From B-modes to quantum gravity ?

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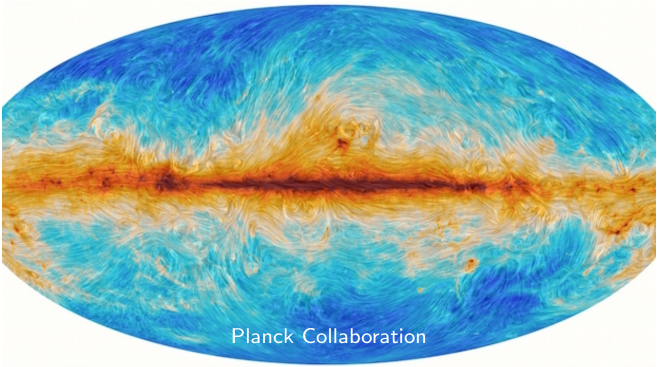


Outline

- An introduction to cosmic inflation
  - motivation
  - background solution
  - imprints on the CMB

- A closer look: Quantum fluctuations during inflation
  - scalar fluctuations
  - tensor fluctuations
  - CMB

[D. Baumann, 0907.5424]
[V. Mukhanov, S. Winitzki (2007)]

- Observing quantum gravity ?
  - counting powers of $\hbar$
  - caveats

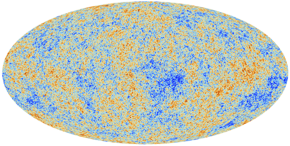
[Wilczek, Krauss 1309.5343]

Cosmic puzzles

FRW metric of expanding universe:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \underbrace{a^2(\tau)[-d\tau^2}_{dt^2} + \underbrace{\frac{dr^2}{1-kr^2} + r^2 d\Omega^2]}_{d\chi^2}$$

Horizon problem:
Why is our universe so homogeneous?



Flatness problem
Why is our universe so flat?

The resolution from Einstein's equation

gravity $\leftrightarrow$ matter/energy

$$\underbrace{G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{gravity, determined by } g_{\mu\nu}} = \underbrace{8\pi G T_{\mu\nu} = 8\pi G \, \mathrm{diag}(\rho, -p, -p, -p)}_{\text{matter / energy : modeled by perfect fluid}}$$

Inserting FRW metric yields Friedman equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2}\,, \quad \dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p)$$

$\rightarrow$ expansion of the universe determined by equation of state $\omega = p/\rho$.
$\omega = 0$: non-relativistic matter, $\omega = 1/3$: radiation
$\rightarrow$ special case $\omega = -1$ (= negative pressure, constant energy density):

$$\omega = -1 \quad \Rightarrow \quad a(t) = a(t_0) e^{H(t - t_0)}$$

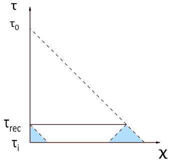
$\rightarrow$ exponentially expanding universe, accelerated expansion, *inflation*

Cosmic puzzles resolved

[Guth 1980]

Horizon problem

In a matter or radiation dominated universe, the co-moving particle horizon $\tau = \int \frac{dt}{a(t)}$ grows with time. *Why is our universe so homogeneous?*



Flatness problem

Today $k/(aH)^2 \simeq 0$. With a growing comoving Hubble horizon $(aH)^{-1}$ we need to require e.g. $|k/(aH)^2| \lesssim \mathcal{O}(10^{-16})$ at BBN. *Why is our universe so flat?*

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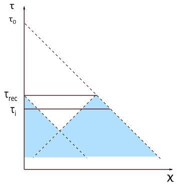
The entire observable universe evolved from an initial patch in causal contact.



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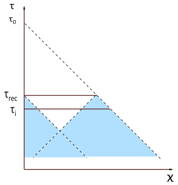

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Today $k/(aH)^2 \simeq 0$. With a growing comoving Hubble horizon $(aH)^{-1}$ we need to require e.g. $|k/(aH)^2| \lesssim \mathcal{O}(10^{-16})$ at BBN. *Why is our universe so flat?*

During inflation $(aH)^{-1}$ decreases, driving the universe towards flatness.

$\rightarrow$ cosmic puzzles resolved if inflation lasted for at least 50 - 60 Hubble times ($N \equiv \int H dt = \ln(a_{\text{end}}/a_{\text{ini}})$).

