Combination and QCD analysis of beauty and charm production cross section measurements in deep inelastic ep scattering at HERA: **paper presentation**

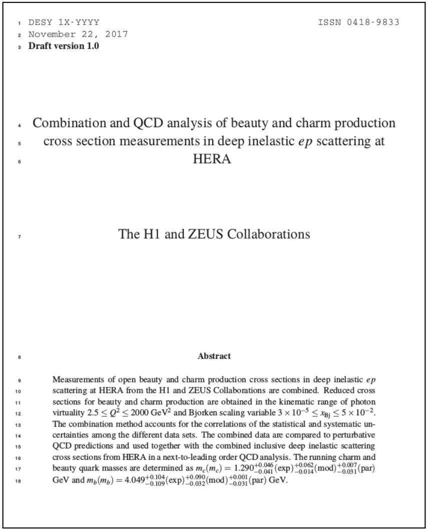
Karin Daum, Oleksandr Zenaiev,
Achim Geiser (for the draft)
(DESY)

Analysis webpage:
`https://www.desy.de/h1zeus/heavy-flavour/cbcomb2017prel`

H1ZEUS plenary, 23.11.2017

Analysis status

- Analysis is done
- Paper draft v0.75 (==v0.5) was circulated among the analysis team, EB and H1ZEUS management on 16 Nov
- Circulation of v1 in H1ZEUS: 23 Nov – 8 Dec
- 1st EB: 13 Dec 2017
- aim to submit paper by QCD Moriond (17 Mar 2018)



DESY 1X-YYYY ISSN 0418-9833
November 22, 2017
Draft version 1.0

Combination and QCD analysis of beauty and charm production cross section measurements in deep inelastic *ep* scattering at HERA

The H1 and ZEUS Collaborations

Abstract

Measurements of open beauty and charm production cross sections in deep inelastic *ep* scattering at HERA from the H1 and ZEUS Collaborations are combined. Reduced cross sections for beauty and charm production are obtained in the kinematic range of photon virtuality $2.5 \leq Q^2 \leq 2000$ GeV^2 and Bjorken scaling variable $3 \times 10^{-5} \leq x_{\mathrm{Bj}} \leq 5 \times 10^{-2}$. The combination method accounts for the correlations of the statistical and systematic uncertainties among the different data sets. The combined data are compared to perturbative QCD predictions and used together with the combined inclusive deep inelastic scattering cross sections from HERA in a next-to-leading order QCD analysis. The running charm and beauty quark masses are determined as $m_c(m_c) = 1.290^{+0.046}_{-0.041}(\mathrm{exp})^{+0.062}_{-0.014}(\mathrm{mod})^{+0.007}_{-0.031}(\mathrm{par})$ GeV and $m_b(m_b) = 4.049^{+0.104}_{-0.109}(\mathrm{exp})^{+0.090}_{-0.032}(\mathrm{mod})^{+0.001}_{-0.031}(\mathrm{par})$ GeV.

Introduction

Why combining data?

- Combined data expected to have ultimate precision
- Easy to use: one dataset, especially if available in open-source package
- Consistency is tested in the combination procedure

Why combining charm+beauty simultaneously?

- We have combined large ZEUS+H1 charm subset [DESY-12-172]: well established procedure, reference/starting point
- DESY-12-172: ≈ 170 citations, extensively used by most modern PDF groups, referred to as "unique data for tests of GMVFNS", as an arbiter of the quality of the description
- A number of beauty measurements by H1 and ZEUS are less "popular": less easy to be used in a quantitative analysis
- Most of beauty measurements done together with charm: accounting for improvement for charm cross sections will automatically improve beauty, consistent combination of c+b will deliver ultimately precise results which can be consistently used together

Combination procedure (same as for preliminary)

- Take measured visible x-section $\sigma_{\rm vis}$ and extrapolate to full phase space $\sigma_{\rm red}$ using consistent NLO setup: $\sigma_{\rm red} = \sigma_{\rm vis}\frac{\sigma_{\rm red}^{\rm NLO}}{\sigma_{\rm vis}^{\rm NLO}}$ [HVQDIS]
- Combine $\sigma_{\rm red}$ accounting for bin-to-bin correlations [HERAverager]

NLO setup for extrapolation as in [DESY-12-172]

- pole masses $m_c = 1.5 \pm 0.15$ GeV, $m_b = 4.5 \pm 0.25$ GeV

  consistent with extracted from data: $m_c = 1.43 \pm 0.04$ GeV, $m_b = 4.35 \pm 0.11$ GeV

  and consistent with PDG: $m_c = 1.67 \pm 0.07$ GeV, $m_b = 4.78 \pm 0.06$ GeV
- $\mu_R = \mu_F = \sqrt{Q^2 + 4m_Q^2}$, varied simultaneously by factor 2
- $\alpha_s^{n_f=3}(M_Z) = 0.105 \pm 0.002$ $(\alpha_s^{n_f=5}(M_Z) = 0.116 \pm 0.002)$
- HERAPDF1.0 FFNS, $n_f = 3$, assign 2% uncor. unc.

  (checked vs HERAPDF2.0: see backup)
- c fragmentation: Kartvelishvili frag. function parametrised as step function with k_T kink (H1, ZEUS meas. [DESY-08-080, DESY-08-209])
- b fragmentation: Peterson $\epsilon_b = 0.0035 \pm 0.0020$ [NP B565 (2000) 245]
- charm fragmentation fractions [EPJ C76 (2016) 397]
- branching ratios PDG2016
- hadronisation effects for data with jets in the final state via corrections from orig. papers

Combination procedure and input measurements

$$\chi^2_{\text{exp}}(\boldsymbol{m},\boldsymbol{b}) = \sum_e \left[\sum_i \frac{\left(m^i - \sum_j \gamma_j^{i,e} m^i b_j - \mu^{i,e}\right)^2}{(\delta_{i,e,stat}\,\mu^{i,e})^2 + (\delta_{i,e,uncorr}\, m^i)^2} \right] + \sum_j b_j^2$$

Data set	Tagging	Q^2 range [GeV^2]	N_c	$\mathcal{L}$ [pb^{-1}]	$\sqrt{s}$ [GeV]	N_b	Systematics	Extrapolation
1 **H1 VTX DESY-09-096 + b**	**VTX**	5 – 2000	29	245	318	12	**DESY-12-172**	**DESY-12-172**[1]
2 **H1 D^* H-I DESY-06-240**	D^{*+}	2 – 100	17	47	318		**DESY-12-172**	**DESY-12-172**
3 **H1 D^* H-II (med. Q^2) DESY-11-066**	D^{*+}	5 – 100	25	348	318		**DESY-12-172**	**DESY-12-172**
4 **H1 D^* H-II (high Q^2) DESY-09-165**	D^{*+}	100 – 1000	6	351	318		**DESY-12-172**	**DESY-12-172**
5 **ZEUS D^* 96-97 DESY-99-101**	D^{*+}	1 – 200	21	37	300		**DESY-12-172**	**DESY-12-172**
6 **ZEUS D^* 98-00 DESY-03-115**	D^{*+}	1.5 – 1000	31	82	318		**DESY-12-172**	**DESY-12-172**
7 **ZEUS D^0 2005 DESY-08-201**	D^0	5 – 1000	9	134	318		**DESY-12-172**	**DESY-12-172**
8 **ZEUS μ 2005 DESY-09-056 + b**	μ	20 – 10000	8	126	318	8	**DESY-12-172**	**DESY-12-172**
9 **ZEUS D^+ H-II DESY-13-028**	D^+	5 – 1000	14	354	318		**new**	**new**
10 **ZEUS D^{*+} HERA-II DESY-13-054**	D^*	5 – 1000	31	363	318		**new**	**new**
11 **ZEUS VTX H-II DESY-14-083 + b**	**VTX**	5 – 1000	18	354	318	17	**in the paper**	**in the paper**
12 **ZEUS e H-II DESY-11-005 (b only)**	e	10 – 1000		363	318	9	**new**	**new**
13 **ZEUS μ + jet H-I DESY-10-047 (b only)**	μ	2 – 3000		114	318	11	**new**	**new**

