Generalized unitarity
&
W +3 jets

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Based on work done with Keith Ellis, Walter Giele, Zoltan Kunszt, Kirill Melnikov

Mini-workshop on fixed order multi-leg automatic NLO calculations, Wuppertal, May 2009

This talk

- Brief reminder of main ideas used in D-dimensional unitarity
- I will concentrate on practical aspects: numerical implementation, efficiency, performance, applications & new results

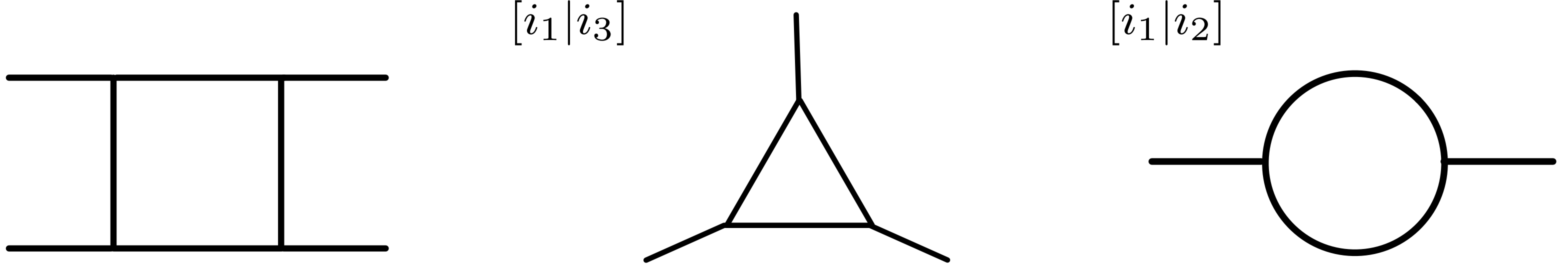
References:
- Ellis, Giele, Kunszt '07 — *[Unitarity in D=4]*
- Giele, Kunszt, Melnikov '08 — *[Unitarity in D≠4]*
- Giele & GZ '08 — *[All one-loop N-gluon amplitudes]*
- Ellis, Giele, Melnikov, Kunszt '08 — *[Massive fermions, ttggg amplitudes]*
- Ellis, Giele, Melnikov, Kunszt, GZ '08 — *[W+5p one-loop amplitudes]*
- Ellis, Melnikov, GZ '09 — *[W+3 jets]*

These papers heavily rely on previous work
- Bern, Dixon, Kosower '94 — *[Unitarity, oneloop from trees]*
- Ossola, Pittau, Papadopoulos '06 — *[OPP]*
- Britto, Cachazo, Feng '04 — *[Generalized cuts]*
- *[....]*

One-loop virtual amplitudes

Cut constructable part can be obtained by taking residues in D=4

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4}\; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3}\; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2}\; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$



Rational part: can be obtained with D ≠ 4

Generic D dependence

Two sources of D dependence

dimensionality of loop momentum D

# of spin eigenstates/ polarization states D_s

Keep D and D_s distinct

$$\mathcal{A}^D \Rightarrow \mathcal{A}^{(D,D_s)}$$

Two key observations

1. External particles in D=4 ⇒ no preferred direction in the extra space

$$\mathcal{N}(l) = \mathcal{N}(l_4, \tilde{l}^2) \qquad \tilde{l}^2 = -\sum_{i=5}^{D} l_i^2$$

$\mathcal{N}$: numerator function

☛ in arbitrary D up to 5 constraints ⇒ get up to pentagon integrals

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2. Dependence of $\mathcal{N}$ on D_s is linear (or almost)

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☛ evaluate at any D_{s1}, D_{s2} ⇒ get $\mathcal{N}_0$ and $\mathcal{N}_1$, i.e. , full $\mathcal{N}$

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Choose D_{s1}, D_{s2} integer ⇒ suitable for numerical implementation

[D_s = 4 - 2ϵ 't-Hooft-Veltman scheme, D_s = 4 FDH scheme]

In practice

- Start from

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

- Use unitarity constraints to determine the coefficients, computed as products of tree-level amplitudes with complex momenta in higher dimensions

- Berends-Giele recursion relations are natural candidates to compute tree level amplitudes: they are very fast for large N and very general (spin, masses, complex momenta)

Berends, Giele '88

Final result

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5} \; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$
$$+ \sum_{[i_1|i_4]} \left(d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} - \frac{D-4}{2} d^{(2)}_{i_1 i_2 i_3 i_4} \; I^{(D+2)}_{i_1 i_2 i_3 i_4} + \frac{(D-4)(D-2)}{4} d^{(4)}_{i_1 i_2 i_3 i_4} \; I^{(D+4)}_{i_1 i_2 i_3 i_4} \right)$$
$$+ \sum_{[i_1|i_3]} \left(c^{(0)}_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} - \frac{D-4}{2} c^{(9)}_{i_1 i_2 i_3} \; I^{(D+2)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b^{(0)}_{i_1 i_2} \; I^{(D)}_{i_1 i_2} - \frac{D-4}{2} b^{(9)}_{i_1 i_2} \; I^{(D+2)}_{i_1 i_2} \right)$$

Cut-constructable part:

$$\mathcal{A}^{CC}_{N} = \sum_{[i_1|i_4]} d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} \; I^{(4-2\epsilon)}_{i_1 i_2}$$

Rational part:

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m^2_{i_1} + m^2_{i_2}}{2} \right) \; b^{(9)}_{i_1 i_2}$$

Vanishing contributions: $\mathcal{A} = \mathcal{O}(\epsilon)$

The F90 Rocket program

Rocket science!

Eruca sativa =Rocket=roquette=arugula=rucola

Recursive unitarity calculation of one-loop amplitudes

So far computed one-loop amplitudes:

- ✓ N-gluons
- ✓ qq + N-gluons
- ✓ qq + W + N-gluons
- ✓ qq + QQ + W
- ✓ tt + N-gluons
- ✓ tt + qq + N-gluons [Schulze]

NB: N is a parameter in Rocket!

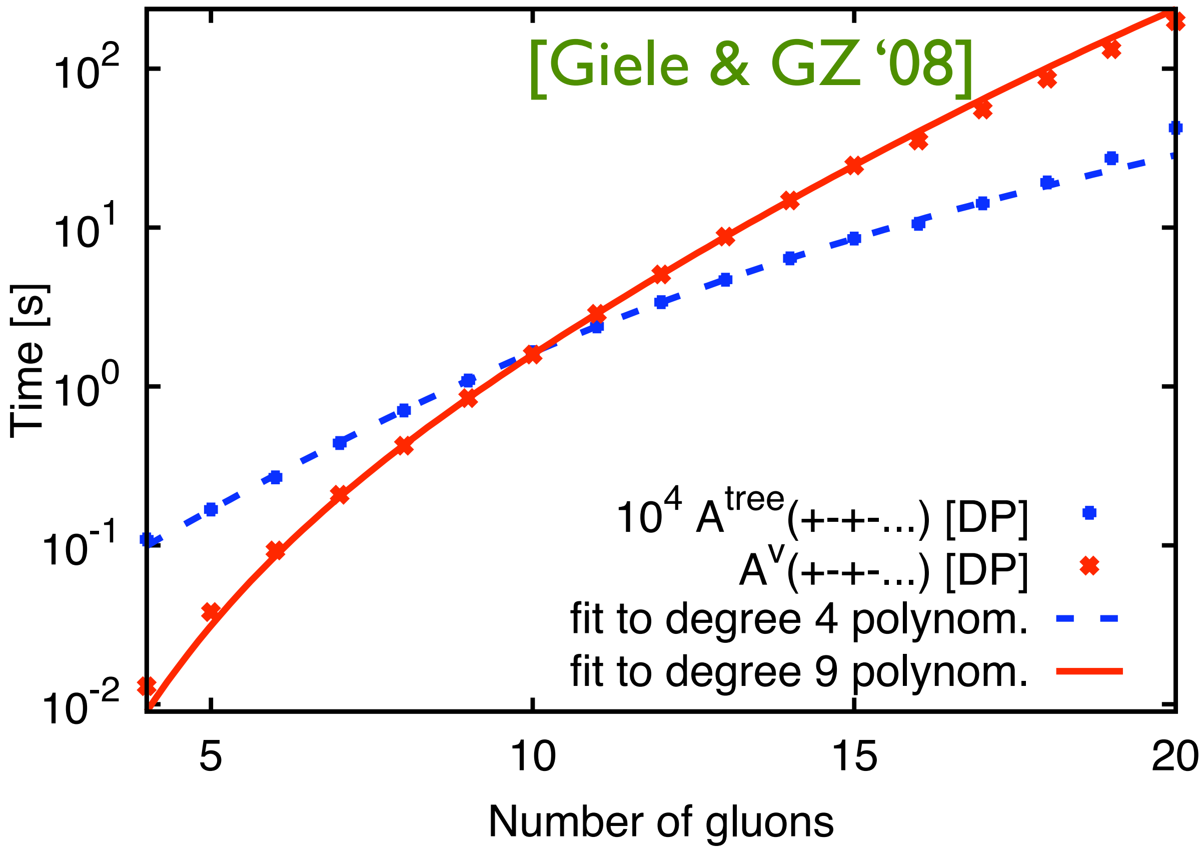
In perspective, for gluons:

N = 6 ⇒ 10860 diags.

N = 7 ⇒ 168925 diags.

Successfully computed up to N=20!

Time for oneloop N-gluon loop amplitudes



- ☛ time ∝ N^9 as expected
- ☛ independent of the helicity configuration

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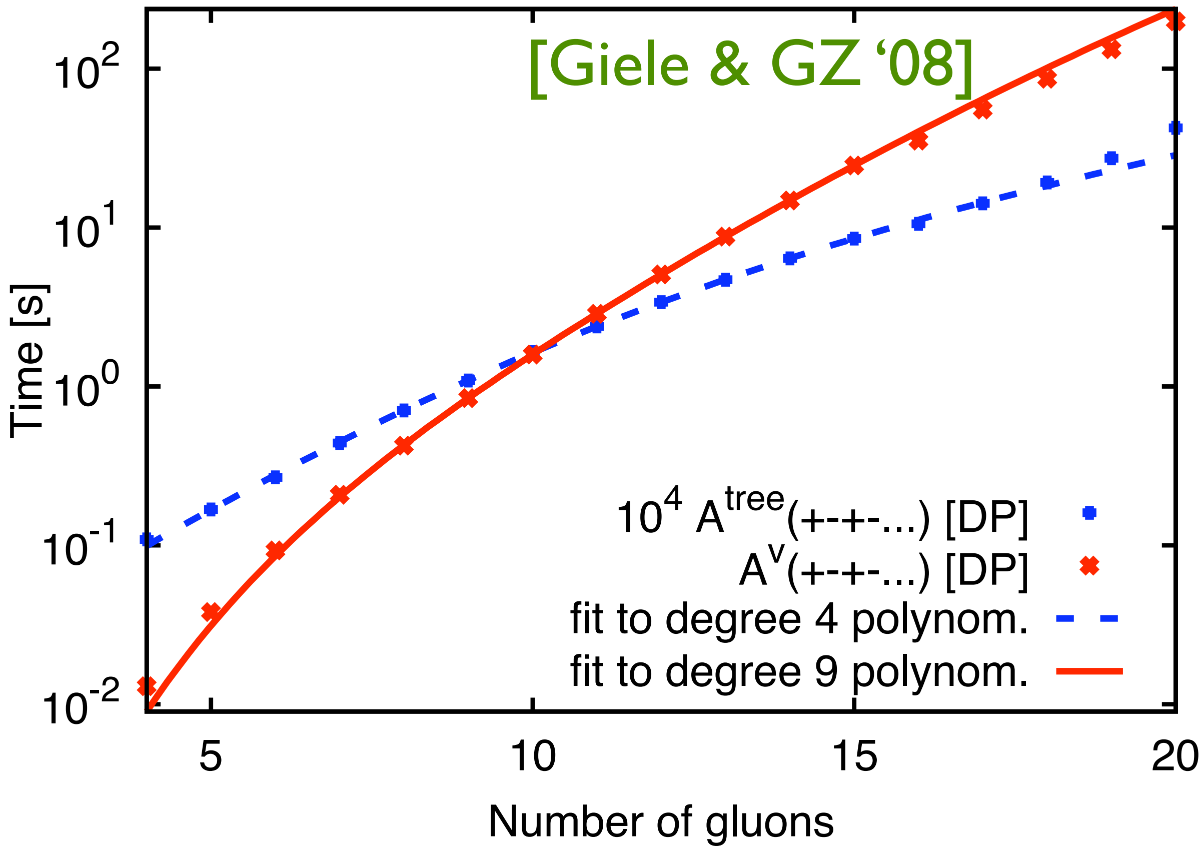

