"Workshop seminar on vacuum energy"

Cosmological evidence for dark energy and alternative interpretations of accelerated expansion

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1. Cosmological evidence for dark energy

1.1. The Cosmological Standard Model

<u>Foundation</u>: The cosmological principle

- On large scales the universe is <u>homogeneous</u> and <u>isotropic</u>

↳ FRW - metric

scale factor

$$ds^2 = -dt^2 + \frac{a(t)^2}{1+kr^2}dr^2 + a(t)^2 r^2 d\tilde{\Omega}^2$$

curvature parameter $-1, 0, +1$

↳ Energy momentum tensor (of matter)

$$(T^{\mu}{}_{\nu})_M = \mathrm{diag}(-\rho_M, p_M, p_M, p_M)$$

comoving frame

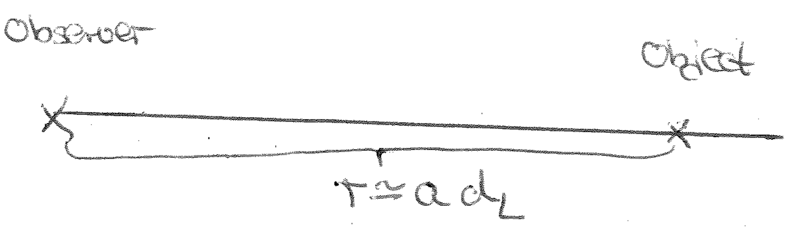
Using $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$, one obtains the Friedmann equations (<u>Evolution equations of the universe</u>)

$\rho_\Lambda = \frac{\Lambda}{8\pi G}$ — Hubble parameter

1. $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda) - \frac{k}{a^2} \equiv H^2$$

2. $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_M + 3p_M - 2\rho_\Lambda) \equiv -\frac{q}{H^2}$$

acceleration parameter



Implications: H and q influence the distance - redshift relation

Redshift: $1 + z \equiv \frac{\lambda_2}{\lambda_1} = \frac{a_0}{a}$

Distance: $d_L = \sqrt{\frac{L}{4\pi F}} = (1+z)\, r(z)\,, \quad F = \frac{L}{4\pi d_L^2}$

luminosity (L), flux (F)

Important are the observables today:

$$\frac{a(t)}{a_0} \overset{TE}{\simeq} 1 + H_0 (t - t_0) - \frac{q_0 H_0^2}{2} (t - t_0)^2 + \dots$$

$$\Rightarrow \boxed{H_0 d_L(z) \simeq z + \frac{1}{2}(1 - q_0) z^2 + \dots}$$

Expansion/Collapse

Acceleration/Decelleration

The parameter H_0: First measured by E. Hubble in 1922|3 [1]

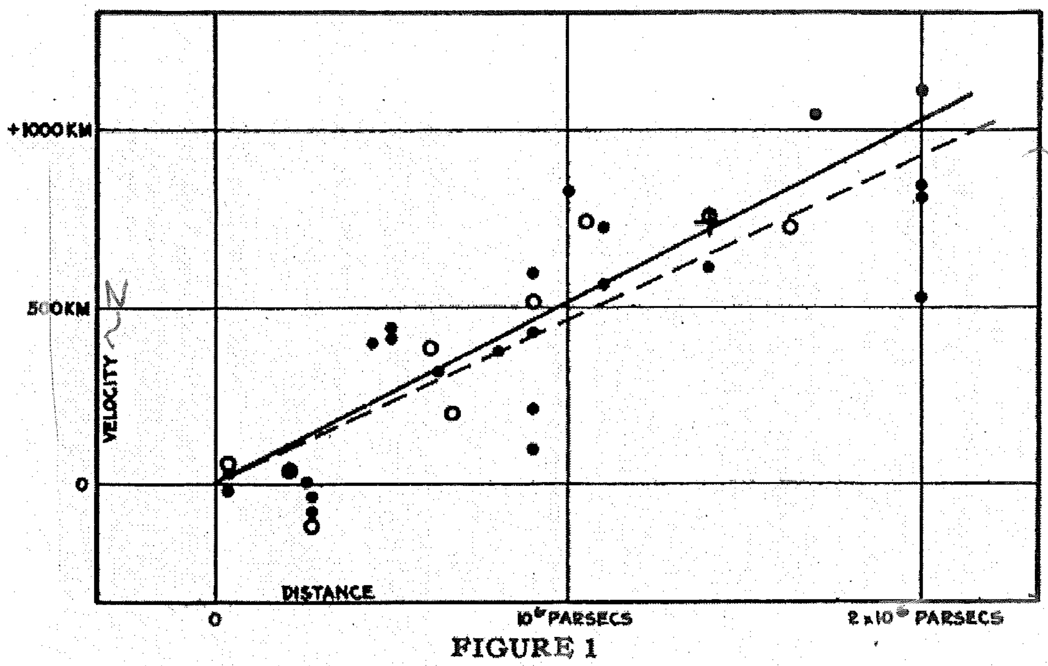


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Expansion

Meets model expectations

Today:
$H_0 = 67{,}3(12)\ \frac{km}{s\,Mpc}$

[PDG]