Particle physics implementation

Consider a scalar field ϕ:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + V(\phi) \right]$$

with $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$:

$$\omega = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \to -1 \quad \text{iff } \dot{\phi}^2/2 \ll V$$

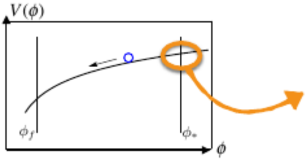
$\rightarrow$ slow-roll inflation:

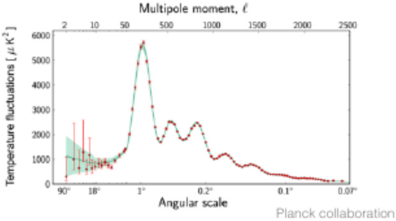
$$3H\dot{\phi} + V'(\phi) \simeq 0 \,, \quad 3H^2 \simeq V(\phi)$$

with slow-roll parameters

$$\epsilon \equiv -\frac{\dot{\phi}}{2H^2} \simeq \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1 \,, \quad \eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \simeq M_P^2 \frac{V''}{V} - \epsilon \ll 1$$

Inflation and the CMB





large vacuum energy $\Rightarrow$ exponential expansion $\Rightarrow$ homogeneity of CMB

quantum fluctuations $\Rightarrow$ become classical $\Rightarrow$ tiny anisotropies

Gravity $\rightarrow$ background evolution ✓
Quantum Gravity ? $\rightarrow$ need to understand fluctuations !

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Cosmological perturbation theory

$$\phi(t,\mathbf{x}) = \bar{\phi}(t) + \delta\phi(t,\mathbf{x})\,, \quad g_{\mu\nu}(t,\mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\mathbf{x})$$

scalar

consider gauge - invariant quantity $\mathcal{R}$: spatial curvature on const.-ϕ hypersurface

vector

not produced in single-field slow-roll, decay $\to$ ignore here

tensor

transverse, symmetric, traceless: $h_{ij} \to h_+, h_\times$ gravitational waves

Goal: compute statistical properties

scalars: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \mathcal{P}_{\mathcal{R}}(k)\,, \quad \Delta_s^2 = \frac{k^3}{2\pi^2} \mathcal{P}_{\mathcal{R}}(k)$

tensors: $\langle h^\lambda_{\mathbf{k}} h^\lambda_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \mathcal{P}_h(k)\,, \quad \Delta_t^2 = \frac{k^3}{\pi^2} \mathcal{P}_h(k)$

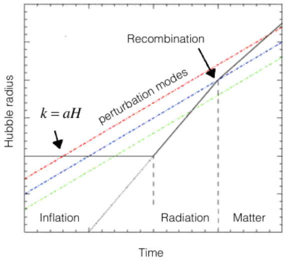
Computation strategy

1. expand action to 2nd order in perturbations
2. derive equations of motion
3. quantize and set initial conditions
4. compute the power spectrum

Note on horizons:

- $1/\lambda \sim k \gg aH$: sub-horizon
- $1/\lambda \sim k \ll aH$: super-horizon

where during inflation $aH = (-\tau)^{-1}$



Scalar fluctuations

1. expand action to 2nd order in perturbations
gauge choice: $\delta\phi = 0, \quad g_{ij} = a^2\left[(1-2\mathcal{R})\delta_{ij} + h_{ij}\right]$

$$\rightarrow \quad S_{(2,\mathcal{R})} = \frac{1}{2}\int d^4x\, a^3 \frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})(\partial^i\mathcal{R})\right]$$

2. derive equations of motion
canonical variable $v \equiv z\mathcal{R}$ with $z^2 = a^2\dot\phi^2/H^2$

$$\rightarrow \quad v''_{\mathbf{k}} + (k^2 - \frac{z''}{z})\, v_{\mathbf{k}} = 0 \quad \text{(Mukhanov equation)}$$

