Dipole subtraction with Sherpa

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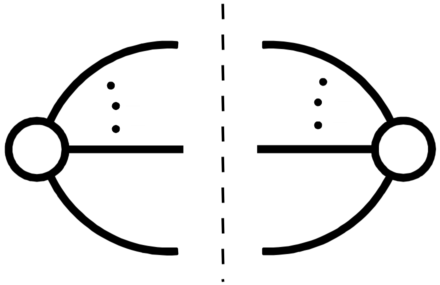
Mini-workshop on fixed order multi-leg automatic NLO calculations, Wuppertal, 2.06.2009

Calculation of cross sections

- LO:

$$\sigma^{LO} = \int_m d\sigma^B$$



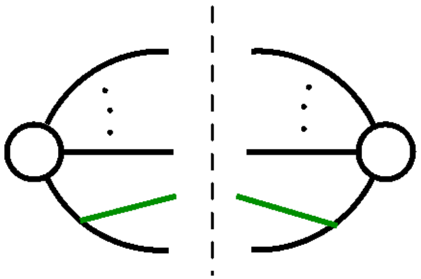
- Finite within the whole phase space (by definition)
- Straightforward to calculate, highly automated tools available:
    - Alpgen
    - Helac/Phegas
    - MadGraph
    - O'Mega/Wizard
    - AMEGIC++
    - Comix
- Typically up to 6-10 final state particles feasible

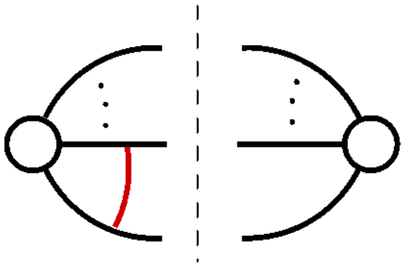
Calculation of cross sections

- NLO:

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^{R} + \int_{m} d\sigma^{V}$$





Real correction:
- Radiation of additional parton
- Divergent in soft and collinear limits

Virtual correction:
- Loop amplitudes
- Infrared poles after integration over loop momentum

➢ Sum is free of divergencies, but cancelation only after integration over phase space!

Calculation of cross sections

Construct finite integrands: **Subtraction method**

- Introduce subtraction term $d\sigma^A$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_m d\sigma^V + \int_{m+1} d\sigma^A$$

$$= \int_{m+1} (d\sigma^R_{\varepsilon=0} - d\sigma^A_{\varepsilon=0}) + \int_m (d\sigma^V + \int_1 d\sigma^A)_{\varepsilon=0}$$

Cancels soft/collinear singularities of the real correction

Simple enough to be integrated analytically over one-parton emission in dimensional regularization:

$$\int_1 d\sigma^A_\varepsilon = \varepsilon^{-2} d\sigma^{(A,2)} + \varepsilon^{-1} d\sigma^{(A,1)} + d\sigma^{(A,0)} + O(\varepsilon)$$

Poles cancel with virtual correction

Calculation of cross sections

Construct finite integrands: **Subtraction method**

- Introduce subtraction term $d\sigma^A$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_m d\sigma^V + \int_{m+1} d\sigma^A$$

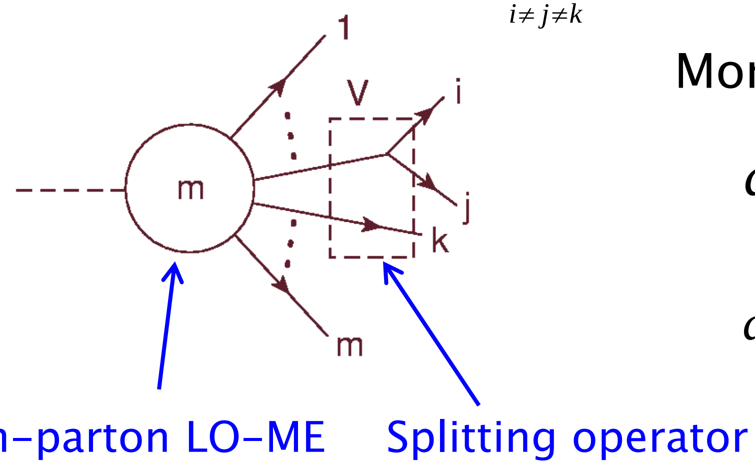
$$= \underbrace{\int_{m+1} (d\sigma^R_{\varepsilon=0} - d\sigma^A_{\varepsilon=0})}_{} + \underbrace{\int_m (d\sigma^V + \int_1 d\sigma^A)_{\varepsilon=0}}_{}$$

