QCD NLO Corrections with BlackHat and Sherpa

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in collaboration with

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Outline

- BlackHat + Sherpa
- BH Recursive rational terms
- BH Numerical stability
- BH+S Application: W+jets @ Tevatron

NLO with Blackhat+Sherpa

NLO cross section

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

- Tasks
  - n-parton PS integration
    - Tree x Tree
    - Virtual x tree
    - Integrated subtraction
  - (n+1)-parton PS
    - Real emission – subtraction

NLO with Blackhat+Sherpa

NLO cross section

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

BlackHat

Sherpa

Sherpa

[Gleisberg,Hoeche,Krauss,Schoenherr,Schumann,Siegert,Winter]

Provides

- Efficient phase space integration
- Event generation
- Analysis framework
- Automated dipole subtraction for the real part (see Tanju's talk) [Catani,Seymour] [Gleisberg,Krauss]
- (and much more)
- Is written in C++

BlackHat

[Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, DM]

- Goal : automate computation of virtual 1-loop amplitudes for QCD processes
- C++ framework
- Cut containing part: 4 Dim, using a combination of Forde's and OPP's methods
  [Ossola,Papadopoulos,Pittau;Forde]
- Rational part:
  - 1- loop recursion (reuse of lower point results)
    [Berger,Bern,Dixon,Forde,Kosower]
  - Rational extraction using D-dim unitarity
    [Bern,Morgan;Bern,Dixon,Dunbar,Kosower;Ellis,Giele,Kunszt,Melnikov; Badger]

Numerical Rational Terms

- Many different techniques
  - Using Specialized Feynman Diagrams
    [Draggiotis, Garzelli, van Hameren, Ossola, Papadopoulos, Pittau]
    Implemented in Phegas/Helac
  - Computing the cuts in D dimensions
    - Numerical method [Ellis,Giele,Kunszt,Melnikov,Zanderighi]
      implemented in Rocket.
    - D-dimensionality can be seen as a mass [Badger]
      numerical adaptation in BlackHat
  - Recursion
    - On-shell recursive approach [Berger,Bern,Dixon,Forde,Kosower]
      implemented in BlackHat

Recursion relations

- Recursion relations allow to compute amplitudes from lower multiplicity amplitudes. [Britto,Cachazo,Feng,Witten]
- Based on the analytic properties of the amplitudes and on the factorization properties on multi-particle poles
- Complex transformation:

$$p_1 \to p_1(z), \qquad p_2 \to p_2(z)$$

- Linear transformation that preserves
  - Onshell properties: $p_1(z)^2 = 0, \qquad p_2(z)^2 = 0$
  - Momentum conservation: $p_1 + p_2 = p_1(z) + p_2(z)$
- A→A(z), physical amplitude is *A(0)*
- Use the analytic properties of *A(z)* to construct *A(0)*

Analytic structure of the amplitude

- Use the analytic properties of the one-loop amplitude to construct the rational term
- Use a complex shift

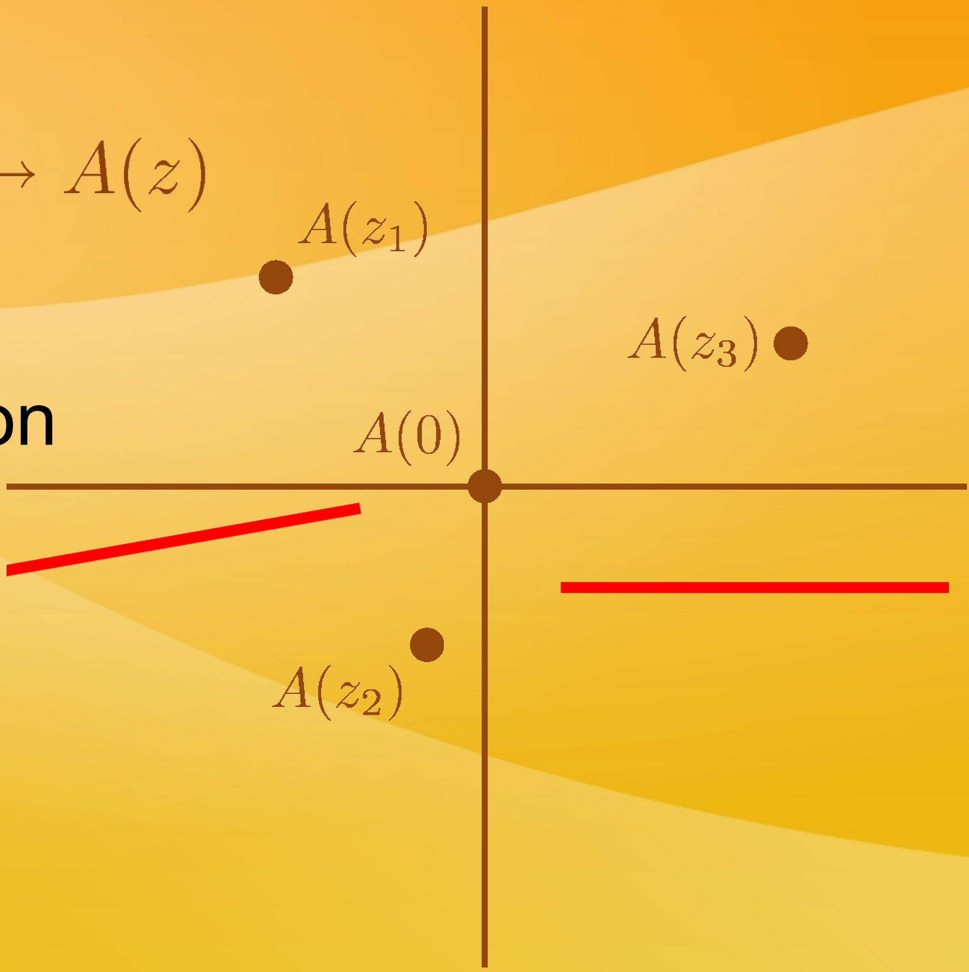
$$p_1 \to p_1(z), p_2 \to p_2(z), \quad A \to A(z)$$