3. quantize and set initial conditions

$$v_{\mathbf{k}} \mapsto \hat v_{\mathbf{k}} = v_{\mathbf{k}}(\tau)\hat a_{\mathbf{k}} + v^*_{-\mathbf{k}}(\tau)\hat a^\dagger_{-\mathbf{k}}\,, \quad \left[\hat a_{\mathbf{k}}, \hat a^\dagger \mathbf{k}'\right] = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')$$

in the infinite past,

$$v''_{\mathbf{k}} + k^2\, v_{\mathbf{k}} = 0 \quad \rightarrow \quad \lim_{\tau\rightarrow-\infty} v_{\mathbf{k}} = \frac{1}{\sqrt{2k}} e^{-ik\tau} \quad \text{(Bunch Davies vacuum)}$$

Scalar fluctuations

$$\Rightarrow v_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

4. compute the power spectrum

$$\mathcal{R} = \frac{H}{\dot\phi}\frac{v}{a} \quad \to \quad \langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \left(\frac{H}{\dot\phi}\right)^2 \underbrace{\frac{|v_k(\tau)|^2}{a^2}}_{\frac{H^2}{2k^3}(1+k^2\tau^2)}$$

$\to$ on super-horizon scales $k\tau \to 0 \to$ 'freeze-out'.
$\to$ power spectrum determined by background *at horizon crossing*:

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \left(\frac{H_*}{\dot\phi_*}\right)^2 \frac{H_*^2}{2k^3}$$

$$\Delta_s^2(k) = \left(\frac{H_*}{\dot\phi_*}\right)^2 \left(\frac{H_*}{2\pi}\right)^2$$

Tensor fluctuations

1. expand action to 2nd order in perturbations

$$S_{2,h} = \frac{M_P^2}{8} \int d\tau \, d^3\mathbf{x} \; a^2 \left[(h'_{ij})^2 - (\partial_l h_{ij})^2 \right]$$

2. derive equations of motion
   canonical variable $v_{\mathbf{k}}^\lambda = \frac{a}{2} M_P h_{\mathbf{k}}^\lambda$ with $\lambda = +, \times$

$$\rightarrow \quad (v_{\mathbf{k}}^\lambda)'' + (k^2 - \frac{a''}{a}) \, v_{\mathbf{k}}^\lambda = 0$$

3. quantize and set initial conditions (Bunch Davies vacuum, as before)
4. compute the power spectrum

$$h_k^\lambda = \frac{2}{M_P} \frac{v_k^\lambda}{a} \quad \rightarrow \quad \langle h_{\mathbf{k}}^\lambda h_{\mathbf{k}'}^\lambda \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \left(\frac{2}{M_P} \right)^2 \frac{H_*^2}{2k^3}$$

$$\Delta_t^2 = \frac{2}{M_P^2} \left(\frac{H_*}{2\pi} \right)^2 , \quad r = \frac{\Delta_t^2}{\Delta_s^2} = 16 \epsilon_*$$

CMB: Connecting to observations

- Scalar and tensor perturbations are imprinted in the density contrast of the primordial plasma
- re-combination: at $T \simeq 0.1$ eV (380.000 years after inflation), neutral hydrogen forms and the Universe becomes transparent to photons $\rightarrow$ cosmological last-scattering surface
- CMB observations measure these photons to deduce properties of last scattering surface

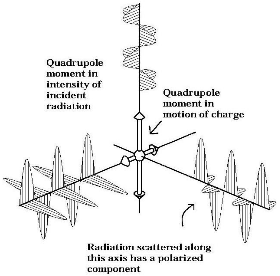
schematically:

$$\underbrace{C_\ell^{XY}}_{\text{CMB}} = \frac{2}{\pi} \int k^2 dk \, \underbrace{P_{\mathcal{R},h}(k)}_{\text{inflation}} \, \underbrace{\Delta_{X\ell}(k) \, \Delta_{Y\ell}(k)}_{\text{transfer functions}}, \quad X, Y = \{T, E, B\}$$

larger scales (super-horizon at re-combination): $\Delta_{X\ell}$ = geometrical projection.
Small scales: take into account sub-horizon evolution after re-entry

