Automating D-Dimensional Unitarity

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Which nlo calculations and why ?

6-particle processes

ttjj	background to ttH
ttbb	background to ttH
VVbb	background to VBF
VVjj	background to VBF
bbbb	susy Higgs, hidden valley
Vjjj(j)	background to new physics

+ A generic framework that can deal with all processes in a uniform manner will be useful

Is the virtual part the hardest ?

- **Not entirely obvious** (qq2ttbb reports that real emission with dipoles is slower than virtual part for same relative precision)

- **Catani-Seymour dipole approach automated**

  - Gleisberg, Krauss arXiv:0709.2881 (based on Amegic++, not yet public)
  - Seymour, Tevlin arXiv:0803.2231 (based on TeVJet)
  - Frederix, Gehrmann, Greiner arXiv:0808.2128 (within madgraph)
  - Hasegawa, Moch, Uwer arXiv:0807.3701 (in progress, MATHEMATICA + MadGraph)
  - Czakon, Papadopoulos, Worek: arxiv 0905.0883 (based on Phegas)

Would the traditional techniques suffice ?

- The number of diagrams explodes.
- The tensor structure can be more complicated.

process	# of F. diagrams
qq2ttbb	188
gg2ttbb	1003
Wjjj	1583
bbbb	2090
gg2gggg	11850

OPP + unitarity

- BlackHat Bern, Dixon, Dunbar, Kosower:9403226,9409265,... (on-shell unitarity) Britto, Cachazo, Feng: 0412103 (cuts with complex momenta), Forde: 07041835 (CC part), on-shell recursion (Rational part) , Ossola, Papadopoulos, Pittau: 0609007 (OPP reduction)

1. Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre: 0803.4180 (BlackHat), 0808.0941(W+3jets, points), Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre: 0902.2760 (W+3jets)

- DsDU (Ds-Dimensional Unitarity) Ellis, Giele, Kunszt, Melnikov:0801.2237(Ds-dimensional Rational part), 0806.3467 (massive fermions),.

1. Rocket: Ellis, Melnikov, Zanderighi: 0901.4101 (W+3jets points), Ellis, Giele, Kunszt, Melnikov, Zanderighi: 0810.2762(W+3jets), Giele, Zanderighi: 0805.2152 (N gluons, points)
2. Winter, Giele: 0902.0094 (N gluons, points)
3. AL: 0812.2998 (N gluons and massless fermions)

- HELAC-1L (based on HELAC Kanaki, Papdopoulos: 0002082 and OPP reduction: Ossola, Papadopoulos, Pittau: 0609007, 0704.1271 (six photons), 0711.3596(CutTools), 0802.1876(Rational terms), Binoth, Ossola, Papadopoulos, Pittau 0804.0350 (VVV), Draggiotis, Garzelli, Papadopoulos, Pittau 0903.0356 (Feynman rules for R2)

1. van Hameren, Papadopoulos, Pittau: 0903.4665 (points for many processes) expanded vertices, colour treatment

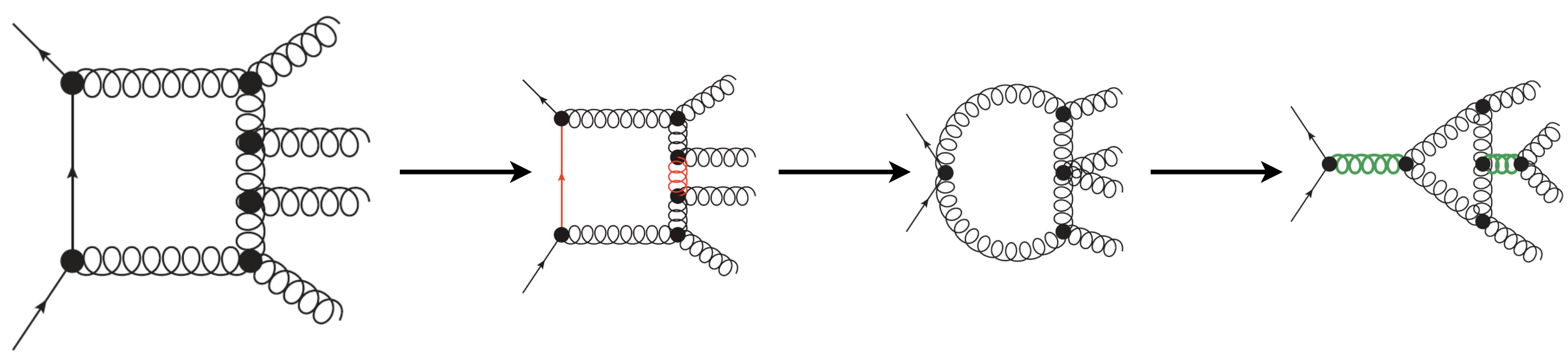
DDU in brief

COLOUR DECOMPOSITION

$$\sum_{\{h\}\{c\}} A_{loop}(\{p_i, h_i, c_i\}) = \sum_{\{h\}\{c\}} \sum_{\sigma} CF(\{c_i\}) A_{PR}(\{p_i, h_i\})$$

gauge invariant subset of colour-stripped graphs

a diagram from which all Feynman diagrams in a primitive can be obtained by pinching and pulling



PARENT is specified by

1) the external particles' ordering
2) the flavour of internal (loop) propagators

DDU in brief

A_{PR} depends on regularization scheme

contains

$$\int [dl] \frac{N(l, \{p_i, h_i\})}{D_a D_b D_c \ldots}$$

$$A_{PR}^{D_s} = A_0 + D_s \cdot A_1$$

$$A^6 = A_0 + 6A_1$$

$$A^8 = A_0 + 8A_1$$

$$\rightarrow \quad A^{D_s} = 4A^6 - 3A^8 + \frac{D_s}{2}(A^8 - A^6)$$

$$\text{In FDH } D_s = 4 \rightarrow A^{FDH} = 2A^6 - A^8$$

DDU in brief

$$A^{D_s} = \sum e_{\{i\}} I^{pent}_{\{i\}} + \sum d_{\{i\}} I^{box}_{\{i\}} + \sum c_{\{i\}} I^{tri}_{\{i\}} + \sum b_{\{i\}} I^{bub}_{\{i\}} + \sum a_{\{i\}} I^{tad}_{\{i\}} + RAT$$

due to higher than 4D loop momentum

processes with massive fermions

$$A_{cc} = \sum_Q \tilde{d}_{Q,0} I_Q + \sum_T c_{T,0} I_T + \sum_B b_{B,0} I_B$$

Terms proportional to $s_e^2 = \sum_{i=4..D_s} (l \cdot n_i)^2$ do not vanish upon integration.

