MadDipole: dipole subtraction method in MadGraph

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Outline

- Motivation
- What is available now in the MadDipole package
- What will be available soon

Motivation...

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[\int_{\mathrm{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

- "Most" of the contributions to a NLO cross section are tree-level like
  - In particular, the Born, real and subtraction terms can be described by tree-level processes
- Many tree-level Monte Carlo event generators available!
  - We use MadGraph/MadEvent
    [*Alwall, Demin, de Visscher, RF, Herquet, Maltoni, Plehn, Rainwater & Stelzer*]

...AND GOAL

$$\sigma^{\rm NLO} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[\int_{\rm loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

- We want a completely automatic Monte Carlo program that calculates all the tree-level like contributions to any NLO computation in QCD
  - Within the MadGraph/MadEvent framework
  - Dipoles for the subtraction terms [*Catani & Seymour*]
  - Massless and Massive final state particles [*Catani, Dittmaier, Seymour & Trocsanyi*]
  - Open source: available to all

What is available now

What is in the package

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[\int_{\mathrm{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

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Normal MadGraph

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Normal MadGraph

Dipole subtraction terms

What is in the package

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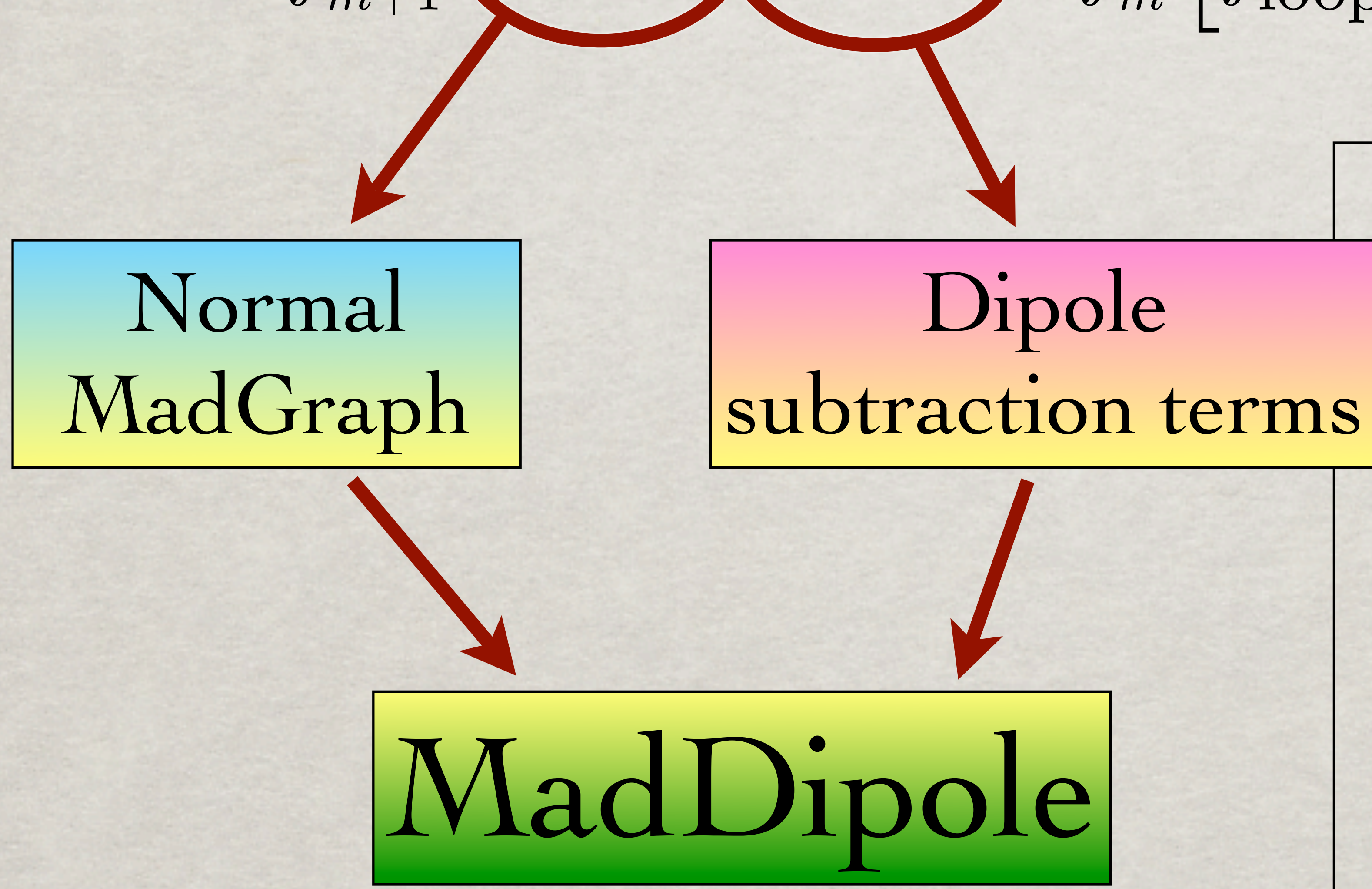
Normal MadGraph

