcal{M}|^2$:

$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 + 2\lambda\Re\mathcal{M}_0\mathcal{M}_1^* + \mathcal{O}(\lambda^2)$$

possible caveat: negative cross section as a consequence of the truncation of the expansion

NCSM Feynman diagrams contributing to $q\bar{q} \longrightarrow W^+W^-$ at first order in λ:



Expansion of $|\mathcal{M}|^2$:

$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 + 2\lambda \Re \mathcal{M}_0 \mathcal{M}_1^* + \mathcal{O}(\lambda^2)$$

possible caveat: negative cross section as a consequence of the truncation of the expansion
possible effects on $d\sigma$:

- azimuthal harmonic oscillation proportional to $\vec{E}_\perp$ and $\vec{B}_\perp$
- corrections proportional to $\vec{E}_\parallel$ and $\vec{B}_\parallel$ independent of ϕ

analytic calculation via FORM, numerical calculations using a modified version of the "optimizing matrix element generator" O'Mega
$\longrightarrow$ observed effects on the cross section for $d\bar{d} \to W^+W^-$:

- cross sections for different combinations of helicities form a strong hierarchy
- effects $\propto \kappa_2$ negligible

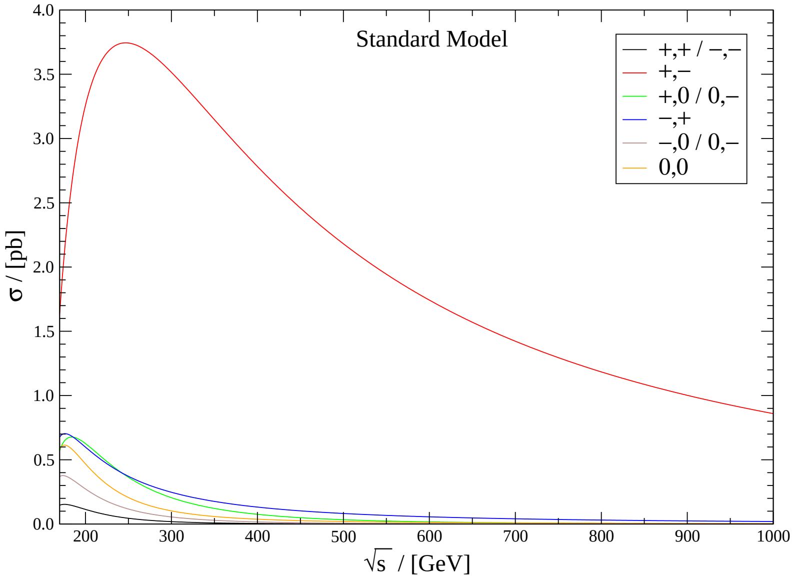
analytic calculation via FORM, numerical calculations using a modified version of the "optimizing matrix element generator" O'Mega
$\longrightarrow$ observed effects on the cross section for $d\bar{d} \to W^+W^-$:

- cross sections for different combinations of helicities form a strong hierarchy
- effects $\propto \kappa_2$ negligible

actual corrections proportional to

$\vec{E}_\perp$	azimuthal oscillation for most combinations of helicities; partial cancellation
$\vec{B}_\perp$	azimuthal oscillations only for suppressed helicities
$\vec{E}_\parallel$	extremely small corrections for all combinations of helicities depending on Γ_Z; negligible
$\vec{B}_\parallel$	very small corrections only for suppressed helicites

integrated cross section for different helicities:



- Monte-Carlo simulation using the “generator-generator” WHiZard / O’Mega
- caveat: very small negative cross section in some regions of phase-space $\longrightarrow$ regularization by the replacement

$$d\sigma \to \max\{d\sigma, 0\}$$

- events binned w.r.t. helicities taken from Monte-Carlo data
- cut 200 GeV $\leq \sqrt{s} \leq$ 1 TeV

