Workshop Seminar
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Wir machen Erkenntnis möglich.

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# The Cosmological Constant Problem

[ and (no) solutions to it ]

original reference: Steven Weinberg, Rev. Mod. Phys. 61 (1989) 1

further reading: (modern, pedagogical lecture notes)
Antonio Padilla, arXiv: 1502.05296

## Outline

1. The cosmological constant
2. The Problem
3. Weinberg's no-go (self-adjustment)
4. Supersymmetry
5. Quantum gravity

"Physics thrives on crisis. [...] Unfortunately, we have run short on crisis lately." (Weinberg, 1989)

## 1. The cosmological constant: an old crisis

Einstein 1915: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G\, T_{\mu\nu}$

Einstein 1917: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G\, T_{\mu\nu}$

* using the Weinberg metric $(-+++)$
* $G$: Newton's constant, $T_{\mu\nu}$: energy-momentum tensor
* static solution universe with dust, $p = 0$

$$\rho = \frac{\lambda}{8\pi G}$$

* size of universe $r = \frac{1}{\sqrt{8\pi \rho G}}$
* mass $M = 2\pi^2 r^3 \rho = \frac{\pi}{4} \lambda^{-1/2} G^{-1}$

Well-motivated assumption:

* homogeneous, isotropic universe

However

* Hubble's expansion (1929 ↳ "größte Eselei")
* de Sitter solution without matter (1917)

$$d\tau^2 = \frac{1}{\cosh^2 Hr}\left[dt^2 - dr^2 - H^{-2}\tanh(Hr)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

with $H = \sqrt{\frac{\lambda}{3}}$ and $\rho = p = 0$.

↳ explains redshift

* other solutions (Friedmann, Lemaître, Robertson, Walker)

$$d\tau^2 = ds^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

↳ comoving coordinates

$$\boxed{\left(\frac{dR}{dt}\right)^2 = -k + \frac{1}{3}R^2\left(8\pi G\rho + \lambda\right)} \quad (*)$$

$\Leftrightarrow$ de Sitter: $k = 0, \rho = 0$

↳ other expanding solutions with $\lambda = 0$ and $\rho > 0$.

## 2. Weinberg's problem

energy density = cosmological constant (!)

* Lorentz invariance: $\langle T_{\mu\nu}\rangle = -\langle \rho \rangle g_{\mu\nu}$

$$\lambda_{eff} = \lambda + 8\pi G\langle\rho\rangle$$

$\Leftrightarrow$ total vacuum energy: $\rho_V = \langle\rho\rangle + \frac{\lambda}{8\pi G} = \frac{\lambda_{eff}}{8\pi G}$

* estimate on $\lambda_{eff}$ from redshift / expanding universe

$$\left[\frac{1}{R}\frac{dR}{dt}\right]_{today} \equiv H_0 \simeq 50 \ldots 100 \,\frac{km}{s\,Mpc} \simeq \left(\tfrac{1}{2} \ldots 1\right) 10^{-10}/yr$$

flat universe: $\frac{|k|}{R^2_{today}} \lesssim H_0^2$

Compare with critical density

$$|\rho - \langle\rho\rangle| \lesssim \frac{3H_0^2}{8\pi G}, \quad \circledast \rightarrow |\lambda_{eff}| \lesssim H_0^2$$

$$\left(\rho_c \simeq 10^{-29}\frac{g}{cm^3} \simeq 10^{-47}\,GeV^4\right)$$

* energy density of empty space (e.g. some field, mass $m$)

$$\langle\rho\rangle = \int_0^\Lambda dk \frac{4\pi k^2}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

↳ cut-off @ Planck scale : $\langle\rho\rangle \simeq 2^{-10}\pi^{-4}G^{-2} = 2\cdot 10^{71}\,GeV^4$
($\Lambda \simeq (8\pi G)^{-1/2}$)

⇒ cancellation in $\left|\langle\rho\rangle + \frac{\lambda}{8\pi G}\right|$ up to 118 digits!

↳ QCD : $\langle\rho\rangle \sim \frac{\Lambda_{QCD}^4}{16\pi^2} \simeq 10^{-6}\,GeV^4 \rightarrow 41$ digits

* other estimates (Zeldovich) : cancel one-loop by $\lambda$
$\Lambda^3$ particles of energy $\Lambda$ per unit volume

$$\langle\rho\rangle \approx \left(\frac{G\Lambda^2}{\Lambda^{-1}}\right)^2\Lambda^3 = G\Lambda^6 \approx 10^{-38}\,GeV \quad (\Lambda = 1\,GeV)$$

* the "real serious worry" : spontaneous symmetry breaking in the ew. sector

$$V = V_0 - \mu^2\phi^+\phi + g(\phi^+\phi)^2, \quad \mu^2>0, g>0$$

@ minimum : $\langle\rho\rangle = V_{min} = V_0 - \frac{\mu^4}{4g}$

with $V(\phi=0) = V_0 = 0 \quad \hookrightarrow \langle\rho\rangle \simeq -g\,(300\,GeV)^4 \simeq 10^6\,GeV^4$
(for $g \approx \alpha^2$)

However, neither $V_0$ or $\lambda$ must vanish
→ cancellation possible

* early times: temperature effects → minimum at $\phi = 0$

↳ $V(\phi) = V_0$ : zero cosmological constant today
⇔ enormous cosmo. const. before PT (drives infl.)