<u>The parameter q_0</u>: Requires measurement of more distant objects (second order effect)

- Prefered method (today): Measure $\underbrace{m-M}_{\text{magnitude}}$ vs. $\underbrace{z}_{\text{redshift}}$

<u>Definition</u>: Apparent and absolute magnitude

- Apparent magnitude:

Measure for the brightness of an object as seen from earth

Vega (zero point) $m_{Vega}=0$

$$\boxed{m \equiv -2.5\, \mathrm{ld}\left(\frac{F}{F_0}\right) \overset{F\sim\frac{1}{d^2}}{=} -5\, \mathrm{ld}\left(\frac{d_0}{d}\right)}$$

$\to m_{sun} \simeq -26.74$

- Absolute magnitude:

Apparent magnitude of an object, if it where located 10pc away from earth

$$\boxed{M - m \equiv -5\, \mathrm{ld}\left(\frac{d}{10\mathrm{pc}}\right)}$$

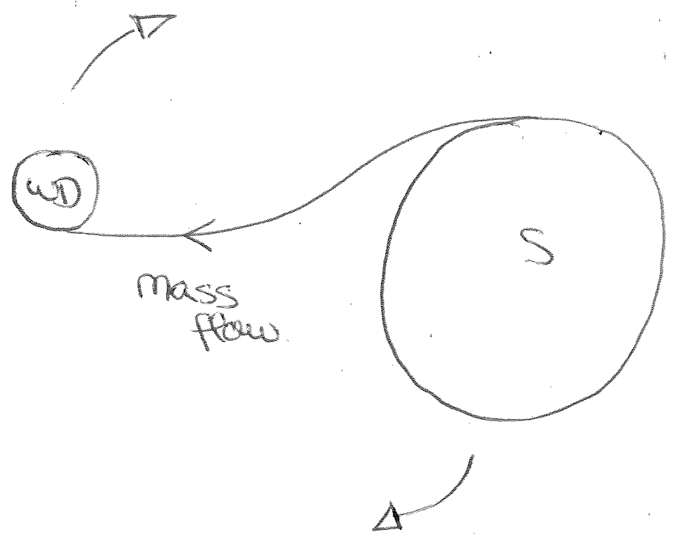
$\to M_{sun} \simeq 4.83 > m_{sun}$

$\to$ All visible stars on the <u>night sky</u> have a magnitude between 1 and 5

Measurement: Type Ia supernovae (SNIa)

Observe binary systems of a white dwarf and a companion star.

Sketch



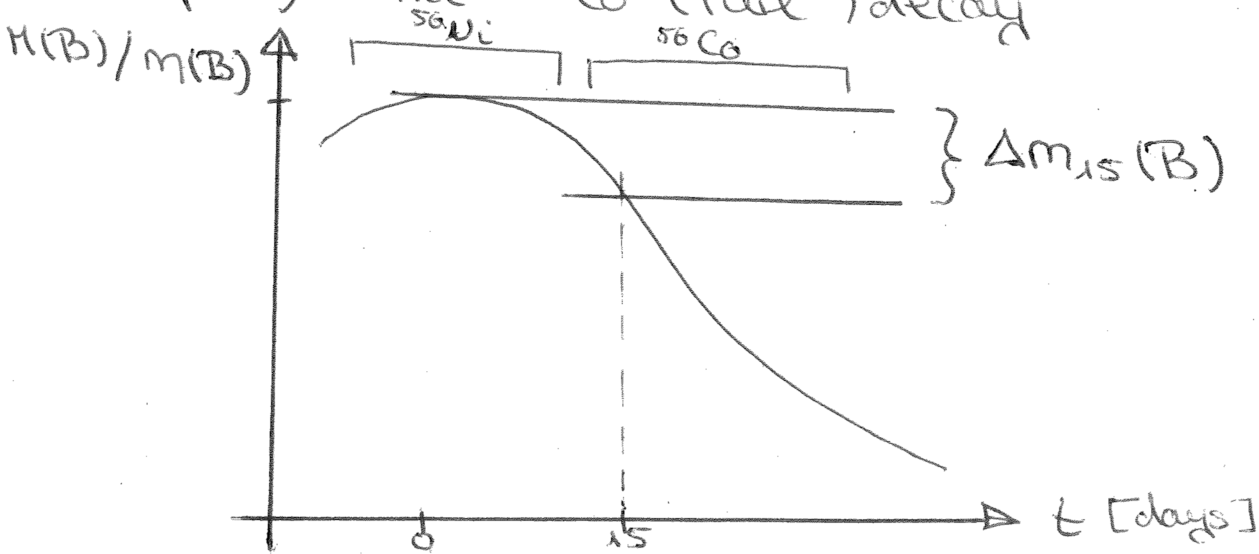
Evolution

- Both objects are orbiting each other
- The companion star spills gas onto th WD, thus increasing its mass
- Once the WD reaches a mass equal to the Chandrasekhar limit of $M \simeq 1.44 M_\odot$, it collapses