Integrands are finite and suitable for numerical integration over phase space

Dipole Subtraction

- Recipe to construct $d\sigma^A$: [S. Catani, M.H. Seymour,1997]
- Based on universal (process independent) soft and collinear limits of QCD matrix elements

Subtraction term: $d\sigma^A = \sum_{i \neq j \neq k} d\sigma^A_{ij,k}$ (for m+1-parton real correction)



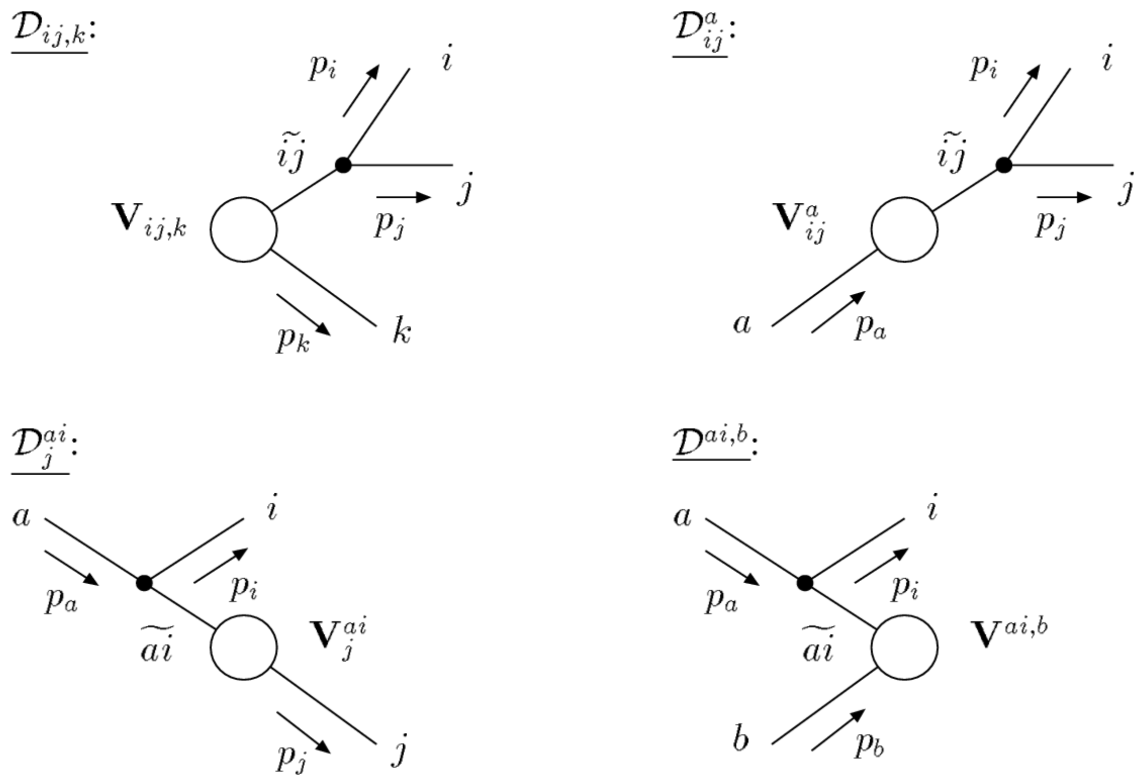
Momentum map: $p_i, p_j, p_k \rightarrow \tilde{p}_{ij}, \tilde{p}_k$

$$d\sigma^A_{ij,k} = d\sigma^{LO}_{\tilde{ij},\tilde{k}} \otimes dV_{ij,k}$$

$$dV_{ij,k} \sim \frac{1}{p_i p_j} \mathbf{T}_{ij} \cdot \mathbf{T}_k \;\; \mathbf{V}_{ij,k} \;\; d\Phi^{(1)}$$

