The running of α_s and m_b in dimensional reduction

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work in collaboration with

D.R.T. Jones, P. Kant, L. Mihaila, M. Steinhauser

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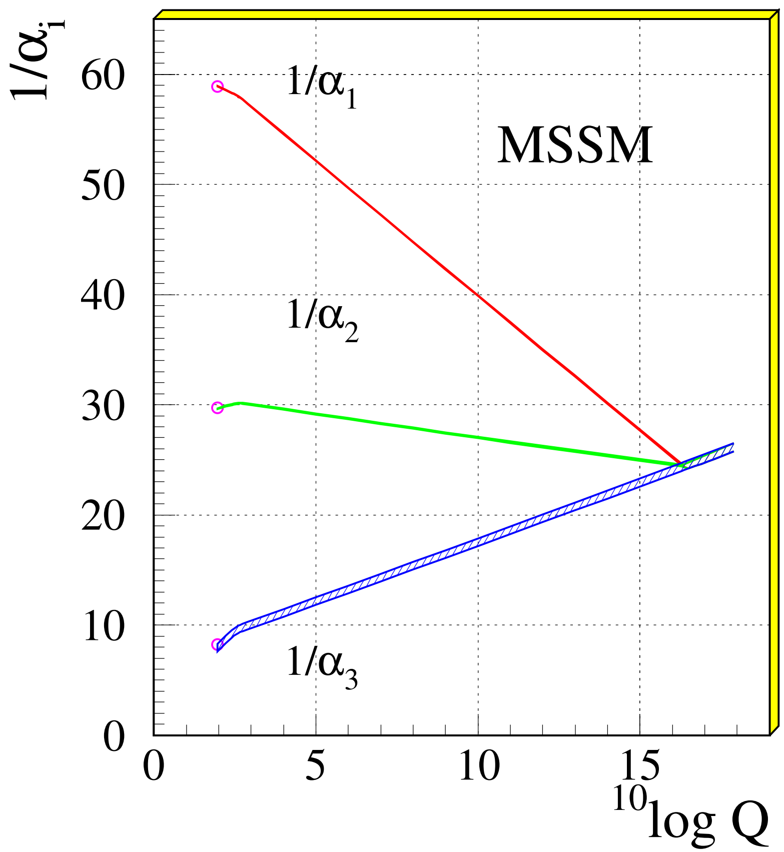

[Amaldi, de Boer, Fürstenau]
[Langacker, Luo]
[Ellis, Kelley, Nanopoulos]

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  SUSY theory

  Need consistent

  - running
  - matching
  - $\overline{\rm MS}$ – $\overline{\rm DR}$ conversion

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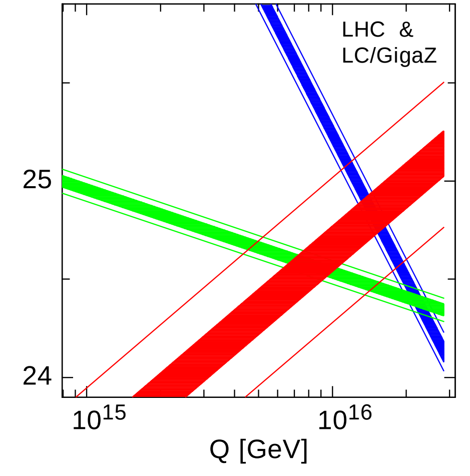
  $\longrightarrow$ SUSY breaking mechanism
  $\longrightarrow$ threshold effects from GUT theory
  $\longrightarrow$ ...



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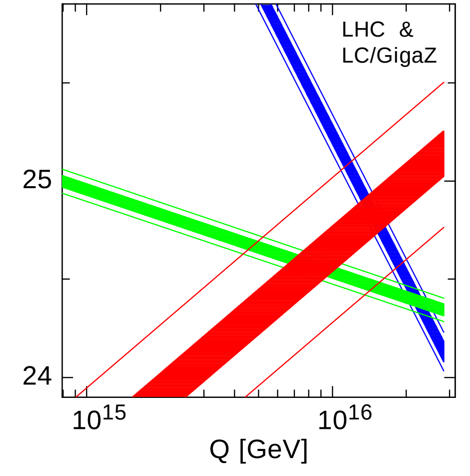