# 3. Weinberg's no-go

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["No-go theorems have a way of relying on apparently technical assumptions that later turn out to have exceptions of great physical interest."]

* the effective cosmological constant term makes it impossible to find solutions of the Einstein equations with constant Minkowski metric: $g_{\mu\nu} = \eta_{\mu\nu}$.

[general covariance cannot ~~just~~ be broken to space-time translations]

* fields preserving translational invariance (i.e. constants)

eom: $\frac{\partial \mathcal{L}}{\partial \varphi_i} = 0 \quad (1), \qquad \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} = 0 \quad (2).$

GL(4) symmetry survives: $\begin{cases} g_{\mu\nu} \to A^{\rho}{}_{\mu} A^{\sigma}{}_{\nu} g_{\rho\sigma} \\ \varphi_i \to D_{ij}(A)\varphi_j \end{cases}$

$\mathcal{L} \to \mathrm{Det}(A)\,\mathcal{L}$

with (1) ↳ unique solution $\mathcal{L} = c\,(\mathrm{Det}\, g)^{1/2}$
but no solutions of (2)

* <u>adjustment mechanism</u>

↳ equilibrium solution in which all fields ($g_{\mu\nu}$ <u>and</u> $\varphi_i$) are constant simultaneously

$$g_{\lambda\nu} \frac{\partial \mathcal{L}(g,\varphi)}{\partial g_{\lambda\nu}} = \sum_{n}^{N} \frac{\partial \mathcal{L}(g,\varphi)}{\partial \varphi_n} f_n(\varphi)$$

[(1) and (2) do not vanish independently!]

↳ symmetry condition for constant fields

$$\delta g_{\lambda\nu} = 2\varepsilon g_{\lambda\nu}, \quad \delta \varphi_n = -\varepsilon f_n(\varphi)$$

leading to $\frac{\partial \mathcal{L}}{\partial \varphi_n} = 0$ at $\varphi_n = \varphi_n^{(0)}$

$\Rightarrow g_{\lambda\nu} \frac{\partial \mathcal{L}}{\partial g_{\lambda\nu}}$ automatically satisfied

With the given assumptions, it is impossible to find a solution for the fields $\psi_n$ (without fine-tuning).

↳ Take $N-1$ fields $\sigma_a$ and one scalar $\phi$:

$$\delta g_{\lambda\nu} = 2\varepsilon g_{\lambda\nu}\,,\quad \delta\sigma_a = 0\,,\quad \delta\phi = -\varepsilon$$

⇒ Lagrangian depends on $g_{\lambda\nu}$ and $\phi$ only as $e^{2\phi} g_{\lambda\nu}$

so $\mathcal{L} = e^{4\phi}(\mathrm{Det}\, g)^{1/2} \mathcal{L}_0(\sigma)$

Source of $\phi$ is trace of energy-momentum tensor

$$\frac{\partial\mathcal{L}}{\partial\phi} = T^\mu{}_\mu (\mathrm{Det}\, g)^{1/2}\,,\quad T^{\mu\nu} = g^{\mu\nu} e^{4\phi}\mathcal{L}_0(\sigma)$$

→ redefine $\hat{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}$

↳ $\phi$ appears only with derivative couplings (does not helps)

* conformal anomalies (Peccei, Solà, Wetterich 1987)

$$\mathcal{L}_{eff} = (\mathrm{Det}\, g)^{1/2}\left[e^{4\phi}\mathcal{L}_0(\sigma) + \phi\, \Theta^\mu{}_\mu\right]$$

now $\frac{\partial\mathcal{L}}{\partial\phi} = (T^\mu{}_\mu + \Theta^\mu{}_\mu)(\mathrm{Det}\, g)^{1/2}$

equilibrium solution $4e^{4\phi_0}\mathcal{L}_0 + \Theta^\mu{}_\mu = 0$ at $\phi = \phi_0$.

↯ not the condition for flat space!

$$0 = \frac{\partial\mathcal{L}_{eff}}{\partial g_{\mu\nu}} \propto e^{4\phi}\mathcal{L}_0 + \phi\, \Theta^\mu{}_\mu$$

$\neq$

↳ Either preserve symmetry → no solution for $\phi$ or break it → no flat space

* changing gravity (maintains general covariance, but determinant of metric is not a dynamical field anymore) ⑥

action for gravity & matter

$$I[\psi, g] = \frac{-1}{16\pi G} \int d^4x \sqrt{g}\, R + \underbrace{I_M[\psi, g]}_{\text{matter action including } -\lambda \int d^4x \sqrt{g} / (8\pi G)}$$

$$\frac{\delta I}{\delta g_{\mu\nu}} - \frac{1}{8\pi G}\left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) + \underbrace{T^{\mu\nu}}_{\delta I_M / \delta g_{\mu\nu}}$$

treat determinant not a dynamical field:

action stationary wrt. variations $g^{\mu\nu}\delta g_{\mu\nu} = 0$

$$R^{\mu\nu} - \frac{1}{4} g^{\mu\nu} R = -8\pi G\left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\lambda}{}_{\lambda}\right) \quad (E')$$

only the traceless part

usual conservation law $T^{\mu\nu}{}_{;\mu} = 0$ ✓

Bianchi identities $\left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right)_{;\mu} = 0$ ✓