- Same method as for previous H1ZEUS data combinations [HERAverager]
- Q^2–x grid for charm as in DESY-12-172; for beauty only subset of points
- Correlations taken into account: systematics, statistical c–b correlations and extrapolation (details available in backup)

Results: combined data

- **NEW combination: χ^2/ndf$= 149/187$ $[+90/93]$**
- H1ZEUS charm combination: χ^2/ndf$= 59/94$ (without D^+ 2005)
- Adding only new charm: χ^2/ndf$= 120/157$ $[+61/63]$
- Adding only beauty: χ^2/ndf$= 84/124$ $[+25/30]$

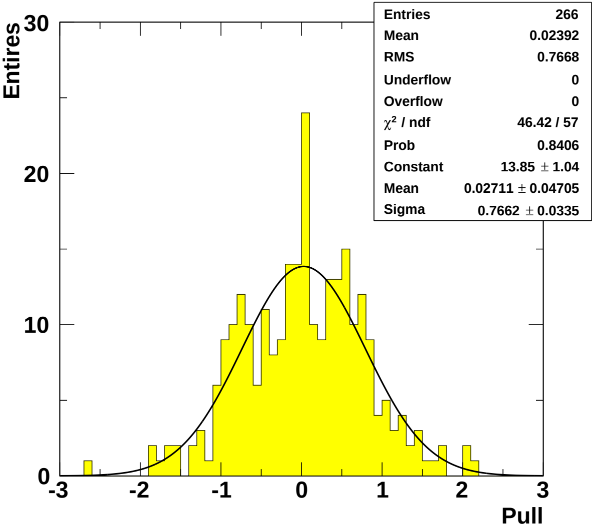
Good consistency!

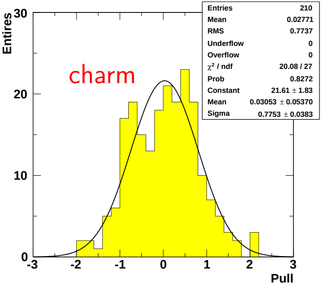
"Partial" χ^2 of newly included data sets:

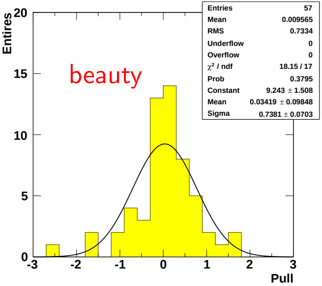
ZEUS D^+ c	+12/14	H1 vtx b	$+8/10$
ZEUS D^* c	+29/31	ZEUS mu b	$+2.9/6$
ZEUS VTX c	+22/18	ZEUS el b	$+3.3/7$
ZEUS VTX b	+14/14	ZEUS mu b HERA-I	$+18/10$