on the full amplitude

- Consider the complex function

$$f(z) = \frac{A(z)}{z}$$

  - Poles, $s_{i\ldots j}(z) \to 0$
  - Branch cuts: $\log(s_{i\ldots j}(z))$



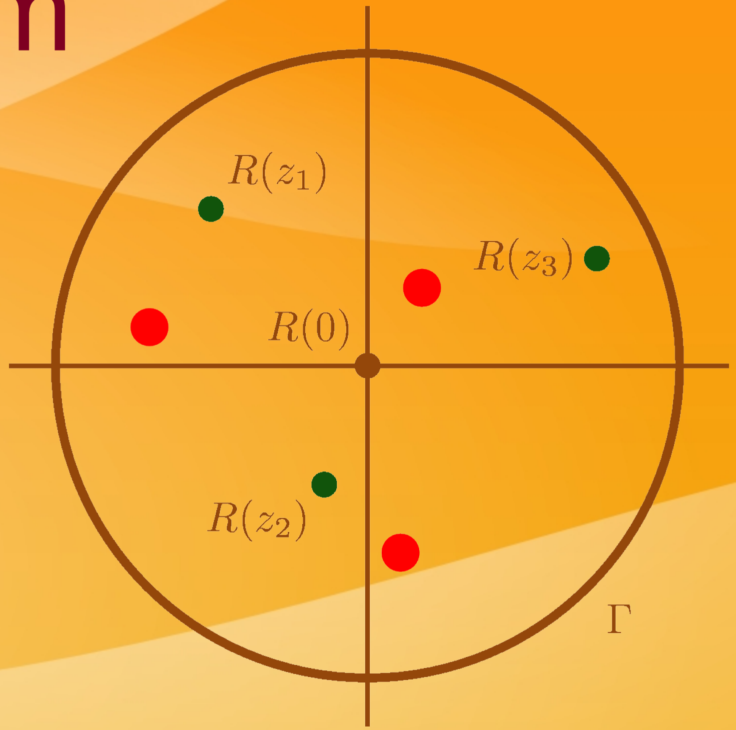
Rational term

- Consider *R(z)*

$$\frac{1}{2\pi i}\int_\Gamma \frac{R(z)}{z} = R_\infty$$

- The value R_∞ of the contour integral at ∞ can be constructed using an auxiliary recursion.

$$R(0) = R_\infty - \sum_{\text{poles}\alpha} \text{Res}_{z=z_\alpha} \frac{R(z)}{z}$$



- Two types of poles: Physical and Spurious

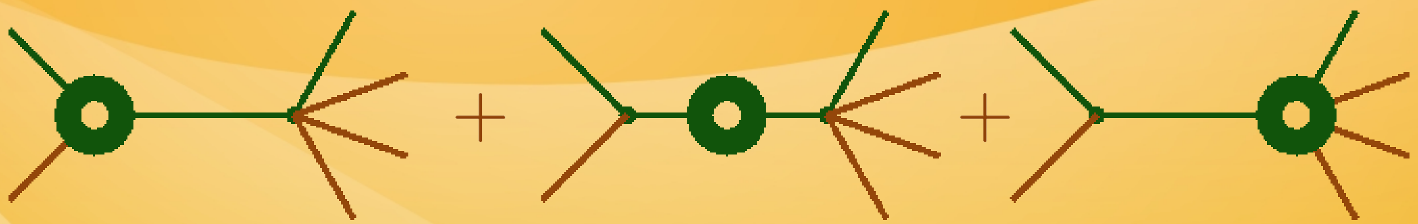
$$R(0) = R_\infty - \sum_{\text{phys}} \text{Res}_{z_p} \frac{R(z)}{z} - \sum_{\text{spur}} \text{Res}_{z_s} \frac{R(z)}{z}$$

Rational Term: Recursive Part

$$R(0) = R_\infty - \sum_{\text{phys}} \mathrm{Res}_{z_p} \frac{R(z)}{z} - \sum_{\text{spur}} \mathrm{Res}_{z_s} \frac{R(z)}{z}$$

R(z) factorizes at the physical pole locations, so that we can use recursion relations. [Bern,Dixon,Kosower]

$$\mathrm{Res}_{z_p} \frac{R(z)}{z} =$$



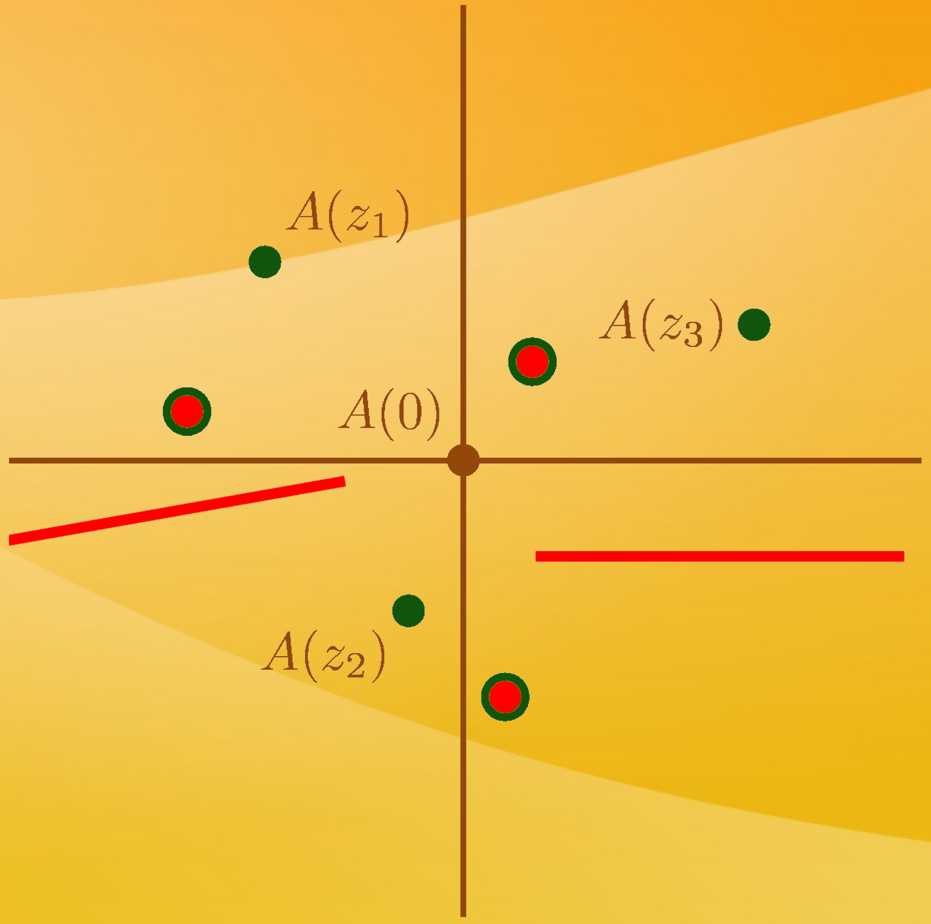
- This part can be constructed from lower point results

$$R_D = - \sum_{z_p} \mathrm{Res}_{z_p} \frac{R(z_p)}{z_p}$$

Recursion for Rational Terms: Spurious Part

- Spurious poles appear in *C(z)* and *R(z)* due to Gram determinants
- The residues of *R(z)/z* and *C(z)/z* at the unphysical poles have to cancel since *A(z)* has no spurious poles.