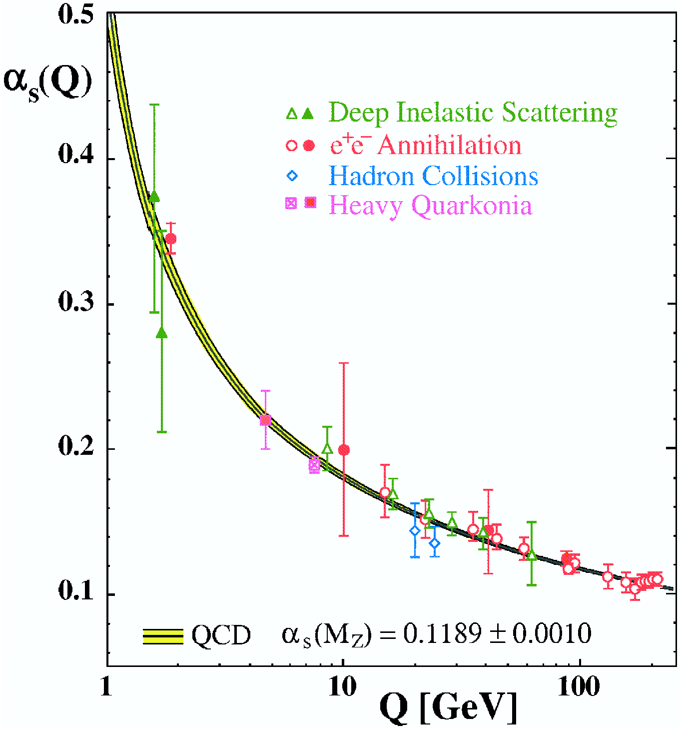
huge activity:
LHC-D BSM, Spectrum Codes, SPA project, ...

Running in QCD

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\,\alpha_s^{(n_f)} = \beta^{(n_f)}(\alpha_s^{(n_f)})$$

β function through 4 loops:

[v. Ritbergen, Larin, Vermaseren 97]
[Czakon 04]



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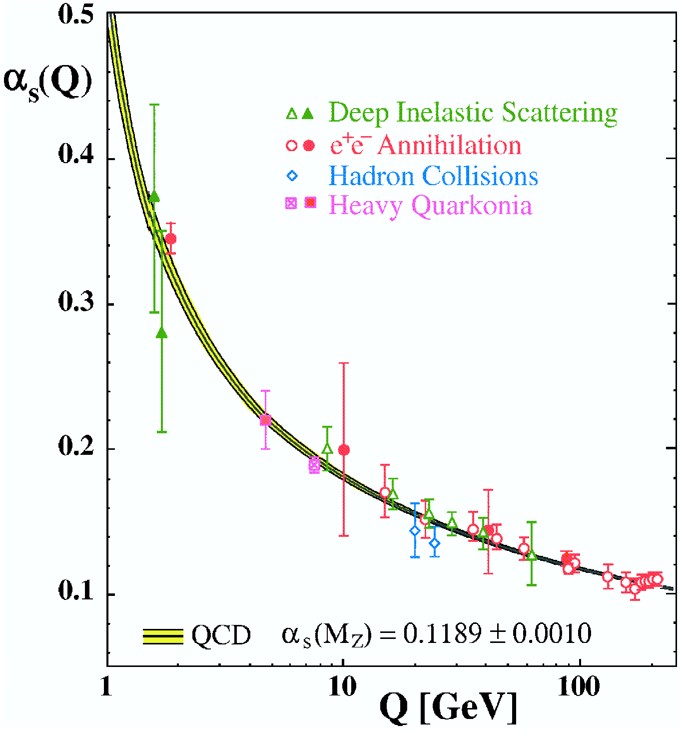

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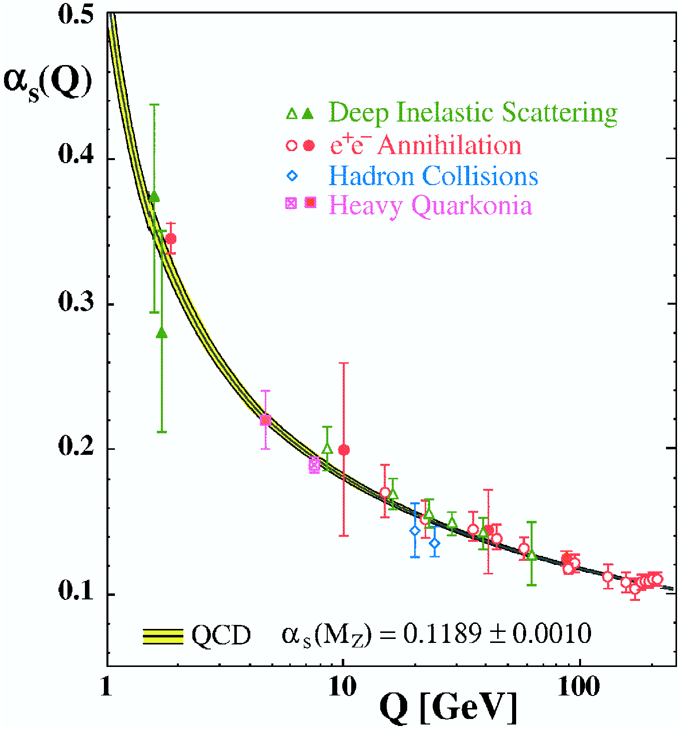
instead:

$$\alpha_s^{(n_f)}(\mu_h) = \zeta_\alpha\, \alpha_s^{(n_f+1)}(\mu_h)$$

ζ_α: decoupling coefficient



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Decoupling coefficients

$$\mathcal{L}_{\rm QCD} = \sum_{i=1}^{6} \bar{q}_i \left(i\not{D} - m_i \right) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