Correlated uncertainties: all shifts within 1.4σ

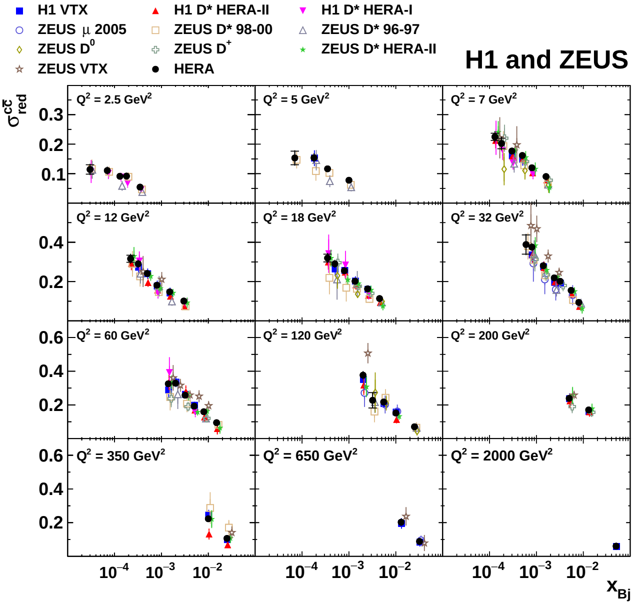
Combined data: pull



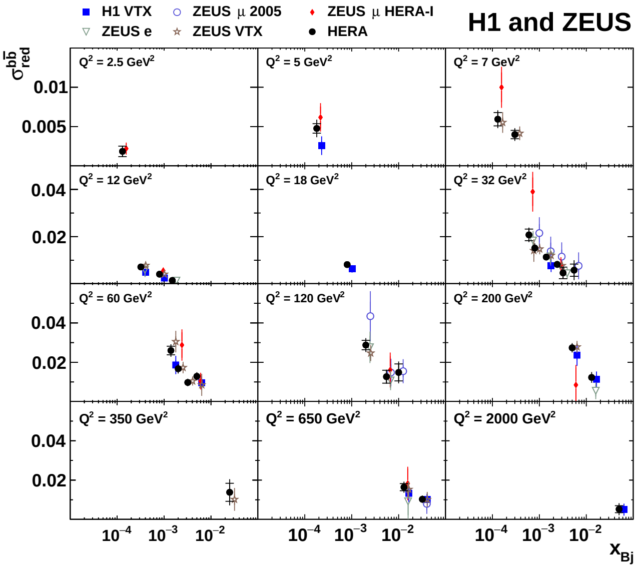




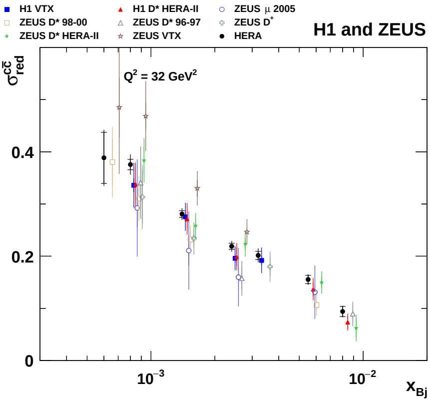
Combined data: charm

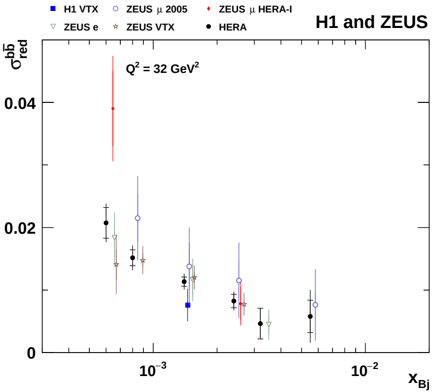


Combined data: beauty

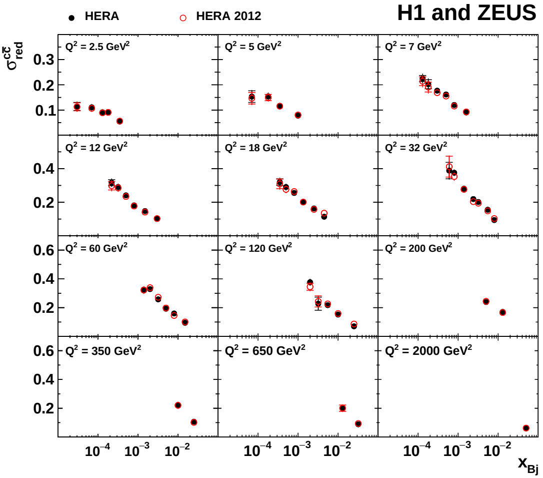


Combined data: charm, beauty $Q^2 = 32$ GeV^2





Combined data: charm compared to HERA2012



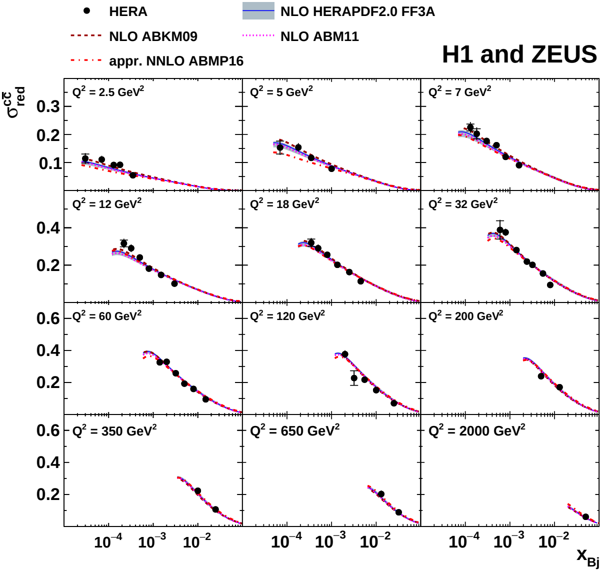
- Consistent with HERA2012 [DESY-12-172], better overall precision
- Marginal improvement at low and very high Q^2, best at medium Q^2
- Very few points: expanded uncertainties due to removed ZEUS D+ 2005 data set
- Some other points: significant uncor. unc. → corr. (correlation with beauty)

Comparison to theoretical predictions

Outline:

- **main focus on FFNS and HERAPDF**
- other FFNS PDFs: ABM, including approximate NNLO
- VFNS:
  - RT OPT: default HF scheme used in HERAPDF
  - FONLL C NNPDF3.1 with and w/out small x resummation: recent calculations aiming to improve description at low x_{Bj} and low Q^2
- summary table with all χ^2 and p-values

Combined data vs FFNS theory: charm

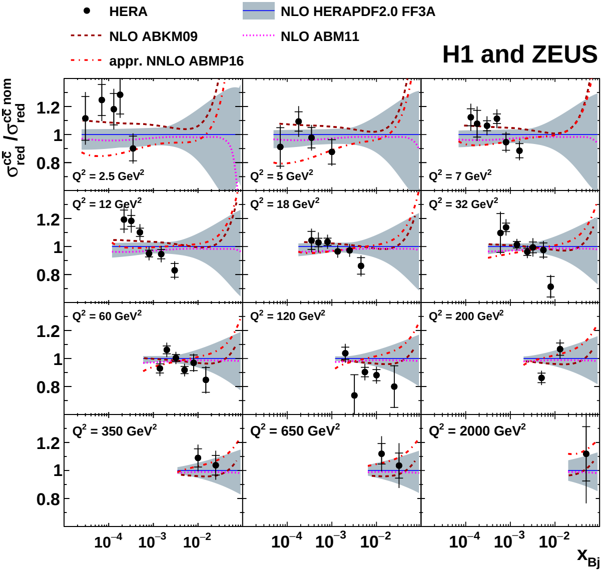


Theory settings:

- NLO FFNS, running HF masses:
  $m_c(m_c) = 1.27 \pm 0.03$ GeV
  $m_b(m_b) = 4.18 \pm 0.03$ GeV
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ (simultaneously varied by factor 2)
- for HERAPDF2.0 FF3A, EIG only in unc. band
- ABM09 provided by authors for HERA 2012 charm paper

Reasonable overall description, but tension in x slope at $7 < Q^2 < 32$ GeV^2 where data are most precise

Combined data vs FFNS theory: charm



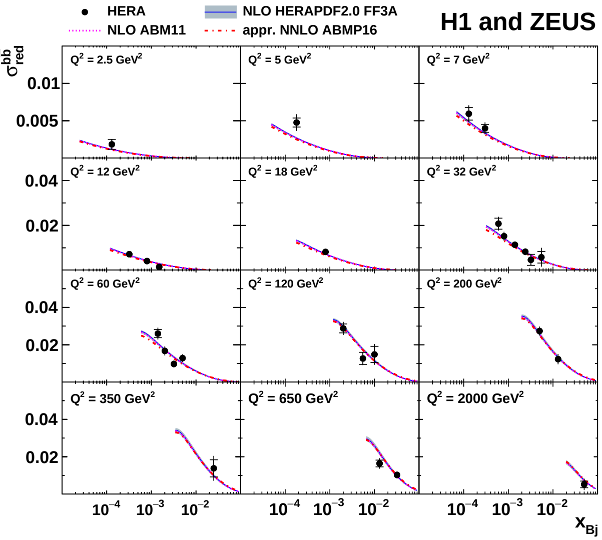
Theory settings:

- NLO FFNS, running HF masses:
  $m_c(m_c) = 1.27 \pm 0.03$ GeV
  $m_b(m_b) = 4.18 \pm 0.03$ GeV
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ (simultaneously varied by factor 2)
- for HERAPDF2.0 FF3A, EIG only in unc. band

Reasonable overall description, but tension in x slope at $7 < Q^2 < 32$ GeV^2 where data are most precise:

- **becomes worser with appr. NNLO and latest PDFs**

Combined data vs FFNS theory: beauty

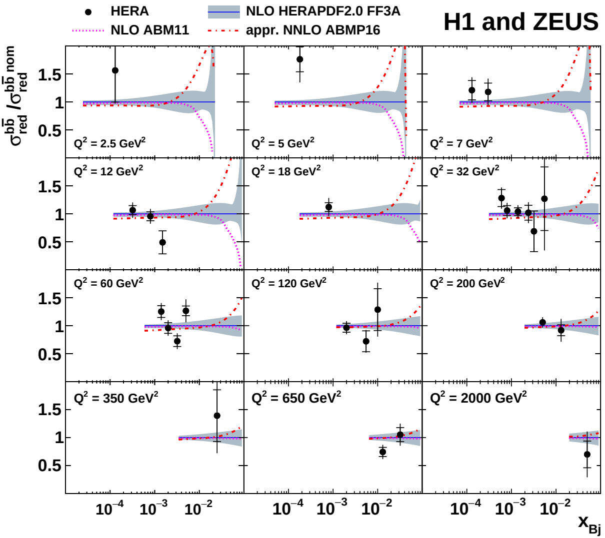


Theory settings:

- NLO FFNS, running HF masses:
  $m_c(m_c) = 1.27 \pm 0.03$ GeV
  $m_b(m_b) = 4.18 \pm 0.03$ GeV
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ (simultaneously varied by factor 2)
- for HERAPDF2.0 FF3A, EIG only in unc. band

Good overall description

Combined data vs FFNS theory: beauty

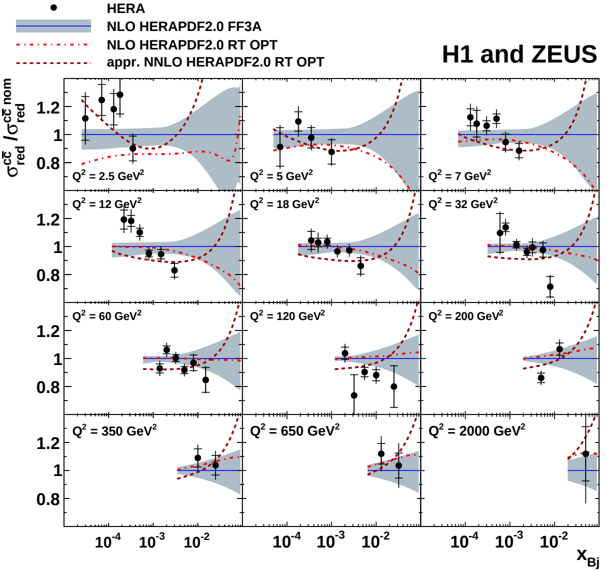


Theory settings:

- NLO FFNS, running HF masses:
  $m_c(m_c) = 1.27 \pm 0.03$ GeV
  $m_b(m_b) = 4.18 \pm 0.03$ GeV
- $\mu_f = \mu_r = \sqrt{Q^2 + 4m_Q^2}$ (simultaneously varied by factor 2)
- for HERAPDF2.0 FF3A, EIG only in unc. band

Good overall description

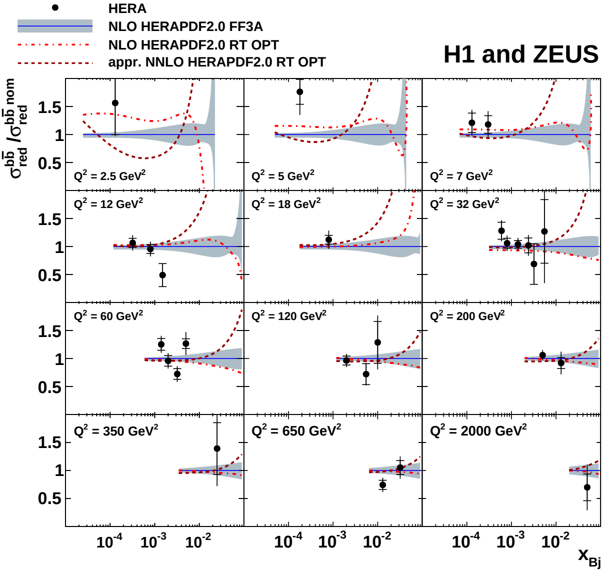
Combined data vs VFNS theory: RT OPT, charm



VFNS:

- HERAPDF2.0 RT OPT (NLO and appr. NNLO): similar to FF3A, except lowest Q^2
- NNPDF predictions provided by authors with recommendation not to use 1st Q^2 bin (avail. in backup, also with PDF unc.)
- NNPDF FONLL C (NNLO for massless, NLO for massive terms): worser data description
- NNPDF FONLL C with small x resummation: does not improve description in whole phase space
- however, both NNPDF pred. provide improved description of x slope

Combined data vs VFNS theory: RT OPT, beauty

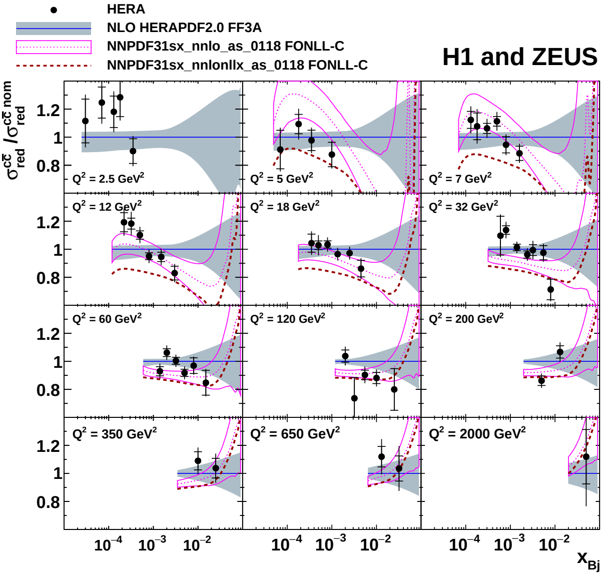


HERAPDF2.0 RT OPT:

- HERAPDF2.0 RT OPT: similar to FF3A, except lowest Q^2
- small differences between NLO and appr. NNLO, except lowest Q^2

Good overall descriptions

Combined data vs VFNS theory: FONLL C, charm



NNPDF3.1 FONLL C:

- NNPDF predictions provided by authors with recommendation not to use 1st Q^2 bin
- better description of x_{Bj} shape
- NNPDF FONLL C (NNLO for massless, NLO for massive terms): worser data description
- NNPDF FONLL C with small x resummation: does not improve description in whole phase space

Combined data vs theory: χ^2

Dataset	PDF	χ^2 [p-value]	χ^2 with PDF unc.
HERA 2012 c	HERAPDF20_NLO_FF3A	59 [0.23]	59
	ABKM09	59 [0.23]	–
	abm11_3n_nlo	62 [0.16]	62
	ABMP16_3_nnlo	70 [0.05]	69
	HERAPDF20_NLO_EIG (RT OPT)	71 [0.04]	70
($N_{\mathrm{dat}} = 52$)	HERAPDF20_NNLO_EIG (RT OPT)	66 [0.09]	65
	NNPDF31sx_nnlo_as_0118 (FONLL-C)	106 [$1.5 \cdot 10^{-6}$]	–
($N_{\mathrm{dat}} = 47$)	NNPDF31sx_nnlonllx_as_0118 (FONLL-C)	71 [0.013]	–
New combined c	HERAPDF20_NLO_FF3A	86 [0.002]	85
	ABKM09	82 [0.005]	–
	abm11_3n_nlo	92 [0.0005]	91
	ABMP16_3_nnlo	109 [$6 \cdot 10^{-6}$]	106
	HERAPDF20_NLO_EIG (RT OPT)	99 [$9 \cdot 10^{-5}$]	98
($N_{\mathrm{dat}} = 52$)	HERAPDF20_NNLO_EIG (RT OPT)	102 [$4 \cdot 10^{-5}$]	99
	NNPDF31sx_nnlo_as_0118 (FONLL-C)	140 [$1.5 \cdot 10^{-11}$]	–
($N_{\mathrm{dat}} = 47$)	NNPDF31sx_nnlonllx_as_0118 (FONLL-C)	114 [$5 \cdot 10^{-7}$]	–
New combined b ($N_{\mathrm{dat}} = 27$)	HERAPDF20_NLO_FF3A	33 [0.20]	33
	abm11_3n_nlo	34 [0.17]	34
	ABMP16_3_nnlo	41 [0.04]	41
	HERAPDF20_NLO_EIG (RT OPT)	33 [0.20]	33
	HERAPDF20_NNLO_EIG (RT OPT)	45 [0.016]	45

QCD analysis settings

Objective: extract HQ masses (not providing PDFs in this paper)

Setup follows HERAPDF2.0 FF, using running HQ mass definition:

- xFitter-1.2.0
- Input data:
  - HERA $e^{\pm}p$ inclusive data, $Q^2_{\mathrm{min}} > 3.5$ GeV^2 [1506.06042]
  - new HERA c and b combined
- FFNS $n_f = 3$ ('FF ABM RUNM'), $O(\alpha_s(F_L)) = O(\alpha_s(F_2))$
- $\alpha_s^{n_f=3}(M_Z) = 0.106$
- free $m_c(m_c)$, $m_b(m_b)$
- DGLAP NLO [QCDNUM]
- PDF parametrisation: 14p HERAPDF at $\mu^2_{f0} = 1.9$ GeV^2, $f_s = 0.4$:

$$\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}\\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}(1+E_{u_v}x^2)\\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}\\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}(1+D_{\bar{U}}x)\\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}
\end{aligned}$$

Additional constrains:

$A_{\bar{U}} = A_{\bar{D}}(1-f_s)$, $B_{\bar{U}} = B_{\bar{D}}$, $C'_g = 25$

$\int_0^1 [\sum_i (q_i(x) + \bar{q}_i(x)) + g(x)] x dx = 1$

$\int_0^1 [u(x) - \bar{u}(x)] dx = 2,$

$\int_0^1 [d(x) - \bar{d}(x)] dx = 1$

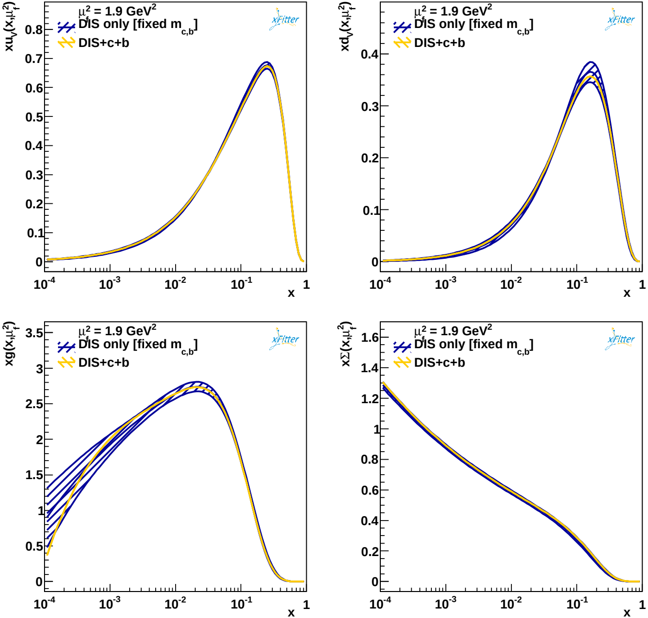
- fit ($\Delta\chi^2 = 1$), model (scales, α_s, f_s, Q^2_{min}) and par. (μ_{f0}, $E_{u_v} = 0$) unc.

Fit results

Dataset (uncor. χ^2 only)	DIS free m	DIS PDG m	DIS+c+b free m	DIS+c+b PDG m
HERA1+2 CCep	49 / 39	55 / 39	54 / 39	55 / 39
HERA1+2 CCem	50 / 42	50 / 42	50 / 42	50 / 42
HERA1+2 NCem	225 / 159	221 / 159	221 / 159	221 / 159
HERA1+2 NCep 820	65 / 70	66 / 70	65 / 70	65 / 70
HERA1+2 NCep 920	415 / 377	420 / 377	418 / 377	419 / 377
HERA1+2 NCep 460	223 / 204	221 / 204	221 / 204	220 / 204
HERA1+2 NCep 575	219 / 254	217 / 254	217 / 254	217 / 254
HERA c	-	-	50 / 52	49 / 52
HERA b	-	-	17 / 27	19 / 27
Correlated χ^2	71	75	125	128
Log penalty χ^2	-12	-6	-4	-6
Total χ^2 / dof	1304 / 1129	1319 / 1131	1435 / 1208	1437 / 1210

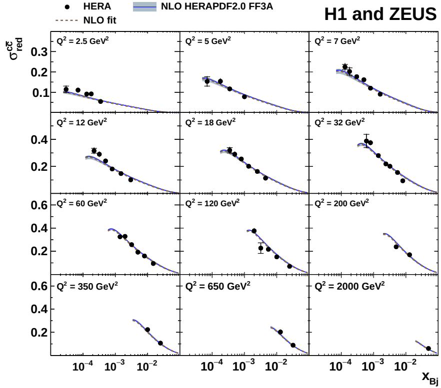
- DIS PDG masses ≈ HERAPDF2.0 FF3A
- DIS+c+b free masses: nominal fit in this paper

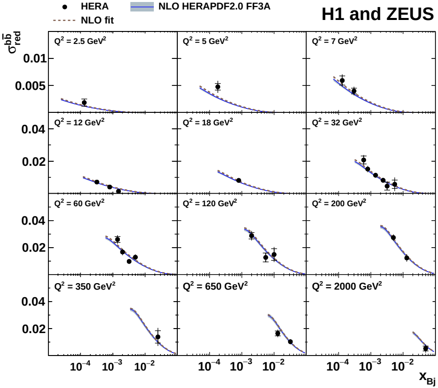
Fitted PDFs



μ_f^2 = 1.9 GeV^2
DIS only [fixed $m_{c,b}$]
DIS+c+b
xFitter
$xu_v(x,\mu_f^2)$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$xd_v(x,\mu_f^2)$
0
0.1
0.2
0.3
0.4
$xg(x,\mu_f^2)$
0
0.5
1
1.5
2
2.5
3
3.5
$x\Sigma(x,\mu_f^2)$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

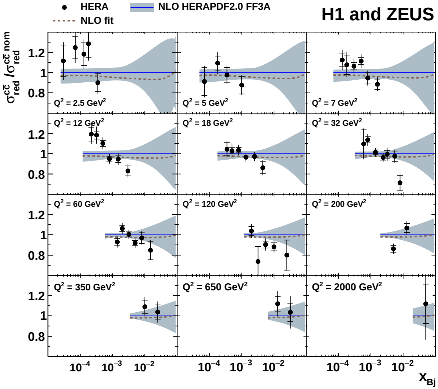
Data to theory comparison in the fit

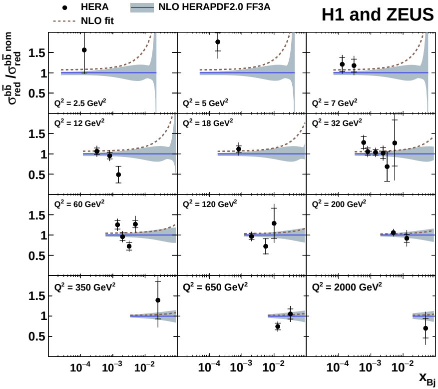




- Observed almost no impact of HF data on PDFs, **except constraining HQ masses**
- Theory predictions from the fit are consistent with HERAPDF2.0

Data to theory comparison in the fit





- Observed almost no impact of HF data on PDFs, **except constraining HQ masses**
- Theory predictions from the fit are consistent with HERAPDF2.0 (constant offsets are explained by changes in the fitted mass values which are $\approx \pm 1\sigma$ of PDG values)

Fit results

Variation	DIS			DIS + c + b		
	χ^2/dof	$m_c(m_c)$ [MeV]	$m_b(m_b)$ [MeV]	χ^2/dof	$m_c(m_c)$ [MeV]	$m_b(m_b)$ [MeV]
Nominal	$1304/1129$	1798^{+144}_{-134}	8450^{+2282}_{-1812}	$1435/1208$	1290^{+46}_{-41}	4049^{+104}_{-109}
$\mu \times 2$	1307	1752 $[-46]$	7794 $[-656]$	1430	1320 $[+30]$	4017 $[-32]$
$\mu \times 0.5$	1305	1804 $[+6]$	8747 $[+297]$	1435	1350 $[+60]$	4139 $[+90]$
$\alpha_s = 0.1075$	1304	1804 $[+6]$	8720 $[+270]$	1437	1276 $[-14]$	4051 $[+2]$
$\alpha_s = 0.1045$	1305	1780 $[-28]$	8041 $[-409]$	1434	1301 $[+11]$	4044 $[-5]$
$f_s = 0.50$	1304	1797 $[-1]$	8375 $[-75]$	1434	1287 $[-3]$	4048 $[-1]$
$f_s = 0.30$	1305	1798 $[+0]$	8474 $[+24]$	1436	1294 $[+4]$	4050 $[+1]$
$Q^2_{\min} = 5.0$ GeV^2	1237/1087	1932 $[+134]$	8300 $[-150]$	1372/1166	1289 $[-1]$	4044 $[-5]$
$Q^2_{\min} = 2.5$ GeV^2	1346/1152	1798 $[-0]$	8509 $[+49]$	1476/1231	1297 $[+7]$	4056 $[+7]$
$\mu^2_{f0} = 2.2$ GeV^2	1305	1798 $[+0]$	8371 $[-79]$	1436	1293 $[+3]$	4048 $[-1]$
$\mu^2_{f0} = 1.6$ GeV^2	1303	1798 $[-0]$	8472 $[+22]$	1434	1289 $[-1]$	4050 $[+1]$
$E_{u_v} = 0$	1319	1450 $[-348]$	3995 $[-4455]$	1440	1259 $[-31]$	4018 $[-31]$