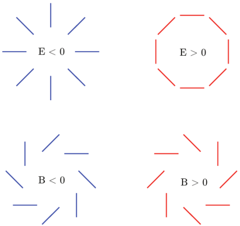
CMB: Polarization

Thomson scattering:
Tensor anisotropies $\rightarrow$ polarized photons



Described by 2×2 symmetric tensor $I_{ij}(\hat{n})$
$\rightarrow$ Stokes parameters:

$$Q = (I_{11} - I_{22})/4\,, \quad U = I_{12}/2$$
$$[T = (I_{11} + I_{22}))/4]$$



$T \rightarrow$ spherical harmonics

$Q, U \rightarrow$ tensor spherical harmonics

$Q, U \mapsto E, B$ with

$$C_\ell^{BB} = (4\pi^2) \int k^2 dk\, P_h(k) \Delta^2_{B\ell}(k)$$

B-modes probe tensor perturbations

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Counting powers of $\hbar$

Do we necessarily need *quantum* metric fluctuations to generated a gravitational wave background during inflation? [Wilczek, Krauss 1309.5343]

Dimensional analysis: [mass] $= M$, [length] $= L$, [time] $= T$

gravitational wave background from fundamental constants $G, H, \hbar, c$:

$[G]$	$[H]^\alpha$	$[\hbar]^\beta$	$[c]^\gamma$	$= [\rho_{\text{grav}}][a^4]$
Newton's const.	Hubble rate	Planck const.	speed of light	grav. rad. in expanding universe, $[E]L$

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first order will dominate if present, $7 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$	expand. universe	encodes quantum origin. $10^{-34} \frac{\text{kg m}^2}{\text{s}^2}$	speed of light , $3 \cdot 10^8 \frac{\text{m}}{\text{s}}$	grav. rad. in expanding universe, $[E]L$

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$L^3 M^{-1} T^{-2}$	$(T^{-1})^\alpha$	$(ML^2T^{-1})^\beta$	$(LT^{-1})^\gamma$	ML^3T^{-2}

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$L^3 M^{-1} T^{-2}$	$(T^{-1})^\alpha$	$(M L^2 T^{-1})^\beta$	$(L T^{-1})^\gamma$	$M L^3 T^{-2}$

$\Rightarrow M$: $\beta = 2$, $\quad L$: $\gamma = -4$, $\quad T$: $\alpha = 2$

$\Rightarrow$ unique solution comes with positive powers of $\bar{h}$!

Counting powers of $\hbar$

$\Rightarrow$ gravitational wave background from inflation $\propto \dfrac{GH^2\hbar^2}{c^4}$

Compare to our earlier result

$$\Delta_t^2 = \frac{2}{M_P^2}\left(\frac{H_*}{2\pi}\right)^2 \text{ w. } M_P^2 = \hbar c/(8\pi G)$$

- powers of H and G match
- powers of $\hbar$ match considering
  Bunch Davies vacuum: $v_k \mapsto \hbar^{1/2}/\sqrt{2k}\, e^{-ik\tau}, \ldots$

A detection of the primordial GW background through CMB tensor modes can be seen as a confirmation of the quantum nature of gravity.

Interpretation

- CMB: Coupling of quantum mechanical phenomenon to classical detector ($=$ expanding universe)
- our classical detector is in fact 'unreasonably large' ($\rightarrow$ Dyson)
- dominant contribution proportional to $\hbar$: there is no classical contribution
- not only *can* the tensor perturbations from inflation be described through quantum effects, they *must* be.
- Although the scalar perturbation spectrum also contains the scalar metric mode, the measurement of the scalar perturbation spectrum in agreement with slow-roll predictions does *not* prove the quantization of gravity (see e.g. [Markkanen, Räsänen, Wahlman '14])

Potential caveats

- r can be very small and we may never detect the primordial GW
- we need to be sure B-modes are of primordial origin
  - consistency relations e.g. $n_T = -r/8$
  - super horizon correlations
- QM in classical GR background is *not* a sign of quantum gravity (but also contains powers of $\hbar$ and G)
- At second order in cosmological perturbation theory, scalar fluctuations can source tensor fluctuations
- Gravitational waves as detected by LIGO are purely classical solutions of Einstein's equations

Conclusions

- Cosmic inflation provides a background solution which solves the horizon and flatness problem
- It predicts scalar and tensor quantum fluctuations, which leave imprints in the CMB
- The B-mode polarization channel is well suited to search for primordial tensor perturbations
- Dimensional analysis indicates that any background of tensor perturbations sourced during inflation has to be of quantum origin