Dipole Subtraction

- 4 different types:



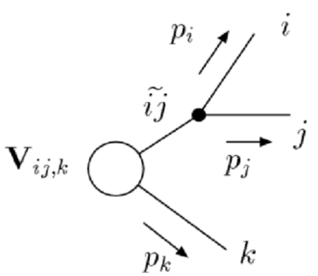
$$d\sigma^A = \left\{ \sum_{i\neq j\neq k} D_{ij,k} + \left[\sum_{i\neq j} D_{ij}{}^{a} + \sum_{k\neq i} D_{k}{}^{ai} + \sum_{i} D^{ai,b} + \langle a \leftrightarrow b \rangle \right] \right\} d\phi$$

Dipole Subtraction

[Z. Nagy 2003]

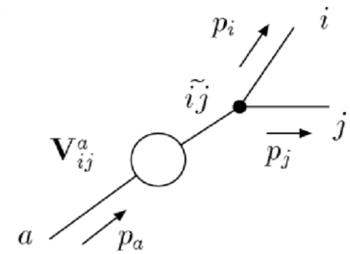
- Simple and very useful modification: restrict dipole phase space to singular region: $D' = D \times \theta(\alpha - y)$

  Cut parameter: $\alpha = 0\ldots1$



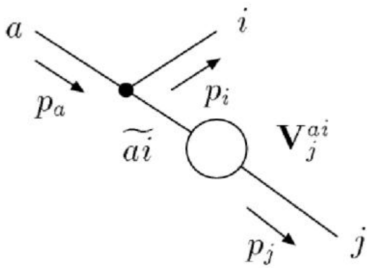
FF:

$$y = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$$



FI:

$$y = \frac{p_i p_j}{p_i p_a + p_j p_a}$$



IF:

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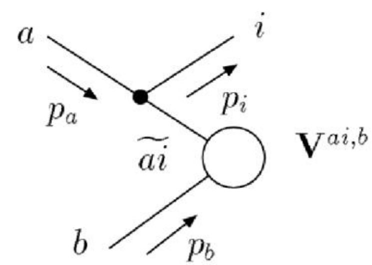


II:

$$y = \frac{p_a p_i}{p_a p_b}$$

$$d\sigma^{A'} = \left\{ \sum_{i \neq j \neq k} D'_{ij,k} + \left[\sum_{i \neq j} D'^{\,a}_{ij} + \sum_{k \neq i} D'^{\,ai}_{k} + \sum_{i} D'^{\,ai,b} + \langle a \leftrightarrow b \rangle \right] \right\} d\phi$$

Dipole Subtraction

- Phase space for one-parton emission factorizes:

$$d\phi(p_i, p_j, p_k; Q) = d\phi(\tilde{p}_{ij}, \tilde{p}_k; Q)\left[dp_i(\tilde{p}_{ij}, \tilde{p}_k) \right]$$

- Integral over dipole terms (performed once and for all)

$$\int_1 d\sigma^A_{ij,k} = d\sigma^{\mathrm{LO}}_{\tilde{ij},\tilde{k}} \otimes \int d^d V_{ij,k} = d\sigma^{\mathrm{LO}}_{\tilde{ij},\tilde{k}} \otimes \mathbf{I}(\varepsilon)$$

$$\mathbf{I}(\varepsilon) = -\frac{\alpha_S}{2\pi} \frac{2}{\Gamma(1-\varepsilon)} \sum_1 \frac{1}{\mathbf{T}_i^2} V_i(\varepsilon) \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{4\pi\mu^2}{2 p_i p_j} \right)^{\varepsilon}$$

$$= \mathbf{A}\varepsilon^{-2} + \mathbf{B}\varepsilon^{-1} + \mathbf{C} + O(\varepsilon)$$