$\longrightarrow$ Supernova

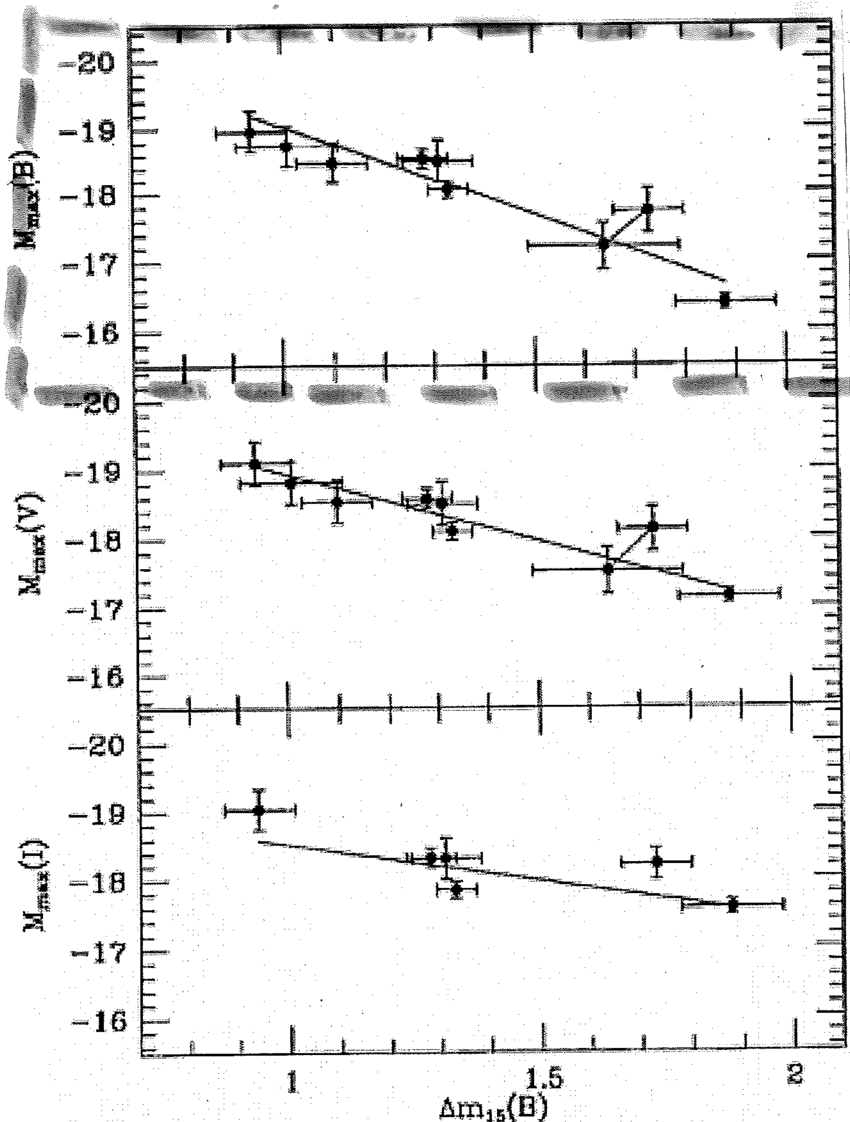
Features:

- SNIa always involve WDs with a well defined mass ($M \simeq 1.44 M_\odot$)
- Light curve predominantly determined by ^{56}Ni (peak) and ^{56}Co (tail) decay



↳ All SNIa are expected to have comparable absolute magnitudes : $\hat{M}_B \in [-20, -19]$
(similar physical properties)

↳ The remaining dispersion can be removed by virtue of the Phillips relation



$\hat{M}(B) \simeq -21.726 + 2.698\, \Delta m_{15}(B)$

[2]

FIG. 1.—Decline rate–peak luminosity relation for the nine best-observed SN Ia's. Absolute magnitudes in B, V, and I are plotted vs. $\Delta m_{15}(B)$, which measures the amount in magnitudes that the B light curve drops during the first 15 days following maximum.