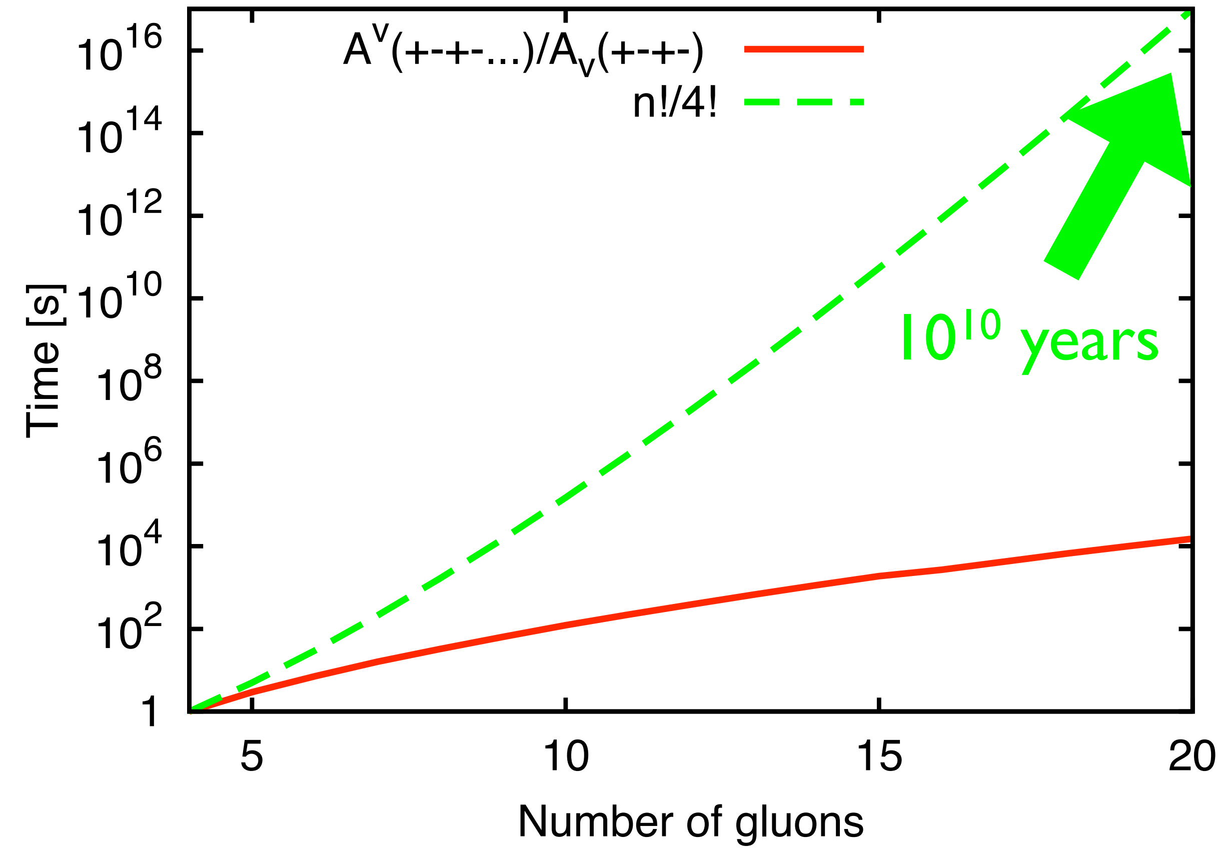

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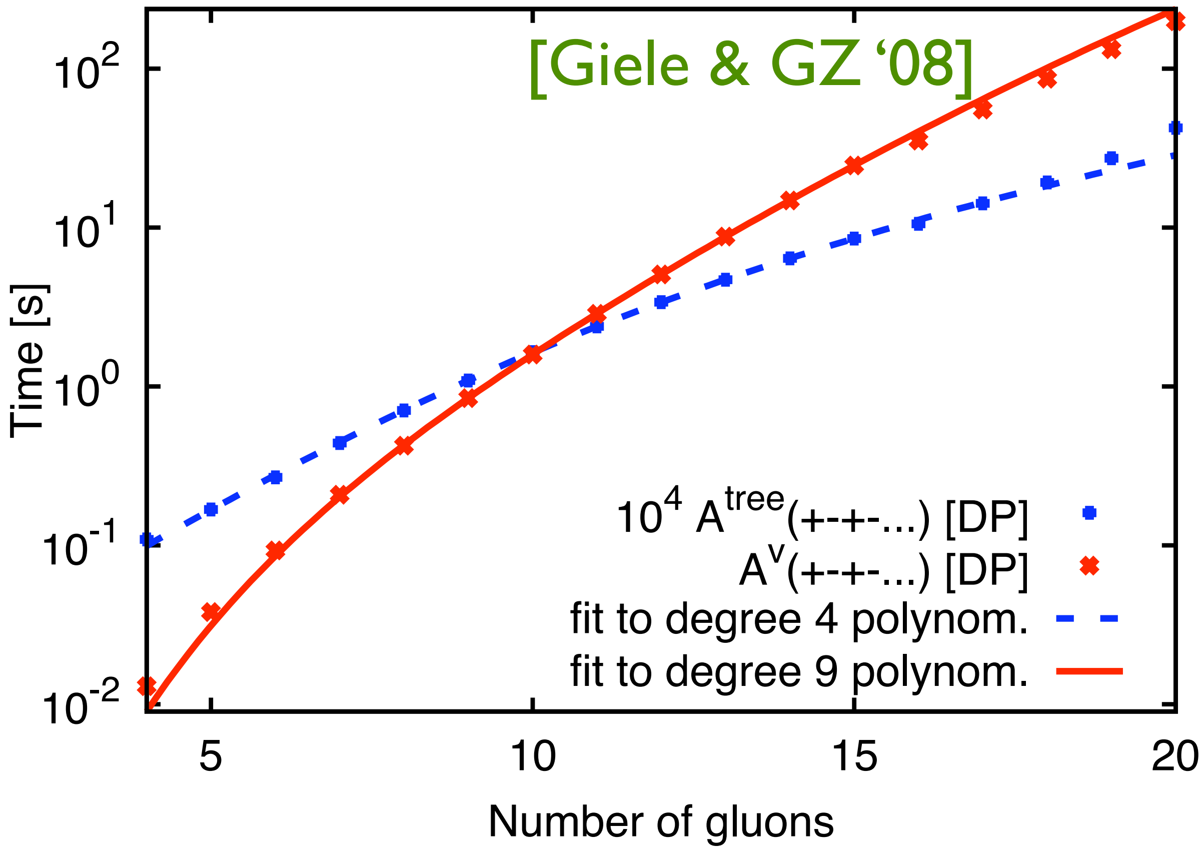

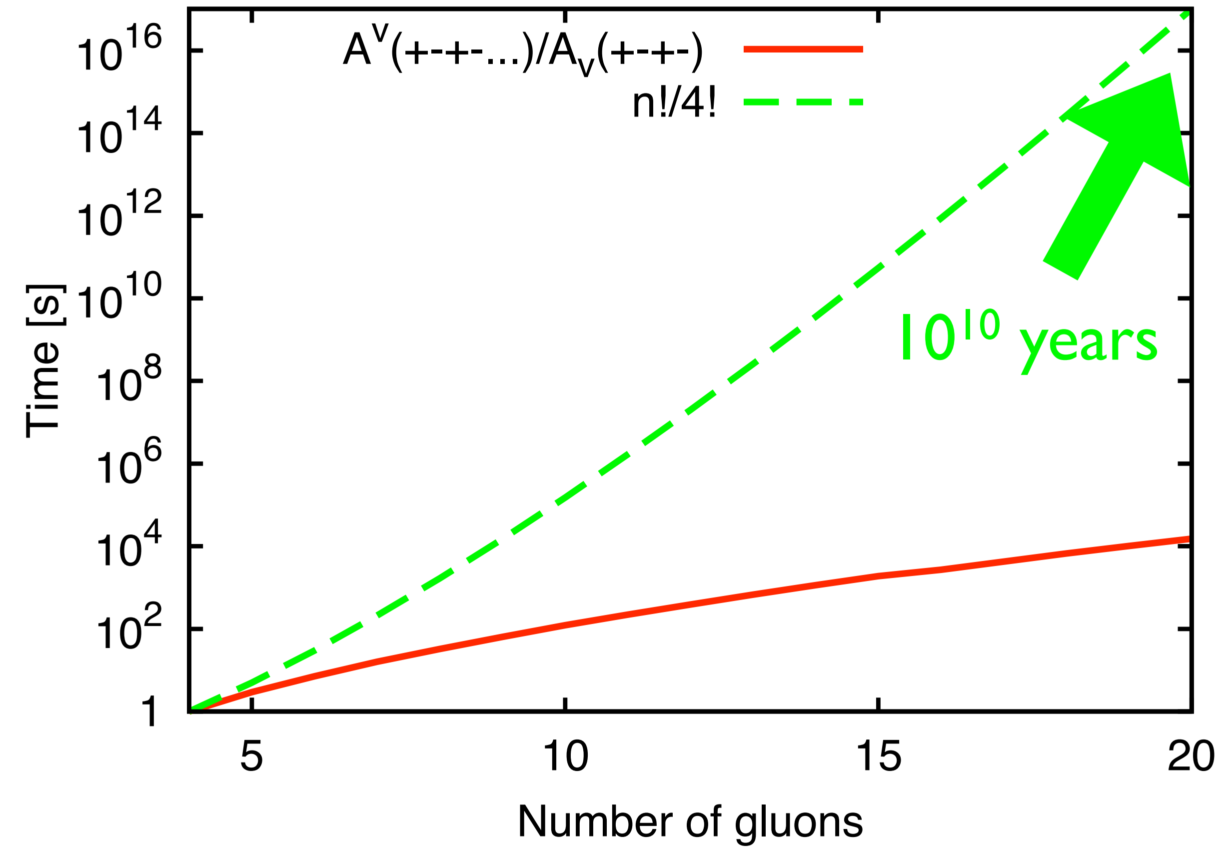

- ☛ time $\propto N^9$ as expected
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- ☛ compare with factorial growth...