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$$q'_i = \zeta_q q_i\,, \qquad m'_i = \zeta_m m_i\,, \qquad g'_s = \zeta_g g_s\,, \qquad \cdots$$

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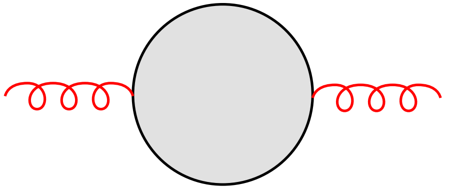
decoupling condition:

$$\Gamma^{(6)}_{\rm light\ fields} = \Gamma^{(5)}_{\rm light\ fields} + \mathcal{O}(\frac{1}{m_t^2})$$

Decoupling coefficients



$$\int \mathrm{d}^4x \mathrm{e}^{ipx} \langle \mathrm{T} A'(x) A'(0) \rangle_{\mathrm{eff}} = \zeta_A^2 \int \mathrm{d}^4x \mathrm{e}^{ipx} \langle \mathrm{T} A(x) A(0) \rangle_{\mathrm{full}} + \mathcal{O}(\frac{p^2}{m_t^2})$$

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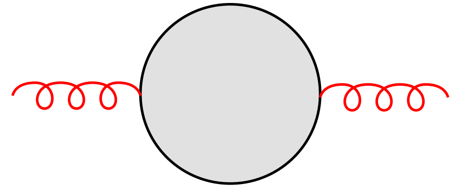

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$$= 1 + \mathcal{O}(\alpha_s) \qquad\qquad = \zeta_A^2 \left(1 + \mathcal{O}(\alpha_s)\right)$$

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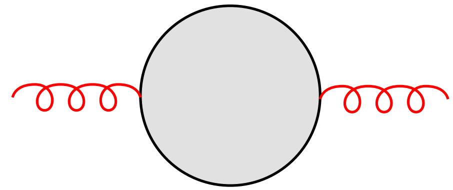

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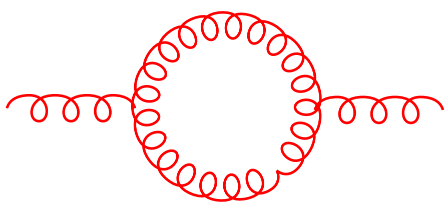
$+$ 

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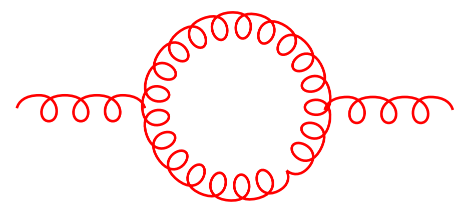
$+$ 

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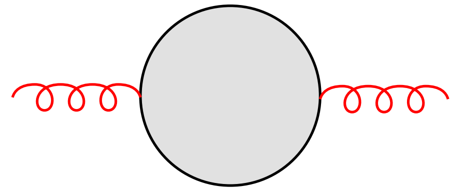

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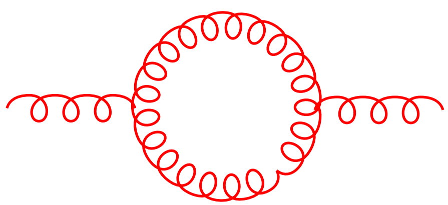
$+$ 

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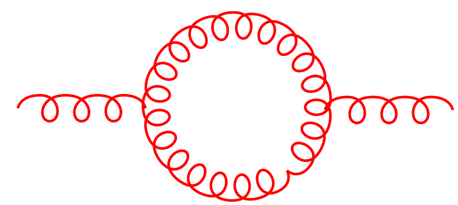
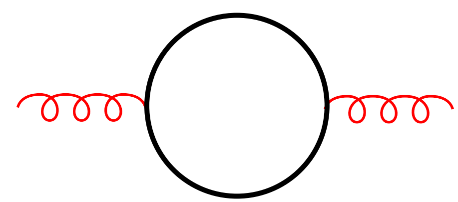
$+$ 

