

Based on a talk given by Freeman Dyson at a conference in honour of his 90th birthday. (on YouTube and there is also a paper)

Consider 3 theoretical hypotheses:

1. Gravity is a quantum field & gravitons exist as free ptcls.
2. Gravity is a quantum field but gravitons only exist as confined ptcls - like quarks, hidden inside composite structures, which we can only observe as classical gravitational fields
3. Gravity is a statistical concept like entropy or temperature and therefore only defined for gravitational effects of matter in bulk and not for effects of individual elementary ptcls.  
- i.e. gravitational field at a point in space-time does not exist.

This talk is concerned w/ testing hypothesis 1.

"Is it in principle possible to detect the quantisation of the gravitational field?"

If the answer to this question is positive, then perhaps we can test 1. However, we need to be careful about the usage of the word "principle". Three meanings:

- a) We can prove a theorem asserting that detection of a graviton would contradict laws of physics.
- b) We have examined a class of possible graviton detectors and demonstrated that they cannot work.
- c) We have examined a class of graviton detectors and demonstrated that they cannot work in the environment provided by the real universe.

The first meaning is the strongest and Dyson explicitly says that he does not make the claim that he can answer the question of "in principle" detectable according to meaning a). He describes one attempt to do so, then moves on to considering 3 classes of detectors. Each appears to fail but w/ varying degrees of certainty.



The plan for the rest of this talk is the following . . .

- § 2 (of Dyson's paper) - The Bohr-Rosenfeld argument for the quantum behavior of the EM field and why it can not be adapted for the gravitational field
- § 3 LIGO-like detectors - measuring effects on geometry of space-time (fails according to measuring b)
- § 4, 5 Detectors using the gravitoelectric effect, the analog of the photoelectric effect famously used by Einstein. (fails according to measuring c)
- § 6, 7 Graviton-photon oscillations in the presence of an extended magnetic field. - much like neutrino oscillations. (fails according to measuring c)  
↳ See Gertsenshtern-Zeldovich process

With varying degrees of certainty, all attempts seem to fail but Dyson says no definite conclusions are reached, that this should be regarded as work in progress and that there is still much work to do.

## The Bohr-Rosenfeld Paper

- Arose in response to Landau and Peierls who argued that the EM field is not measurable in a domain where quantum and relativistic effects are important.
- Famously hard to read paper "... even by Bohr's standards this paper is very difficult to penetrate.... it is a very good paper that one does not have to read. You just have to know it exists" - Pais

In their paper "On the question of the measurability of the Electromagnetic field strengths" BR show that in a class of experiments, that the EM effects of finite-sized test bodies can be minimised to the extent dictated by the commutation relations of the theory.

⇒ It is mathematically inconsistent to have a classical EM field interacting w/ a QM measuring apparatus.



$$\text{e.g. } \Delta E_n(1) \Delta E_n(2) \sim t \left| A(1,2) - A(2,1) \right| \quad (1)$$

LHS: product of uncertainties of measurements of two averages of  $\hat{E}_x$ -component of electric field, averaged over two space-time regions (1) & (2).

RHS:  $A(1,2)$  is double average over regions (1) & (2) of the retarded electric field produced in (2) by a unit dipole charge in (1).

(1) is precisely the uncertainty reln implied by the commutation rules of quantum electrodynamics.

The question then is whether the BR argument applies to gravitational fields. If yes, then the gravitational field must be quantum in nature and this is in principle detectable.

Unfortunately the answer is negative, due to a crucial feature of the apparatus that makes it inapplicable to gravitational fields.

"In order to disturb the electromagnetic field to be measured as little as possible during the presence of the test body system, we shall imagine placed beside each electric or magnetic component particle another exactly oppositely charged neutralizing particle."

- Say we observe the movement of an object w/ charge or current  $J$  in order to measure the local electric or magnetic field, we need a compensating object w/ charge/current  $-J$  in order to counter the production of an additional EM field.
- The strategy therefore fails for measurements of the gravitational field since there exists no negative masses to compensate the fields produced by the test objects.

The conclusion is that the quantisation of the gravitational field is not a logical consequence of the quantum behavior of the measuring apparatus.



## Can LIGO Detect a graviton

A classical gravitational wave may be considered a coherent superposition of a large number of gravitons. The energy density of this wave is

$$E = \left( \frac{c^2}{32\pi G} \right) \omega^2 f^2 \quad \begin{matrix} \leftarrow \text{strain amplitude } f = \frac{\delta}{D} \\ \uparrow \text{angular frequency} \\ \text{energy density} \end{matrix} \quad (2)$$

LIGO can detect a wave w/ strain  $f = 10^{-21}$  ( $\delta = 10^{-18} \text{ m}$ ) (LIGO functions in the  $\omega = 10^2 - 10^3$  range but Dyson does not mention this.)