Standard candles

"Roughly":
Measure Δm_{15} and $\hat{m}$ to obtain

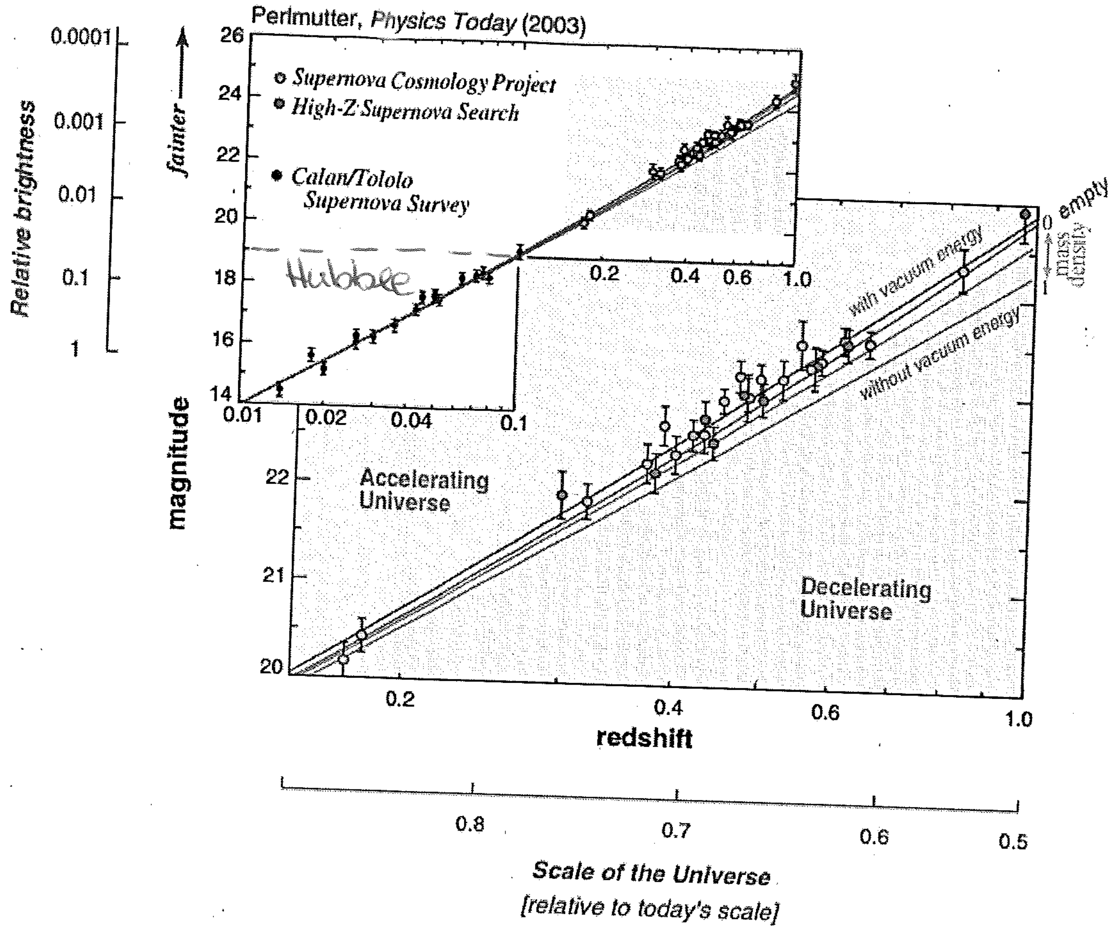
$$\hat{m} - \hat{M}$$

(However, the reality is a little bit more complicated)

→ Templates are used in the fit

~ 15% error

Results of the measurement [3]:



→ Expansion seems to be accelerating
(Universe appears to be „more empty than empty")

Historically: Postulate existence of additional energy density with

$$\boxed{\rho_\Lambda = -P_\Lambda}$$

→ dark energy

This way a good fit is archieved!

$\chi^2/_{ndof} \simeq 1.1$

$$\boxed{\Omega_\Lambda \equiv \frac{8\pi G}{3H_0^2}\rho_\Lambda \simeq 0.7} \qquad \boxed{\Omega_M \equiv \frac{8\pi G}{3H_0}\rho_{M,0} \simeq 0.3}$$

1.2. Cosmic microwave background (CMB) [cf. 10]

- At the time of recombination, the universe becomes transparent (photons decouple and free-stream until detection)

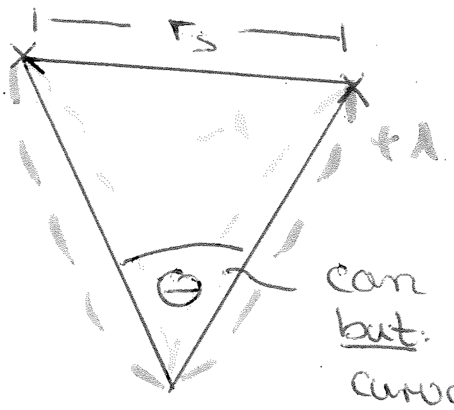
CMB $\triangleq$ Image of the universe at the time of recombination ($z_{rec} \simeq 1100$)