Dipole subtraction terms

- Dipole splitting functions
- Color linked Borns
- 'Off-diagonal helicity' terms
- Subroutine: DIPOLESUM(P, DIP)

What is in the package

$$\sigma^{\rm NLO} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[\int_{\rm loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$



- Dipole splitting functions
- Color linked Borns
- ‘Off-diagonal helicity’ terms
- Subroutine: **DIPOLESUM(P, DIP)**

In practice

- Download the MadDipole package from any of the MadGraph web pages, e.g., http://madgraph.phys.ucl.ac.be
- Edit the *./Cards/proc_card.dat* and give the real radiation (n+1) process, e.g., *gg>tt~bb~g*
- Run *./bin/newprocess* and all necessary files will be created in the *./SubProcesses/P1_gg_ttxbbxg/* directory
- Exactly the same syntax as running the normal MadGraph code!

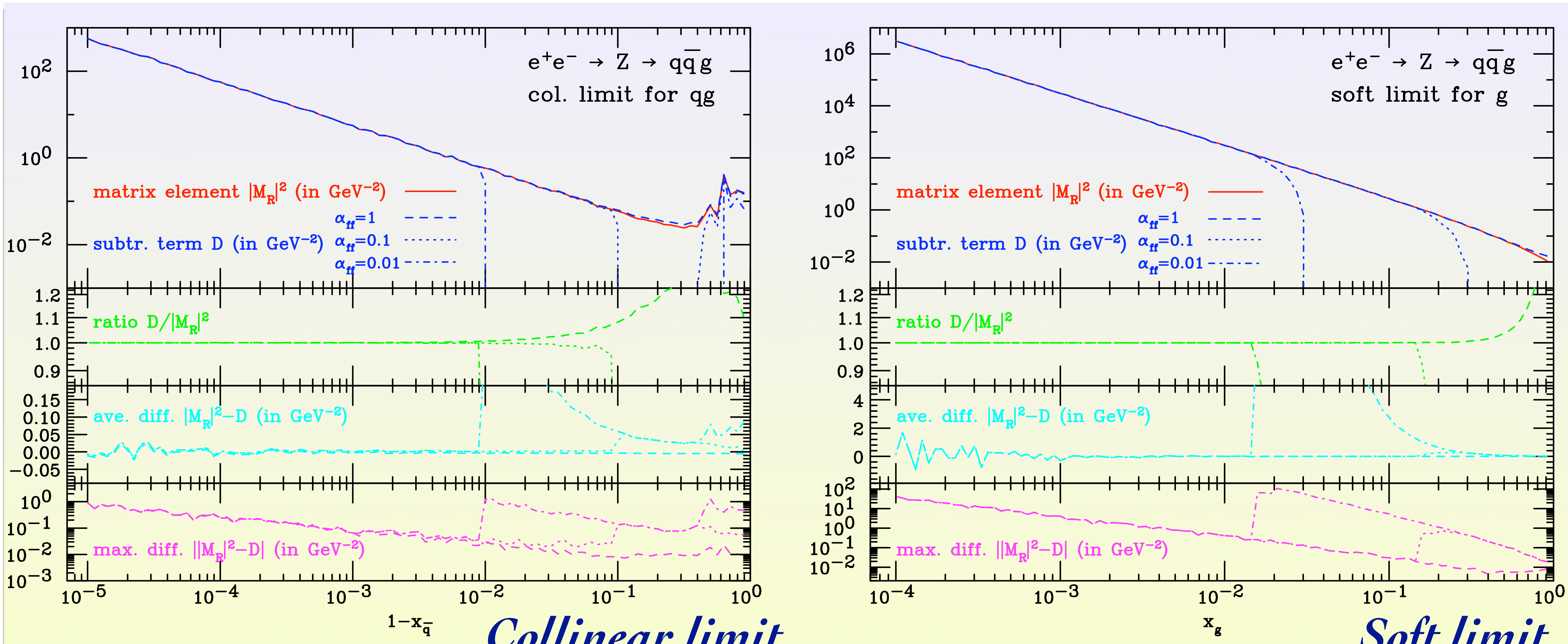
Finite dipoles due to heavy particles

- In processes with top or bottom quarks there are dipoles for which the unresolved parton is massive
- In this case the collinear singularities are regulated by the mass. However, it could develop potentially large logarithms
- Our code calculates all dipoles, but puts the non-divergent into a separate subroutine: DIPOLESUMFINITE(...)
- This subroutine can easily be included to cancel the potentially large logarithms