$+$  $)$

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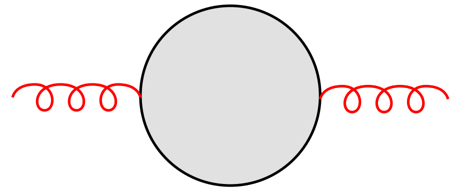

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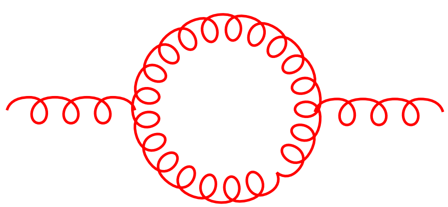
$+$ 

$p^2 = 0$

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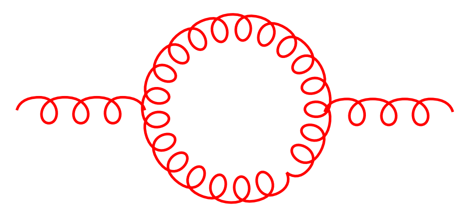
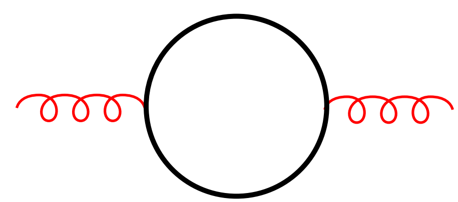
$+$ 

$+$  $)$

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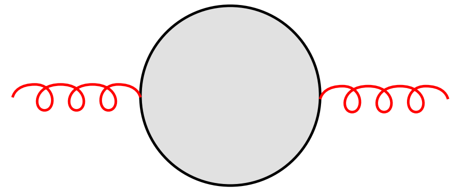

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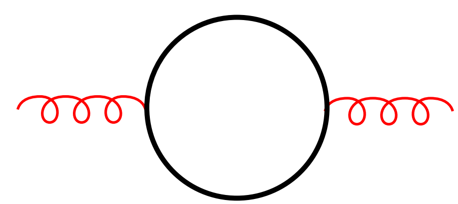

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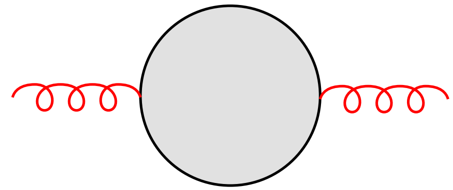

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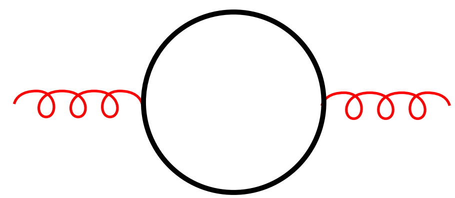
$\Rightarrow \qquad \zeta_A^2 = 1 -$  $\Big|_{p^2=0}$

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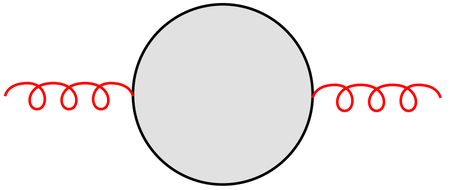

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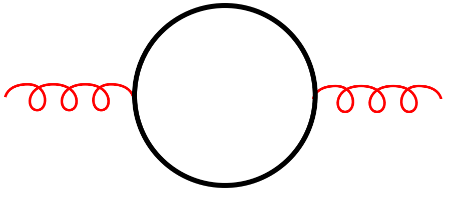
$$\Rightarrow \qquad \zeta_A^2 = 1 -$$  $\Big|_{p^2=0}$