For  $f = 10^{-21}$  &  $\omega = 1 \text{ kilohertz}$ , (2) gives  $E = 10^{-10} \text{ ergs/cm}^3$  ( $1 \text{ erg} = 1 \text{ g cm}^2 \text{ s}^{-2} = 10^{-7} \text{ Joules} = 6 \times 10^{-11} \text{ eV}$ )

A single graviton w/ angular freq.  $\omega$  cannot be confined to a region smaller than linear dimension  $\lambda = c/\omega$ . Hence energy density of a single graviton is at most

$$E_s = \hbar \omega \frac{\omega^3}{c^3} = \frac{\hbar \omega^4}{c^3} \quad (3)$$

So a gravitational wave detectable by LIGO has at least  $3 \times 10^{37}$  gravitons

So, ignoring noise, we need an improved sensitivity by a factor of order  $3 \times 10^{37}$ .

To assess if a detection of a single graviton is in principle possible (meaning b)) w/ a LIGO-like experiment, we now analyse Super Sensitive LIGO-SSLIGO

SSLIGO — For a rough estimate of required sensitivity, equate (2) & (3)

$$\left( \frac{c^2}{32\pi G} \right) \omega^2 f^2 = \frac{\hbar \omega^4}{c^3} \Rightarrow f = 32\pi L_p^2 \frac{\omega^2}{c^2} \quad L_p = \left( \frac{G\hbar}{c^3} \right)^{\frac{1}{2}} \quad (4)$$

$D$  can not exceed  $\frac{c}{\omega}$ . Optimum detectability  $\Rightarrow$  when  $D = \frac{c}{\omega}$

$$\Rightarrow \delta = (32\pi)^{\frac{1}{2}} L_p \quad (6)$$



$$\delta = (32\pi)^{\frac{1}{2}} L_p$$

(6)

$\Rightarrow$  Required precision of measurement of separation between the two mirrors  $\propto \mathcal{O}(L_p)$  --- Is this possible in principle?

Heisenberg uncertainty reln between position and momentum of freely floating objects gives the lower bound

$$ML_p^2 \geq \hbar T \leftarrow \text{duration of measurement}$$

↑ mass of each object

Require  $T > D$ , the time taken to communicate between the two mirrors

$$\Rightarrow ML_p^2 \geq \hbar \frac{D}{c}$$

$$\Rightarrow D \leq \frac{MC}{c^2} < r_s$$

The experimental apparatus is doomed to collapsing into a black hole.

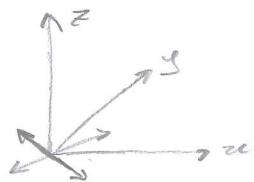
Next Dyson goes on to consider mirrors that are clamped in position. In his case the limitation comes from the quantum fluctuations of the rigid structure. The conclusion is the same.

The conclusion is that SShIGO is impossible due to laws of physics conspiring to forbid measurements w/ distance error less than  $L_p$ .

## Graviton Detectors

The simplest type of graviton detector is an electron in an atom. When the frequency is  $10^{15}$  Hz or higher, a single graviton can kick out an electron from an atom — this is the gravito-electro effect — analogous to the photoelectric effect Einstein used to infer the existence of quanta of the electromagnetic field — photons.





To explore this possibility we need to know two quantities:

- the cross-section for interaction of a graviton w/ atom
- the intensity of possible natural or artificial sources of high energy gravitons

Choose coordinate axes s.t. graviton propagates in z-dir w/ two orthogonal modes of polarization @  $45^\circ$  to x- and y-axes  $\rightarrow$  wave amplitude  $\propto xy$  and  $x^2 - y^2$

Steps:

1. Compute the matrix element for the electron to absorb the graviton and move from its ground state  $a$  to another state  $b$ . He does this using the quadrupole approximation

$$D_{ab} = m \int \psi_b^* xy \psi_a d\tau \quad \uparrow \text{volume element}$$

2. Compute the logarithmic average of the cross-section

$$\begin{aligned} S_a &= \int \sigma(\omega) \frac{d\omega}{\omega} \\ &= 4\pi^2 L_p^2 Q \end{aligned}$$

where  $Q$  is a numerical factor depending on the initial state of the electron

3. Consider different initial states for electron  
 $\rightarrow$  finds  $Q$  is always of order 1.

He concludes that the cross-section for absorption of a graviton by any kind of ptd will be of the same magnitude

$$S_a \approx 4\pi^2 L_p^2 = \frac{4\pi^2 G \hbar}{c^3} = 8 \times 10^{-65} \text{ cm}^2$$

spread over a range of graviton energies extending from the binding-energy of the ptd to a few times the binding-energy. For any macroscopic detector composed of ordinary matter, the absorption cross-section per gram will be of the order  $10^{-41} \text{ cm}^2 \text{ g}^{-1}$   
(Avogadro =  $6.022 \times 10^{23}$ )



## Thermal Gravitons

Dyson writes: "we have an splendid natural generator of gravitons in the kilovolt range, producing far more gravitons than any artificial source. It is called, the sun."