- In the limit of zero mass (or infinite energy), including the non-divergent dipoles is the same as generating the code with zero masses from the start

Phase-space restriction

- Subtraction is only needed when integrating close to the singular region
  - Include the dipoles only when approaching the collinear of soft limits, using the "alpha parameter" [*Nagy & Trocsanyi*]
  - For the subtraction terms to the reals, a simple theta function is sufficient
  - Four parameters: `alpha_ff`, `alpha_fi`, `alpha_if` & `alpha_ii` adjustable by the user

Checks



- Divergences are cancelled by the subtraction terms: ratio approaches one
- Left-over rise in difference is an integrable singularity

Further checks against MCFM

process	subprocesses (n+1)
Drell-Yan (W)	$q\bar{q}' \to W^+(\to e^+\nu_e)g$ $qg \to W^+(\to e^+\nu_e)q'$
Drell-Yan (Z)	$q\bar{q} \to Z(\to e^+e^-)g$ $qg \to Z(\to e^+e^-)q$
Drell-Yan (Z+jet)	$q\bar{q} \to Z(\to e^+e^-)q'\bar{q}'$ $q\bar{q} \to Z(\to e^+e^-)q\bar{q}$ $q\bar{q} \to Z(\to e^+e^-)gg$ $q\bar{g} \to Z(\to e^+e^-)qg$ $g\bar{g} \to Z(\to e^+e^-)q\bar{q}$
top quark pair ($t\bar{t}$)	$q\bar{q} \to t(\to bl^+\nu_l)\bar{t}(\to \bar{b}l^-\bar{\nu}_l)g$ $qg \to t(\to bl^+\nu_l)\bar{t}(\to \bar{b}l^-\bar{\nu}_l)q$ $gg \to t(\to bl^+\nu_l)\bar{t}(\to \bar{b}l^-\bar{\nu}_l)g$
t-channel single top with massive b-quark [63]	$gg \to t\bar{b}q\bar{q}'$ $qq' \to t\bar{b}q'q''$ $qq' \to t\bar{b}q'q''$ $qg \to t\bar{b}q'g$