- Before recombination photons are tightly coupled (Thompson scattering)
- Maximum distance photons could have travelled (sound horizon)

$$r_s \approx \frac{(1+z_{rec})\, c_s}{H z_{rec}} \simeq 145\,\mathrm{Mpc}$$

↳ typical correlation length for temperature fluctuations

↳ typical angular size



can be measured
but: depends on curvature / geometry of the universe (as encoded in k)

A measurement of Θ allows for deducing

$$\frac{k}{H_0^2 a_0^2} = \Omega_M + \Omega_\Lambda - 1$$

<u>Result:</u> $\boxed{\Omega_M + \Omega_\Lambda \simeq 1}$ (compatible with the SNIa measurements)

→ <u>Most people believe that DE exists!</u>

1.3. Criticism and open questions

- A rigorous analysis of the JLA catalogue (750 SNIa) only shows marginal evidence for cosmic acceleration (only 3σ) [4]
- A different physical effect might cause distant supernovae to appear dimmer (axion-photon conversion) [5]
- The cosmological principle is only an assumption and might be wrong. Thus the universe might be
  - anisotropic [6,7]
  - inhomogeneous [8,9]

→ All of these scenarios are not yet fully excluded! <u>Yet ΛCDM is highly favored!</u>

2. Alternative explainations for accelerated expansion

2.1. Lemaître – Tolman – Bondi models

[cf. 11]

Toy model for a universe with radial inhomogeneities

Most general metric (Generalization of FRW)

$$ds^2 = -dt^2 + X^2(r,t)\,dr^2 + A^2(r,t)\,d\tilde{\Omega}^2$$

Energy momentum tensor:

comoving frame

$$T^\mu{}_\nu = -\rho_M(r,t)\,\delta^\mu{}_0\,\delta^0{}_\nu - \underbrace{\rho_\Lambda\,\delta^\mu{}_\nu}_{\text{for generality, still allow for a cosmological constant}}$$

- Using the $\mu=0$, $\nu=1$ component of the Einstein equations ($G_{01}=0$), one obtains

$$\dot{A}' = A'\,\frac{\dot{X}}{X} \Rightarrow X = C(r)\,A'$$
$$\equiv \frac{1}{\sqrt{1-k(r)}}\,A'$$

Thus

$$\boxed{ds^2 = -dt^2 + \frac{(A'(r,t))^2}{1-k(r)}\,dr^2 + A^2(r,t)\,d\Omega}$$

(matches FRW metric for $A(r,t) = a(t)\,r$ and $k(r) = kr^2$

...ng this metric, the remaining evolution ...ations are

$$\frac{\dot{A}^2+k}{A^2} + \frac{2\dot{A}\dot{A}'+k'}{AA'} = 8\pi G(\rho_M+\rho_\Lambda) \quad (1)$$

$$\dot{A}^2 + 2A\ddot{A} + k = 8\pi G \rho_\Lambda A^2 \quad (2)$$

<u>Derivation of Friedmann-like equations:</u>

- From eq. (2), one finds (multiplied by $\dot{A}$)

$$\underbrace{\dot{A}^2\dot{A} + 2A\ddot{A}\dot{A}}_{=\frac{d}{dt}(\dot{A}^2A)} + k\dot{A} = 8\pi G \rho_\Lambda A^2\dot{A} \quad \Big| \int dt$$

$$\Leftrightarrow \quad \dot{A}^2A + k\dot{A} = 8\pi G \rho_\Lambda \tfrac{1}{3}A^3 + \underbrace{F(r)}_{\text{integration constant}}$$

$$\Leftrightarrow \quad \frac{\dot{A}^2}{A^2} = \frac{F(r)}{A^3} + \frac{8\pi G}{3}\rho_\Lambda - \frac{k(r)}{A^2} \quad (3)$$

determined by boundary conditions at $t=t_0$

- Differentiation of $(3)\cdot A^3$ with respect to r and using (1) yields:

$$F' = 8\pi G \rho_M A' A^2$$

$$\Rightarrow F = 8\pi G \int_0^r \rho_M A' A^2 dr' \equiv 8\pi G \bar{\rho}_M \int_0^r A' A^2 dr'$$

$$= \boxed{\frac{8\pi G}{3}\bar{\rho}_M A^3 = F}$$

Consequently, (3) takes the form

$$H^2 = \boxed{\left(\frac{\dot{A}}{A}\right)^2 = \frac{8\pi G}{3}\left(\bar{\rho}_M + \rho_\Lambda\right) - \frac{k}{A^2}}$$

Similar/analogue to first Friedmann equation (definition of Hubble parameter)

- Using (2) to express k, k' by $\dot{A}, \dot{A}', \dots$ etc and plugging it into (1) yields

$$\boxed{\frac{2}{3}\frac{\ddot{A}}{A} + \frac{1}{3}\frac{\ddot{A}'}{A'} = -\frac{4\pi G}{3}\left(\bar{\rho}_M - 2\rho_\Lambda\right)}$$

similar/analogue to second Friedmann equation (acceleration equation)

→ The total acceleration might also be caused by radial acceleration ($\ddot{A}' > 0$)
Thus the very notion of acceleration becomes ambigous!