They can be rewritten in terms of integrals in D+2,D+4. They contribute to the rational part.

$$A_R = -\sum_Q \frac{d_{Q,4}}{6} - \sum_T \frac{c_{T,9}}{2} - \sum_B \frac{b_{B,9}}{6}$$

DDU in brief

<u>OPP TO FIND THE COEFFICIENTS</u>

$$A^{D_s} = \sum_i \int [dl] \frac{\bar{e}_Q(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_i \int [dl] \frac{\bar{d}_Q(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_i \int [dl] \frac{\bar{c}_T(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_i \int [dl] \frac{\bar{b}_B(l)}{d_{i_1} d_{i_2}} + \sum_i \int [dl] \frac{\bar{a}_S(l)}{d_{i_1}}$$

Thanks to the particular functional form of coefficients, e.g.

$$\bar{d} = d_0 + d_1 s_1 + d_2 s_e^2 + d_3 s_1 s_e^2 + d_4 s_e^4$$

we can build NxN linear system of equations (choosing N different values for the loop momentum vector that solve the “unitarity constraints”) and solve for d_0, d_1, d_2, d_3, d_4

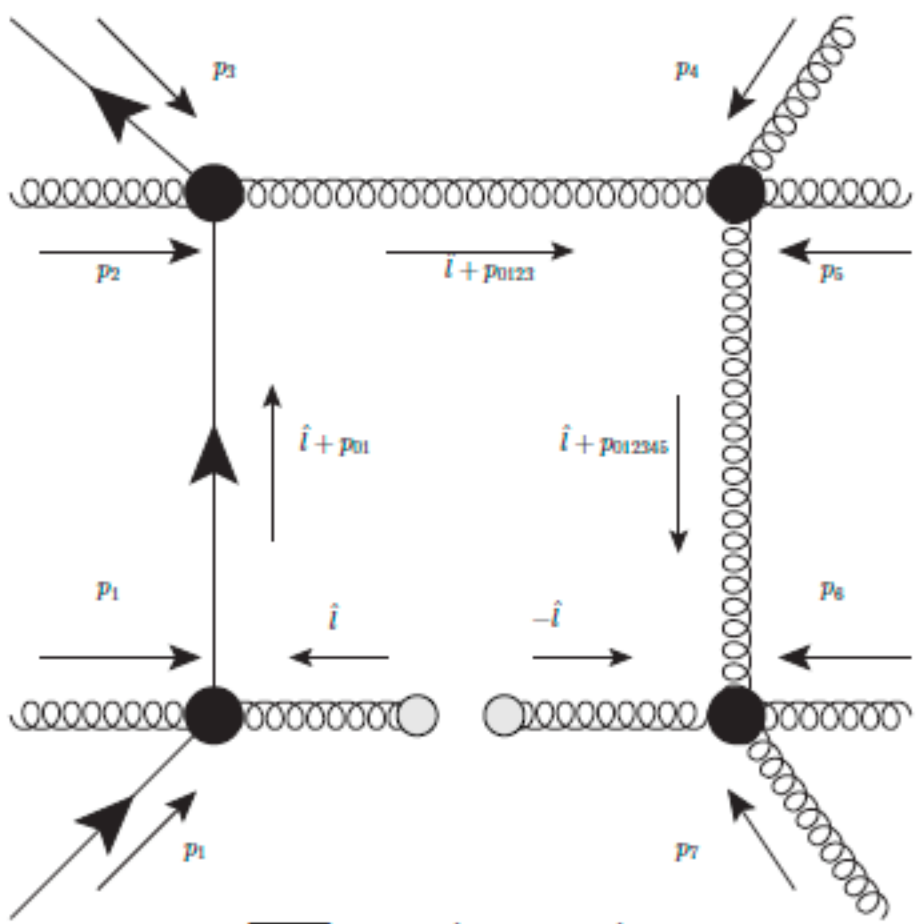
<u>OPP: RESIDUE CALLS PER CUT</u>

5-cut	1 (0+1)
4-cut	5 (2+3)
3-cut	10 (7+3)
2-cut	10 (9+1)