Comparison with other methods: time roughly comparable

Berger, Bern, Cordero, Dixon, Forde, Ita, Kosower, Maitre '08
Giele & Winter '09
Lazopoulos '09

Issues of automated one-loop

- checks of the results
  - ☛ poles, ward identities, independence of choice of D_1 and D_2, independence of the choice of the solution of the unitarity constraints, independence from choice of auxiliary vectors (gauge)
- numerical instabilities at special points
  - ☛ efficient procedure for identification of special points, than run in quadruple precision. Checked that target accuracy is reached.
- numerical efficiency
  - ☛ polynomial scaling for any NLO amplitude (N^9 for gluons)
- practicality: computation of realistic LHC processes
  - ☛ *first application: W + 3 jets*

First application: W + 3 jets

1. W + 3 jets measured at the Tevaton, but LO varies by more than a factor 2 for reasonable changes in scales

	$W^{\pm}$, TeV	W^{+}, LHC	W^{-}, LHC
σ [pb], $\mu = 40$ GeV	74.0 ± 0.2	783.1 ± 2.7	481.6 ± 1.4
σ [pb], $\mu = 80$ GeV	45.5 ± 0.1	515.1 ± 1.1	316.7 ± 0.7
σ [pb], $\mu = 160$ GeV	29.5 ± 0.1	353.5 ± 0.8	217.5 ± 0.5

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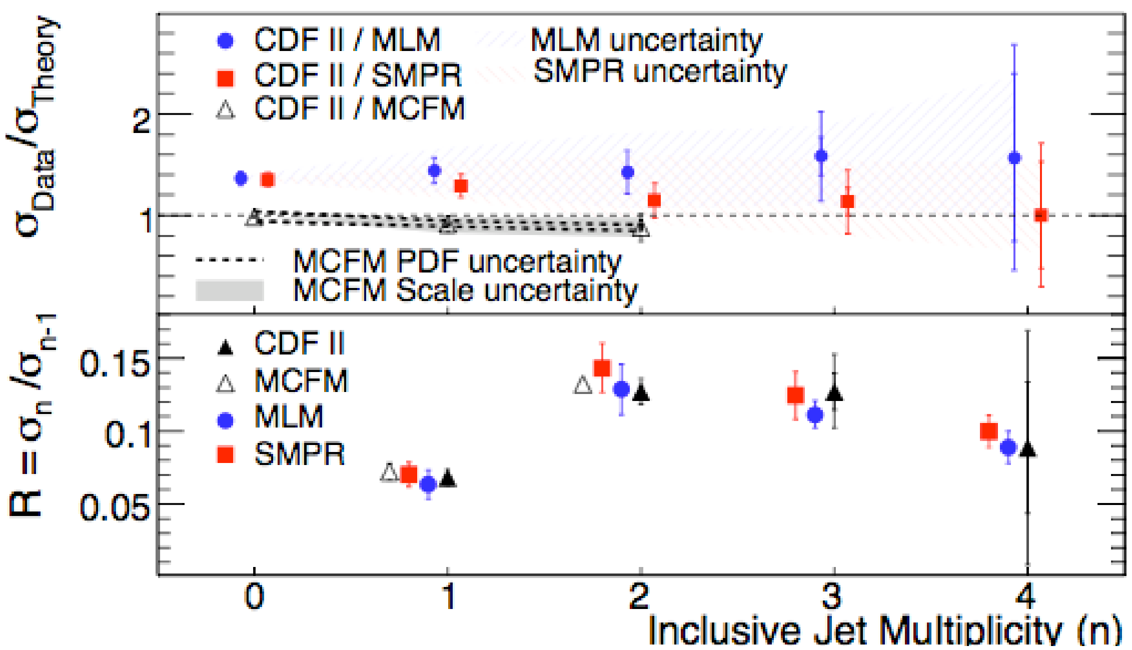
II. Measurements at the Tevaton: for W + n jets with n=1,2 data is described well by NLO QCD

$\Rightarrow$ verify this for 3 and more jets



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III. W + 3 jets of interest at the LHC, as one of the backgrounds to model-independent new physics searches using jets + MET

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IV. Calculation highly non-trivial optimal testing ground

$$0 \rightarrow \bar{u}\, d\, g\, g\, g\, W^+$$ ⇒ 1203 +104 Feynman diagrams

$$0 \rightarrow \bar{u}\, d\, \bar{Q}\, Q\, g\, W^+$$ ⇒ 258 +18 Feynman diagrams

Color decomposition

$$0 \to \bar{q} + q + (n-2)\ \text{gluons} + W$$

<u>Tree level:</u>

$$\mathcal{A}_n^{\text{tree}}(1_{\bar{q}}, 2_q, 3_g, \ldots, n_g) \;=\; g^{n-2} \sum_{\sigma \in S_{n-2}} (T^{a_{\sigma(3)}} \ldots T^{a_{\sigma(n)}})_{i_2}^{\;\bar{\imath}_1}\; A_n^{\text{tree}}(1_{\bar{q}}, 2_q; \sigma(3)_g, \ldots, \sigma(n)_g)$$

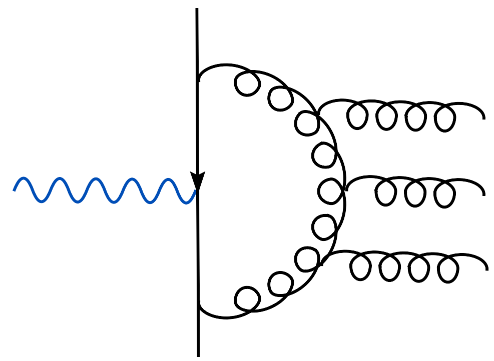
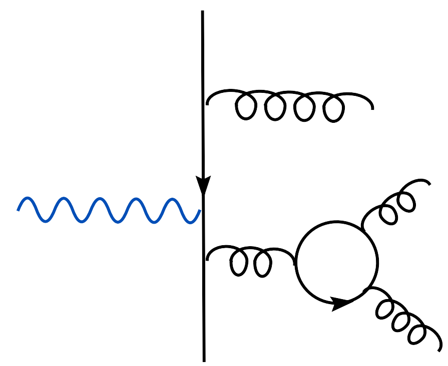
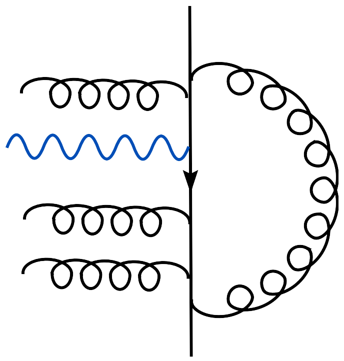
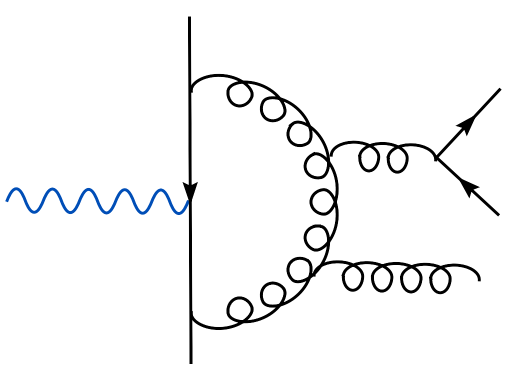
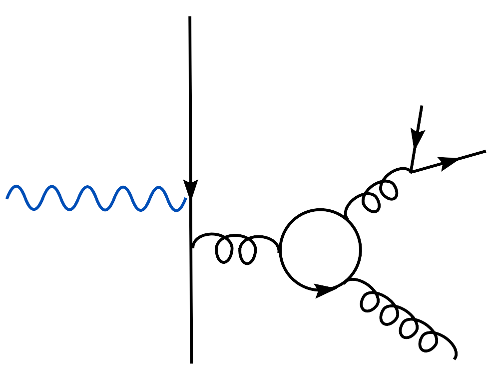
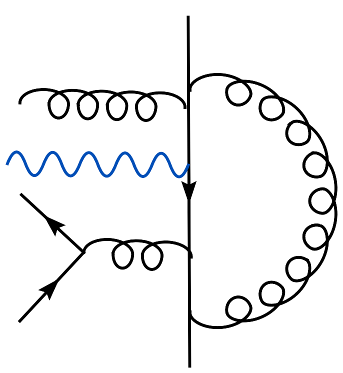
<u>One-loop decomposition into primitive amplitudes:</u>

Bern, Dixon, Kosower ’94

$$\begin{aligned}\mathcal{A}_n^{\text{1-loop}}(1_{\bar{q}}, 2_q, 3_g, \ldots, n_g) = g^n \Bigg[&\sum_{p=2}^{n} \sum_{\sigma \in S_{n-2}} (T^{x_2} T^{a_{\sigma_3}} \cdots T^{a_{\sigma_p}} T^{x_1})_{i_2}^{\;\bar{\imath}_1} (F^{a_{\sigma_{p+1}}} \cdots F^{a_{\sigma_n}})_{x_1 x_2} \\ &\times (-1)^n A_n^L(1_{\bar{q}}, \sigma(p)_g, \ldots, \sigma(3)_g, 2_q, \sigma(n)_g, \ldots, \sigma(p+1)_g) \\ &+ \frac{n_f}{N_c} \sum_{j=1}^{n-1} \sum_{\sigma \in S_{n-2}/S_{n;j}} \text{Gr}_{n;j}^{(\bar{q}q)}(\sigma_3 \ldots, \sigma_n) A_{n;j}^{[1/2]}(1_{\bar{q}}, 2_q; \sigma(3)_g, \ldots, \sigma(n)_g) \Bigg]\end{aligned}$$

Knowledge of (gauge invariant) primitives specifies one-loop amplitude.
One highest level N-point function per primitive.

Primitive amplitudes: color structures

	Leading color	Fermion loops	Subleading color
2-quark 3-gluon			
	$\mathrm{LC} \equiv (N_c^2 - 1)N_c^3$	$\mathrm{LC} \cdot \frac{n_f}{N_c}$	$\cdots$
4-quark 1-gluon			
	$\mathrm{LC} \cdot \frac{n_f}{N_c}$	$\cdots$	$\cdots$

Rules of the game

Procedure:

- order all SU(3) particles & allow all orderings of colorless particles

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Explicitly for W + 3 jets:

❶ ❷ ❸ ❹ ❺

u_1 g_2 g_3 g_4 d_5 + W

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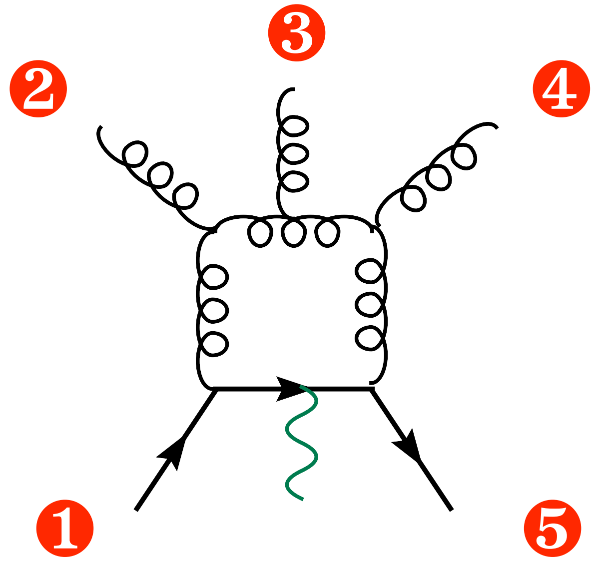

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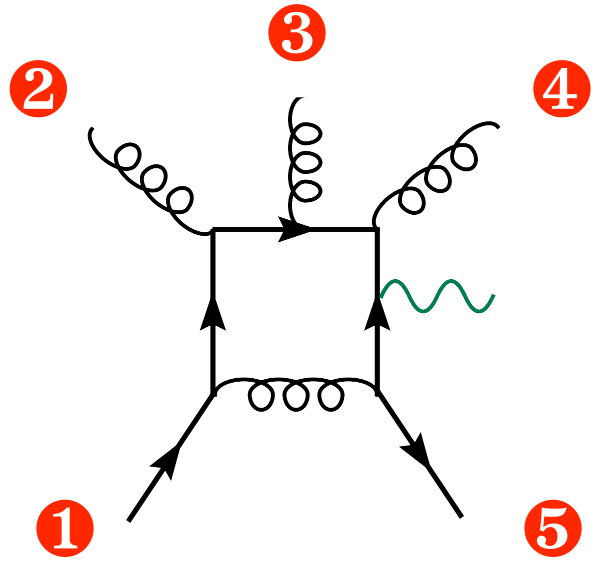