$$\frac{A(z)}{z} = \frac{R(z)}{z} + \frac{C(z)}{z}$$



$$\mathrm{Res}_{z_s} \frac{R_S(z)}{z} = -\mathrm{Res}_{z_s} \frac{C(z)}{z}$$

Numerical extraction

- We compute numerically

$$\sum_{\mathrm{spur}} \mathrm{Res}_{z_s} \frac{C(z_s)}{z_s}$$

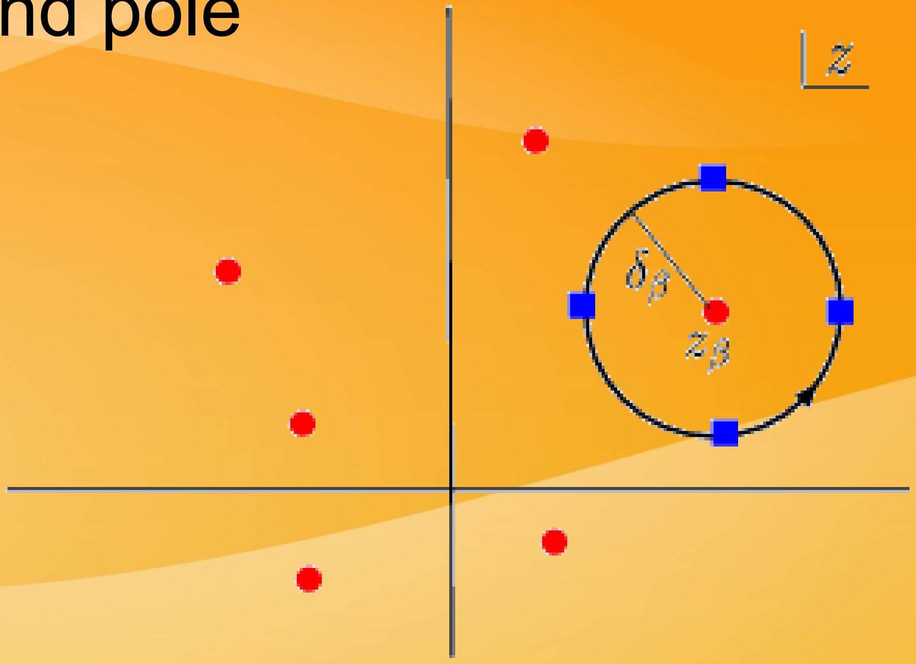
- Numerical spurious extraction is tricky, but possible because
  - Precise cut part input
  - Location of the spurious poles is known a priori
  - Only need to evaluate a small part of *C(z)* around the pole
  - Only need rational part of the expansion of the integral functions around vanishing Gram determinant

Spurious pole extraction

- Choose complex values around pole

$$z_\beta^j = z_\beta + \delta_\beta e^{2\pi i j/m}$$

$$\mathrm{Res}_{z_\beta} f(z) \simeq \frac{1}{m} \sum_{j=1}^{m} \delta e^{2\pi i j/m} f(z_\beta^j)$$



- Choose δ_β separately for each spurious pole, for each phase space point
- Adapt its value when requested
  - Too close: large cancellation between points
  - Too far: influence from other poles and lower/higher powers

Numerical Stability

- Use high precision libraries QD [Bailey,Hida,Li]
  - Use it only when necessary
  - Can use it either
    - only for the badly behaved part (cut, spurious pole)
    - for the full amplitude (large cancellation)
  - Automatic reevaluation when necessary
  - No need for a priori knowledge of when the precision is going to be insufficient
  - For free: High precision targets for comparison
  - Run time cost higher

Numerical Precision

- Different types of tests
  - On-line
    - For every phase-space point
    - For every process
    - Low (averaged) run-time cost
  - Off-line
    - Checks/prove accuracy of the method
    - No need for a very large number of phase-space points
    - Can have a higher run-time cost

Numerical Accuracy (on-line)

- The precision of the computed amplitude can be assessed using the known infrared structure of the amplitude
- Cut Part

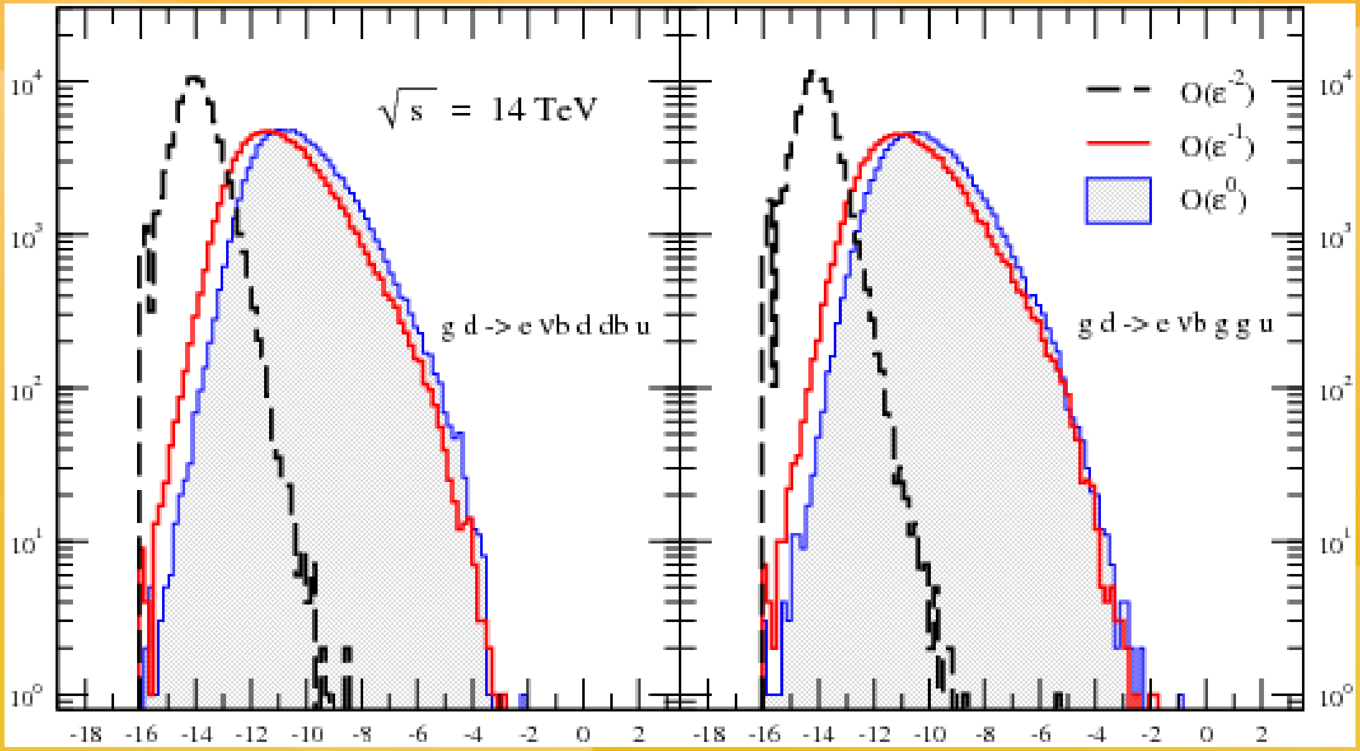
$$A_n^{\rm oneloop}|_{1/\epsilon,\,{\rm non-log}} = \frac{1}{\epsilon}\sum_k b_k = -\left[\frac{1}{\epsilon}\left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N_c}\right)\right] A_n^{\rm tree}$$