- Monte-Carlo simulation using the “generator-generator” WHiZard / O’Mega
- caveat: very small negative cross section in some regions of phase-space $\longrightarrow$ regularization by the replacement

$$d\sigma \to \max\{d\sigma, 0\}$$

- events binned w.r.t. helicities taken from Monte-Carlo data
- cut 200 GeV $\leq \sqrt{s} \leq$ 1 TeV

boost from lab frame into parton CMS mixes up transverse components of $\vec{E}$ and $\vec{B}$

$$E_1 \xrightarrow{\Lambda} \gamma(E_1 - \beta B_2) \qquad B_1 \xrightarrow{\Lambda} \gamma(B_1 + \beta E_2)$$

$$E_2 \xrightarrow{\Lambda} \gamma(E_2 + \beta B_1) \qquad B_2 \xrightarrow{\Lambda} \gamma(B_2 - \beta E_1)$$

$\longrightarrow$ influence of $\vec{B}$ greatly enhanced in the hadronic process!

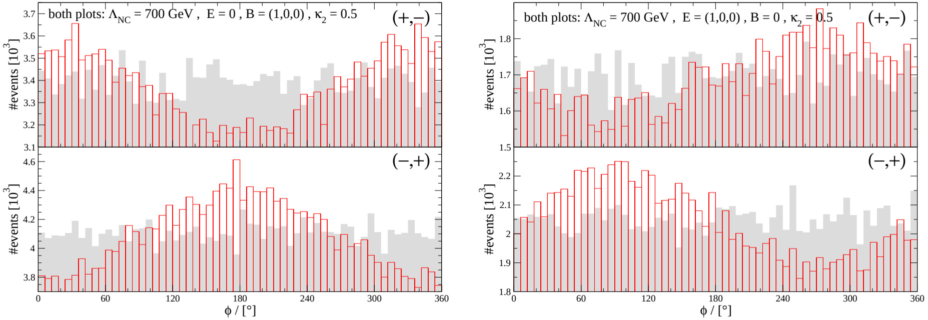
Azimuthal oscillation for $(-,+)$ and $(+,-)$ proportional to $\vec{B}_\perp$, but:

cancellation

of the oszillation $\propto \vec{E}_\perp$ between events with $\bar{q}$ from negative x_3 direction and those with $\bar{q}$ from positive direction

possible cuts:

$$0^\circ \leq (\theta^+ + \theta^-) \leq 180^\circ \qquad \text{or} \qquad 180^\circ \leq (\theta^+ + \theta^-) \leq 360^\circ$$



differences between distributions for the two different polar cuts

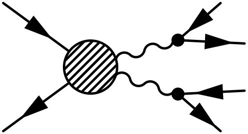
- $\propto \vec{E}_\perp$: azimuthal distribution shifted by π
- $\propto \vec{B}_\perp$: nothing changes

independent measurement of $\vec{E}_\perp$

1. obtain azimuthal distributions for the complementary polar cuts
2. add them shifted by π

$\longrightarrow$ oscillation $\propto \vec{B}_\perp$ cancels out!

Consider cascade-type diagrams for W production and subsequent decay (just like LEP2)



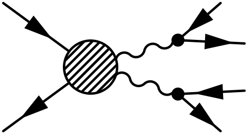
$\longrightarrow$ decomposition of the cross section into sum over helicities:

$$\frac{d\sigma}{d^3k_+\, d^3k_-\, d\bar{\theta}_+\, d\bar{\theta}_-} \propto \sum_{rs} \frac{d\sigma_{rs}}{d^3k_+\, d^3k_-} P_r\left(\cos\bar{\theta}_+\right) P_s\left(\cos\bar{\theta}_-\right)$$

with angular distributions

$$P_r(x) = \begin{cases} \frac{1}{2}(1+x)^2 & , r = 1 \\ \frac{1}{2}(1-x)^2 & , r = -1 \\ 1 - x^2 & , r = 0 \end{cases}$$

Consider cascade-type diagrams for W production and subsequent decay (just like LEP2)