Dipole Subtraction

- Cross section with hadronic initial states:

$$\sigma(p,p') = \sum_{a,b} \int_0^1 d\eta\, f_a(\eta,\mu_F) \int_0^1 d\eta'\, f_b(\eta',\mu_F) \left[\sigma^{\mathrm{LO}}(\eta p, \eta' p') + \sigma^{\mathrm{NLO}}(\eta p, \eta' p', \mu_F)\right]$$

$$\sigma^{\mathrm{NLO}}(\eta p, \eta' p', \mu_F) = \int_{m+1} d\sigma^R_{ab}(p_a,p_b) + \int_m d\sigma^V_{ab}(p_a,p_b) + \int_m d\sigma^C_{ab}(p_a,p_b,\mu_F)$$

  - Collinear counterterm $d\sigma^C_{ab}$ contains collinear IR pole terms and factorization scale & scheme dependent finite terms (to cancel with similar dependence in PDFs)
  - Final result structure of counter-/subtraction terms:

$$\int_{m+1} d\sigma^A_{ab}(p_a,p_b) + \int_m d\sigma^C_{ab}(p_a,p_b,\mu_F) = \int_m \left[d\sigma^B_{ab}(p_a,p_b) \otimes \mathbf{I}(\varepsilon)\right]$$

$$+ \sum_{a'} \int_0^1 dx \int_m \left[\left(\mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(xp_a, x, \mu_F)\right) \otimes d\sigma^B_{a'b}(xp_a,p_b)\right] + \langle a \leftrightarrow b \rangle$$

(all poles contained in $\mathbf{I}(\varepsilon)$-term)

Implementations of CS subtraction

- Published so far:
  - TG, Krauss, 2007 [0709.2881]
  - Seymour, Tevlin, 2008 [0803.2231]
  - Hasegawa, Moch, Uwer, 2008 [0807.3701]
  - Frederix, Gehrmann, Greiner, 2008 [0808.2128]
  - Czakon, Papadopoulos, Worek, 2009 [0905.0883]

Implementation in SHERPA

[TG, F. Krauss, 2007]

- Main ingredient for dipole subtraction terms:

  tree-level matrix elements

- Basis: automated tree-level ME generator AMEGIC++ (the built-in ME generator of SHERPA)

$$|M|^2 = {}_m\langle 1,\ldots,m|1,\ldots,m\rangle_m = \sum_{i,j}\left[A_i(\{p_k\})A_j^*(\{p_k\})\right]\left[C_iC_j^\dagger\right]$$

  - Feynman amplitudes, computed in a helicity method
  - Automatic analytic simplification, storage as C++ libraries
  - Automatic generation of phase-space maps
  - Provides structures to generate all parton-level processes contributing to a jet-XS at once
  - Implemented models: SM, MSSM, ADD, …

Automatic Dipole Subtraction in SHERPA

$$\sigma = \int_m d\sigma^B + \int_{m+1} \left[d\sigma^R - d\sigma^A \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]$$

- Extensions of AMEGIC++
  - Automated generation of real correction and all dipole subtraction terms
  - Coefficients for $\varepsilon^{-2}, \varepsilon^{-1}$ and ε^{0} of integrated subtraction terms

⇒ Guarantees independence of details of the subtraction method

  - Phase space integration methods for real and virtual corrections
  - Automatic organization of parton level subprocesses
  - Completely general implementation for massless partons
  - Massive version is implemented but $\int_1 d\sigma^A$ needs more checks

Automatic Dipole Subtraction in SHERPA

$$\sigma = \int_m d\sigma^B + \int_{m+1}\left[d\sigma^R - d\sigma^A\right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A\right]$$

$$|M_{(1\text{-loop})}(\{p_i\})|^2 \, d\phi$$

Interface:

$$\{p_1, p_2, \ldots, p_m\}$$

1-loop codes, i.e

BlackHat

Rocket

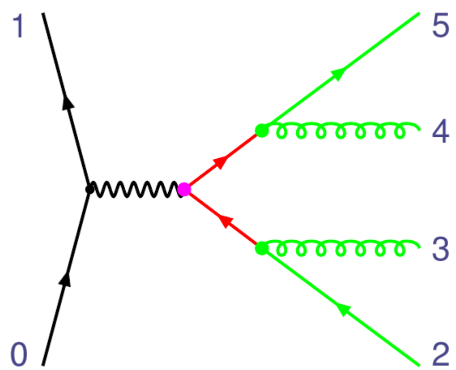
...