Define (index 0 indicates present time $t = t_0$)

$$\Omega_M = \underbrace{\frac{8\pi G}{3H_0^2}}_{=1/\hat{\rho}} \bar{\rho}_M \frac{A^3}{A_0^3} = \frac{F}{H_0^2 A_0^3}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\hat{\rho}}, \quad \Omega_c = \frac{k}{H_0^2 A_0^2}$$

$$\Rightarrow \boxed{H^2 = H_0^2\left[\Omega_M\left(\frac{A_0}{A}\right)^3 + \Omega_\Lambda + \Omega_c\left(\frac{A_0}{A}\right)^2\right]} = \left(\frac{\dot{A}}{A}\right)^2$$

Integrating this equation finally yields

$$\boxed{t_0 - t = \frac{1}{H_0}\int_{\frac{A}{A_0}}^{1} \frac{dx}{\sqrt{\Omega_M x^{-1} + \Omega_\Lambda x^2 + \Omega_c}}}$$

which determines $A(r,t)$ and all its derivatives (depending on the initial conditions: $k(r)$, $\bar{\rho}_M(t_0, r)$)

The distance-redshift relation:

Geodesic equation (incoming light rays) ; $ds^2 = 0$, $d\Omega^2 = 0$

$$\Rightarrow 0 = -dt^2 + \frac{A'^2}{1-k} dr^2$$

$$\frac{dt}{du} = -\frac{dr}{du}\frac{A'}{\sqrt{1-k}}$$

Consider two incoming light rays with $t_1 = t(u)$ and $t_2 = t(u) + \lambda(u)$

$$\frac{dt_1}{du} = -\frac{dr}{du}\frac{A'}{\sqrt{1-k}}$$

$$\frac{dt_2}{du} = \frac{dt}{du} + \frac{d\lambda}{du} = -\frac{dr}{du}\frac{A'}{\sqrt{1-k}} + \frac{d\lambda}{du}$$

$$= -\frac{dr}{du}\frac{A'(r, t+\lambda)}{\sqrt{1-k}} \simeq -\frac{dr}{du}\frac{A' + \dot{A}'\lambda}{\sqrt{1-k}}$$

$$\Rightarrow \frac{d\lambda}{du} \simeq -\frac{dr}{du}\frac{\dot{A}'\lambda}{\sqrt{1-k}}$$

Redshift $\quad z = \frac{\lambda_0 - \lambda}{\lambda}$

$$\Rightarrow \frac{dz}{du} = \frac{dr}{du} \frac{(1+z)\dot{A}'}{\sqrt{1-k}}$$

Thus one finds:

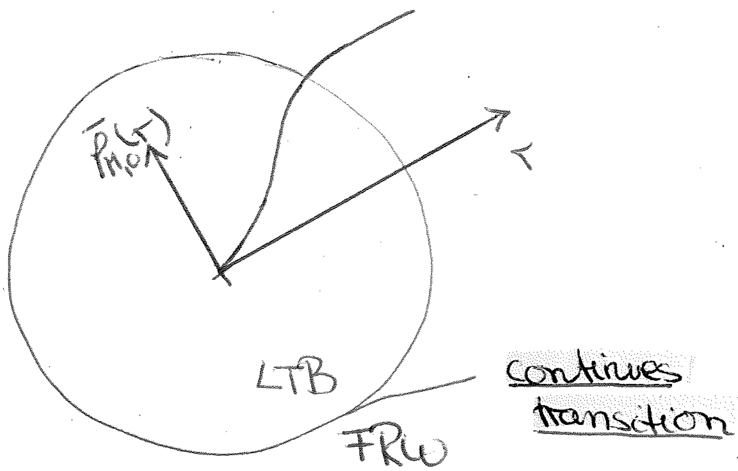
$$\boxed{\frac{dt}{dz} = -\frac{A'}{(1+z)\dot{A}'} \quad ; \quad \frac{dr}{dz} = \frac{\sqrt{1+H_0^2(1-\Omega_M-\Omega_\Lambda)A_0^2}}{(1+z)\dot{A}'}}$$