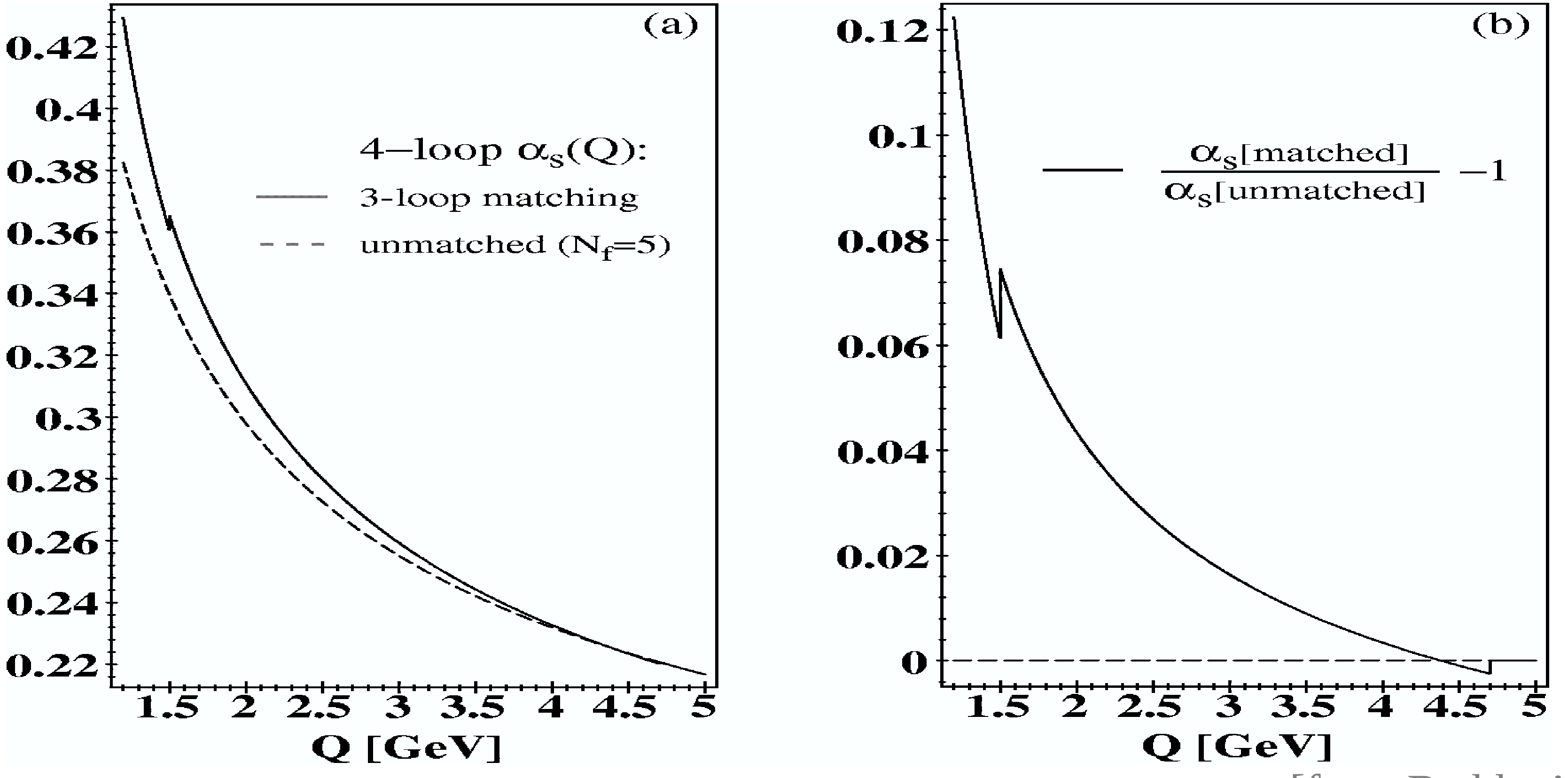
calculate through 3 loops using

FORM, MINCER, MATAD, EXP, . . .

[Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker; R.H.; . . .]

Matching

consistency: n-loop running $\leftrightarrow$ $(n-1)$-loop matching



[from Bethke ’06]

4-loop running in QCD: [v. Ritbergen, Larin, Vermaseren 97][Czakon 04]
3-loop matching in QCD: [Chetyrkin, Kniehl, Steinhauser 97]

Higher orders in SUSY

- Dimensional Regularization (DREG) ['t Hooft, Veltman 72]
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  finite SUSY-restoring CT's required

- alternative(?): Dimensional Reduction (DRED) [Siegel 79]
  - keep vector fields 4-dimensional
  - compactify space-time to $d = 4 - 2\epsilon < 0$
  - seems consistent with SUSY so far (in practical calc's)
  - but: restricted algebraic operations (inconsistencies with $\epsilon_{\mu\nu\rho\sigma}$)
    [Siegel 80][Stöckinger 05]

Dimensional Reduction

- 4-dimensional (vector-)fields on d-dimensional space, $d = 4 - 2\epsilon < 4$:

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$$g^{(\epsilon)}_{\mu\nu} g^{(\epsilon),\mu\nu} = 2\epsilon\,, \qquad g^{(d)}_{\mu\nu} g^{(\epsilon),\mu\nu} = 0$$

- 4-vector $v^{(4)}_\mu$:

$$v^{(d)}_\mu = g^{(d)}_{\mu\nu} v^{(4),\nu}\,, \qquad v^{(\epsilon)}_\mu = g^{(\epsilon)}_{\mu\nu} v^{(4),\nu}\,,$$

$$v^{(4)}_\mu = v^{(d)}_\mu + v^{(\epsilon)}_\mu$$

Dimensional Reduction

$$\mathcal{L}^{(4)}(A_\mu^{(4)}, \psi, \ldots) = \mathcal{L}^{(d)}(A_\mu^{(d)}, \psi, \ldots) + \mathcal{L}^{(\epsilon)}(A_\mu^{(d)}, A_\mu^{(\epsilon)}, \psi, \ldots)$$

$$A_\mu^{(4)}(x) = A_\mu^{(d)}(x) + A_\mu^{(\epsilon)}(x)$$

- $A_\mu^{(\epsilon)}(x)$: "epsilon scalar"