Quote these results as:

DIS (not sensible for HQ mass extraction):

$m_c(m_c) = 1798^{+144}_{-134}(\text{fit})$ MeV

$m_b(m_b) = 8450^{+2282}_{-1812}(\text{fit})$ MeV

‘$E_{u_v} = 0$’: $m_c(m_c) = 1450$ MeV, $m_c(m_c) = 3995$ MeV (dominant par. unc.)

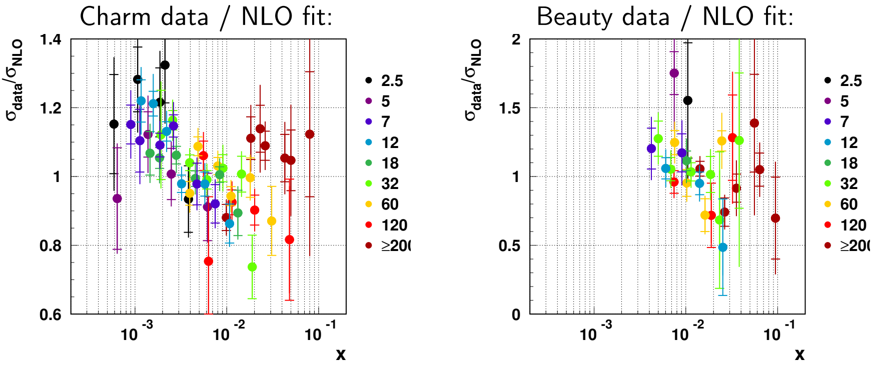
DIS + c + b:

$$\boldsymbol{m_c(m_c) = 1290^{+46}_{-41}(\text{fit})^{+62}_{-14}(\text{mod})^{+7}_{-31}(\text{par})}$$

$$\boldsymbol{m_b(m_b) = 4049^{+104}_{-109}(\text{fit})^{+90}_{-32}(\text{mod})^{+1}_{-31}(\text{par})}$$

Model unc. are dominated by scale var., par. unc. are dominated by ‘$E_{u_v} = 0$’.

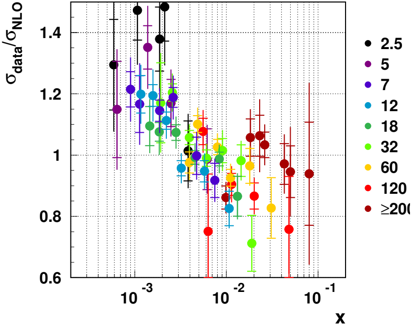
Further studies of data and theory agreement



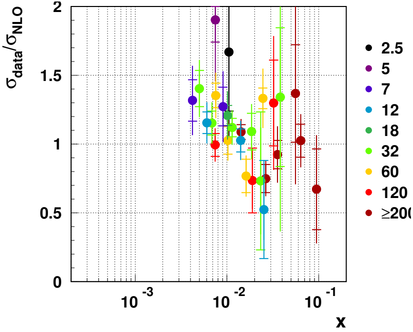
- x is average partonic momentum fraction calculated at NLO
- Charm: apparent x trend in wide Q^2 range fir $0.0005 \lesssim x \lesssim 0.01$
- Beauty: similar trend in the same x range within larger uncertainties
- These are paper plots

Further studies of data and theory agreement

Charm data / ABMP16 ap. NNLO:

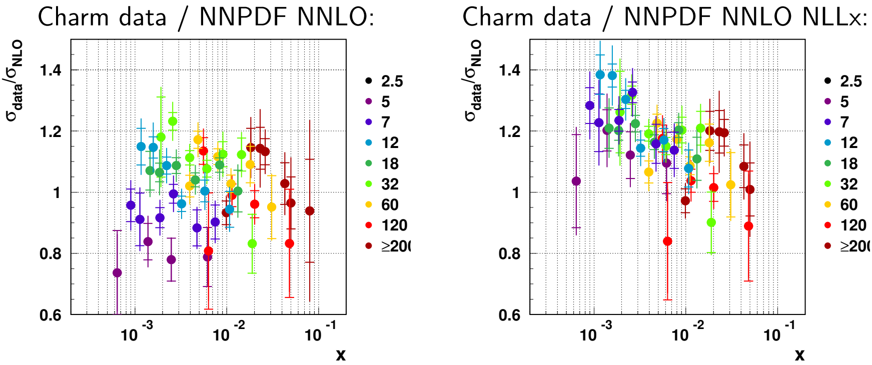


Beauty data / ABMP16 ap. NNLO:



- Worser description of x at appr. NNLO
- These are **not** paper plots

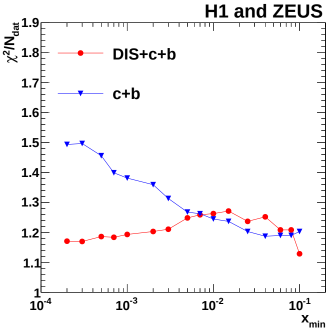
Further studies of data and theory agreement



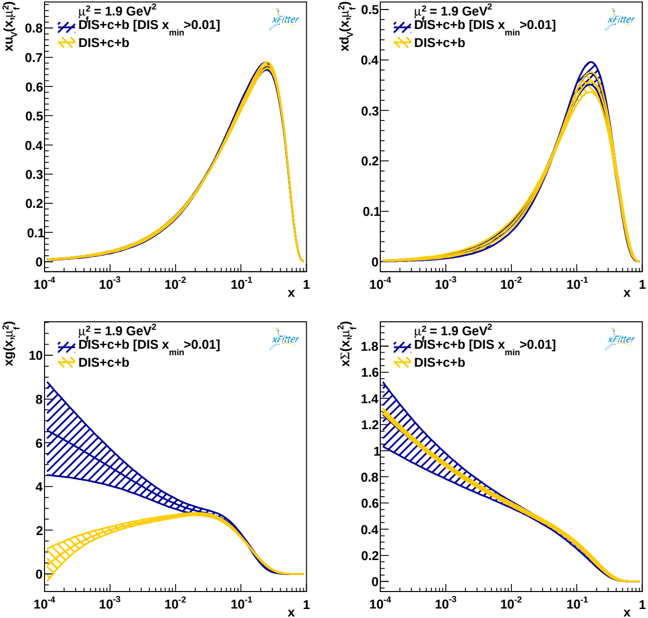
- Better description of x w/out small x resummation, but worser description of Q^2 shape
- With small x resummation: data above theory
- These are **not** paper plots