- Spurious poles

$$A_n^{\rm oneloop}(z_s)|_{1/\epsilon,\,{\rm non-log}} = \frac{1}{\epsilon}\sum_k b_k(z_s) = 0$$

Numerical Accuracy (off-line)

- Off-line
  - Compare with known formulae
  - Compare with higher precision results
  - Check combination of amplitudes



Application:

W + jets

W+jets

- W/Z+jets processes are important
  - For SM physics (Higgs, $t\bar{t}$, single top)
  - Background to new physics
  - Luminosity determination
- So far
  - MCFM [John Campbell, Keith Ellis]
    - NLO W+1 jet (Feynman diagrams)
    - NLO W+2 jets (amplitudes from (early) unitarity methods)
  - Leading color primitive amplitudes (2q3gW) [BlackHat]
  - All primitive amplitudes [Ellis,Giele,Kunszt,Melnikov,Zanderighi]
  - Leading color W+3 jets (2q3gW) [Ellis,Melnikov,Zanderighi]
  - Leading color W+3 jets (all subprocesses) [BlackHat]

W+jets @ Tevatron

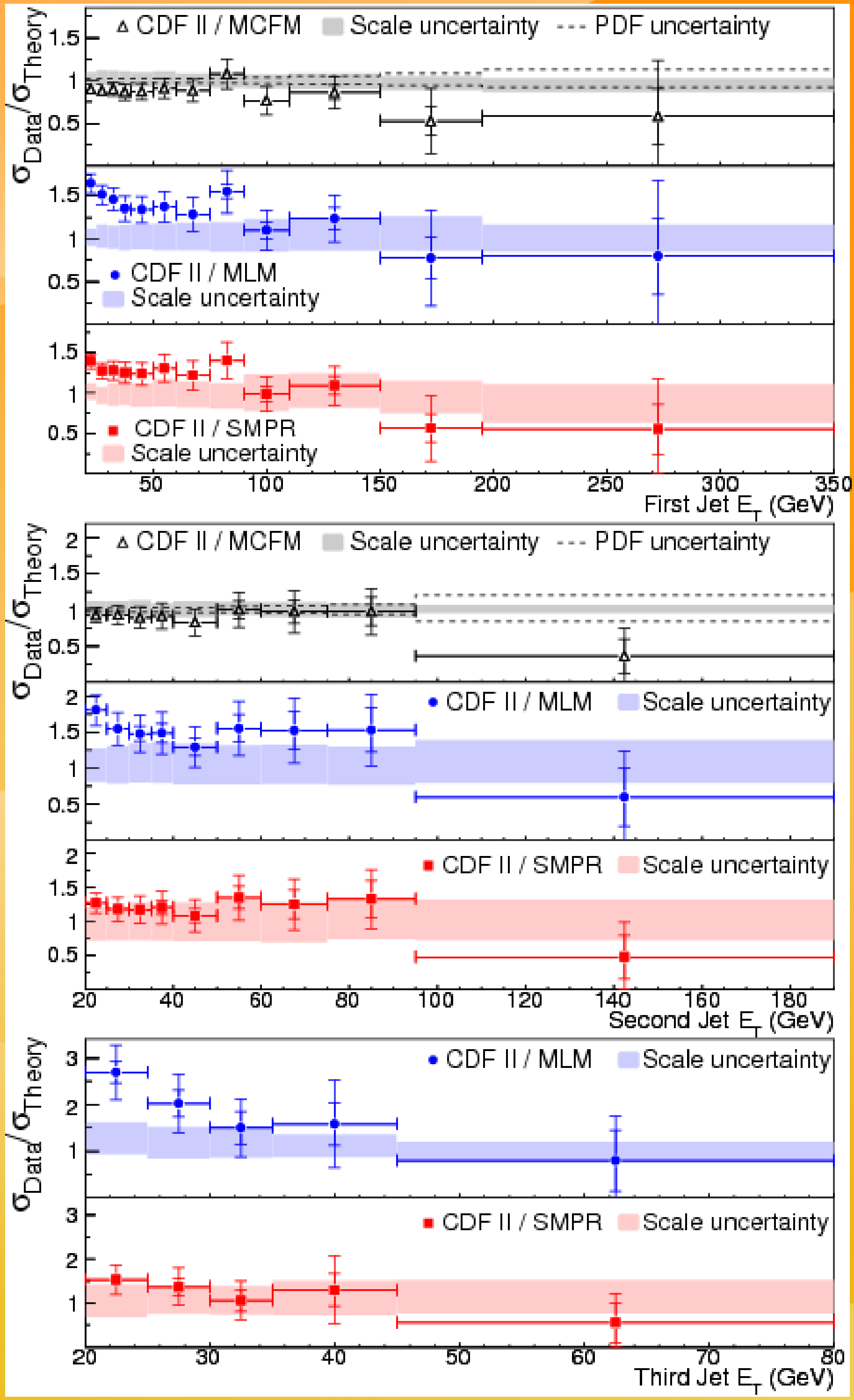
- CDF Collaboration
  - $320pb^{-1}$
  - Corrected for comparison with particle level
  - Comparison with
    - NLO: MCFM
    - MLM = Alpgen+Herwig
    - SMPR = Madgraph+Pythia