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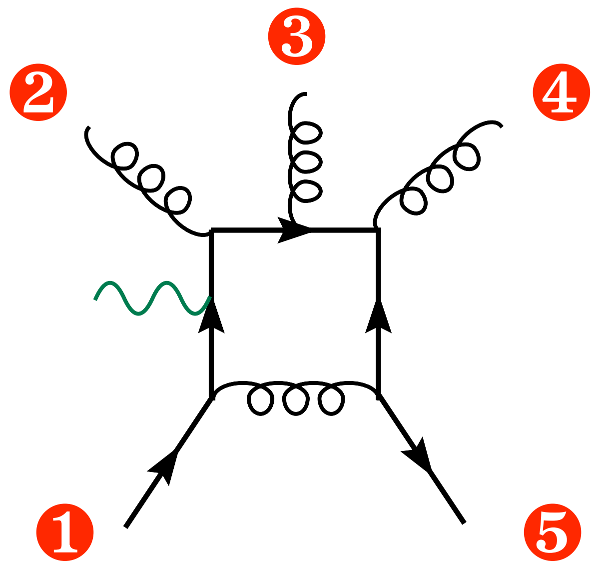

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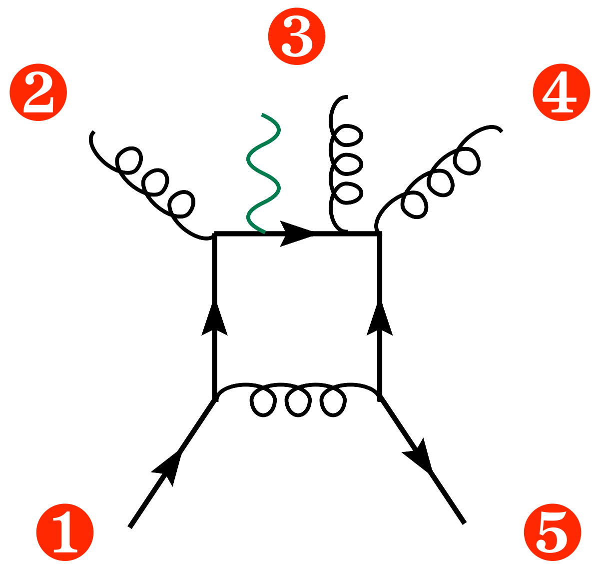

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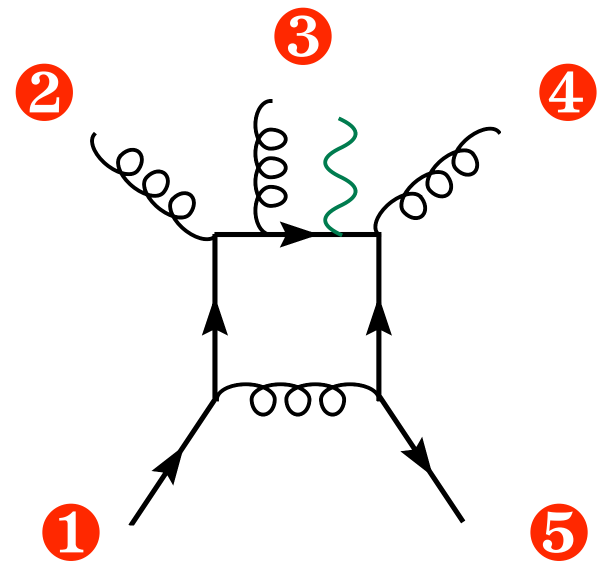

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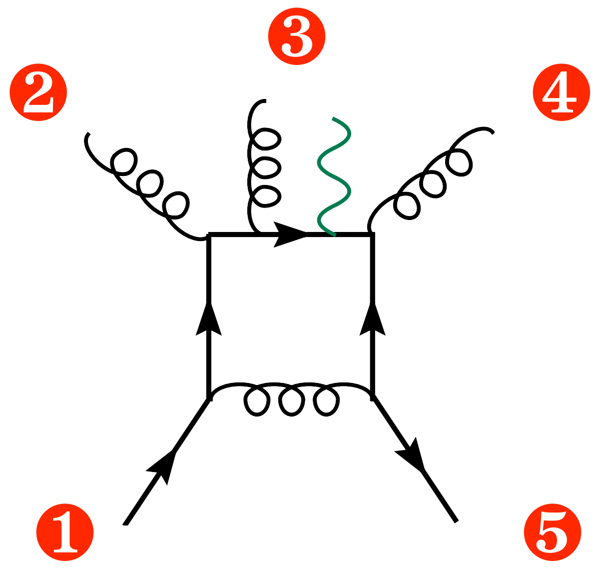

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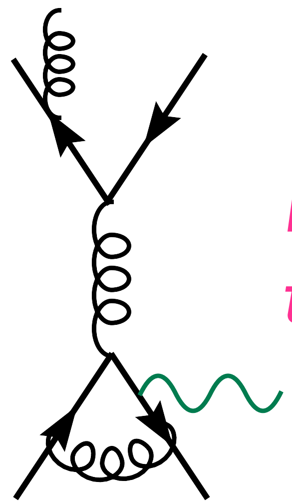

How does this work?

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Explicitly for W+3jets:

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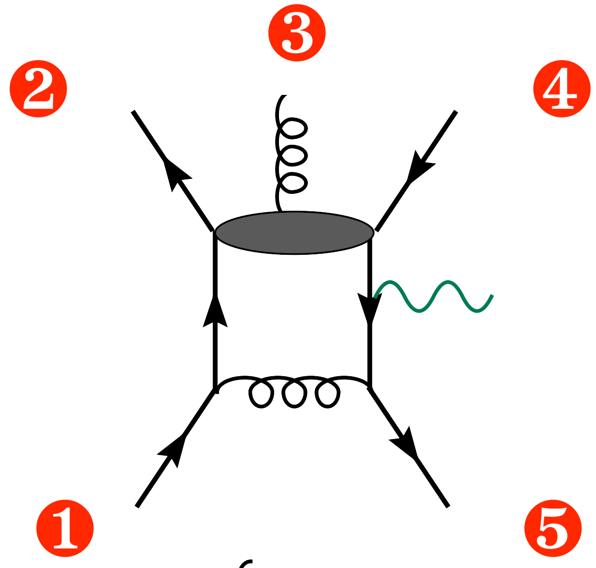
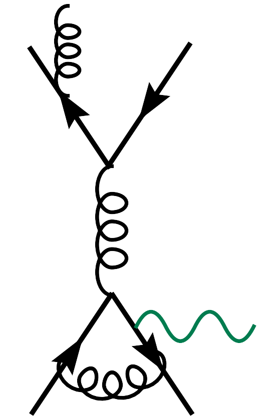


Refers e.g. to:



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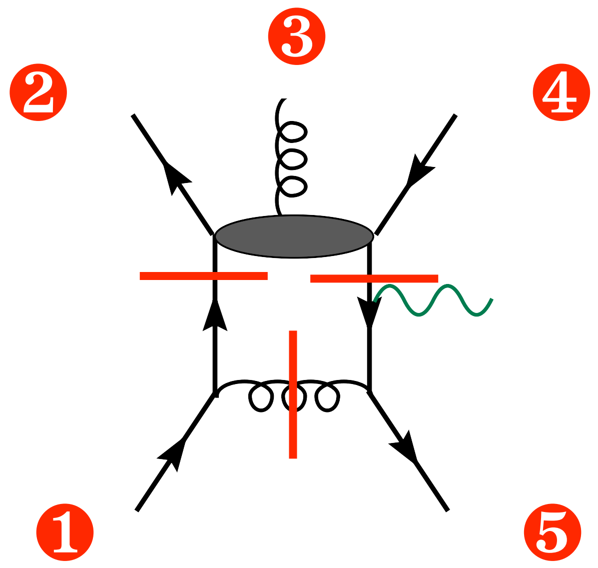

✓ *accept*

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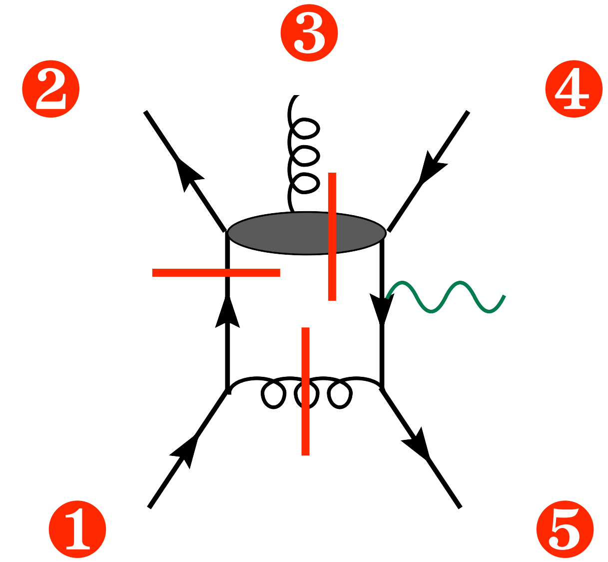

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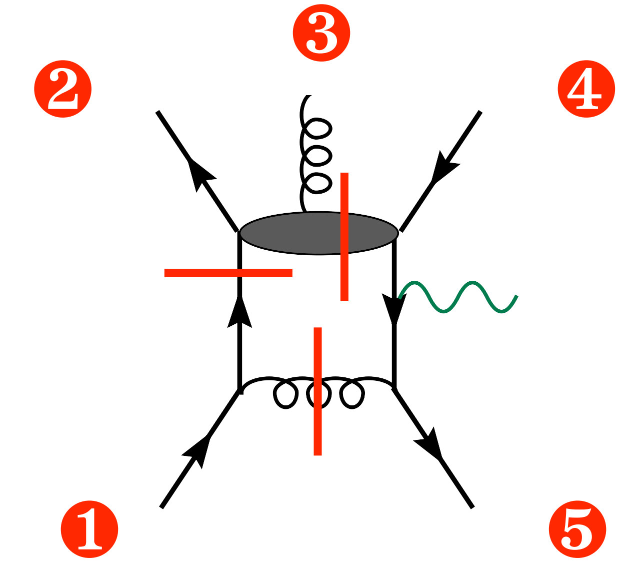
- N-point case: parent must be 1PI N-point, use dummy lines if needed
- consider all cuts and throw away those involving dummy lines
- process each cut use standard D-dimensional unitarity

Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

u_1 q_2 g_3 q_4 $d_5 + W$



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Rules of the game

Procedure:

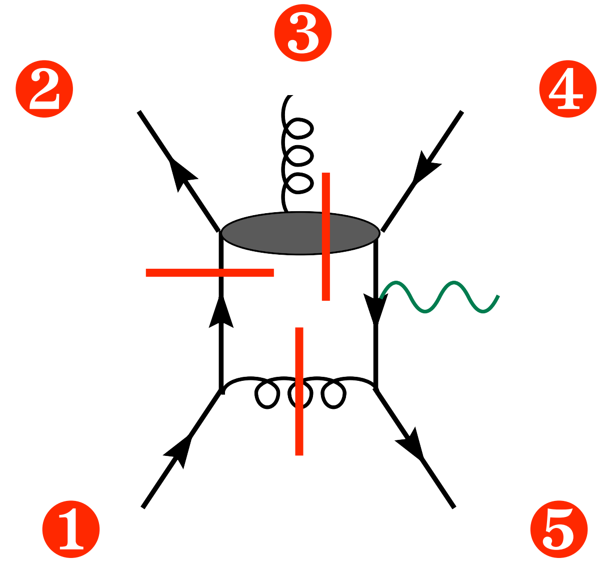
- order all SU(3) particles & allow all orderings of colorless particles
- draw the parent diagram so that the loop is in the fixed position compared to the external fermion line [L/R]
- N-point case: parent must be 1PI N-point, use dummy lines if needed
- consider all cuts and throw away those involving dummy lines
- process each cut use standard D-dimensional unitarity
- tree-level amplitudes are computed via color stripped Feynman rules

Bern, Dixon, Kosower '94

Explicitly for W+3jets:

❶ ❷ ❸ ❹ ❺

$u_1 \; q_2 \; g_3 \; q_4 \; d_5 + W$



✗ *reject*

Sample results

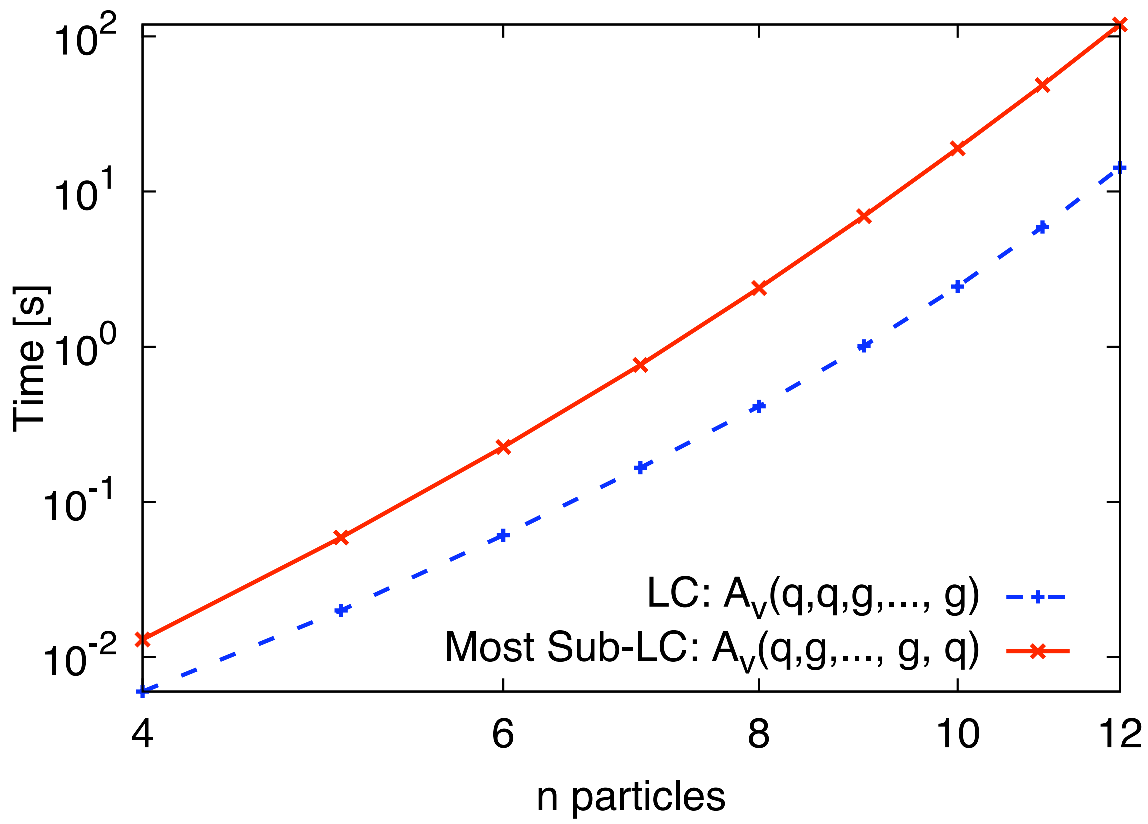
Helicity	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$A^{\rm tree}(1_{\bar q}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^+\, 6_{\bar l}^+\, 7_l^-)$			$-0.006873 + i\,0.011728$
$r_L^{[1]}(1_{\bar q}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^+\, 6_{\bar l}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$5.993700 - i\,19.646278$
$A^{\rm tree}(1_{\bar q}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^-\, 6_{\bar l}^+\, 7_l^-)$			$0.010248 - i\,0.007726$
$r_L^{[1]}(1_{\bar q}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^-\, 6_{\bar l}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-14.377555 - i\,37.219716$
$A^{\rm tree}(1_{\bar q}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^+\, 6_{\bar l}^+\, 7_l^-)$			$0.495774 - i\,1.274796$
$r_L^{[1]}(1_{\bar q}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^+\, 6_{\bar l}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.039489 - i\,30.210418$
$A^{\rm tree}(1_{\bar q}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^-\, 6_{\bar l}^+\, 7_l^-)$			$-0.294256 - i\,0.223277$
$r_L^{[1]}(1_{\bar q}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^-\, 6_{\bar l}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.444709 - i\,26.101951$

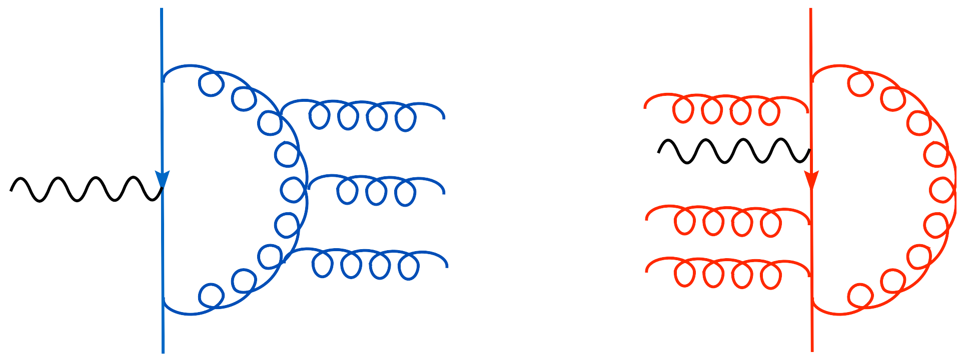
$$r_L^{[j]}(1,2,3,4,5,6,7) = \frac{1}{c_\Gamma}\frac{A_L^{[j]}(1,2,3,4,5,6,7)}{A^{\rm tree}(1,2,3,4,5,6,7)}, \quad c_\Gamma = \frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)^2}{(4\pi)^{2-\epsilon}\Gamma(1-2\epsilon)},$$

Leading color amplitudes in 0808.0941
[Berger, Bern, Cordero, Dixon, Forde, Ita, Kosower, Maitre]

All amplitudes in 0810.2542
[Ellis, Giele, Kunszt, Melnikov, GZ]

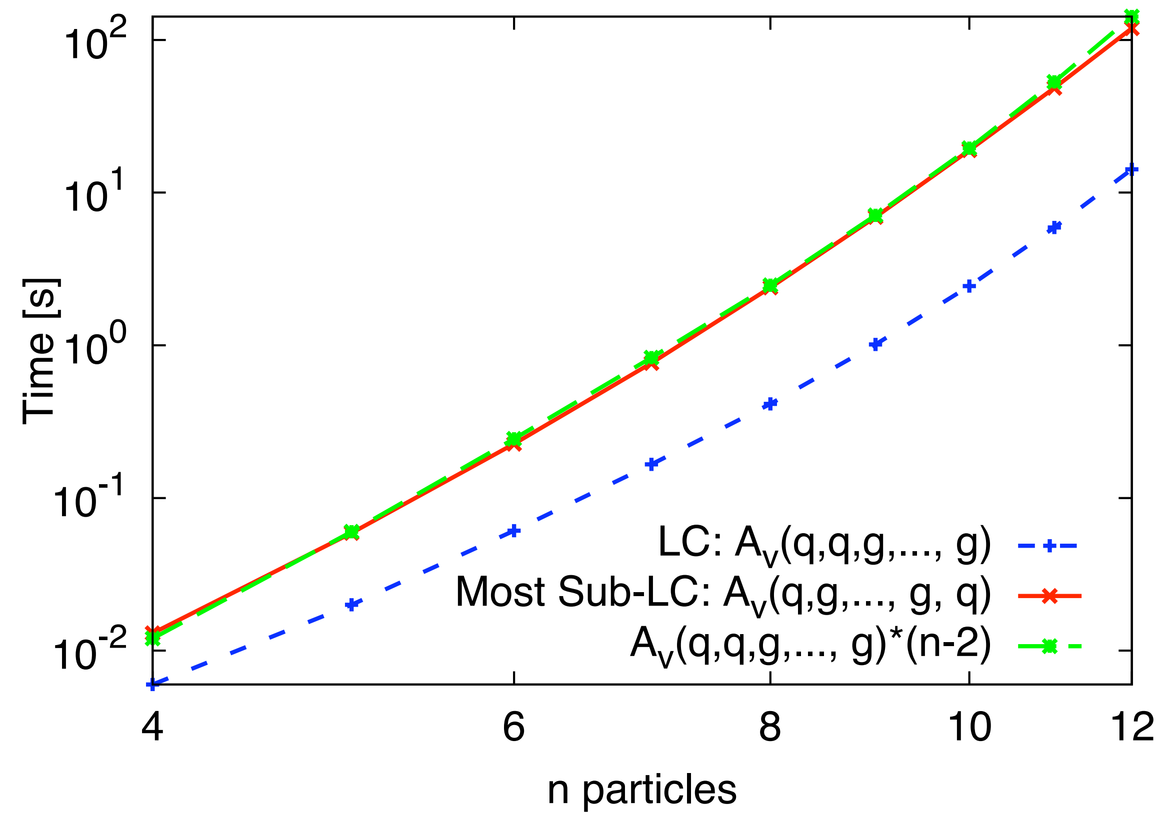
Time dependence of qq + W + n gluons

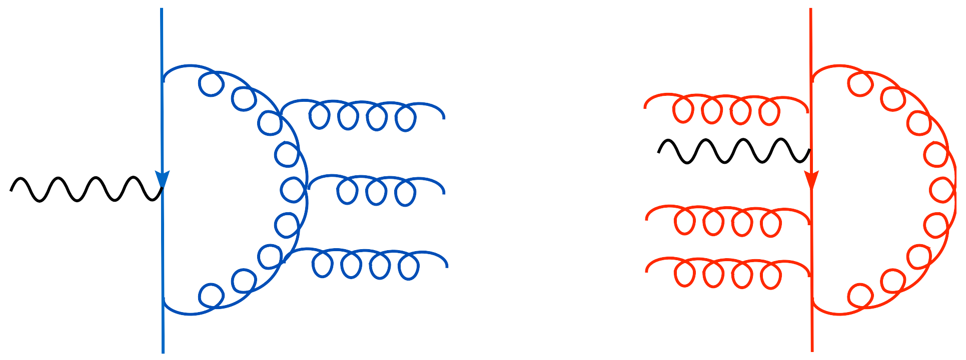




# of cuts: N_{cuts} $N_{\text{cuts}} \cdot (n-2)$

Time dependence of qq + W + n gluons





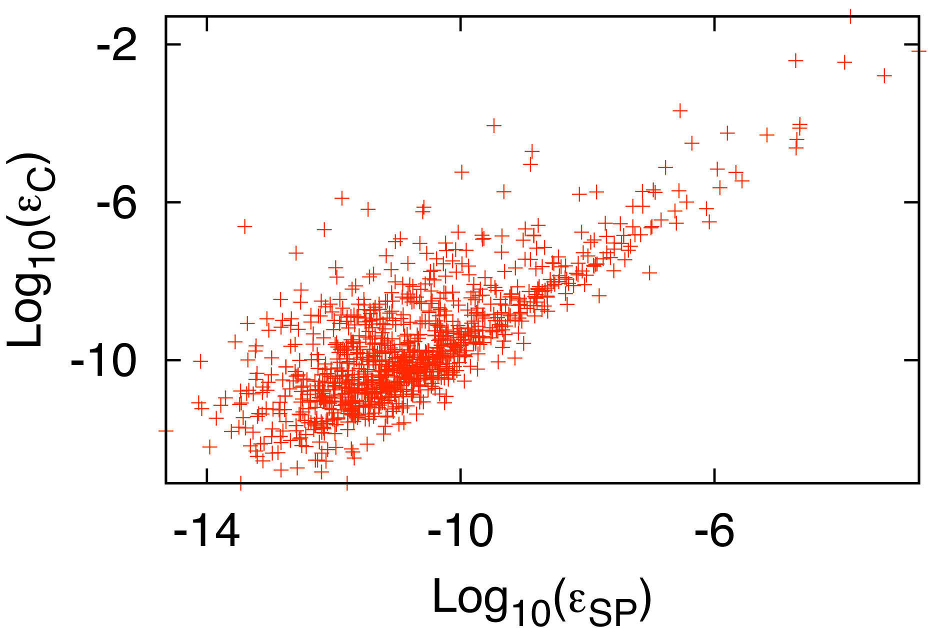
\# of cuts: N_{cuts} $\quad$ $N_{\text{cuts}} \cdot (n-2)$