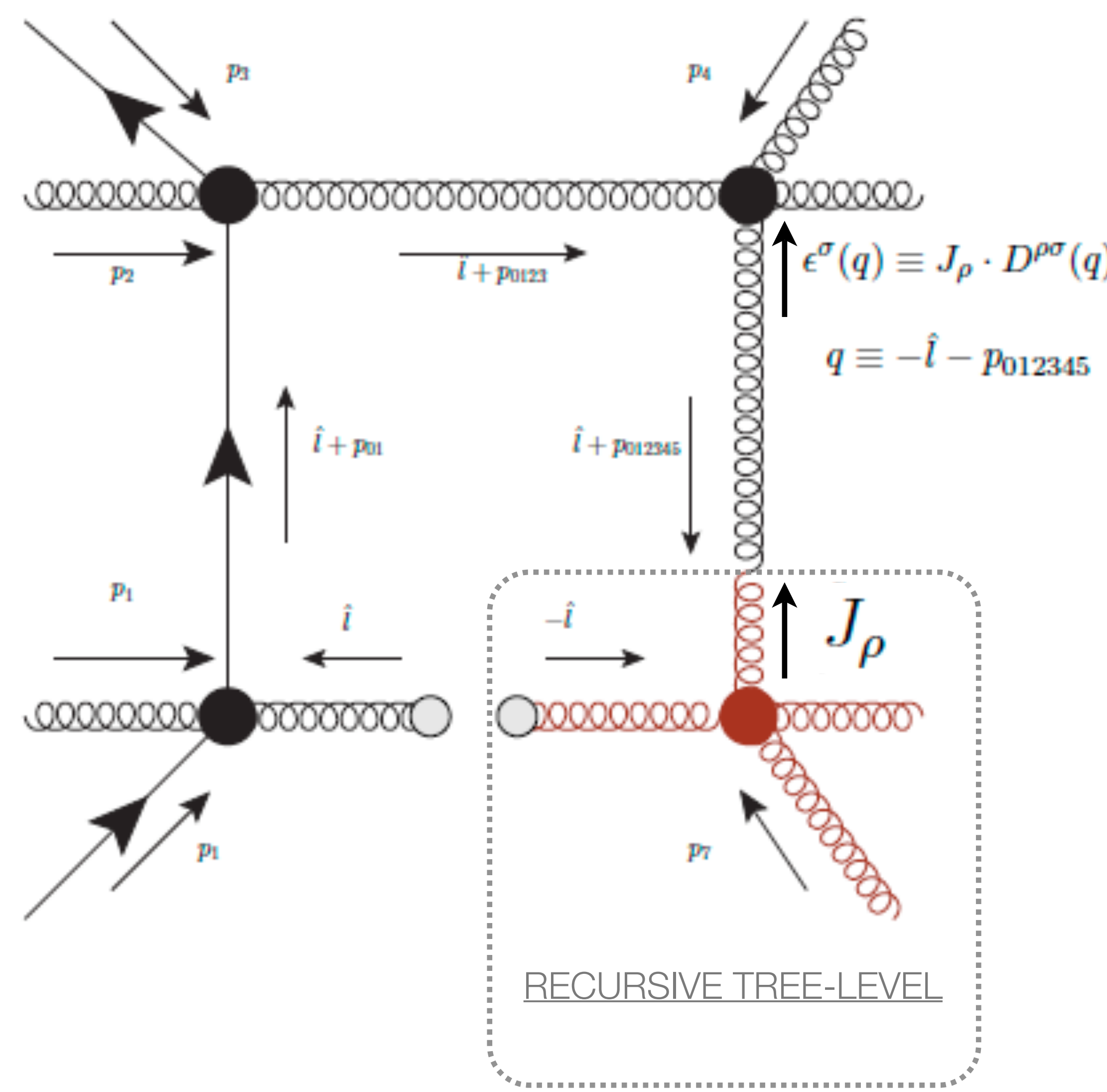
Minimal set (one can always implement redundancy)

DDU in brief

“...that solve the unitarity constraints”



$$\sum_h \epsilon_h^{\mu}(\hat{l})^* \epsilon_h^{\nu}(\hat{l})$$



DDU in brief

		Rings evaluated
5-cut	1 (0+1)	6, 8
4-cut	5 (2+3)	4x2, 6x3, 8x3
3-cut	10 (7+3)	4x7, 6x3, 8x3
2-cut	10 (9+1)	4x9, 6x1, 8x1

In the case N=6 one needs 251 4d rings, 120 6d rings and 120 8d rings.

Fermions in DDU

$$u_h(p) = \frac{\hat{p}}{\sqrt{2p\cdot n}}\xi_h(n) \qquad \sum_h \xi_{h;a}\bar{\xi}_{h;b} = \hat{n}_{ab}$$

$$\bar{u}_h(p) = \bar{\xi}_h(n)\frac{\hat{p}}{\sqrt{2p\cdot n}} \qquad \hat{p}_{ab} \equiv p_\mu \Gamma^\mu_{ab} \;\; \mu = 0\ldots D_s$$

$$D_s = 6: \quad \Gamma^{\mu=0\ldots3} = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}_{8\times8} \quad \Gamma^4 = \begin{pmatrix} 0 & \gamma_5 \\ -\gamma_5 & 0 \end{pmatrix}_{8\times8} \quad \Gamma^5 = \begin{pmatrix} 0 & i\gamma_5 \\ i\gamma_5 & 0 \end{pmatrix}_{8\times8}$$

$$h = 1\ldots4 \qquad \xi_1 = \begin{pmatrix} \phi_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \xi_2 = \begin{pmatrix} 0 \\ \phi_2 \\ 0 \\ 0 \end{pmatrix} \quad \xi_3 = \begin{pmatrix} 0 \\ 0 \\ \phi_1 \\ 0 \end{pmatrix} \quad \xi_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \phi_2 \end{pmatrix}$$

$$D_s = 8: \quad \Gamma^{\mu=0\ldots5} = \begin{pmatrix} \Gamma^\mu & 0 \\ 0 & \Gamma^\mu \end{pmatrix}_{16\times16} \quad \Gamma^6 = \begin{pmatrix} 0 & \Gamma_5 \\ -\Gamma_5 & 0 \end{pmatrix}_{16\times16} \quad \Gamma^7 = \begin{pmatrix} 0 & i\Gamma_5 \\ i\Gamma_5 & 0 \end{pmatrix}_{16\times16}$$

$$h = 1\ldots8 \qquad \xi_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \phi_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \xi_6 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \phi_2 \\ 0 \\ 0 \end{pmatrix} \quad \xi_7 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \phi_1 \\ 0 \end{pmatrix} \quad \xi_8 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \phi_2 \end{pmatrix}$$

80% of the time spent in calculating

ordered trees

Speed / efficiency : ordered trees

tree	t(μs)	tree	t(μs)	tree	t(μs)	tree	t(μs)
4g	5.7	uugg	5	uudd	5	-	-
5g	11	uu3g	10	uuddg	9	-	-
6g	20	uu4g	17	uudd2g	15	uuddcc	13
7g	35	uu5g	29	uudd3g	25	uuddccg	22
8g	60	uu6g	46	uudd4g	40	uuddccgg	36