$$|M_{(1\text{-loop})}|^2 = a\varepsilon^0 + b\varepsilon^{-1} + c\varepsilon^{-2}$$

Note to phase space integration

- For real (subtracted) correction:

the (m+1)-parton phase space is generated directly multi-channeling over parametrizations obtained from the (Feynman diagramatic) structure of the real ME



•Generate a channel for each diagram using a few building blocks

$$\sim P_0(23)P_0(45)D(23,45)D(2,3)D(4,5)$$

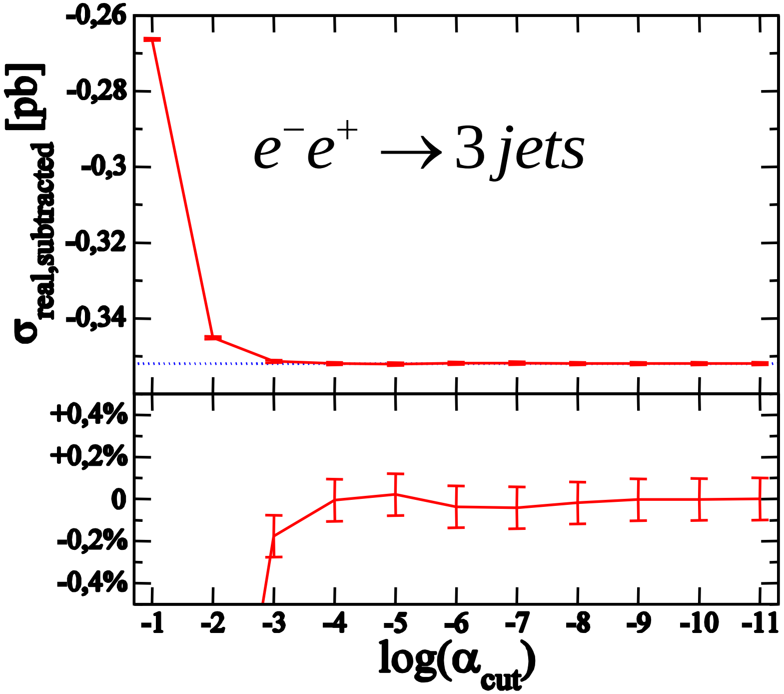
•Adapt to the full structure of the integrand by relative weights of single channels and a VEGAS grid for each map

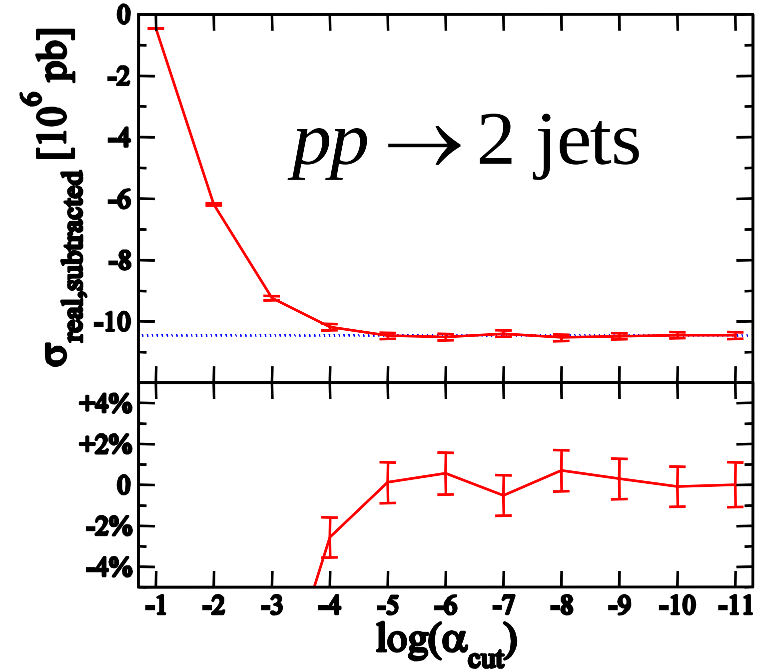
Convergence and consistency checks

Cutoff dependence of subtracted real correction:

$$\alpha_{\text{min}} = \min(\alpha_{\text{dipole}}) < \alpha_{\text{cut}}$$

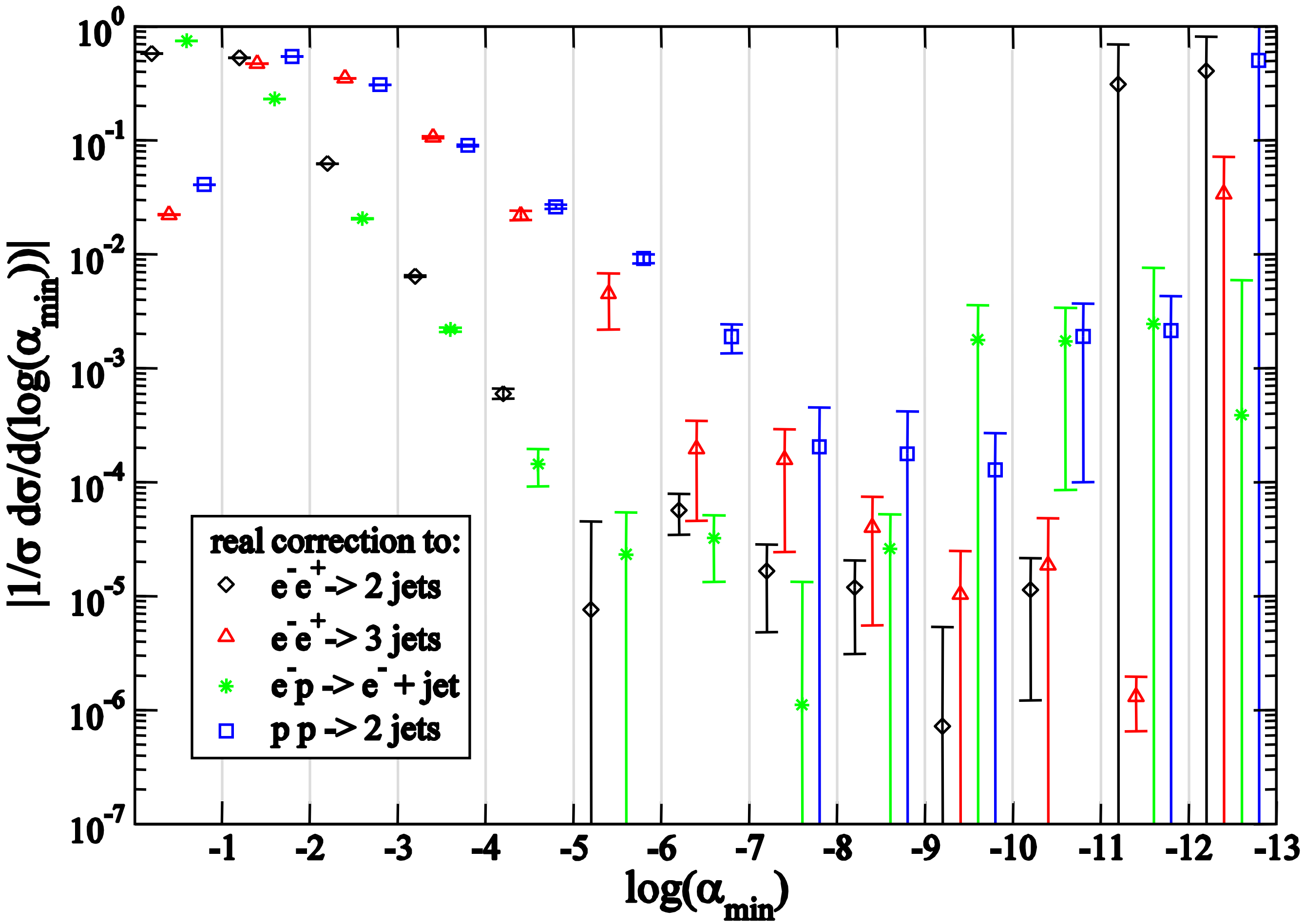
i.e. FF-dipole: $\alpha_{\text{dipole}} = y_{ij,k} = \dfrac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$