$\longrightarrow$ decomposition of the cross section into sum over helicities:

$$\frac{d\sigma}{d^3k_+\, d^3k_-\, d\bar\theta_+\, d\bar\theta_-} \propto \sum_{rs} \frac{d\sigma_{rs}}{d^3k_+\, d^3k_-} P_r\left(\cos\bar\theta_+\right) P_s\left(\cos\bar\theta_-\right)$$

with angular distributions and projectors

$$P_r(x) = \begin{cases} \frac{1}{2}(1+x)^2 & ,r=1 \\ \frac{1}{2}(1-x)^2 & ,r=-1 \\ 1-x^2 & ,r=0 \end{cases} \qquad Q_s(x) = \begin{cases} -\frac{1}{2}+x+\frac{5}{2}x^2 & ,s=1 \\ -\frac{1}{2}-x+\frac{5}{2}x^2 & ,s=-1 \\ 2-5x^2 & ,s=0 \end{cases}$$

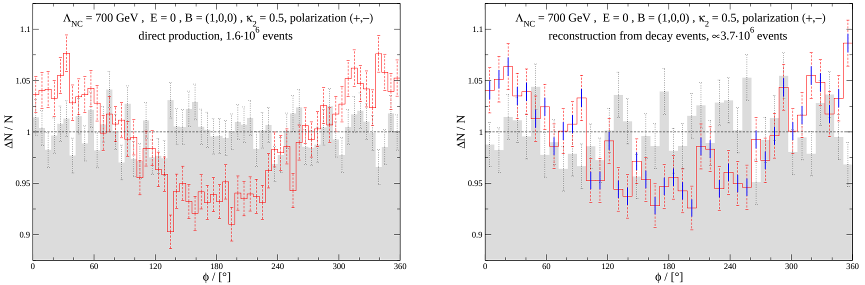
polynomials Q_s are orthogonal to the P_r

$$\int_{-1}^{1} dx\, Q_s(x) P_s(x) = \delta_{rs} \int_{-1}^{1} dx\, P_r(x)^2$$

$\longrightarrow$ reconstruction of the polarized production cross section by folding the unpolarized cross section with the Q_s.

caveats

- fermion charge neccesary to discriminate between transverse polarizations; very hard in the case of hadronic decay
- neutrino momentum must be reconstructed in the case of leptonic decay $\rightarrow$ pair production with semileptonic decay is favourable
- statistical error signisficantly larger $\sqrt{N}$



Reconstruction of the neutrino momentum:

transverse components p^1, p^2 from momentum conservation, mass shell condition defines an hyperbola in the p^0-p^3 plane

$$p^{0^2} - p^{3^2} - |p_\perp|^2 = 0$$

mass shell condition of the intermediary $W \longrightarrow$ straight line

$$(p+q)^2 = 2\left(p^0 q^0 - p^3 q^3 - \vec{p}_\perp \vec{q}_\perp\right) = m_W^2$$

$\longrightarrow$ in general two points of intersection corresponding to “valid” momenta

choice: select the momentum that minimizes $\cos(\theta_+ - \theta_-)$

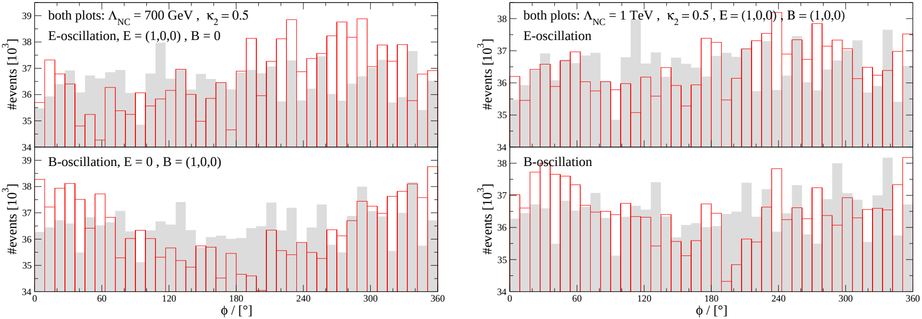
- diagrams decompose into two gauge equivalence classes; implementation of the class containing the cascade diagrams
- additional cut $70\,\text{GeV} \leq m_+ \leq 90\,\text{GeV}$ and acceptance cuts $5^\circ \leq \theta \leq 175^\circ$ (not on neutrino momentum)
- integrated luminosity $\int dt\, \mathcal{L} = 400\,\text{fb}^{-1}$, sum over 3 fermion flavours and 2 quark generations