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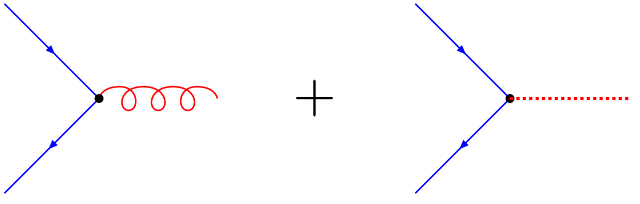
- $A_\mu^{(\epsilon)}(x)$: "epsilon scalar"
- example:

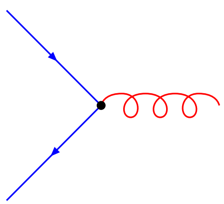
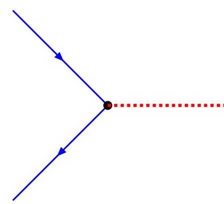
$$A_\mu^{(4)}\bar\psi\psi = A_\mu^{(d)}\bar\psi\psi + A_\mu^{(\epsilon)}\bar\psi\psi$$



$\rightarrow$ additional Feynman rules for epsilon scalars

Renormalization

- SUSY: $Z^{(d)} \stackrel{!}{=} Z^{(\epsilon)}$
- non-SUSY: $Z^{(d)} \neq Z^{(\epsilon)}$ in general

 $\longrightarrow \alpha_s$  $\longrightarrow \alpha_e$

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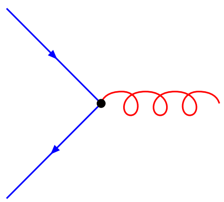
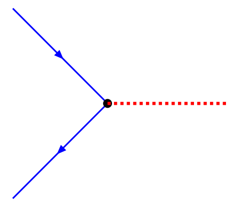
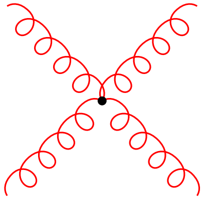


$\longrightarrow \alpha_s$

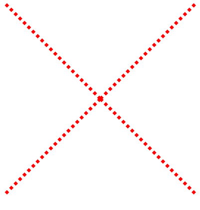


$\longrightarrow \alpha_e$

even worse:



$\longrightarrow \alpha_s \, f^{abe} f^{cde}$



$\longrightarrow \lambda_1 \, f^{abe} f^{cde}, \quad \lambda_2 \, \delta^{ab} \delta^{cd}, \quad \lambda_3 \, d^{abe} d^{cde}$

Running of α_s in SUSY

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  - consider various scenarios:

    (A) $\ldots$

    (B) $\ldots$

    (C) $M_{\rm SUSY} \gg m_t \gg m_b$: $\quad \alpha_s^{(5)} \to \alpha_s^{(6)} \to \alpha_s^{(\rm full)}$

    (D) $M_{\rm SUSY}, m_t \gg m_b$: $\quad \alpha_s^{(5)} \to \alpha_s^{(\rm full)}$

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    (C) $M_{\rm SUSY} \gg m_t \gg m_b$: $\alpha_s^{(5)} \to \alpha_s^{(6)} \to \alpha_s^{(\rm full)}$

    (D) $M_{\rm SUSY}, m_t \gg m_b$: $\alpha_s^{(5)} \to \alpha_s^{(\rm full)}$

- note: input is $\alpha_s^{(5)}(M_Z)$ in $\overline{\rm MS}$ scheme!

  need conversion: $\alpha_s^{\overline{\rm MS}} \leftrightarrow \alpha_s^{\overline{\rm DR}}$

$\overline{\mathrm{MS}} - \overline{\mathrm{DR}}$ *conversion*

value of α_s in *physical scheme* independent of regularization:

$$\alpha_s^{\mathrm{ph}} = z_{\overline{\mathrm{MS}}}^{\mathrm{ph}}\,\alpha_s^{\overline{\mathrm{MS}}}\,, \qquad \alpha_s^{\mathrm{ph}} = z_{\overline{\mathrm{DR}}}^{\mathrm{ph}}\,\alpha_s^{\overline{\mathrm{DR}}}\,,$$

$$\Rightarrow \quad \alpha_s^{\overline{\mathrm{DR}}} = \left(z_{\overline{\mathrm{MS}}}^{\mathrm{ph}}/z_{\overline{\mathrm{DR}}}^{\mathrm{ph}}\right)\alpha_s^{\overline{\mathrm{MS}}}\,.$$

$$\alpha_s^{\overline{\mathrm{DR}}} = \alpha_s^{\overline{\mathrm{MS}}}\left[1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi} + \frac{11}{8}\left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right)^2 - \frac{n_f}{12}\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{\alpha_e}{\pi} + \ldots\right]$$