- Point-by-point in phase-space check against MCFM [*Campbell & Ellis*]
- No discrepancies found
- This shows that all possible FI, IF and II dipoles (massless and massive) are okay

CHECK PROGRAM

- With the package a small code, called **CHECK_DIP**, is provided
- It calculates all the collinear and soft limits for the given process
- It prints the matrix element and subtraction terms for points closer and closer to the limit on the screen

Check program

$$e^+e^- \to Z \to u\bar{u}g$$

Collinear limit: p(4).p(5) goes to zero

p(4).p(5)/s(1,2)	sqrt(s(4, 5))	|M|^2	|Sub.term|^2	|M|^2/|Sub.term|^2
0.315364E+00	0.561573E+03	0.500612E-07	0.863461E-07	0.579774E+00
0.737460E-01	0.271562E+03	0.512878E-06	0.511343E-06	0.100300E+01
0.537588E-01	0.231859E+03	0.118507E-05	0.120238E-05	0.985603E+00
0.332833E-01	0.182437E+03	0.105116E-05	0.109007E-05	0.964305E+00
0.143302E-01	0.119709E+03	0.940775E-05	0.922832E-05	0.101944E+01
0.113982E-01	0.106762E+03	0.387269E-05	0.390105E-05	0.992730E+00
0.101978E-01	0.100984E+03	0.377575E-04	0.378739E-04	0.996928E+00
0.573922E-02	0.757576E+02	0.409948E-04	0.407579E-04	0.100581E+01
0.253423E-02	0.503411E+02	0.194822E-04	0.196108E-04	0.993440E+00
0.251508E-02	0.501505E+02	0.113841E-03	0.113628E-03	0.100188E+01
0.192366E-02	0.438595E+02	0.532619E-03	0.530903E-03	0.100323E+01
0.479893E-03	0.219065E+02	0.908399E-04	0.903827E-04	0.100506E+01
0.317967E-03	0.178316E+02	0.355093E-03	0.354674E-03	0.100118E+01
0.168300E-03	0.129730E+02	0.579438E-03	0.581225E-03	0.996925E+00
0.145372E-03	0.120571E+02	0.284682E-03	0.285169E-03	0.998294E+00
0.609018E-04	0.780396E+01	0.969336E-02	0.969342E-02	0.999994E+00
0.259173E-04	0.509091E+01	0.107919E-02	0.107984E-02	0.999399E+00
0.186237E-04	0.431552E+01	0.914959E-02	0.914747E-02	0.100023E+01
0.178298E-04	0.422253E+01	0.149607E-02	0.149572E-02	0.100023E+01
0.910116E-05	0.301681E+01	0.436849E-02	0.437017E-02	0.999616E+00
0.156505E-05	0.125102E+01	0.277892E-01	0.277948E-01	0.999800E+00
0.152407E-05	0.123453E+01	0.287201E-01	0.287121E-01	0.100028E+01
0.140303E-05	0.118449E+01	0.236653E-01	0.236693E-01	0.999831E+00
0.353217E-06	0.594321E+00	0.128877E+00	0.128881E+00	0.999970E+00