$E_T^e > 20\ \mathrm{GeV} \qquad E_T^{\mathrm{jets}} > 20\ \mathrm{GeV}$

$|\eta^e| < 1.1 \qquad \not{E}_T > 30\ \mathrm{GeV}$

$|\eta^{\mathrm{jets}}| < 2 \qquad E_T^{\mathrm{jets}} > 20\ \mathrm{GeV}$

$M_T^W > 20\ \mathrm{GeV}$



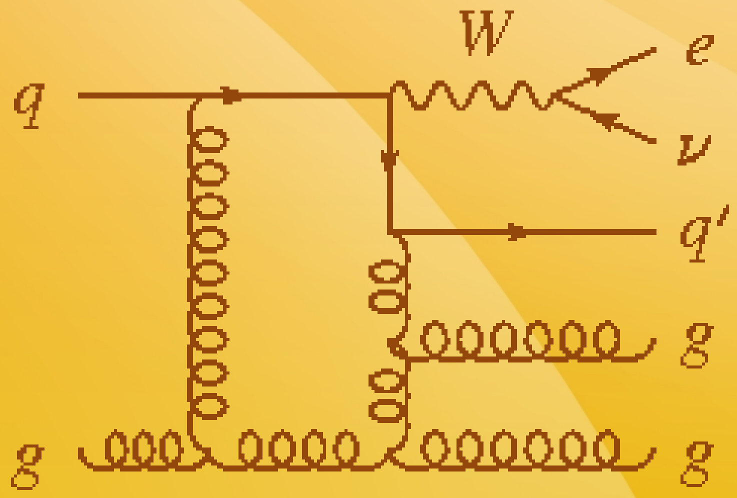
[CDF Collaboration PRD 77 011108, Arxiv:0711.404]

Leading color approximation

- Neglect terms of order

  $\frac{1}{N_c^2}$ (subleading color), $\frac{N_f}{N_c}$ (closed fermion loop)

  in finite part of the virtual amplitude
- Works for W+1,2 jets within 3%



LC Approximation

- Validity proven to 3% for 1,2,3 jets
- Total cross section ($E_T^{n\mathrm{th-jet}} > 25\ \mathrm{GeV}$)

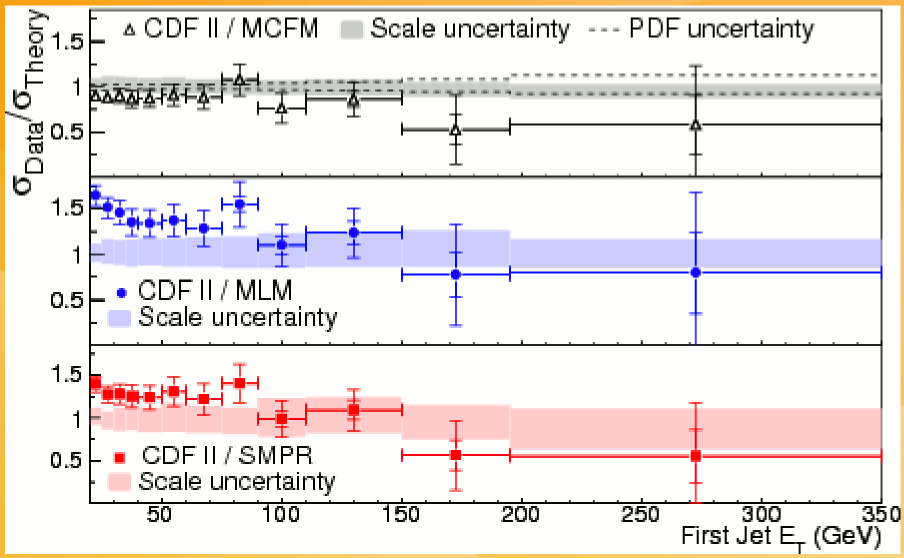
number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.908^{+0.044}_{-0.142}$	$0.882(5)^{+0.057}_{-0.138}$