[R.H., Kant, Mihaila, Steinhauser 06]

even 3-loop: [R.H., Jones, Kant, Mihaila, Steinhauser 06]

$\alpha_s^{(5)}(M_Z) \to \alpha_s(M_{\rm GUT})$

- Example:

$$\alpha_s^{(5),\overline{\rm MS}}(M_Z) \to \alpha_s^{(5),\overline{\rm MS}}(M_{\rm SUSY})$$ — QCD running in $\overline{\rm MS}$

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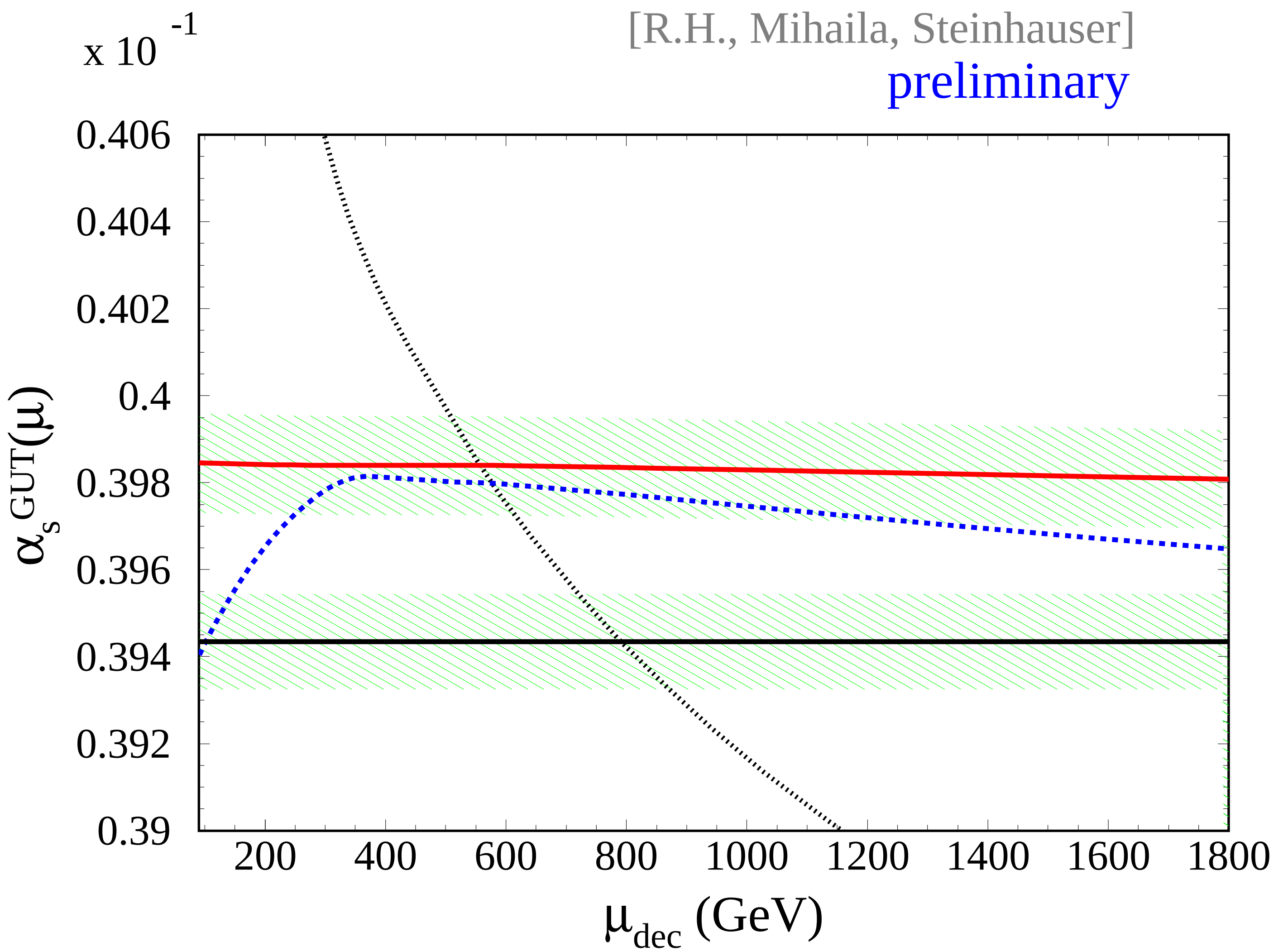
$\alpha_s(M_{\mathrm{GUT}})$ from $\alpha_s(M_Z)$