Weinberg long ago calculated the graviton luminosity of the sun, caused by gravitational bremsstrahlung. Gould later corrected the calculation."

For electron-ion collisions w/ energy E, the differential cross-section  $\rho(\omega)$  for producing a graviton of energy  $\omega$  is divergent at low energies. The total cross-section has no meaning. The physically meaningful quantity is the integral of the differential cross-section multiplied by the energy of the graviton.

$$\int \rho(\omega) \hbar \omega d\omega = \frac{320}{9} Z^2 \alpha^2 L_p^2 E \quad \begin{matrix} \text{EM fine structure constant} \\ \downarrow \\ \text{charge of ion} \end{matrix}$$

Including a similar contribution from electron-electron collisions, the total luminosity is found to be

$$L_g = 79 \text{ Megawatts} \sim 10^{24} \text{ gravitons/second} \\ \text{w/ energy in kilovolt range}$$

This gives a flux at earth of

$$F_g = 4 \times 10^{-4} \text{ gravitons per cm}^2 \text{ per second}$$

If we imagine the whole mass of earth to be available as raw material for the manufacture of graviton detectors

$$(\text{cross section per gram}) \times (\text{mass of earth}) \times \text{Flux}$$

$$10^{-41} \times 5.9 \times 10^{27} \times 4 \times 10^{-4} = 2.4 \times 10^{-17} \text{ gravitons/second}$$

If the experiment runs for the lifetime of the sun  
 - 5 billion years =  $1.58 \times 10^{17}$  s

$$2.4 \times 10^{-17} \times 1.58 \times 10^{17} \rightarrow 4 \text{ gravitons!}$$

Dyson concludes: The experiment barely succeeds, but in principle it can detect gravitons.



How can we tweak the experiment to improve the graviton count?

According to Gould there are stronger sources of gravitons in the universe - white dwarfs at the beginning of their lives, and hot neutron stars.

White dwarfs  $\sim 10^4 L_\odot$  neutron stars  $\sim 10^{10} L_\odot$   
(luminosities proportional to central densities)

However lifetimes are shorter.  $10^7$  years WD  
 $10^4$  years NS

Total lifetime output of gravitons, roughly 100 and 105 times that of the sun

Next, we make the detector the size of the sun and orbiting at a distance of 0.01 astronomical unit. (period of 8 hours)  
Expected # of gravitons detected WD:  $10^{13}$ , NS:  $10^{16}$

- Detection of  $\sim 1/\text{min}$  for WD and  $3 \times 10^4 / \text{s}$  for NS

Dyson concludes that gravitons are in principle detectable, if we disregard background noise.

Dyson reckons that neutrinos are probably the biggest problem.

- For the sun, about  $10^{14}$  neutrinos per graviton are emitted
- The cross-section for neutrino-electron scattering is about  $10^{20}$  times the cross-section for graviton absorption

$$10^{14} \times 10^{20} = 10^{34} \text{ neutrino background events for each grav. event}$$

The situation is even worse for WD & NS since neutrino production increases more rapidly than graviton production w/ temp.

Before jumping to the conclusions, we should explore possible ways to remove this neutrino background

Dyson concludes that a shield of ordinary matter would need to be  $10^{16}$  km thick and mass so large it would collapse to BH.  
He also considers trying to detect each neutrino after the scattering event.  
- this suffers from the same problem as the shield.

Dyson concludes that w/ known materials and known physical processes detection of thermal gravitons appears to be impossible in a noisy universe.



## Nonthermal Gravitons

Next we consider the Gertsenshtein process — photon graviton oscillations due to passage of photons through an extended classical magnetic field. The interaction energy of a weak gravitational field in the presence of an electromagnetic field  $B$ :

$$I = \left( \frac{8\pi G}{c^4} \right) h_{ij} T_{ij} \quad (25)$$

↑ Energy momentum tensor of  
gravitational field

EM field.

Suppose  $h_{ij}$  is field of a graviton moving in  $z$  direction and  $T_{ij}$  contains both the photon magnetic field and a fixed classical background field.

$$T_{ij} = \left( \frac{1}{4\pi} \right) (B_i + b_i)(B_j + b_j) \quad (26)$$

↑ ↑ background magnetic field  
magnetic field of photon

$$\Rightarrow I \propto \left( \frac{4G}{c^4} \right) h_{zz} B_z b_z$$

which is bilinear in the photon and graviton fields. The effect of this term is to mix the photon and graviton fields.

→ there is an oscillation between graviton and photon states, just like the oscillations between neutrino states that causes changing of flavour.