Check program

$$e^+e^- \to Z \to u\bar{u}g$$
$$\alpha_{ff} = 0.1$$

Limit: p(5) goes soft

p(0, 5)^2/s(1,2)	|M|^2	|Sub.term|^2	|M|^2/|Sub.term|^2
0.869211E-03	0.109575E-06	0.000000E+00	INF
0.688787E-03	0.193565E-06	0.000000E+00	INF
0.421426E-03	0.699189E-06	0.000000E+00	INF
0.387974E-03	0.659780E-06	0.000000E+00	INF
0.336094E-03	0.154721E-05	0.000000E+00	INF
0.185406E-03	0.376282E-05	0.265364E-05	0.141799E+01
0.137483E-03	0.818395E-05	0.816317E-05	0.100255E+01
0.103854E-03	0.269806E-04	0.269632E-04	0.100064E+01
0.637535E-04	0.314766E-04	0.314798E-04	0.999898E+00
0.376601E-04	0.122914E-03	0.122984E-03	0.999433E+00
0.191145E-04	0.780061E-03	0.778950E-03	0.100143E+01
0.179479E-04	0.565700E-03	0.565041E-03	0.100117E+01
0.129797E-04	0.111247E-02	0.111223E-02	0.100021E+01
0.105788E-04	0.203953E-02	0.203945E-02	0.100004E+01
0.774787E-05	0.346362E-02	0.346388E-02	0.999927E+00
0.563720E-05	0.610302E-02	0.610181E-02	0.100020E+01
0.343292E-05	0.235165E-01	0.235160E-01	0.100002E+01
0.909122E-06	0.347171E+00	0.347161E+00	0.100003E+01

What will be available soon

Integrated dipoles

$$\sigma^{\text{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[\int_{\text{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

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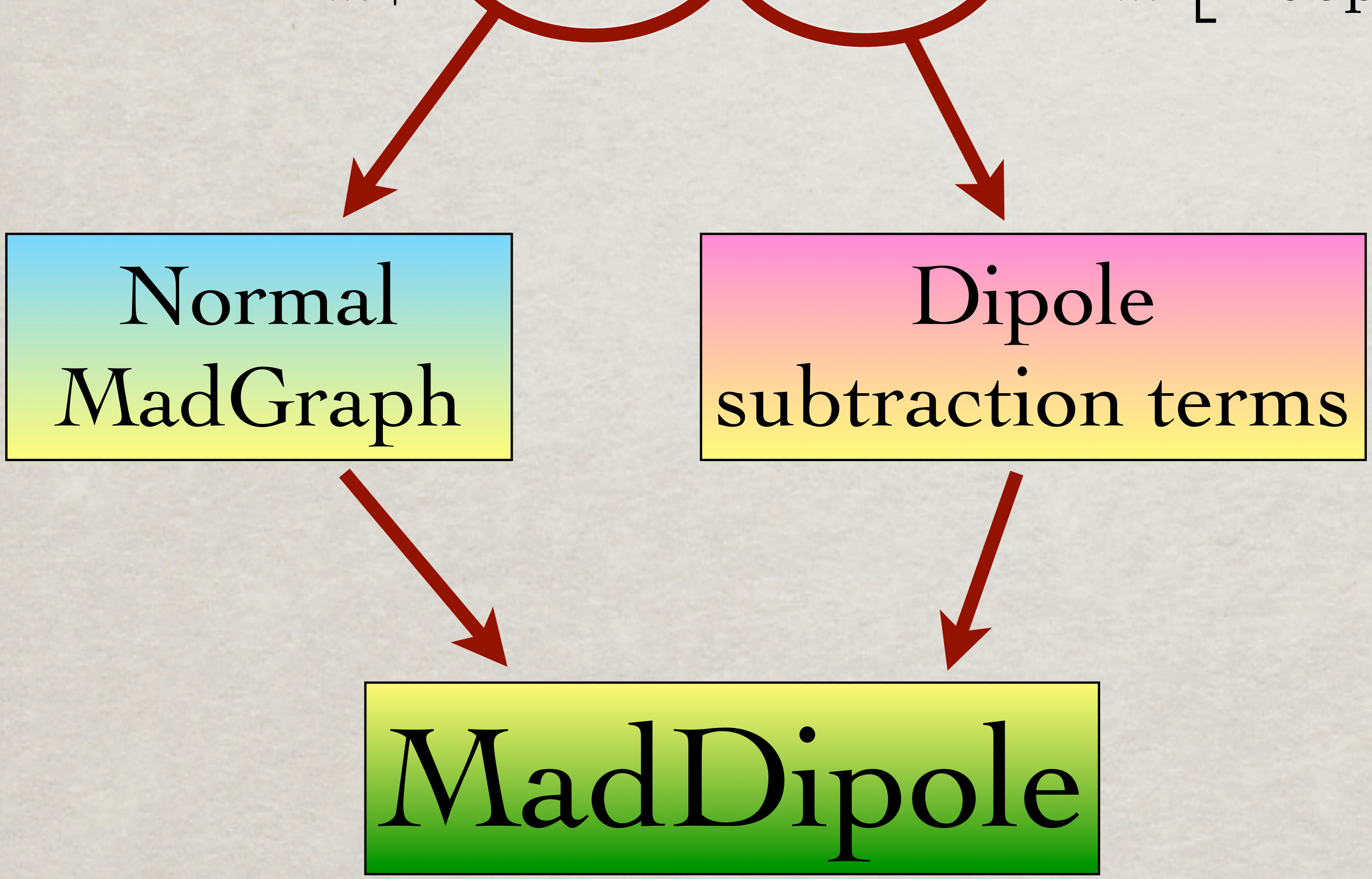