Similar plots for qq + n gluons

Finding instabilities

1. Correlation in the accuracy of single pole and constant part

⇒ if the accuracy on the poles is worse than X use higher precision
This does not check the rational part



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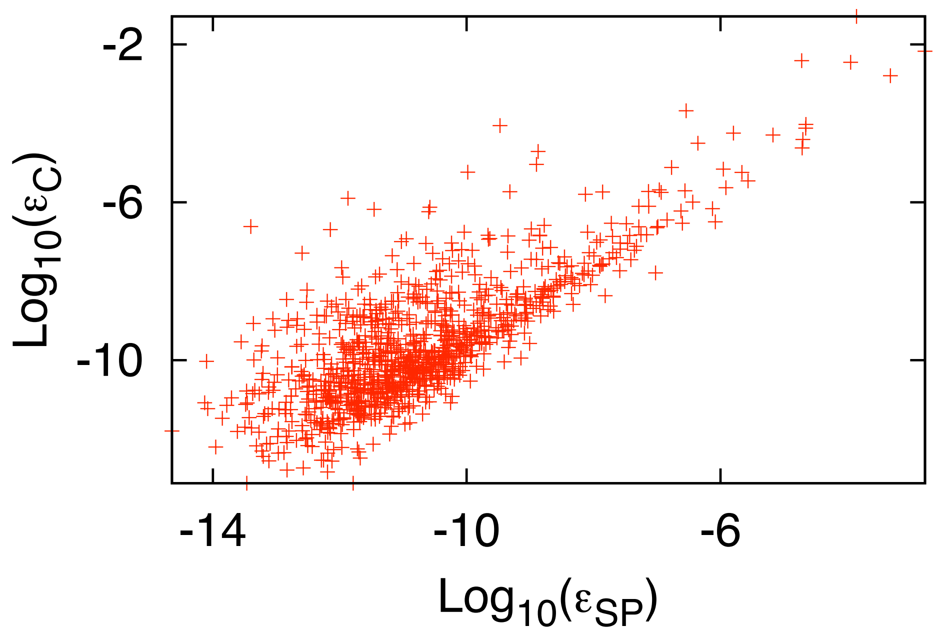
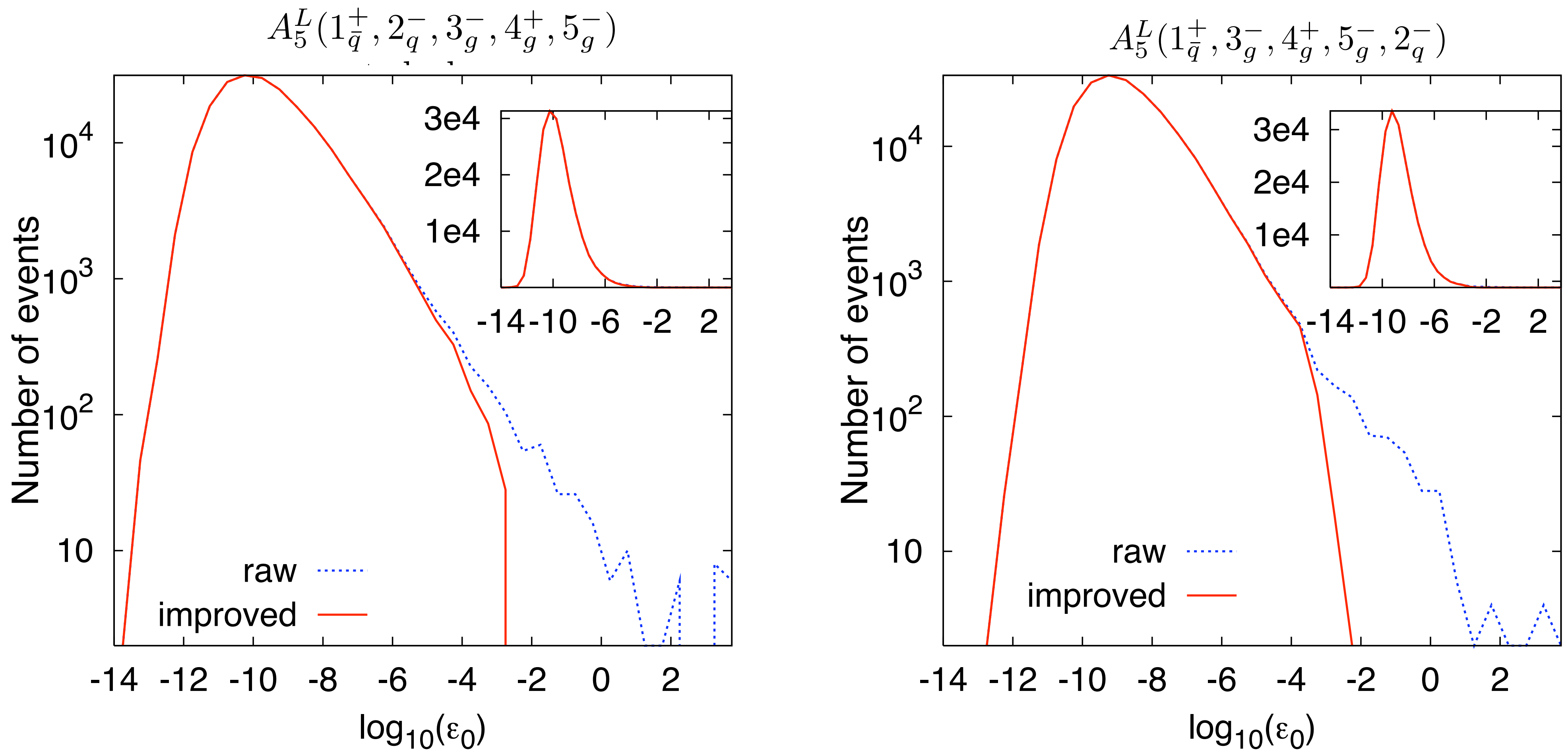


2. How good is the system of equations solved ?

Look at how well residues are reconstructed using the coefficients
In practice: choose a random loop momentum and for a given cut
- compute the residue as linear combination of coefficients
- compute the residue directly

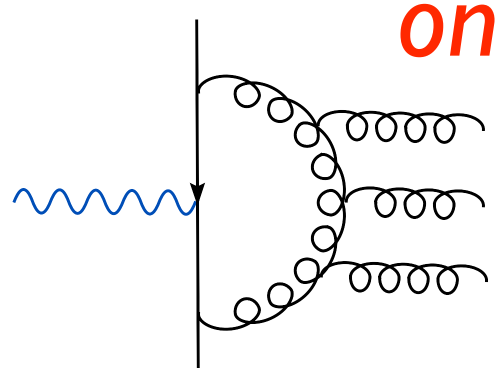
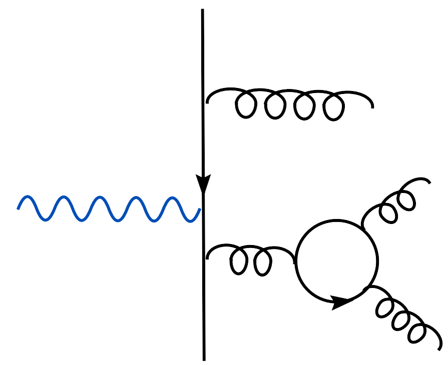
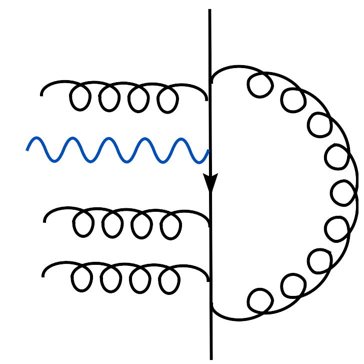
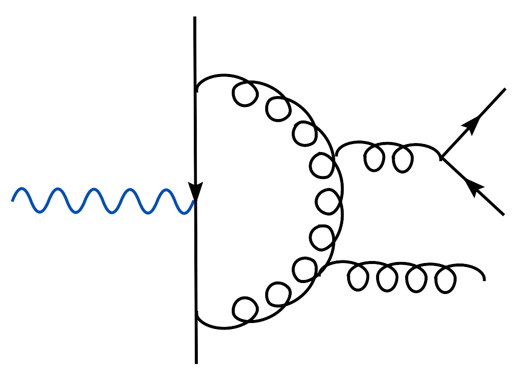
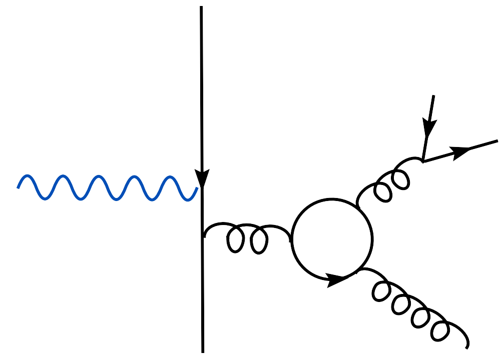
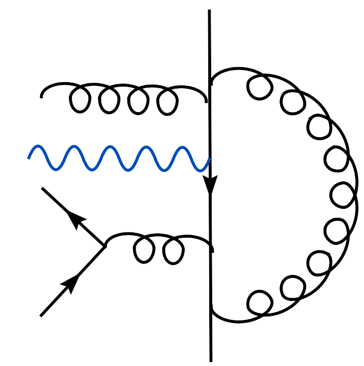
⇒ if the results differ more than X use higher precision

Instabilities and accuracy



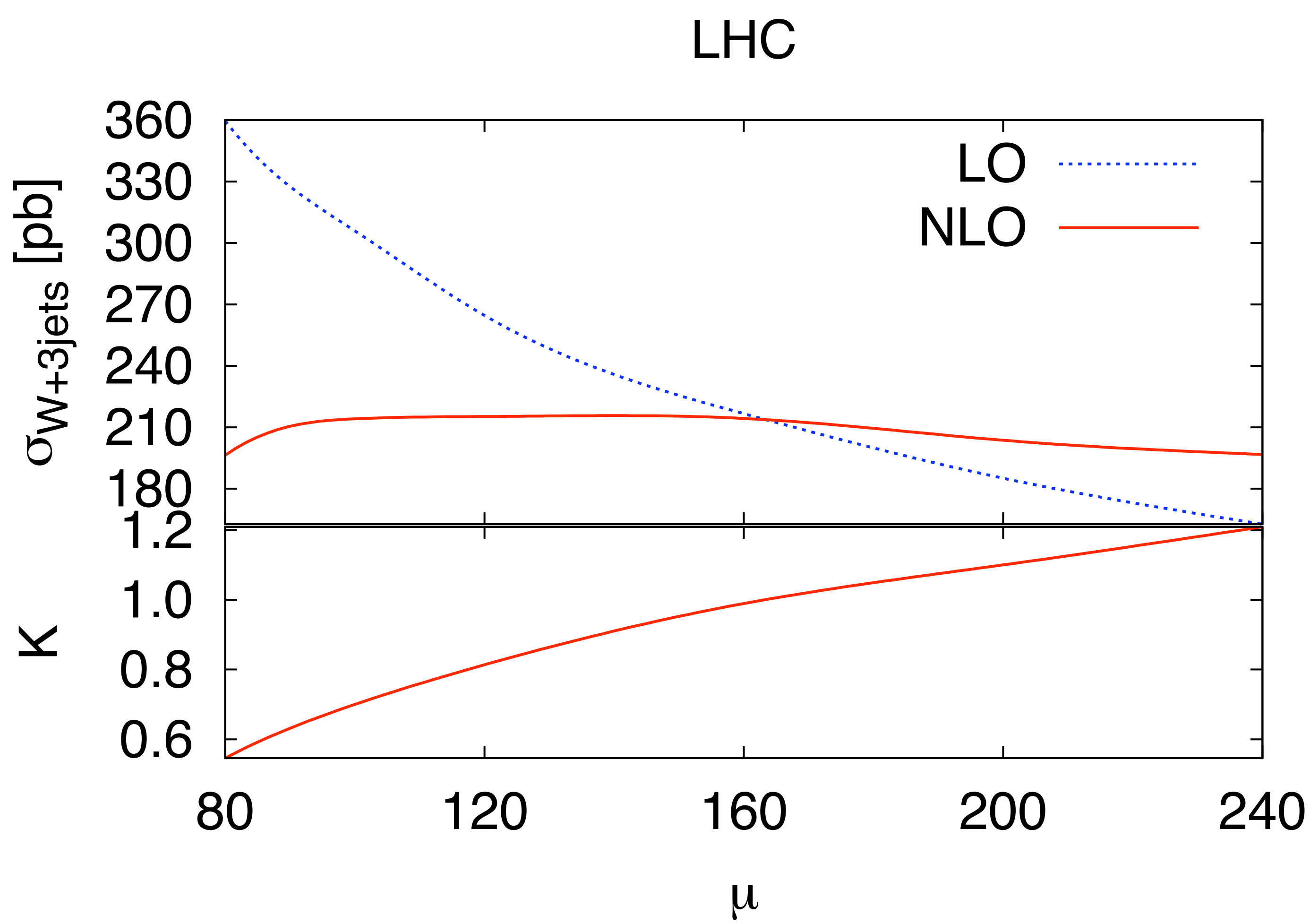
⇒ *All instabilities detected and cured with quadruple precision*