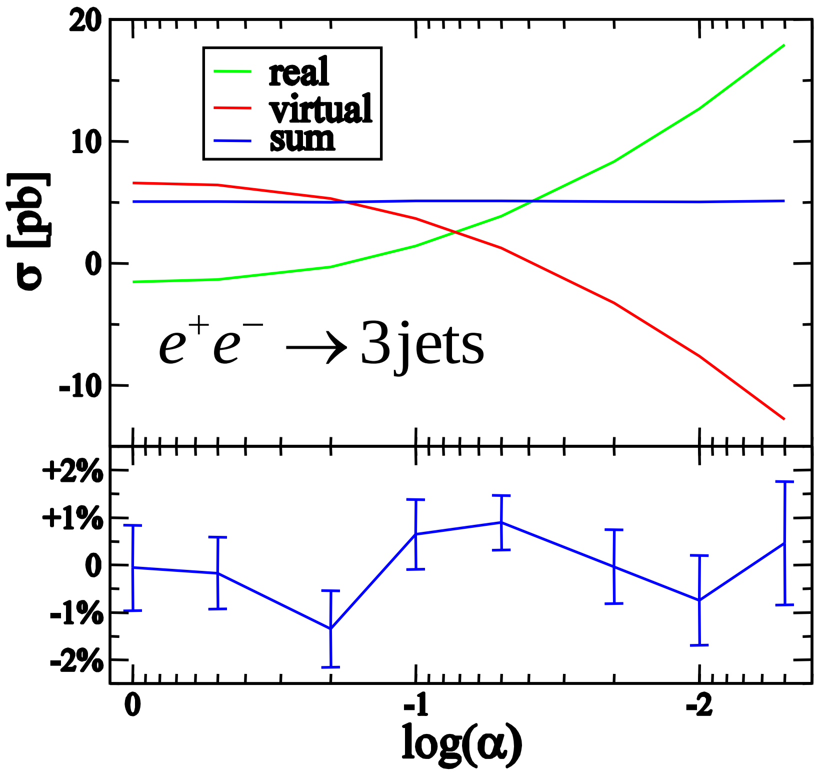
Convergence and consistency checks

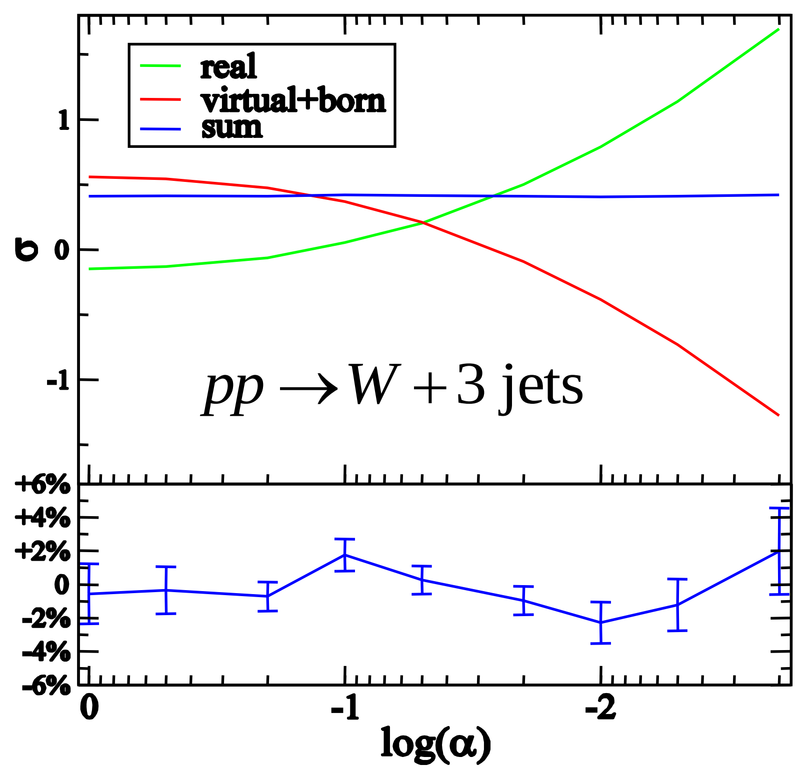
Cutoff dependence of subtracted real correction:



Convergence and consistency checks

Consistency check with modified dipole terms: $d\sigma^{A'} = d\sigma^{A}\theta(\alpha - y)$





Comparisons with other codes

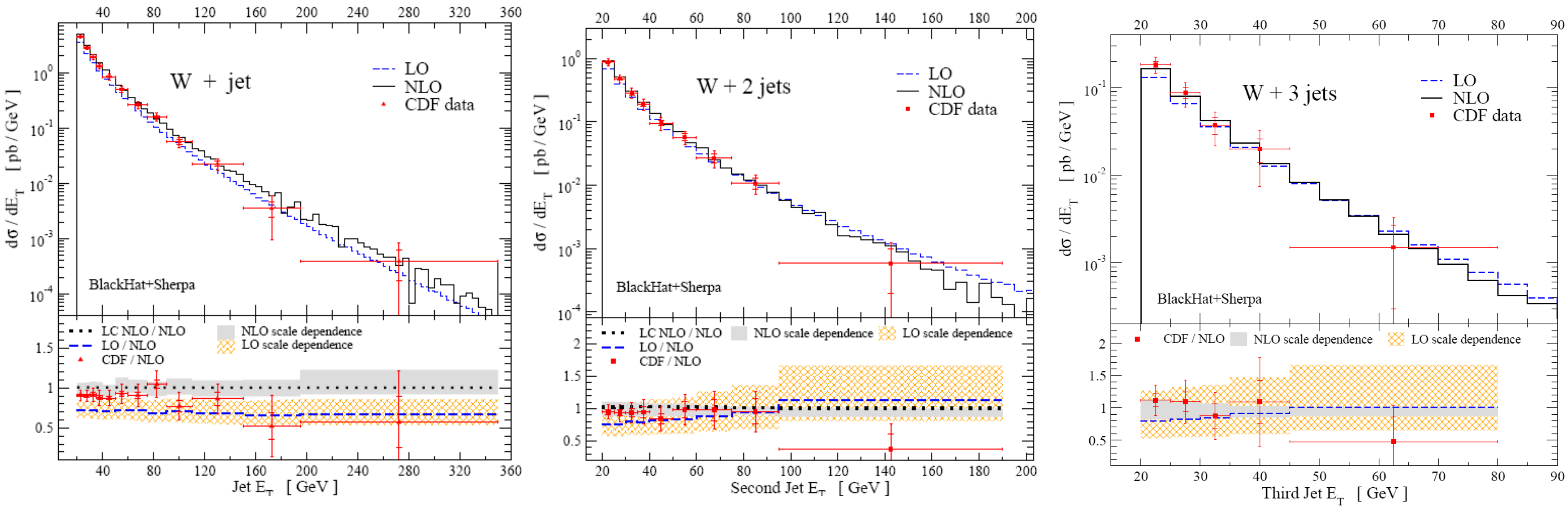
- With M. Seymour's code DISENT:

$$e^- e^+ \to 3\text{jets},\ \ e^- p \to e^- + 2\text{jets}$$

  - Explicit comparison of dipole terms
  - Comparison of the insertion operators **I, K, P**
  - Checks of integrated results for real (subtracted) correction and integrated dipole terms
- Comparison of dipole terms for $pp \to ZZ + \text{jet}$ with a code by N. Kauer
- Checks of full NLO results for a number of processes with MCFM

Results with BlackHat

W+jet @ Tevatron [BlackHat+Sherpa, 2009]



For more details see talks by Daniel and David.

- Soon to be released: (~end of June)

SHERPA v1.2

- Includes:
  - Dipole subtraction
  - Analysis framework for NLO events
  - A revised implementation of (LO) CKKW merging to work with multiple ME generators and Parton Showers [Hoeche, Krauss, Schumann, Siegert, 2009]
  - The ME generator COMIX [TG, Hoeche, 2008]
  - A new Parton shower, based on CS dipoles [Schumann, Krauss, 2009]
  - And more…

www.sherpa-mc.de