- diagrams decompose into two gauge equivalence classes; implementation of the class containing the cascade diagrams
- additional cut 70 GeV $\leq m_+ \leq$ 90 GeV and acceptance cuts 5° $\leq \theta \leq$ 175° (not on neutrino momentum)
- integrated luminosity $\int dt\, \mathcal{L} = 400\,\text{fb}^{-1}$, sum over 3 fermion flavours and 2 quark generations

data analysis

1. reconstruct the neutrino momentum
2. weight events with the Q_s and bin them to obtain the helicity distributions
3. add $(-,+)$ and $(+,+)$ to obtain $(\mp,+)$; similarily for $(\pm,-)$
4. add $(\mp,+)$ and $(\pm,-)$ with a phase of π
5. no polar cuts $\rightarrow \vec{B}_\perp$-analysis; shift and add complementary polar cuts for $\vec{E}_\perp$-analysis

results:



- oscillation phase contains information how $\vec{E}_\perp$ and $\vec{B}_\perp$ are aligned in the x_1-x_2-plane
- magnitude of the oscillations contains information about the absolute value

- very distinct signal (azimuthal oscillation) due to breaking of lorentz invariance
- sensitivity limit somewhere around 1 TeV — not ideal for initial probing of the NCSM, but:
- observables allowing for the independent measurement of $\vec{E}_\perp$ and $\vec{B}_\perp$
- no bounds on κ_2 or $\vec{E}_\parallel$ and $\vec{B}_\parallel$

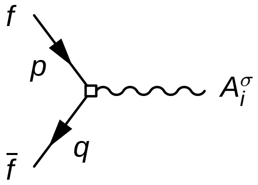
experimental challenges:

- reconstruction of the helicity distributions produces large error bars $\longrightarrow$ statistics is critical
- ambiguity in the determination of the neutrino momentum $\longrightarrow$ better criterion for choosing between solutions?

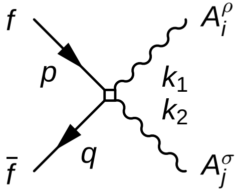
Seiberg-Witten-Maps to first order in λ

$$\hat{\tau}(\tau, A) = \tau + \frac{\lambda}{4}\theta^{\mu\nu}\left\{\partial_\mu \tau, A_\nu\right\} + \mathcal{O}(\lambda^2)$$
$$\hat{A}^\mu(A) = A^\mu - \frac{\lambda\theta^{\rho\sigma}}{4}\left\{A_\rho, \partial_\sigma A^\mu + F_\sigma{}^\mu\right\} + \mathcal{O}(\lambda^2)$$
$$\hat{\Psi}(\Psi, A) = \Psi + \frac{\lambda\theta^{\mu\nu}}{4}\left(iA_\mu A_\nu \Psi - 2A_\mu \partial_\nu \Psi\right) + \mathcal{O}(\lambda^2)$$