Further studies of data and theory agreement

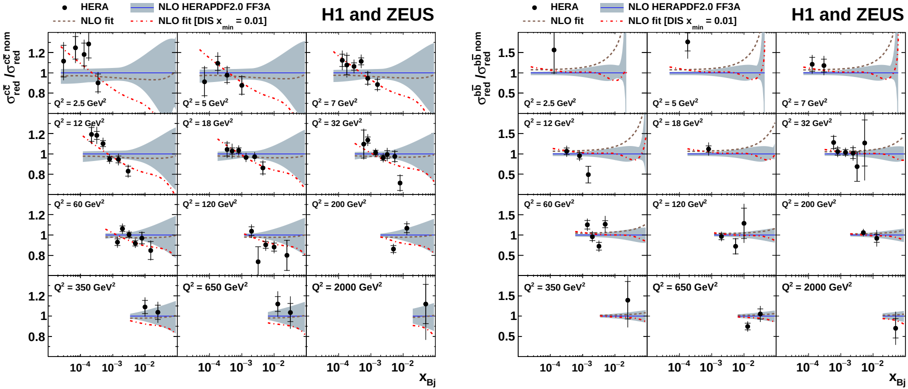
- Apparent difference in x slope suggests that the HF data prefer a steeper low x gluon
- Gluon distribution is **not directly** constrained by inclusive data: relies on many assumptions (parametrisation, sum rules etc.)
- HF data are **directly** sensitive to the gluon distribution, however quantitatively they can not compete with inclusive data and the latter dominate in the fit
- *to increase the weight of HF data*, we apply $x > x_{\min}$ cut on inclusive data in the PDF fit and monitor changes by looking at χ^2 of HF data



PDFs with $x_{\mathrm{min}} = 0.01$ cut on inclusive data

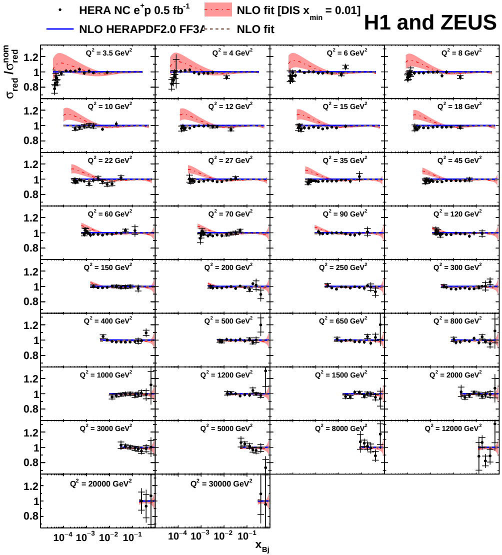


Data to theory comparison in the fit



- Fit with $x_{\min} = 0.01$ provides a steeper low-x gluon and **better description of HF data:**
  - $\chi^2 = 98$ vs 116 in nominal fit per 79 degrees of freedom
  - $p = 0.07$ vs $p = 0.004$
- $x_{\min} = 0.01$: $m_c(m_c) = 1400^{+45}_{-51}$ MeV, $m_b(m_b) = 4.11^{+0.10}_{-0.11}$ GeV

Data to theory comparison in the fit



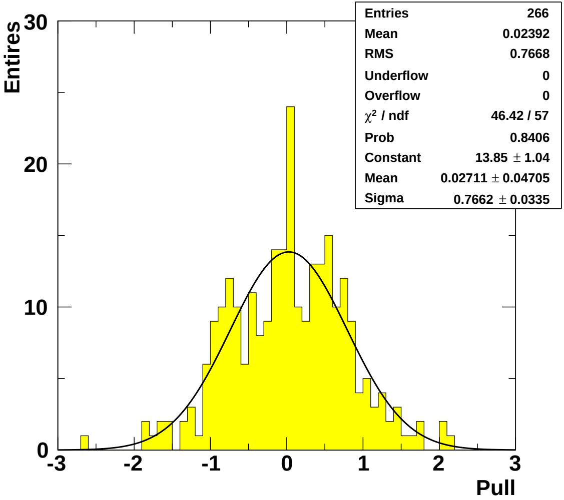
- However as expected, description of inclusive data **excluded from the fit** is worser: $\chi^2 = 2874$ vs 1435 in the nominal fit per 1224 data points

Summary

- Combination of charm beauty production measurements:
  - all input measurements are consistent
  - improved precision compared to previously published charm
  - first combination of beauty
- Combined data are compared to FFNS theoretical predictions:
  - overall reasonable description, but revealed tensions in x slope
  - worser description by approximate NNLO
  - worser description by VFNS predictions
  - predictions with small x resummation does not provide good description of data

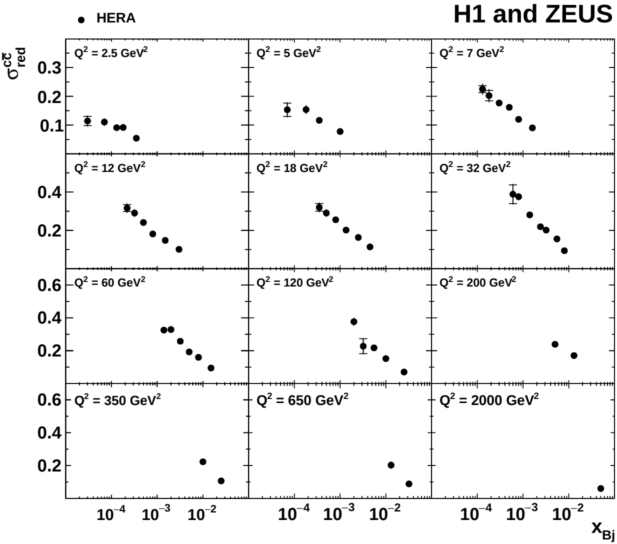
    ⇒ revealed tension in data to theory comparison is not due to FFNS
- Extracted HQ masses:
  - attempts to extract from inclusive data only are sensible
  - $m_c(m_c) = 1290^{+46}_{-41}(\text{fit})^{+62}_{-14}(\text{mod})^{+7}_{-31}(\text{par})$ MeV
  - $m_b(m_b) = 4049^{+104}_{-109}(\text{fit})^{+90}_{-32}(\text{mod})^{+1}_{-31}(\text{par})$ MeV
- Further studies revealed that HF data are described better by PDFs with a steeper gluon:
  - in the adopted PDF framework, it appears in tension with the gluon distribution extracted from inclusive HERA data

Plots proposed for the paper

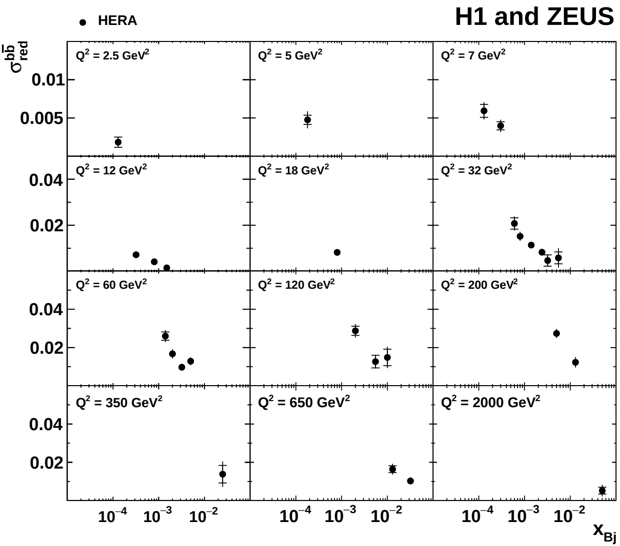


Entries	266
Mean	0.02392
RMS	0.7668
Underflow	0
Overflow	0
χ^2 / ndf	46.42 / 57
Prob	0.8406
Constant	13.85 ± 1.04
Mean	0.02711 ± 0.04705
Sigma	0.7662 ± 0.0335