Luminosity distance

$$d_L(z) \overset{[10]}{=} (1+z)^2 A(r(z), t(z))$$

Concrete models & implications

- H. Alnes et. al. (2006) [12]
  The earth is located near the centre of an underdense bubble



$$\bar{\rho}_{M,0}(r) \sim 1 - \Delta\rho\left(\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{r-r_0}{2\Delta r}\right)\right) \quad ; \quad \rho_\Lambda = 0$$
$$k(r) \sim 1 - \Delta k \text{ ——————— }$$

Best fit: $\chi^2/_{ndof} \simeq 1.1$

$\Delta\rho = -\Delta k \simeq 0.9$ $\Delta r / r_0 \simeq 0.4$

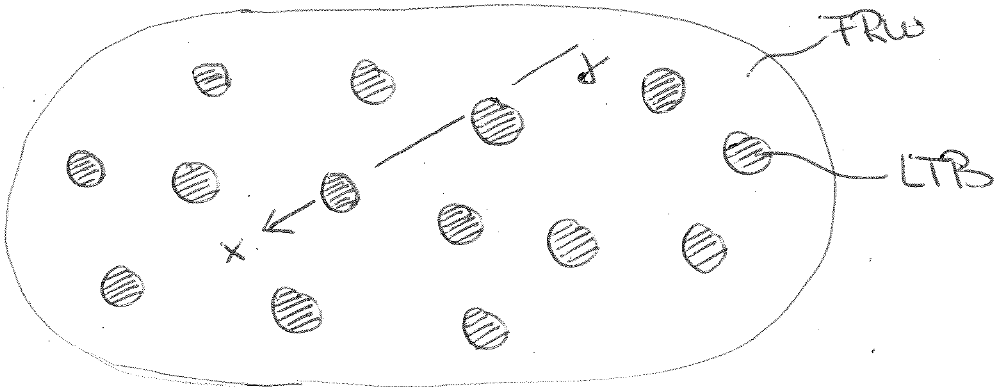
$r_0 = 1.35\,\mathrm{Gpc}$

→ Compatible with supernova data

But: Highly finetuned!

⑥ V. Marra et. al. (Swiss - Cheese universe, 2007) [13]

The universe is a „FRW cheese with LTB holes"



Result: The holes cause the photon path to deviate from the FRW case

→ Possibility to partly mimic the effects of dark energy

But: Only toy model to study the effects of inhomogeneities

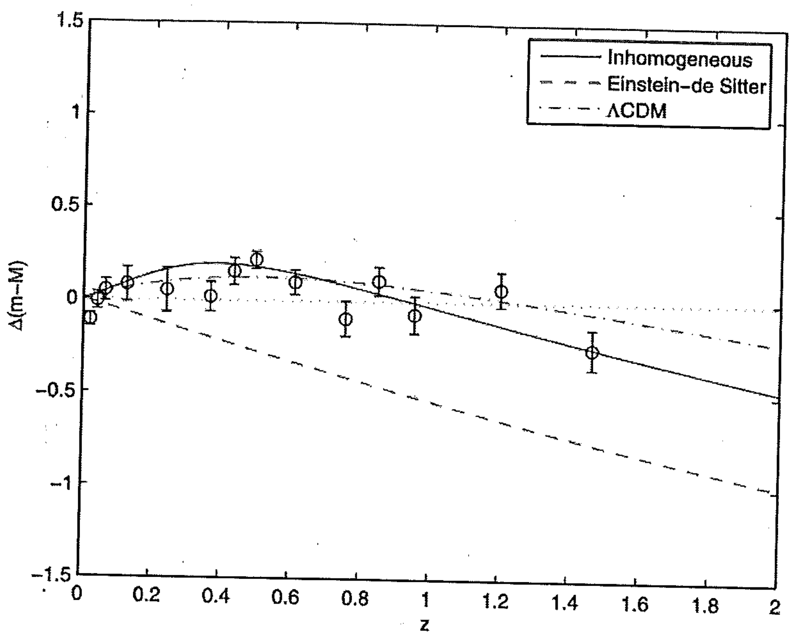


FIG. 3 (color online). Distance modulus vs redshift for our standard model together with supernova observations.