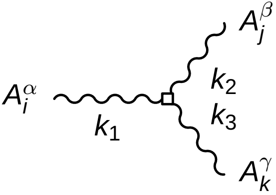
Feynman rules:



$$= \frac{\lambda}{2} T_i p_\mu q_\nu \left(\theta^{\mu\nu}\gamma^\sigma + \theta^{\sigma\mu}\gamma^\nu + \theta^{\nu\sigma}\gamma^\mu\right)$$



$$= \frac{\lambda}{2}\Big(\not p\,\theta^{\sigma\rho}\left[T_i, T_j\right] + \left(\not k_1\theta^{\sigma\rho} + p\theta^{\sigma}\gamma^{\rho} + q\theta^{\rho}\gamma^{\sigma}\right) T_i T_j + \left(\not k_2\theta^{\rho\sigma} + p\theta^{\rho}\gamma^{\sigma} + q\theta^{\sigma}\gamma^{\rho}\right) T_j T_i + \left(k_1\theta^{\sigma}\gamma^{\rho} + k_2\theta^{\rho}\gamma^{\sigma}\right)\left\{T_i, T_j\right\}\Big)$$

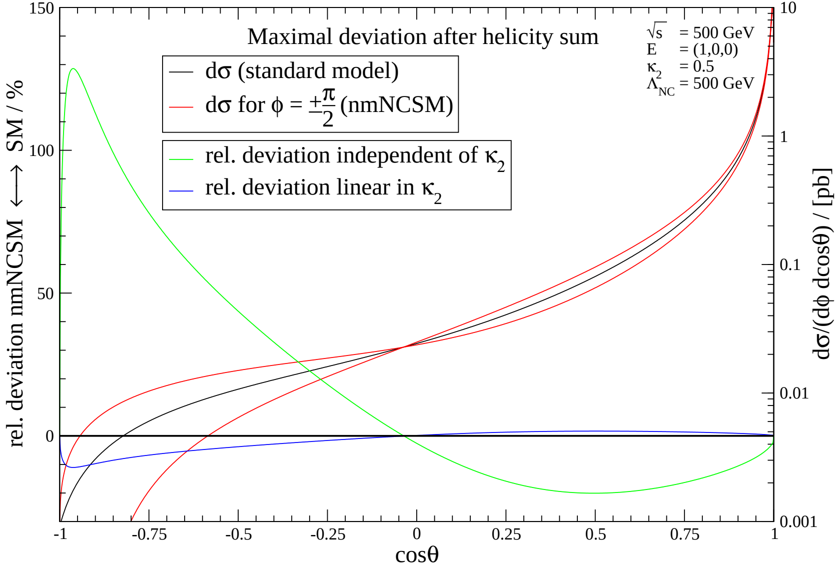


$$= -\lambda T_{i\{jk\}}\,\Theta_3\left((k_1,\alpha),(k_2,\beta),(k_3,\gamma)\right)$$

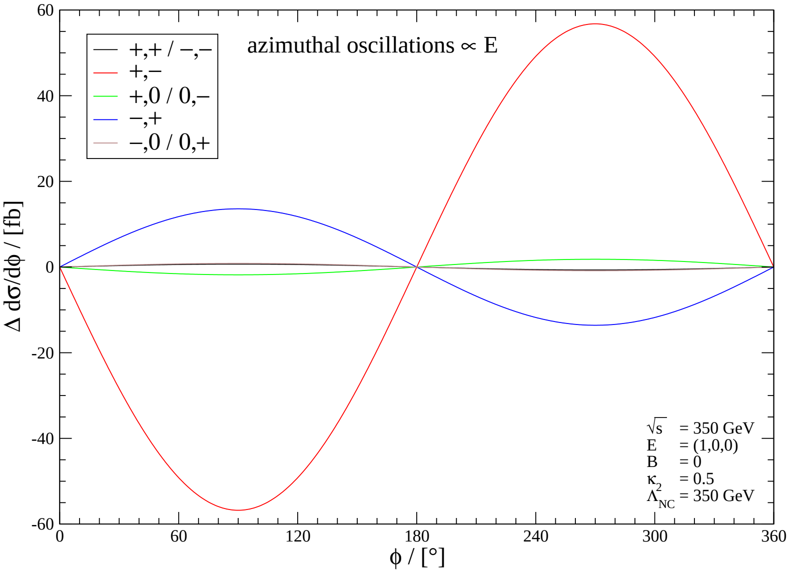
with the group factor

$$T_{i\{jk\}} = \mathbf{Tr}\,T_i\left\{T_j, T_k\right\} = \frac{1}{3}\sum_{\text{Perm.}} \mathbf{Tr}\,T_i T_j T_k$$