Preliminary

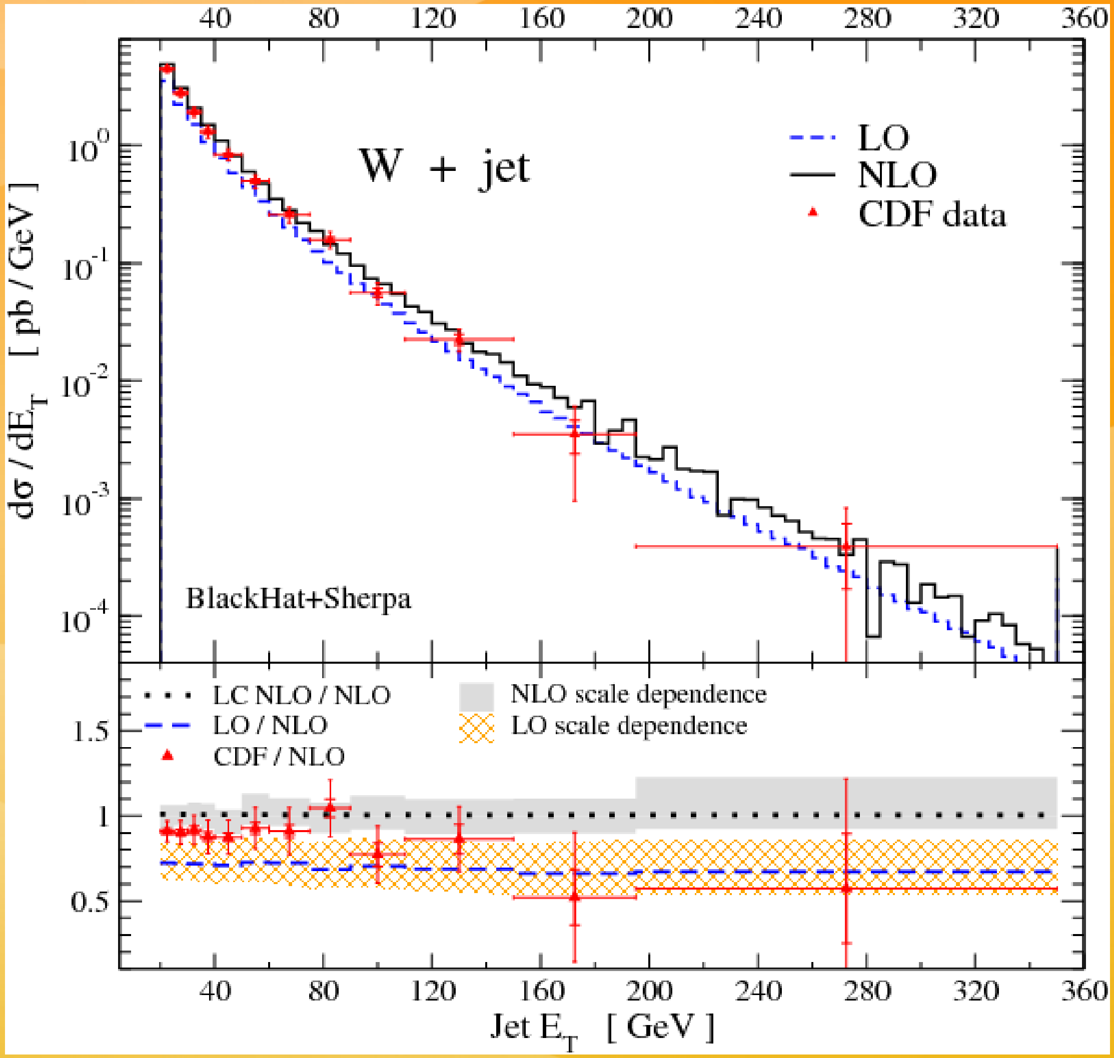
+ Allow for faster computation

- Can be more difficult to combine with real part

W+1 jet @ Tevatron



[CDF Collaboration PRD 77 011108, ArXiv:0711.4044]