Integrated dipoles

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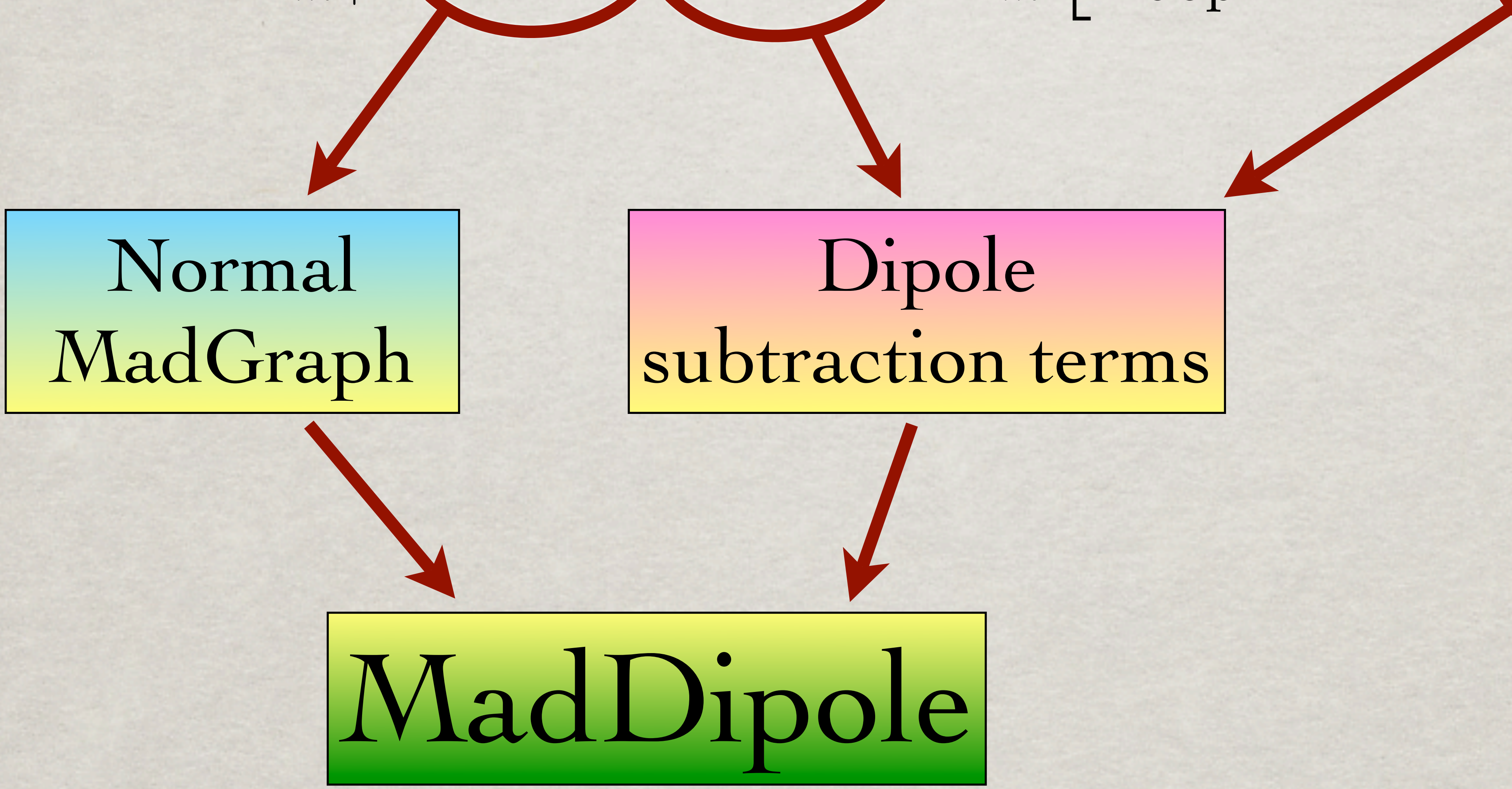