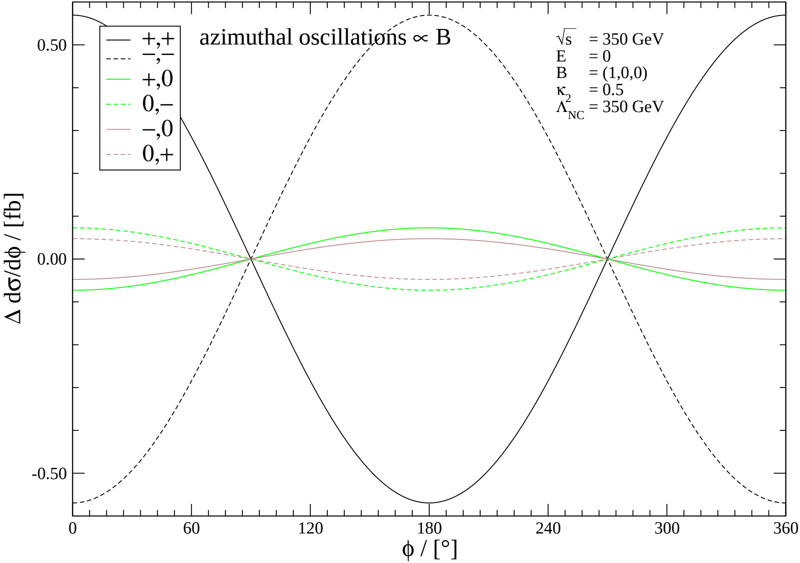
deviation Standard Model vs. NCSM, summed over helicities