INTEL XEON X5450 @3.0GHz

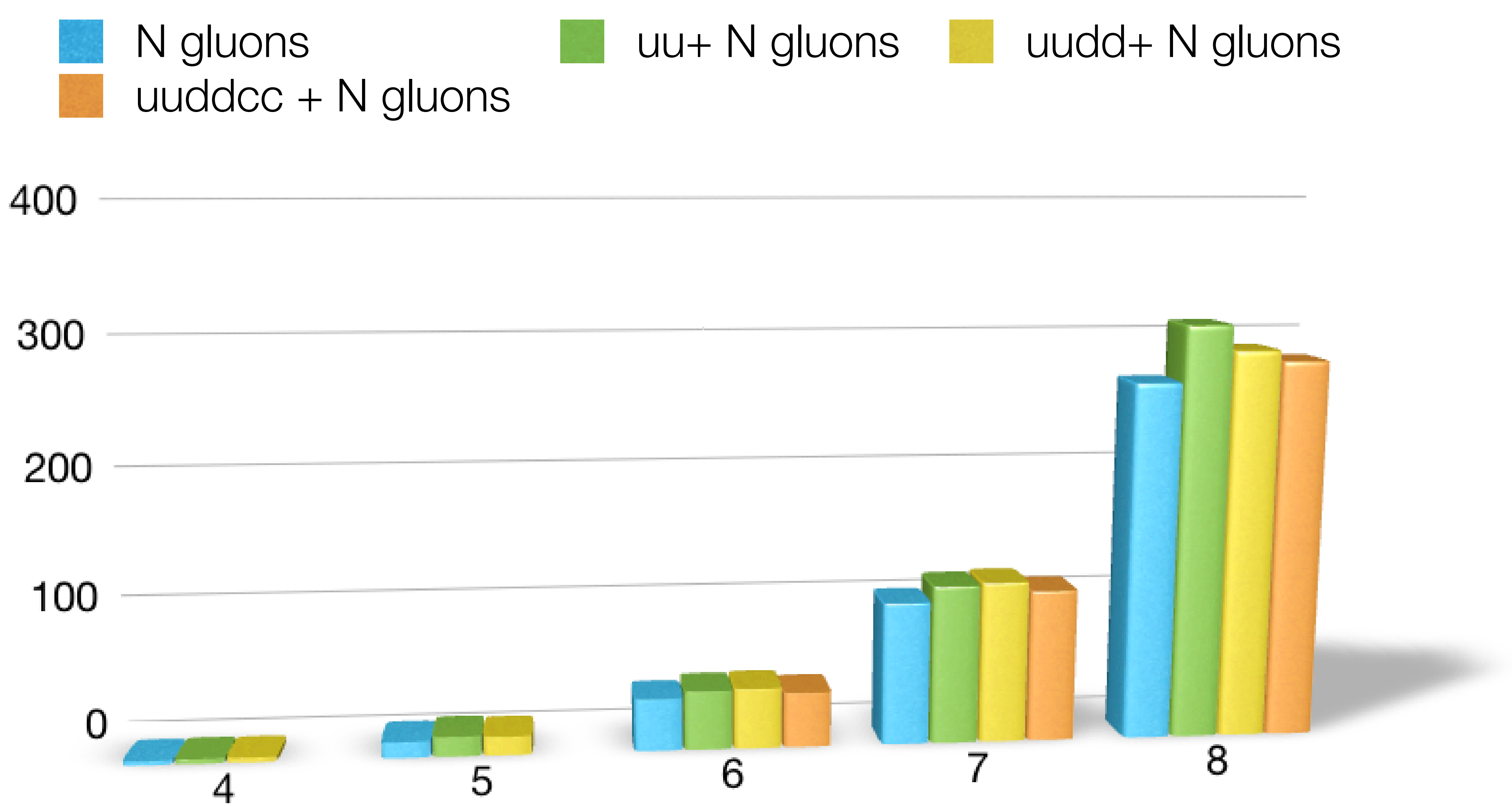
Speed / efficiency : primitives

cpu time for primitives

primitive	t(ms)	primitive	t(ms)	primitive	t(ms)	primitive	t(ms)
4g	3.1	uugg	3.6	uudd	3.5	-	-
5g	12	uu3g	15	uuddg	14	-	-
6g	39	uu4g	44	uudd2g	45	uuddcc	41
7g	107	uu5g	119	uudd3g	121	uuddccg	114
8g	268	uu6g	310	uudd4g	291	uuddccgg	283

INTEL XEON X5450 @3.0GHz

Speed / efficiency : primitives



Accuracy - stability

- agreement in ε-poles only checks CC part .
- redundancy in OPP systems.
- exploit freedom in the solution of unitarity constraints.

}

Resort to Quadruple or Multiple precision

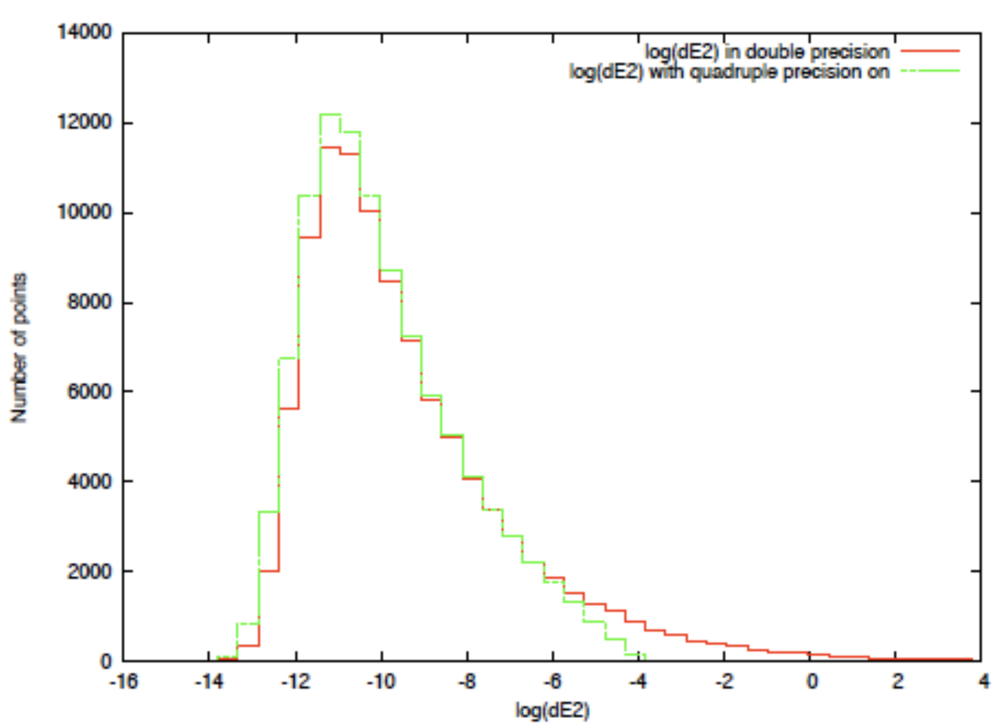
QD/ARPREC by Bailey et al.
http://crd.lbl.gov/~dhbailey/mpdist/