If a photon travels a distance  $D$  through a uniform transverse magnetic field  $B$ , it will emerge as a graviton w/ probability

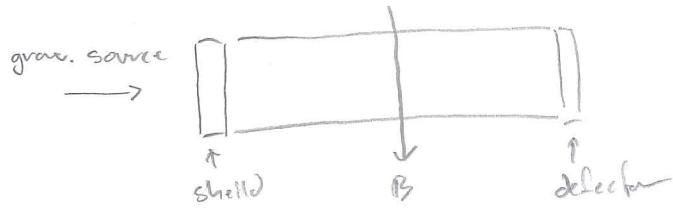
$$P = \sin^2 \left( \frac{G^{1/2} B D}{2c^2} \right) = \sin^2 \left( \frac{D}{L} \right)$$

— Mixing length  $L = \left( \frac{2c^2}{G^{1/2} B} \right) = \left( \frac{10^{25}}{B} \right)$  (N.B. indep. of wavelength)

Assuming  $D \ll L \Rightarrow P \approx \frac{GB^2 D^2}{4c^4}$  (30)

It's the quadratic dependence of  $P$  on  $D$  which makes this interesting.





Can also think of Gertsenshtern process as the basis of a graviton detector

$$\text{If } D \gg 1 \text{ astronomical unit} = 10^{13} \text{ cm} \text{ then } P = \left(\frac{10^{13}}{10^{25}}\right)^2 B^2 = 10^{-24} \text{ Js}^2$$

$\Rightarrow$  Requires very strong magnet. C.f. search for axion-photon conversion being searched for at CERN w/ a surplus magnet from LHC. - The magnet  $\rightarrow$  quarters and magnetic field  $9 \times 10^{14}$  Gauss

However, Dyson finds a problem — previous studies neglected the nonlinearity of the EM field caused by  $e^+e^-$  pair production in the vacuum. The fourth-order term in the EM field energy density

$$\left(\frac{\alpha}{360\pi^2 H_c^2}\right) [(E^2 - H^2)^2 + 7(E \cdot H)^2]$$

$$H_c = \left(\frac{m^2 c^3}{e\hbar}\right) = 5 \times 10^{13} \text{ Gauss}$$

$\rightarrow$  is the critical magnetic field strength at which electron-positron pair fluctuations become non-negligible

$$\text{now } I \supset \frac{\alpha}{360\pi^2 H_c^2} (4(B \cdot E)^2 + 7(B \cdot H)^2)$$

(35)  
Electric fields of  
the photon

It follows that the photon velocity  $\rightarrow$  reduced by a fraction

$$g = 1 - \frac{v}{c} = \left(\frac{k\alpha B^2}{360\pi^2 H_c^2}\right)$$

w/  $k=4, 7$  depending on if photon is polarized w/ e-field or b-field  $\parallel$  to  $B$ .  $\rightarrow$  choose  $k=4$  since that is more favourable

The photon and graviton waves will loose coherence after traveling for a distance

$$L_c = \left(\frac{c}{g\omega}\right) = \left(\frac{90\pi^2 c H_c^2}{k\alpha B^2 \omega}\right) = \frac{10^{43}}{B^2 \omega}$$

if  $D > L_c$ , the Gertsenshtern process fails, a necessary condition is therefore

$$DB^2 \omega \leq 10^{43}$$



$$(30) \quad P = \frac{B^2 D^2}{10^{50}}$$

$$(31) \quad DB^2 w \leq 10^{43}$$

$$\Rightarrow P \leq \frac{B^2}{10^{50}} \frac{10^{86}}{B^4 w^2} = \frac{10^{86}}{B^2 w^2} = \frac{10^{-4}}{B^2} \quad \text{so far 100 kilowatts } w = 10^{20}$$

$$D \leq \frac{10^{23}}{B^2}$$

(40)

Dyson considers 2 situations:

1) Pulsars  $B = 10^{12}$   $D = 10^6 \Rightarrow$  Gertsenshtern process fails contradicting previous results

2) Pipe detector — tiny  $B = 10^5$ ,  $10^{13} \text{ cm} \rightarrow P = 10^{-14}$

Increasing  $B$  requires decreasing  $D$ , making the situation worse.

Dyson concludes that the detector works in principle but fails for practical reason in the real universe.

## Conclusions

Examined 3 possible kinds of graviton detector  
(low energy gravitons)

- 1) LIGO-like — ineffective as a consequence of laws of physics
- 2) Gravitoelectric effect — kilovolt gravitons — ineffective due to background noise from neutrinos in the real universe.
- 3) High energy gravitons using Gertsenshtern process, also does not seem to work but Dyson says this is only because of practical limits to the size of detectors.

We are a long way from settling the question of whether gravitons exist. But the question whether gravitons are in principle detectable is also interesting and may be easier to decide.



(11)