Covariant derivative of $(E')$

$$\frac{1}{4}\partial_\mu R = 8\pi G \frac{1}{4}\partial_\mu T^{\lambda}{}_{\lambda} \Leftrightarrow R - 8\pi G T^{\lambda}{}_{\lambda} \equiv \underbrace{-4\Lambda}_{=\text{const.}}$$

$$\Rightarrow \boxed{R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \Lambda g^{\mu\nu} = -8\pi G T^{\mu\nu}}$$

↳ Einstein equations with cosmological constant

↳ integration constant instead of vacuum fluctuations

↳ no cosmological constant & vacuum fluctuations cancel in $(E')$ → there are flat solutions in the absence of matter and radiation

↳ holds in (simple) quantization

⇒ $\lambda$ as constant of integration → state vector superposition

# 4. Supersymmetry (Gives a symmetry argument for a vanishing cosmological const.)

Simple argument

$$\{ Q_\alpha , Q^{+}_{\dot\beta} \} = (\sigma_\mu)_{\alpha\dot\beta} P^\mu \quad \text{anti-commutator}$$

unbroken SUSY : $Q_\alpha |0\rangle = Q^{+}_{\dot\alpha} |0\rangle = 0$

$\Leftrightarrow \langle 0 | P^\mu | 0 \rangle \cancel{(\sigma_\mu)_{\alpha\dot\beta}} \equiv 0$

scalar potential $V(\phi, \phi^+)$ given by superpotential

$$V(\phi, \phi^+) = \sum_i \left| \frac{\partial W(\phi)}{\partial \phi^i} \right|^2$$

Condition for unbroken SUSY : $\langle S \rangle = V_{min} = 0$

☹ SUSY is broken in the real world

* global supersymmetry + gravity
= local supergravity

$\Rightarrow$ models with $V = 0$ and $D_i W \neq 0$ (broken SUSY)
with Kähler potential of the type

$$K = -3 \ln \left[ T + T^* - h(C^a, C^{a*}) \right] / (8\pi G) + \tilde{K}(S^m, S^{m*})$$

and superpotential $W = W_1(C^a) + W_2(S^m)$

$T, C^a, S^n$ chiral superfields
$h$ and $\tilde{K}$ are arbitrary real functions

## 5. Short look on Quantum Cosmology

* effective action with 3-form gauge field $A_{\mu\nu\lambda}$

$$\Gamma_{eff}[A,g] = \frac{\lambda}{8\pi G}\int \sqrt{g}\, d^4x + \frac{1}{16\pi G}\int \sqrt{g}\, R\, d^4x + \frac{1}{48}\int d^4x \sqrt{g}\, F_{\mu\nu\lambda\rho}F^{\mu\nu\lambda\rho} + \dots$$

where $F_{\mu\nu\rho\sigma} = \partial_{[\mu}A_{\nu\rho\sigma]}$

↳ Stationary in $A_{\mu\nu\lambda}$ → $F_{\mu\nu\lambda\rho}$ vanishing covariant derivative

$$\Gamma_{eff} = \frac{\lambda(c)}{8\pi G}\int \sqrt{g}\, d^4x + \frac{1}{16\pi G}\int \sqrt{g}\, R\, d^4x + \dots$$

with $c$ defined via $F^{\mu\nu\rho\sigma} = c\,\varepsilon^{\mu\nu\rho\sigma}/\sqrt{g}$

and $\lambda(c) = \frac{c^2}{2} + \lambda$

↳ Stationary in $g_{\mu\nu}$: Einstein eqns. with $\lambda(c)$, $R = -4\lambda(c)$

$$\Gamma_{eff} = -\frac{\lambda(c)}{8\pi G}\int \sqrt{g}\, d^4x$$

↳ solution of Einstein eqns. with $\lambda(c) > 0$ (Hartle–Hawking boundary cond.)

4-sphere with $r = \sqrt{\frac{3}{\lambda(c)}}$

probability density $\sim e^{-\Gamma_{eff}} = \exp\left(\frac{3\pi}{G\lambda(c)}\right)$

↳ for $\lambda(c) < 0$: solution compact with periodicity cond.

all $\Gamma_{eff} \geq 0$.

probability density has infinite peak for $\lambda(c) \to 0+$

$$P(c) = \delta(c - c_0) \quad \text{with } \lambda(c = c_0) = 0$$

* furthermore: wormholes and baby universes, loss of quantum coherence by observers

again: probability distribution peaks @ $\lambda_{eff} \to 0+$