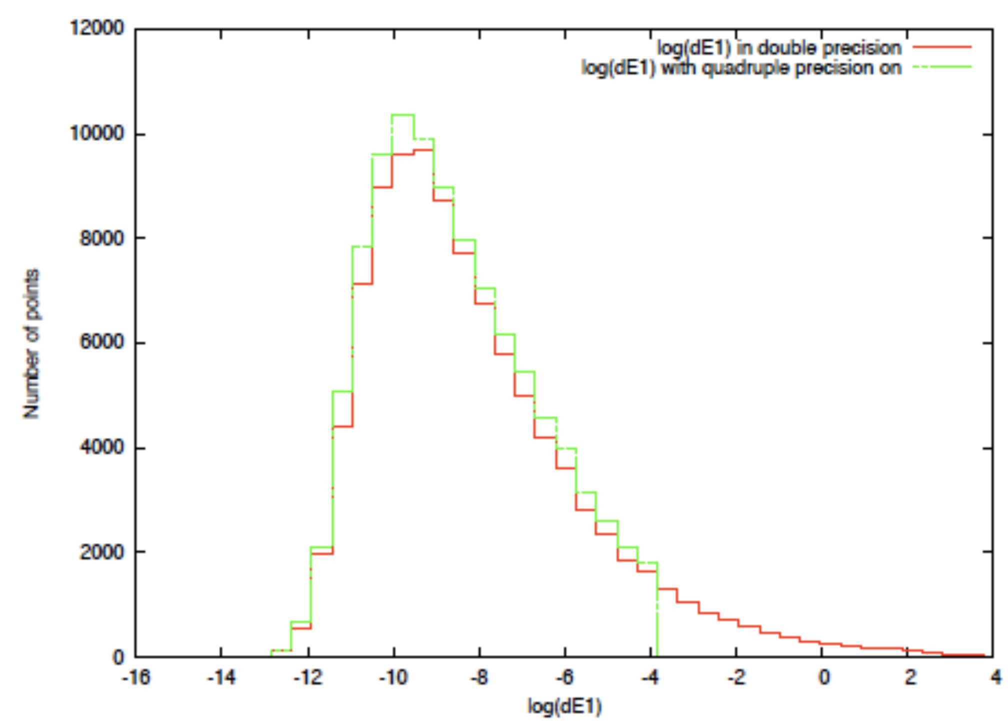
Gluonic study

- Results similar with Rocket and J. Winter

- With reasonable cuts on gluons and not very unfortunate choices in loop momentum freedom, problematic points are of the order of 5%.

- Pentagon coefficient recombination helps.





Colour treatment

- Explicit summation over colour-ordered amplitudes (more flexible, leading colour approximation, symmetrization in some cases)

- Monte Carlo over colour configurations and summation over connections (a la HELAC).