Integrated dipoles: some details

- For integrated splitting function with initial state particles contains plus-distributions and delta function: we need the Born's at x and also at x=*1*. (With x the parton energy fraction)
- Non-trivial dependence on the alpha parameter
  → New integrals were needed
- Results depend on regularization scheme: both 't Hooft-Veltman and Dimensional Reduction implemented
- Common factors in loop integrals can be excluded
  → Two implementations: with and without expanding this term:

$$\mathcal{V}_{ij,k} = \int \left[dp_i(\widetilde{p}_{ij},\widetilde{p}_k)\right] \frac{1}{2p_i\cdot p_j} < \boldsymbol{V}_{ij,k} > \equiv \frac{\alpha_S}{2\pi} \underbrace{\frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2}{s_{ij,k}}\right)^{\epsilon}}_{=1+\left(\log\left(\frac{4\pi\mu}{s_{ij,k}}\right)-\gamma\right)\epsilon+O(\epsilon^2)} \mathcal{V}_{ij}(\epsilon)$$

Integrated dipoles: in practice

- One new subroutine: INTDIPOLES(P,X, EPSSQ,EPS,FINITE)
- Momenta and parton energy fractions as input, and returns three 5-dimensional vectors with

  1) $\delta(1\text{-}x)$ terms

  2) terms regular in x

  3) plus-distribution (singular in x=1)

  - and for the massive case ($x_+ = 1\text{-}4m_f^2/Q^2$)

  4) $\delta(x_+\text{-}x)$ terms

  5) x_+-distributions

- Implementation done → needs heavy testing

Integrated dipoles: in practice

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  - and for the massive case (x

  4) $\delta(x_{+}-x)$ terms

  5) x_{+}-distributions

Divergent terms okay
Alpha dependence okay
Overall normalization is in progress...

- Implementation done → needs heavy testing

Phase space integration

- The phase space integration over subtracted NLO matrix elements is more complicated than at LO
- Not a problem for simple $2 \rightarrow 2$ (or $2 \rightarrow 3$) processes, but becomes an issue in more complicated cases
- We are working on a fully automatic phase space generator that can efficiently and in a parallel nature integrate the subtracted matrix elements

Beyond the SM

- MadGraph knows how to do BSM physics...
- However, in New Physics models with new colored particles different types of dipoles are needed
  - For example, SUSY squarks are heavy color triplet scalars
- The MadDipole code is modular enough to easily deal with these new structures, but the dipoles need to be implemented, tested, ...

Conclusions

- MadDipole is a package that calculates any real matrix element, including all dipole subtraction terms
- It calculates also the contributions to be added to the virtual corrections
- The phase space integration of the subtracted matrix elements is still work in progress
- An automatic NLO package that only needs the virtual corrections as external input is within reach

Conclusions

- MadDipole is a package that calculates any real matrix element, including all dipole subtraction terms
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wiki-page (accessible from any of the MadGraph web pages):
http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Software/MadDipole