Primitive amplitudes: color structures

	Leading color	Fermion loops	Subleading color
2-quark 3-gluon	*only*  $\mathrm{LC} \equiv (N_c^2-1)N_c^3$	 $\mathrm{LC} \cdot \frac{n_f}{N_c}$	 $\cdots$
4-quark 1-gluon	 $\mathrm{LC} \cdot \frac{n_f}{N_c}$	 $\cdots$	 $\cdots$

At tree level: leading color works up to O(10%), 4-quark processes O(30%)

Scale variation: W^+ + 3 jets



[Cuts and input defined in Ellis, Melnikov, GZ ’09]

- very strong dependence at LO, remarkable independence at NLO
- LO = NLO at scales ~ 160 GeV
- W + 3 jets similar to W + 2 jets, however the price to pay for an infelicitous choice of scales is higher now
- similar results at the Tevatron

Second W + 3 jet calculation

More recently, similar calculation for W + 3 jets done in Blackhat+Sherpa

C. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower, D. Maitre [0902.2760]

In the above paper: still leading color approximation in virtual (not real), all subprocesses included (but no fermion loops)

Next step: inclusion of all subprocesses and comparison with Berger et al.

CDF cuts

$$p_{\perp,j} > 20\text{GeV} \qquad p_{\perp,e} > 20\text{GeV} \qquad E_{\perp,\text{miss}} > 30\text{GeV}$$

$$|\eta_e| < 1.1 \qquad M_{\perp,W} > 20\text{GeV}$$

$$\mu_0 = \sqrt{p^2_{\perp,W} + M^2_W} \qquad \mu = \mu_R = \mu_F = [\mu_0/2, 2\mu_0]$$

- CDF uses JETCLU with R = 0.4, but this is not infrared safe, use SIScone with the same R
  Difference small in inclusive cross-section [more in distributions]

  SIScone ⇒ Salam & Soyez '06

- CDF applies lepton-isolation cuts. This is a O(10%) effect. Lepton-isolation and detector acceptance cuts are believe to cancel out
  No lepton isolation applied

- PDFs: cteq6l1 and cteq6m, all other input as in 0902.2760
  NB: diagonal CKM O(1-2%) effect relative to Cabibbo rotated one

Leading color adjustment

Define

$$\mathcal{R}_{\mathcal{O}} = \frac{\int \mathcal{O}(p)\mathrm{d}\sigma^{\mathrm{FC}}_{LO}(\mu, p)}{\int \mathcal{O}(p)\mathrm{d}\sigma^{\mathrm{LC}}_{LO}(\mu, p)}$$

Leading color adjustment

Define

$$\mathcal{R}_{\mathcal{O}} = \frac{\int \mathcal{O}(p) \mathrm{d}\sigma_{LO}^{\mathrm{FC}}(\mu, p)}{\int \mathcal{O}(p) \mathrm{d}\sigma_{LO}^{\mathrm{LC}}(\mu, p)}$$

This turns out to be independent of factorization/renormalizaion and on the observable (e.g. bin of distribution)

$$\mathcal{R}_{\mathcal{O}}(\mu) \Rightarrow r$$

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Define our best approximation to the NLO result as

$$\mathcal{O}^{\mathrm{NLO}} = r \cdot \mathcal{O}^{\mathrm{NLO,LC}}$$

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$$\mathcal{R}_{\mathcal{O}}(\mu) \Rightarrow r$$

Define our best approximation to the NLO result as

$$\mathcal{O}^{\mathrm{NLO}} = r \cdot \mathcal{O}^{\mathrm{NLO,LC}}$$

Leading color adjustment tested in W+2jets: OK to few %

Cross-section at the Tevatron

$$\sigma_{W+3j}(p_{\perp,j} > 25\,\mathrm{GeV}) = (0.84 \pm 0.24)\,\mathrm{pb}$$

CDF

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$$\sigma_{W+3j}(p_{\perp,j} > 25\,\mathrm{GeV}) = (0.84 \pm 0.24)\,\mathrm{pb}$$

CDF

LO^{LC}						
$0.89^{+0.55}_{-0.31}$						

Cross-section at the Tevatron

$$\sigma_{W+3j}(p_{\perp,j} > 25\,\mathrm{GeV}) = (0.84 \pm 0.24)\,\mathrm{pb}$$

CDF

$\mathrm{LO}^{\mathrm{LC}}$	$\mathrm{LO}^{\mathrm{FC}}$					
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$					

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CDF

$\mathrm{LO}^{\mathrm{LC}}$	$\mathrm{LO}^{\mathrm{FC}}$	$r = \frac{\mathrm{LO}^{\mathrm{FC}}}{\mathrm{LO}^{\mathrm{LC}}}$				
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91				

Cross-section at the Tevatron

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CDF

$\mathrm{LO}^{\mathrm{LC}}$	$\mathrm{LO}^{\mathrm{FC}}$	$r = \frac{\mathrm{LO}^{\mathrm{FC}}}{\mathrm{LO}^{\mathrm{LC}}}$	$\mathrm{NLO}^{\mathrm{LC}}$ (prelim)			
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91	$1.005^{+0.054}_{-0.165}$			

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$\mathrm{LO}^{\mathrm{LC}}$	$\mathrm{LO}^{\mathrm{FC}}$	$r = \frac{\mathrm{LO}^{\mathrm{FC}}}{\mathrm{LO}^{\mathrm{LC}}}$	$\mathrm{NLO}^{\mathrm{LC}}$ (prelim)	$r \cdot \mathrm{NLO}^{\mathrm{LC}}$ (prelim)		
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91	$1.005^{+0.054}_{-0.165}$	$0.914^{+0.050}_{-0.150}$		

'Our best shot'

Cross-section at the Tevatron

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CDF

$\mathrm{LO}^{\mathrm{LC}}$	$\mathrm{LO}^{\mathrm{FC}}$	$r = \frac{\mathrm{LO}^{\mathrm{FC}}}{\mathrm{LO}^{\mathrm{LC}}}$	$\mathrm{NLO}^{\mathrm{LC}}$ (prelim)	$r \cdot \mathrm{NLO}^{\mathrm{LC}}$ (prelim)	Berger et al. (LC, v3)	
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91	$1.005^{+0.054}_{-0.165}$	$0.914^{+0.050}_{-0.150}$	$0.908^{+0.044}_{-0.142}$	

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CDF

LO^{LC}	LO^{FC}	$r = \frac{\mathrm{LO^{FC}}}{\mathrm{LO^{LC}}}$	NLO^{LC} (prelim)	$r \cdot NLO^{LC}$ (prelim)	Berger et al. (LC, v3)	Berger et al. (FC, prelim)
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NB: errors are standard scale variation errors, statistical errors smaller

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$\Rightarrow$ agreement between independent calculations to within 3%

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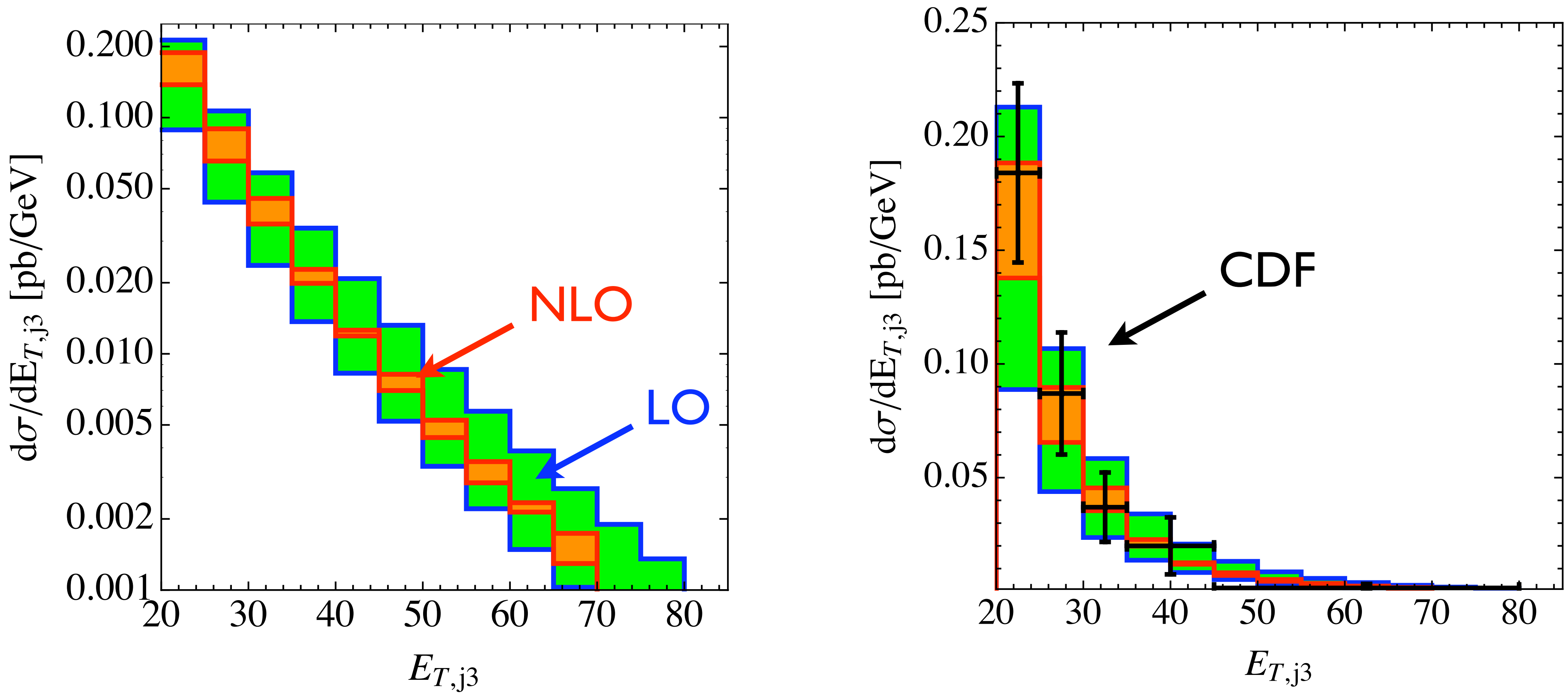
$\Rightarrow$ agreement between independent calculations to within 3%

$\Rightarrow$ leading color approximation works very well. After leading color adjustment procedure it is good to 3% (nothing with ≥ 3jets can be measured better than that at the LHC)