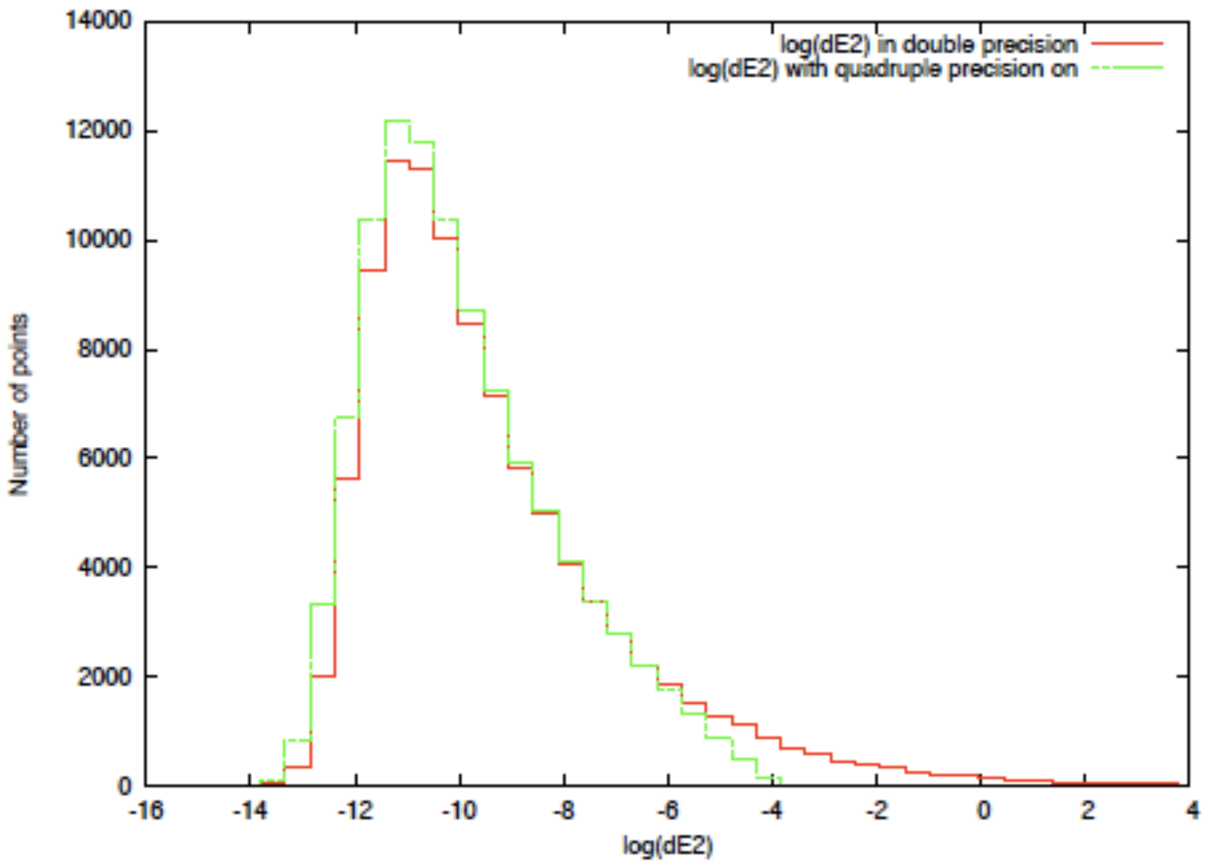
Next steps

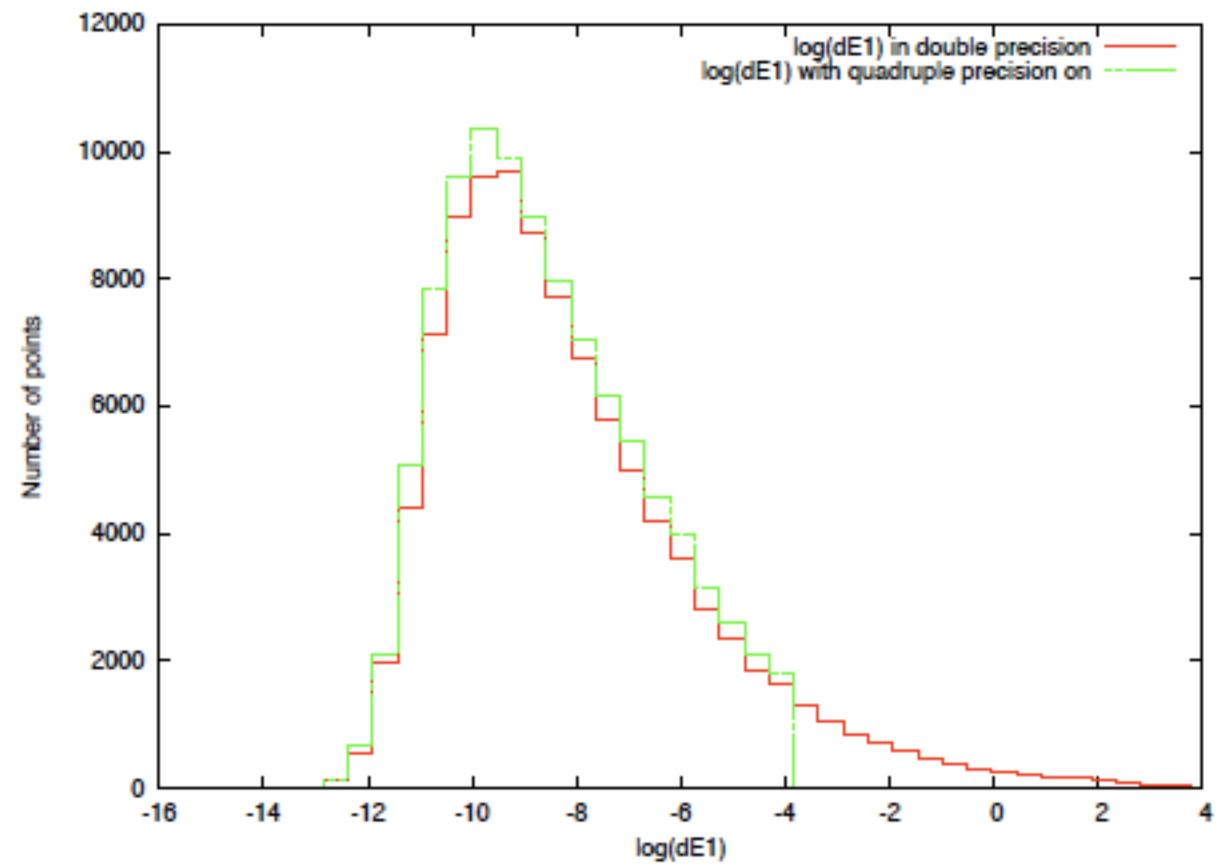
- Quadruple (multiple) precision (almost there)
- Massive fermions
- E/W Vector bosons
- Colour treatment

Outlook

- Approaching the times of automatic NLO matrix element generators.
- As in tree level matrix element generators, eventually generality, reliability, portability, user-friendliness will be more important than speed...
- ... as long as generic codes can deliver in reasonable times.

precision





When pole coefficients don't agree with analytic formula QP is switched on

D. H. Bailey, Y. Hida, K. Jeyabalan, X. S. Li and B. Thompson, "ARPREC (C++/Fortran-90 arbitrary precision package)" (http://crd.lbl.gov/~dhbailey/mpdist/)

experimentalist's ghost

An experimenter's wishlist

Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

WWj: Dittmaier, Kallweit, Uwer / arXiv:0710.1577
Campbell, Ellis, Zanderighi / arXiv:0710.1832

ttj:Dittmaier, Uwer, Weinzierl
hep-ph/0703120

W+3j: Ellis,Melnikov, Zanderighi
arXiv:0901.4101

W+3j: BlackHat
arXiv:0902.2760

ZZZ: AL, Melnikov, Petriello, hep-ph/0703273
WWZ: Hankele, Zeppenfeld, arXiv:0712.3544

ttZ: AL, McElmurry,Melnikov, Petriello /arXiv:
0804.2220

VVV:Binoth , Ossola, Papadopoulos, Pittau arXiv:0804.0350

the ‘upgraded experimentalist’s wish list’

The NLO multileg working group summary report (Les Houches 2007) 0803.0494

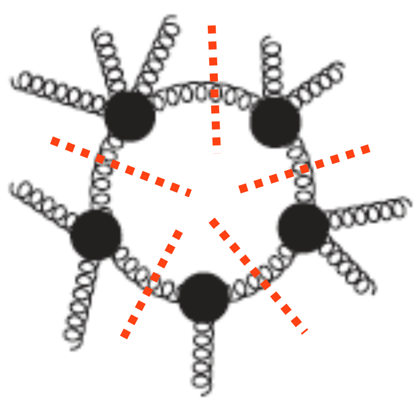
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\, b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\, b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures

Bredenstein, Dener, Dittmaier, Pozzorini:0905.0110

W+3j: Ellis,Melnikov, Zanderighi
arXiv:0901.4101

W+3j: BlackHat
arXiv:0902.2760

Pen-tuple cuts



$$\chi^{D_s}_{\alpha\beta\gamma\delta\epsilon}(\hat{l}) = \bar{e}_{\alpha\beta\gamma\delta\epsilon}(\hat{l})$$

$$d_\alpha = l^2 \quad d_\beta = (l+Q_1)^2 \quad d_\gamma = (l+Q_1+Q_2)^2 \quad d_\delta = (l+Q_1+Q_2+Q_3)^2 \quad d_\epsilon = (l+Q_1+Q_2+Q_3+Q_4)^2$$

$$\hat{l}^\mu = V^\mu + \sqrt{-V^2} n_5^\mu \qquad V^\mu = x_i v_i^\mu \qquad v_i \cdot Q_j = \delta_{ij}$$

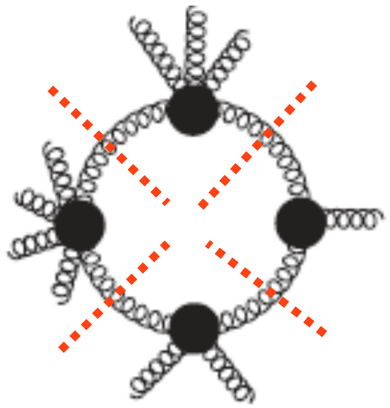
Vermaseren - Van Neerven

$$\bar{e}(l) = e_0$$

One $\hat{l}$ for pen-tuple cut

Scalar pentagon reduced to boxes

Quadruple cuts



$$\bar{d}_{\alpha\beta\gamma\delta}(\hat{l}) = \mathcal{X}^{D_s}_{\alpha\beta\gamma\delta}(\hat{l}) - \sum_{\epsilon\neq\alpha\beta\gamma\delta} \frac{e_{\alpha\beta\gamma\delta\epsilon}(\hat{l})}{d_\epsilon(\hat{l})}$$

$$d_\alpha = l^2 \quad d_\beta = (l+Q_1)^2 \quad d_\gamma = (l+Q_1+Q_2)^2 \quad d_\delta = (l+Q_1+Q_2+Q_3)^2$$

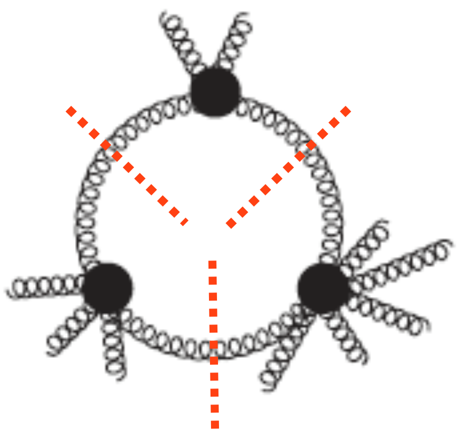
$$\hat{l}^\mu = V^\mu + a_1 n_1^\mu + a_5 n_5^\mu$$

$$a_1^2 + a_5^2 + V^2 = 0$$

$$\bar{d} = d_0 + d_1 s_1 + d_2 s_e^2 + d_3 s_1 s_e^2 + d_4 s_e^4$$

Five $\hat{l}$ for quadruple cut, of which three in D=5

Triple and Double cuts



$$\bar{c}_{\alpha\beta\gamma}(\hat{l}) = \mathcal{X}^{D_s}_{\alpha\beta\gamma}(\hat{l}) - \sum_{\delta,\epsilon\neq\alpha\beta\gamma} \frac{\bar{e}_{\alpha\beta\gamma\delta\epsilon}(\hat{l})}{d_\epsilon(\hat{l})d_\delta(\hat{l})} - \sum_{\delta\neq\alpha\beta\gamma} \frac{\bar{d}_{\alpha\beta\gamma\delta}(\hat{l})}{d_\delta(\hat{l})}$$

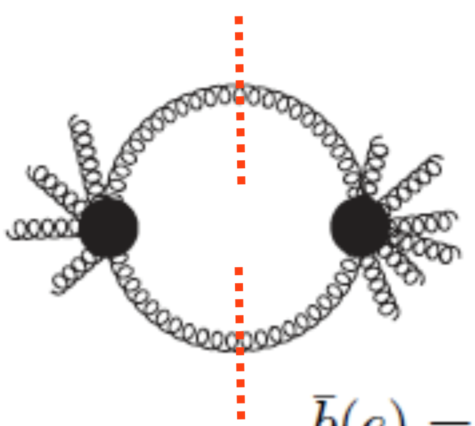
$$s_i = l \cdot n_i$$

$$s_e^2 = \sum_{i=4..D} (l \cdot n_i)^2$$

$$\hat{l}^\mu = V^\mu + a_1 n_1^\mu + a_2 n_2^\mu + a_5 n_5^\mu$$

$$\bar{c}(l) = c_0 + c_1 s_1 + c_2 s_2 + c_3(s_1^2 - s_2^2) + s_1 s_2(c_4 + c_5 s_1 + c_6 s_2) + c_7 s_1 s_e^2 + c_8 s_2 s_e^2 + c_9 s_e^4$$

10 $\hat{l}$ for triple cut of which three in D=5



$$\bar{b}_{\alpha\beta}(\hat{l}) = \mathcal{X}^{D_s}_{\alpha\beta}(\hat{l}) - \sum_{\gamma\delta,\epsilon\neq\alpha\beta} \frac{\bar{e}_{\alpha\beta\gamma\delta\epsilon}(\hat{l})}{d_\epsilon(\hat{l})d_\delta(\hat{l})d_\gamma(\hat{l})} - \sum_{\gamma\delta\neq\alpha\beta} \frac{\bar{d}_{\alpha\beta\gamma\delta}(\hat{l})}{d_\delta(\hat{l})d_\gamma(\hat{l})} - \sum_{\gamma\neq\alpha\beta} \frac{\bar{c}_{\alpha\beta\gamma}(\hat{l})}{d_\gamma(\hat{l})}$$

$$l^\mu = V^\mu + a_1 n_1^\mu + a_2 n_2^\mu + a_3 n_3^\mu + a_5 n_5^\mu$$

$$\bar{b}(e) = b_0 + b_1 s_1 + b_2 s_2 + b_3 s_3 + b_4(s_1^2 - s_3^2) + b_5(s_2^2 - s_3^2) + b_6 s_1 s_2 + b_7 s_1 s_3 + b_8 s_2 s_3 + b_9 s_e^2$$