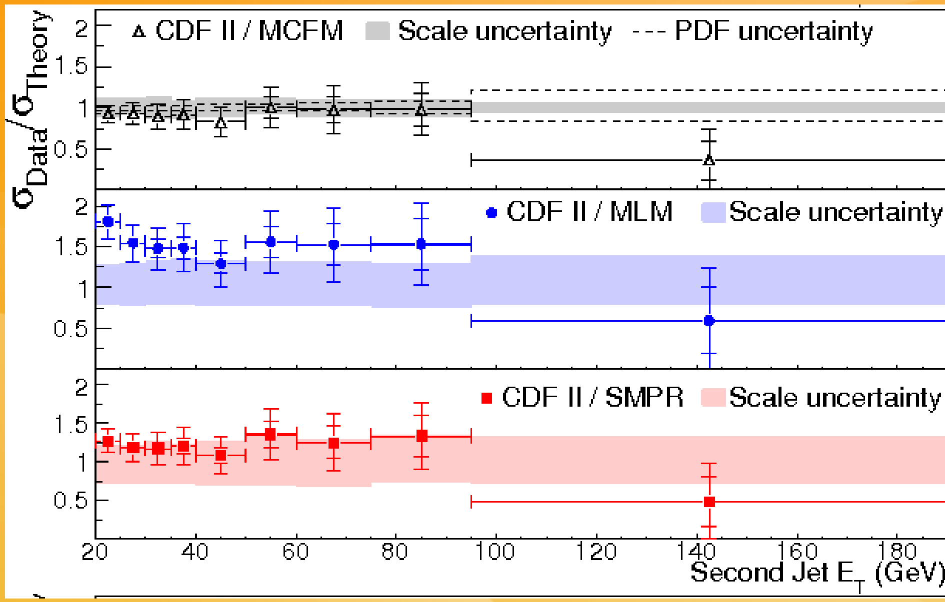


$$\mu = \sqrt{m_W^2 + p_T^2(W)}$$

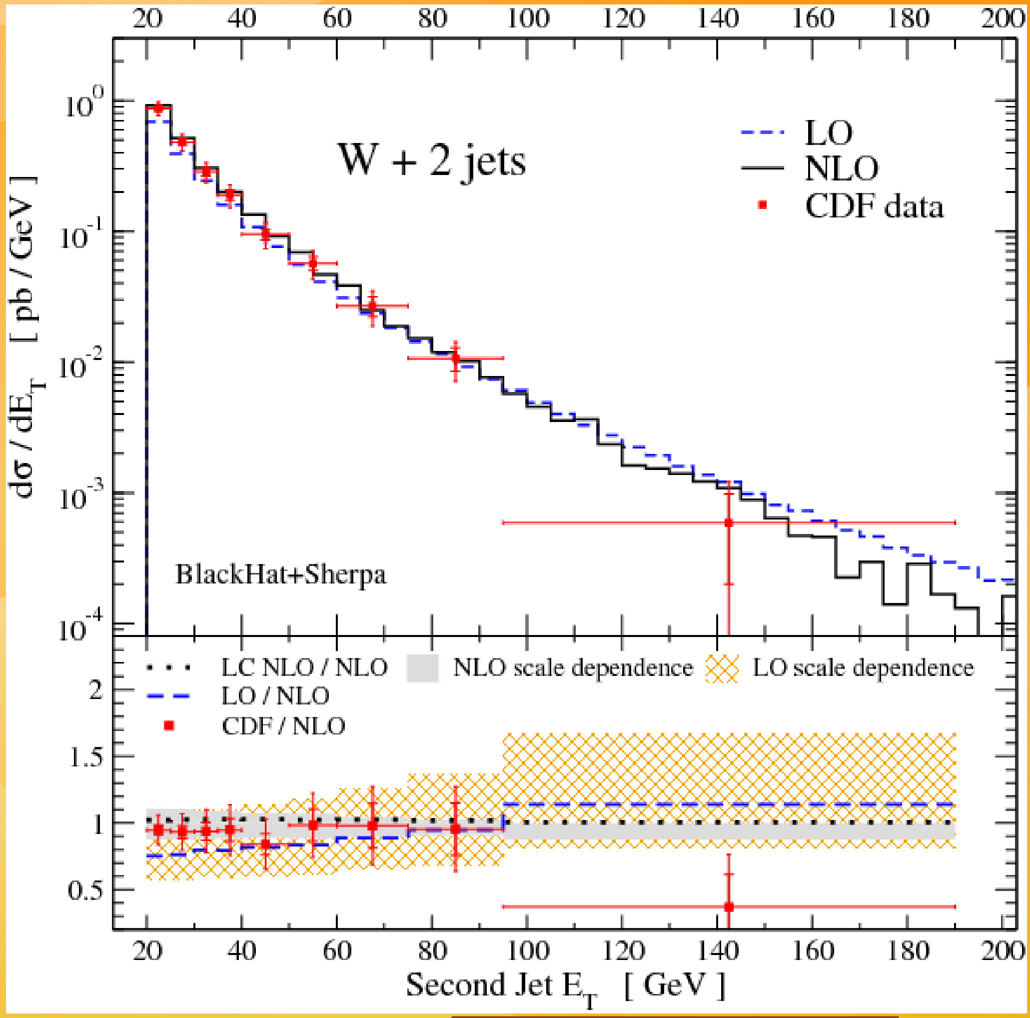
PDF: CTEQ6M

Jet algorithm: SISCone [Salam,Soyez]

W+2 jets @ Tevatron



[CDF Collaboration PRD 77 011108, ArXiv:0711.4044]

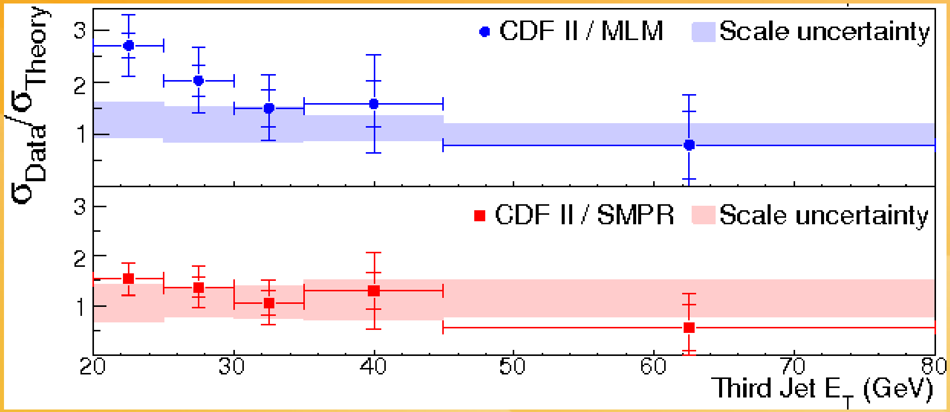


$$\mu = \sqrt{m_W^2 + p_T^2(W)}$$

PDF: CTEQ6M

Jet algorithm: SISCone [Salam,Soyez]

W+3 jet @ Tevatron



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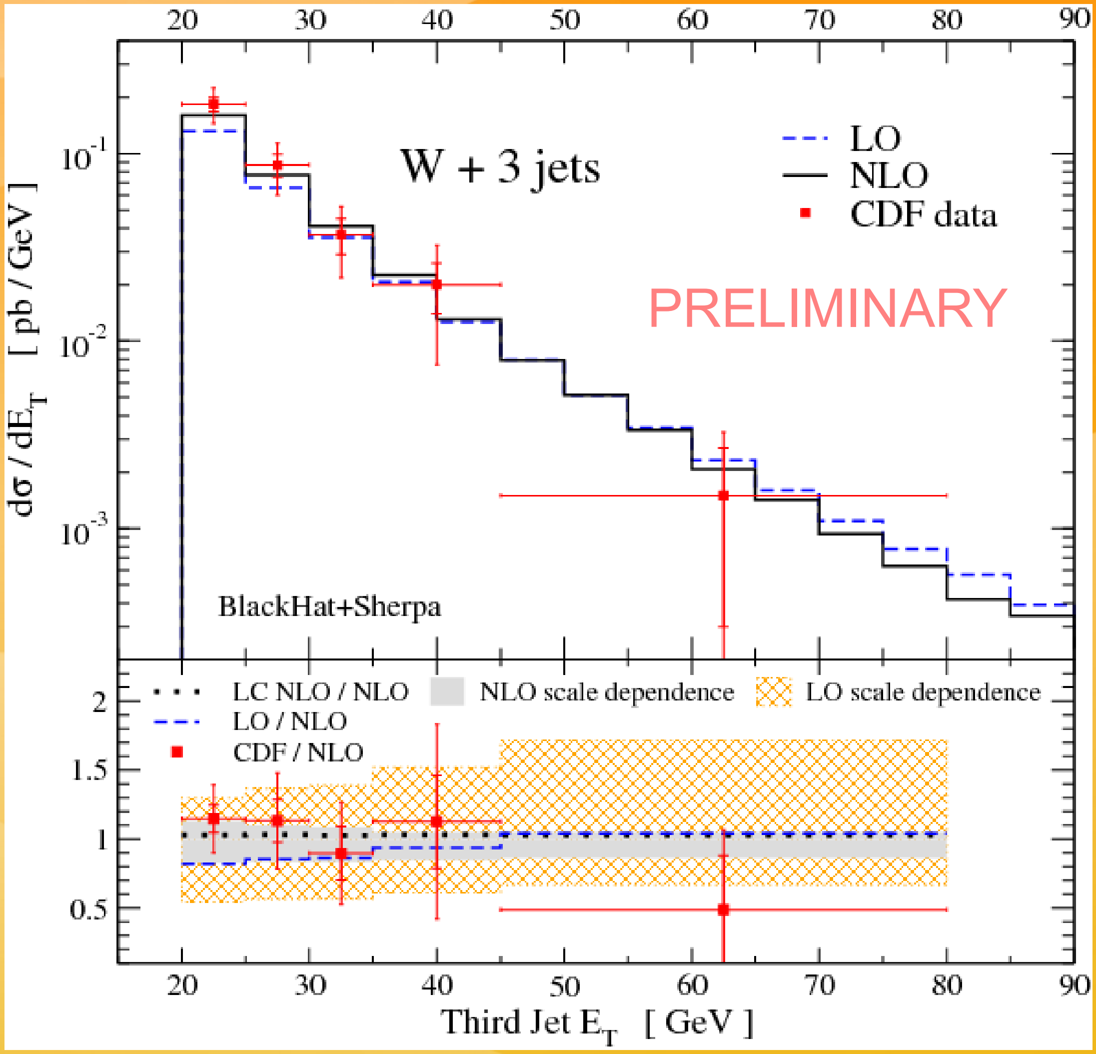