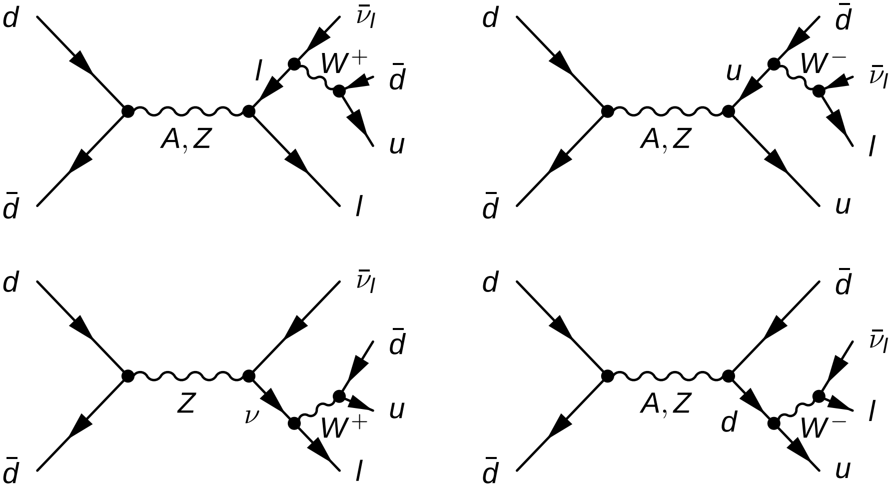


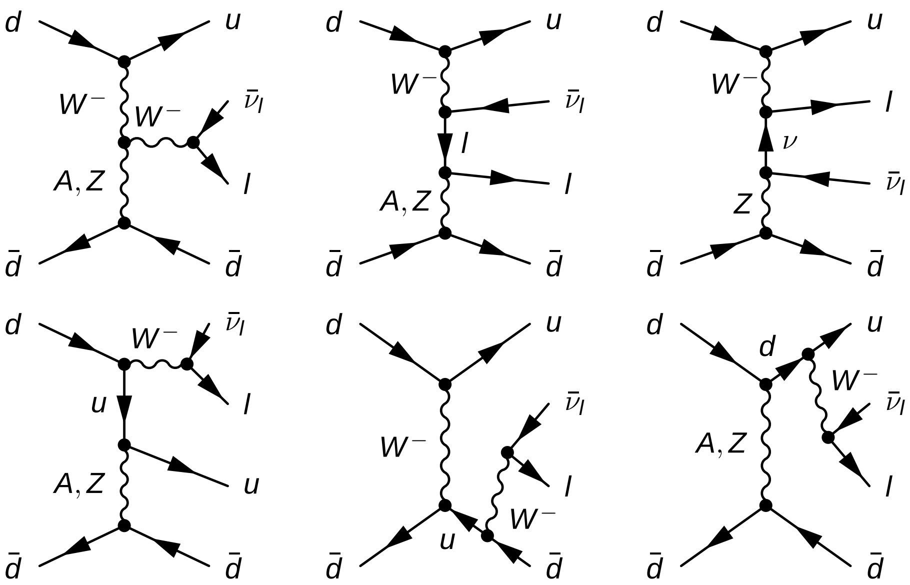
azimuthal oscillation $\propto \vec{E}_\perp$

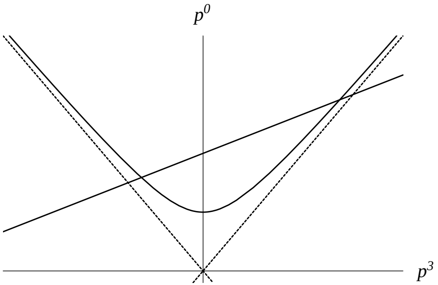


azimuthal oscillation $\propto \vec{B}_\perp$




d
$\bar{d}$
A, Z
l
W^+
$\bar{\nu}_l$
$\bar{d}$
u
l
d
$\bar{d}$
A, Z
u
W^-
$\bar{d}$
$\bar{\nu}_l$
l
u
d
$\bar{d}$
Z
ν
W^+
$\bar{\nu}_l$
$\bar{d}$
u
l
d
$\bar{d}$
A, Z
d
W^-
$\bar{d}$
$\bar{\nu}_l$
l
u


d
u
W^-
W^-
$\bar{\nu}_l$
l
A, Z
$\bar{d}$
$\bar{d}$
d
u
W^-
$\bar{\nu}_l$
l
l
A, Z
$\bar{d}$
$\bar{d}$
d
u
W^-
l
ν
$\bar{\nu}_l$
Z
$\bar{d}$
$\bar{d}$
d
W^-
$\bar{\nu}_l$
l
u
u
A, Z
$\bar{d}$
$\bar{d}$
d
u
W^-
$\bar{\nu}_l$
l
W^-
u
$\bar{d}$
$\bar{d}$
d
u
d
W^-
$\bar{\nu}_l$
A, Z
l
$\bar{d}$
$\bar{d}$



asymptotes (quark mass shell):

$$p_0 = \pm 1$$

slope (W mass shell):

$$\left|\frac{q^3}{q^0}\right| = \frac{\sqrt{{q^0}^2 - q_\perp^2}}{q^0} < 1$$

$\longrightarrow$ two points of intersection

$$p^0_{1/2} = \frac{{q^0}^2\left(m_W^2 + 2\vec{p}_\perp\vec{q}_\perp\right) \pm q^3 A}{2q^0\left({q^0}^2 - {q^3}^2\right)} \qquad p^3_{1/2} = \frac{2q^3\vec{p}_\perp\vec{q}_\perp \pm A}{2\left({q^0}^2 - {q^3}^2\right)}$$

$$A = q^0\sqrt{\left(m_W^2 + 2\vec{q}_\perp\vec{p}_\perp\right)^2 + 4\left|q_\perp\right|^2\left({q^3}^2 - {q^0}^2\right)}$$