10 $\hat{l}$ for double cut of which one in D=5

The result for fixed Ds

$$A_{cc} = \sum_Q \tilde{d}_{Q,0} I_Q + \sum_T c_{T,0} I_T + \sum_B b_{B,0} I_B$$

$$A_R = -\sum_Q \frac{d_{Q,4}}{6} - \sum_T \frac{c_{T,9}}{2} - \sum_B \frac{b_{B,9}}{6}$$

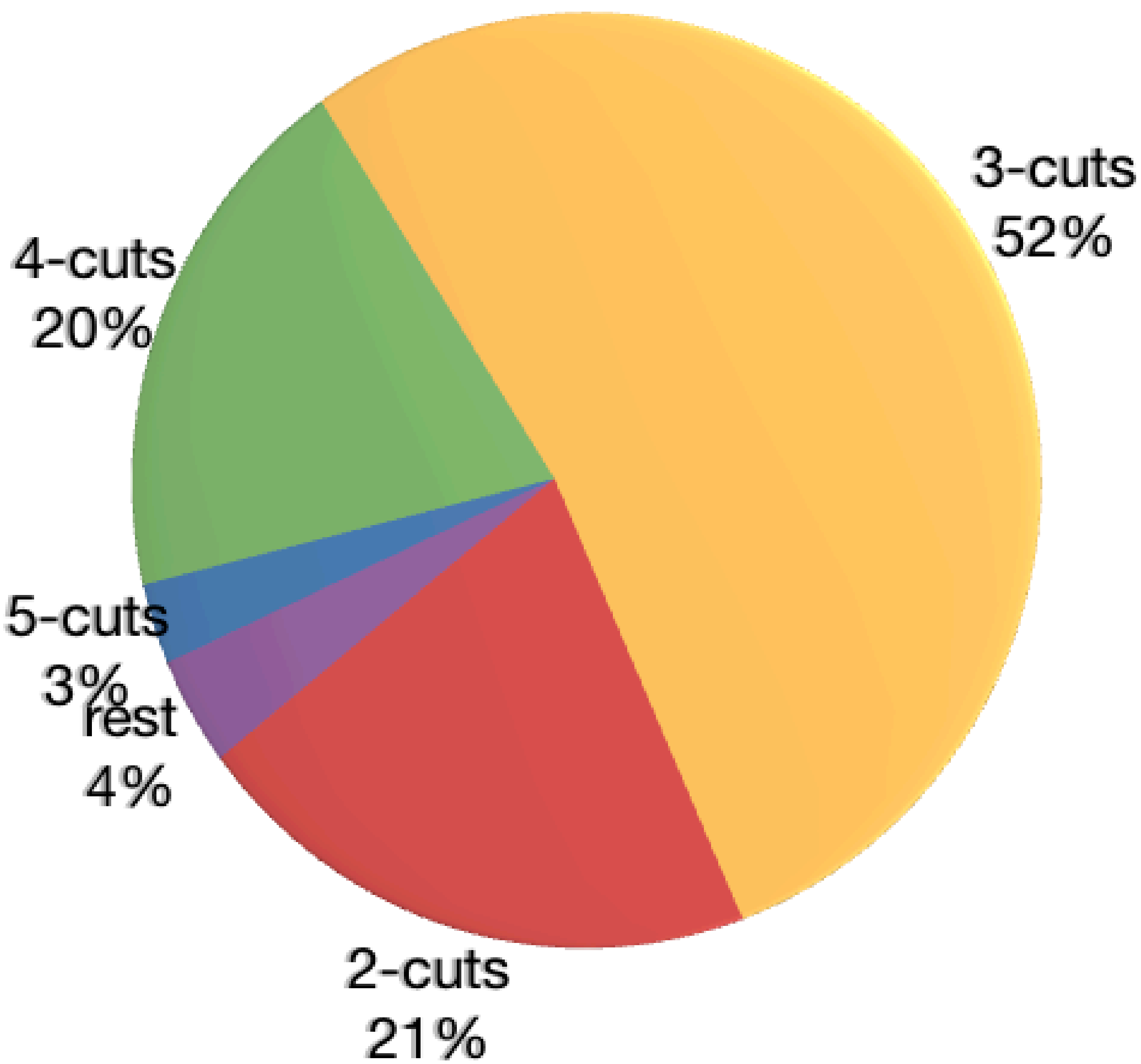
Terms proportional to $s_e^2 = \sum_{i=4,D_s} (l \cdot n_i)^2$ do not vanish upon integration. They can be rewritten in terms of integrals in D+2,D+4. They contribute to the rational part.

Necessary ingredient: the one-loop scalar integrals

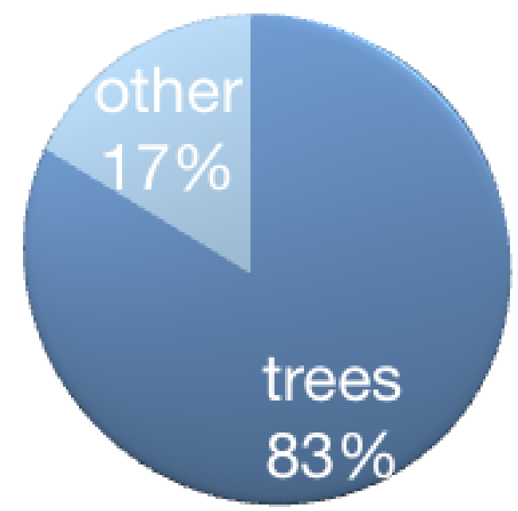
Scalar Integrals: QCDLoop-1.4 Ellis, Zanderighi 0712.1851

The 6 gluon case

CPU time share for 6 gluons



CPU time share of tree-level building blocks



3	1150
4	850
5	520
6	280