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$\alpha_s^{\overline{\rm DR}}$ *in QCD*

- coupled differential equations:

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- evaluate $m_b^{\overline{\rm DR}}(M_Z)$ from $m_b^{\overline{\rm MS}}(m_b)$ in two different ways:

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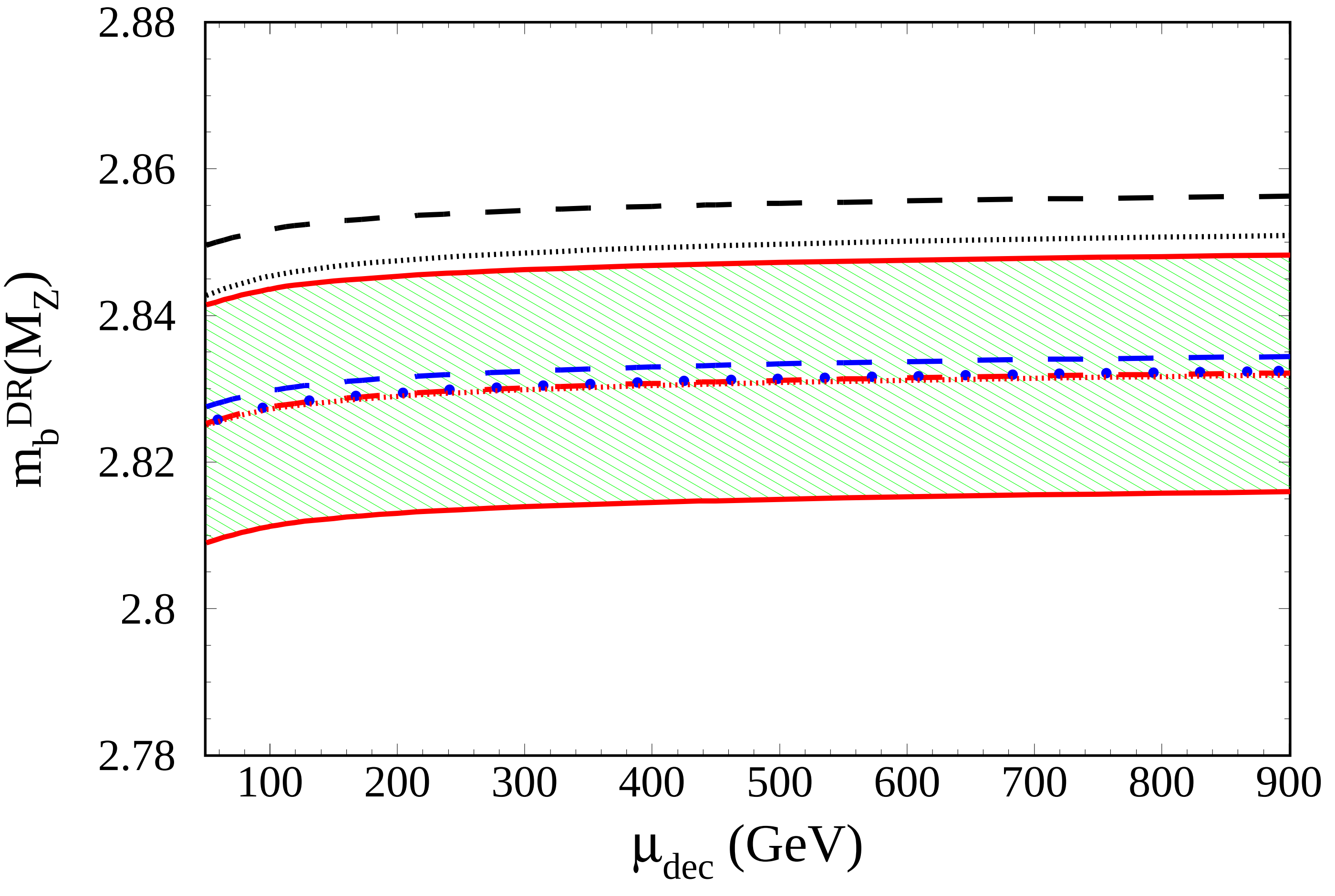
$\to$ consistent?

$m_b^{\overline{\mathrm{DR}}}(M_Z)$ *from* $m_b^{\overline{\mathrm{MS}}}(m_b)$

[R.H., Mihaila, Steinhauser]
preliminary



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