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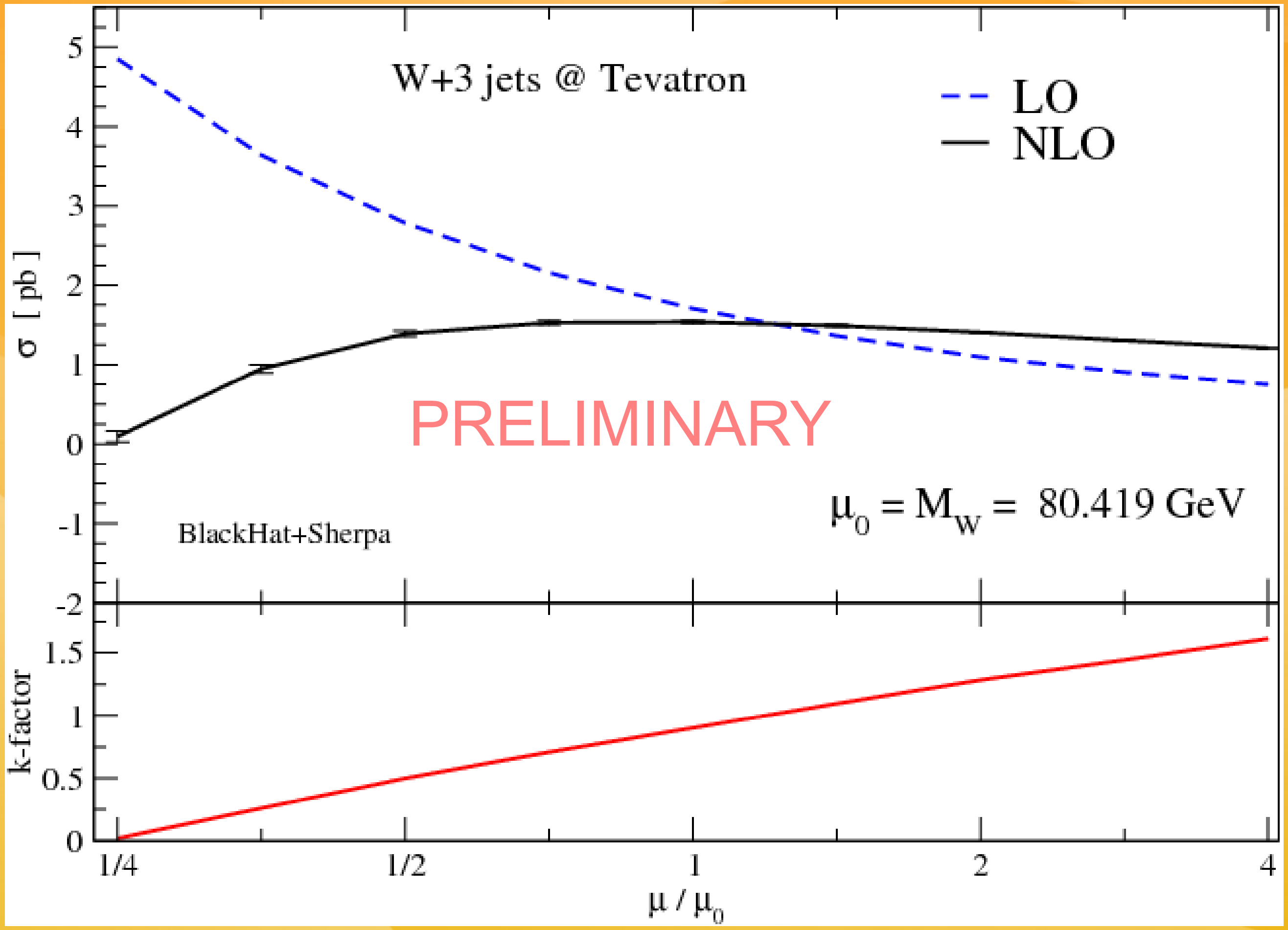
Scale dependence

- NLO scale dependence smaller than at LO

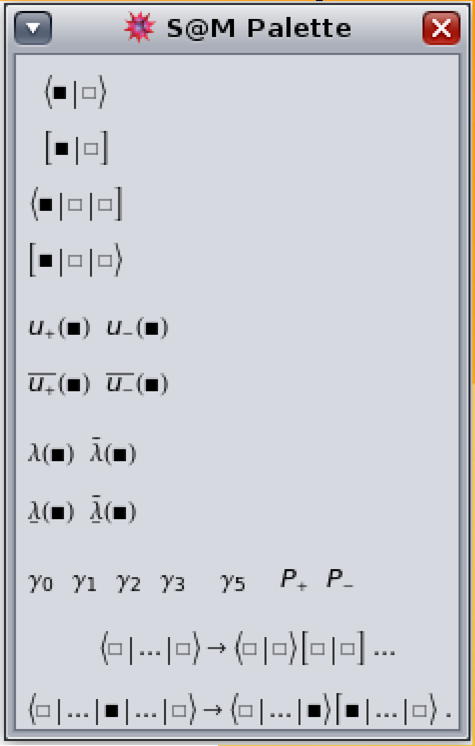


Conclusion

- Numerical accuracy is well under control by (dynamically) using high precision arithmetic
- Numerical implementations of unitarity+on-shell recursion can produce phenomenologically useful results
- First comparison of NLO W+3 jets and experimental data from the Tevatron
- Presented (preliminary) full color results for NLO W+3 jets at the Tevatron
- Shows potential of unitarity techniques

S@M [DM,P. Mastrolia arXiv:0710.5559]

- Mathematica implementation of the spinor-helicity formalism
- Numerical evaluation
- Complex shifts
- 2-dim 4-dim spinors
- < > and [ ] notation
- ...



In[2]:=

$$\text{Spaa}[1,2]^4 / (\text{Spaa}[1,2]\,\text{Spaa}[2,3]\,\text{Spaa}[3,4]\,\text{Spaa}[4,5]\,\text{Spaa}[5,1]) - \frac{\text{Spab}[1,2,4]}{\text{Spbb}[1,3,5,2]}$$

Out[2]=

$$-\frac{\langle 1\,|\,2\rangle^3}{\langle 1\,|\,5\rangle\,\langle 2\,|\,3\rangle\,\langle 3\,|\,4\rangle\,\langle 4\,|\,5\rangle} + \frac{\langle 1\,|\,2\,|\,4]}{[2\,|\,5\,|\,3\,|\,1]}$$

Google: Mathematica spinor package