Sample distribution: $E_{t,j3}$

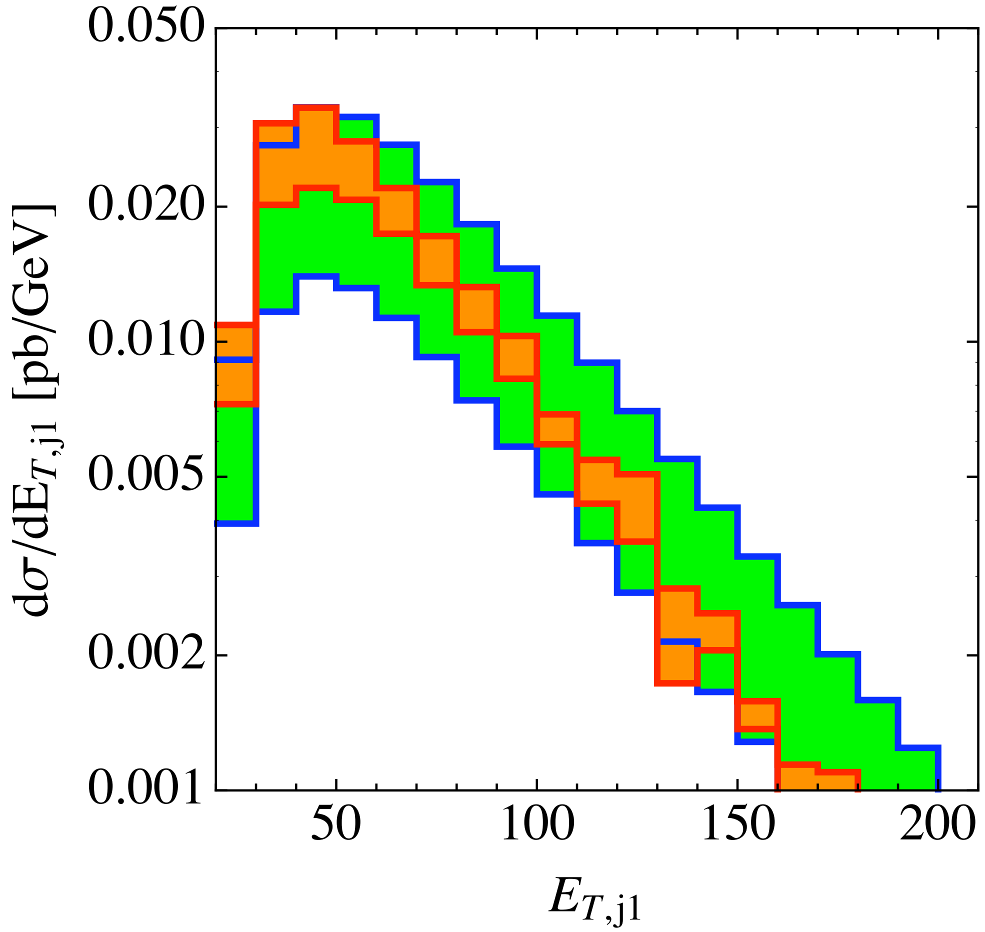


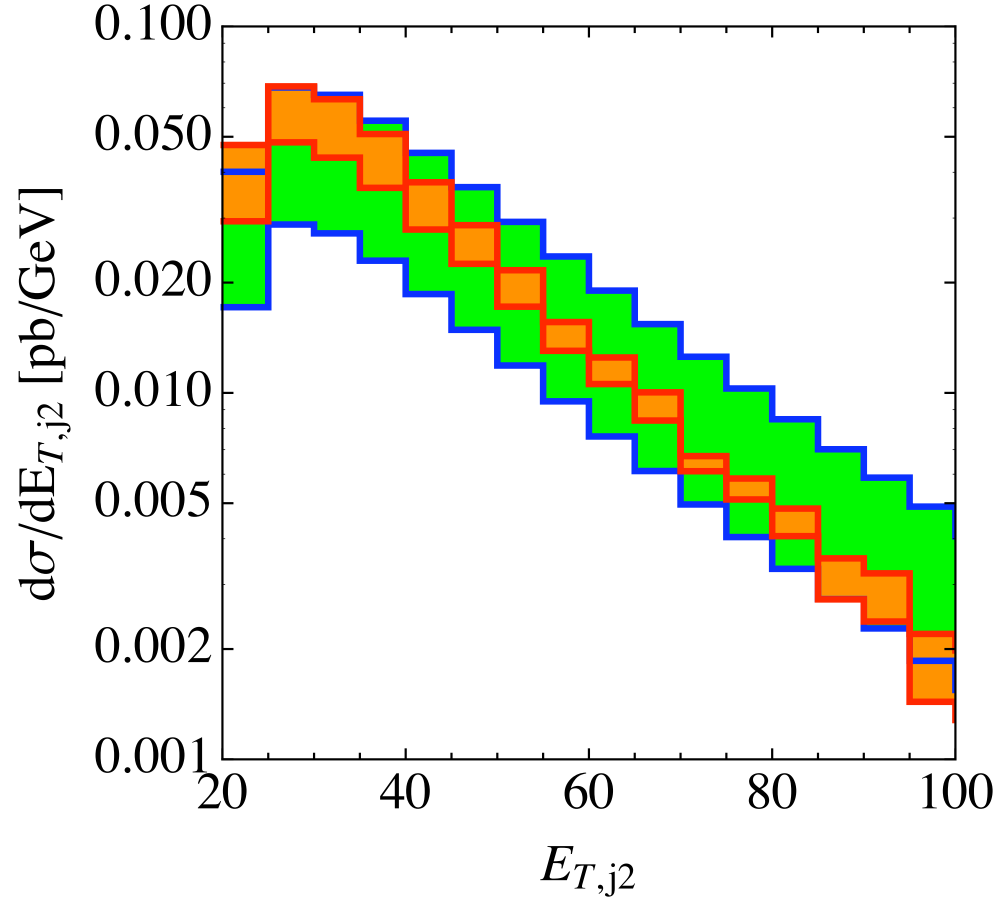
EMZ, prelim.

Comparison to data

- OK within large experimental errors
- even with reduced exp. errors, accurate comparison not possible because of difference jet-algorithm used

Sample distribution: $E_{t,j1}$ and $E_{t,j2}$



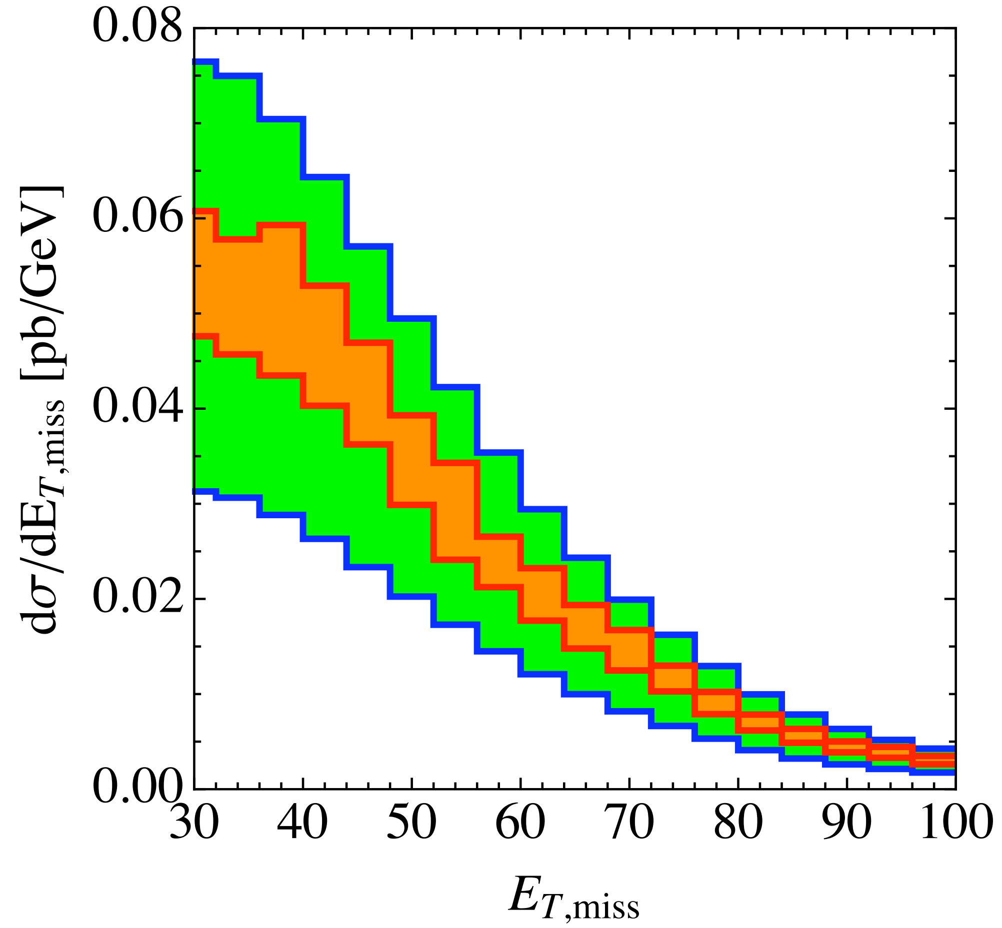


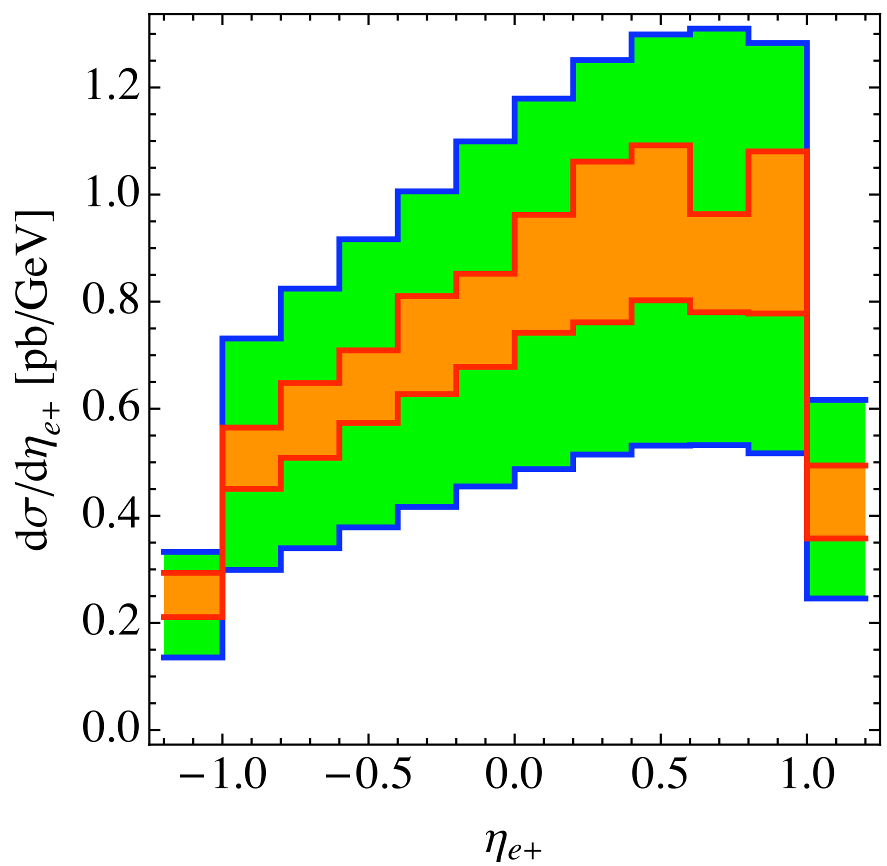
EMZ, prelim.

Hadronic observables:

- scale reduction (factor 4)
- change in shape

Sample distribution: $E_{t,j1}$ and $E_{t,j2}$





EMZ, prelim.

Leptonic observables:

- scale reduction (factor 4)
- inclusive K-factor works very well

Final remarks

Generalized D-dimensional unitarity

- ✗ general Berends-Giele recursion for tree level amplitudes: numerically efficient (large N), general (D, spins, masses)
- ✗ simple method, suitable for automation
- ✗ universal method (general masses, spins) and unified approach, no ‘special’ cases, no exceptions
- ✗ speed: numerical performance as expected (polynomial)
- ✗ transparent: full control on all parts

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Maturity reached for cross-sections calculations?
Demonstrated by first explicit calculation of W + 3 jets
(but still room for further improvements)