2.2. Axion-photon conversion (Backup) [cf. 5]

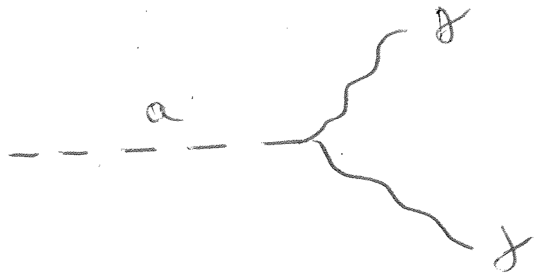
Idea: Photons propagating in the extragalactic magnetic field might oscillate into axions

$\longrightarrow$ Distant supernovae appear dimmer than they actually are

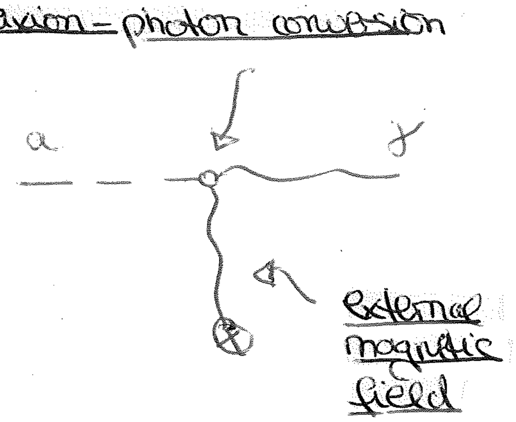
Model:
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial^\mu a\,\partial_\mu a - \frac{1}{2}m_a a^2$$
$$-\frac{1}{4}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Rightarrow \mathcal{L}_{int} = -\frac{1}{4}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu} = +\frac{a}{f}\vec{E}\cdot\vec{B}$$

Interaction vertex:



or



Incorporate external magnetic field

$$F_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow \underbrace{F^{x}_{\mu\nu}\tilde{F}^{\mu\nu}}_{\underbrace{-4\vec{E}_x\cdot\vec{B}}_{=0}} + \underbrace{F_{\mu\nu}\tilde{F}^{\mu\nu}_{x}}_{-4\vec{E}\cdot\vec{B}_x}$$

$$\Rightarrow L_{int} = \frac{a}{f}\,\vec{E}\cdot\vec{B}_x \underset{\text{gauge}}{\overset{\text{Coulomb}}{=}} -\frac{a}{f}\,(\partial_t\vec{A})\cdot\vec{B}$$

Conversion probability

travel distance

$$P_{\gamma\to a}(r) = \frac{1}{3}\left[1-\exp\left(-\frac{3P_0 r}{s}\right)\right] \xrightarrow{r\to\infty} \frac{1}{3}$$

domain size

$$\Rightarrow F \longrightarrow (1-P_{\gamma\to a})F$$

$$\Rightarrow d_L \longrightarrow d_L / \underbrace{(1-P_{\gamma\to a})^2}$$

Distant supernovae appear dimmer

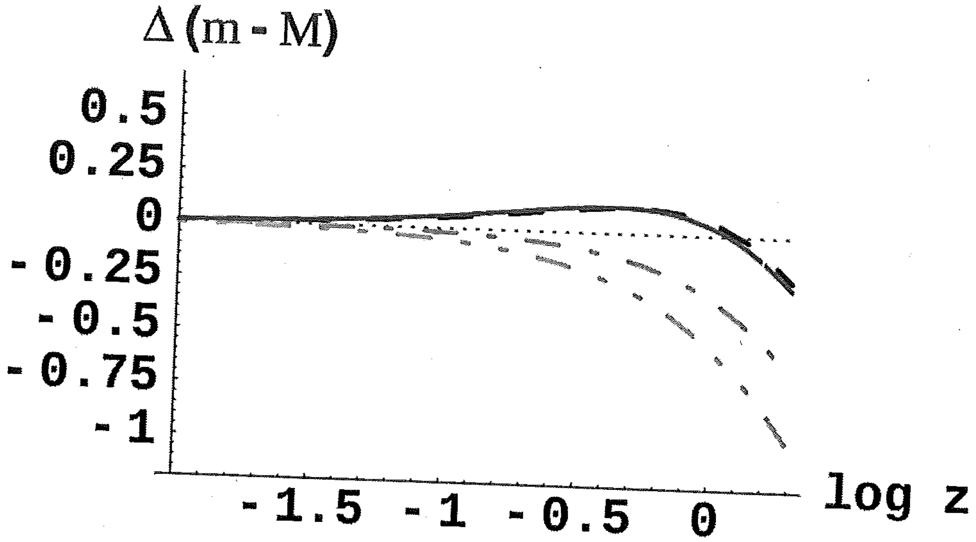


Figure 1: The luminosity-distance vs. redshift curve for several models, relative to the curve with $\Omega_{tot} = 0$ (dotted horizontal line). The dashed curve is a best fit to the supernova data assuming the Universe is accelerating ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$); the solid line is the oscillation model with $\Omega_m = 0.3$, $\Omega_S = 0.7$, $M = 4 \cdot 10^{11}$ GeV, $m = 10^{-16}$ eV; the dot-dashed line is $\Omega_m = 0.3$, $\Omega_S = 0.7$ with no oscillations, and the dot-dot-dashed line is for $\Omega_m = 1$ again with no oscillations.