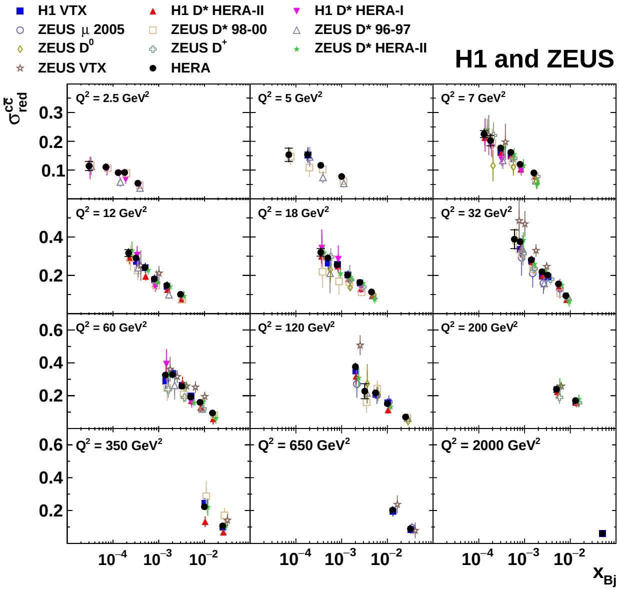
Plots proposed for the paper



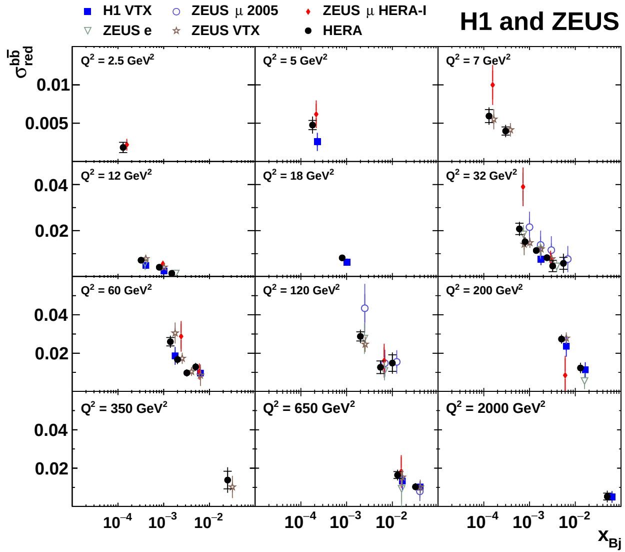
Plots proposed for the paper



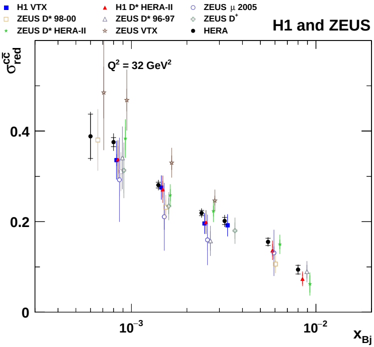
Plots proposed for the paper

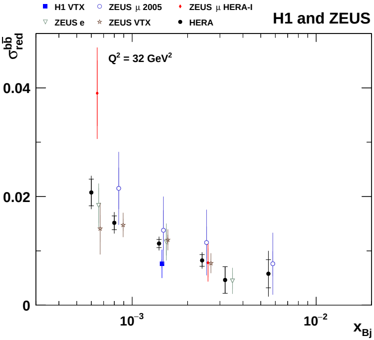


Plots proposed for the paper

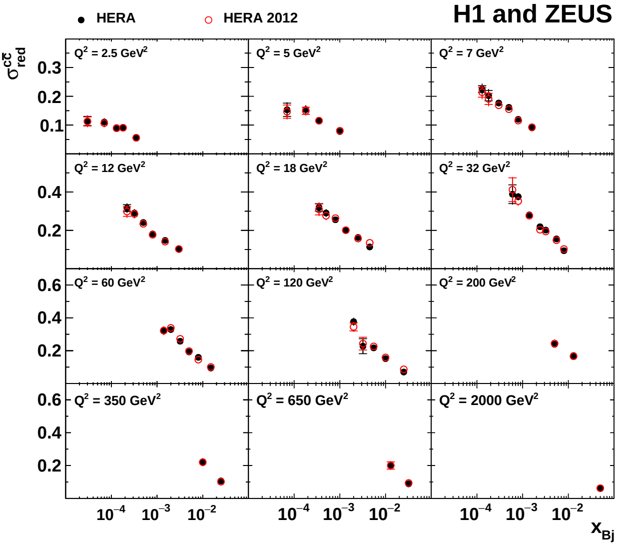


Plots proposed for the paper

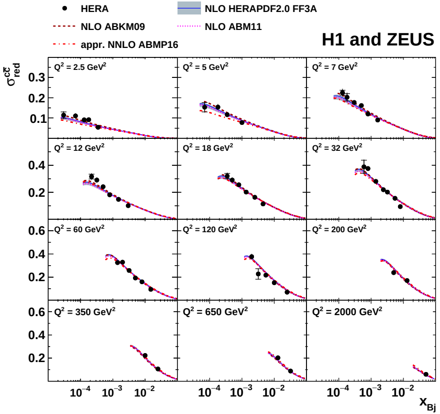


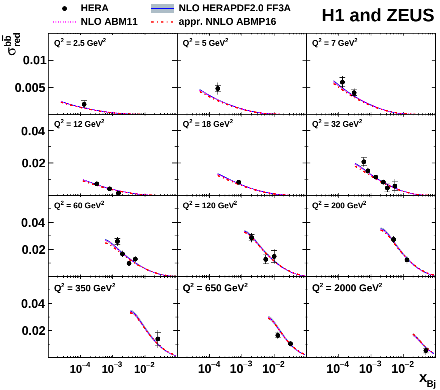


Plots proposed for the paper

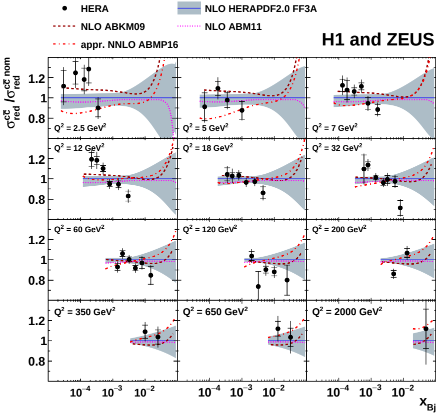


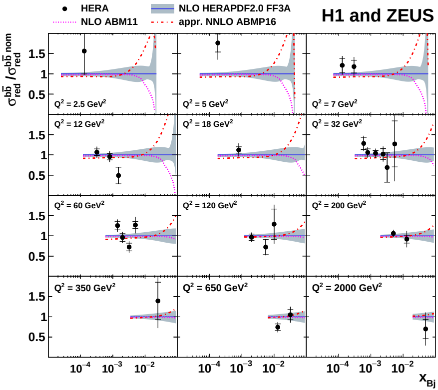
Plots proposed for the paper



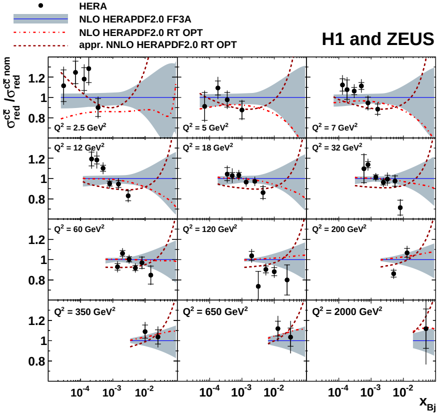


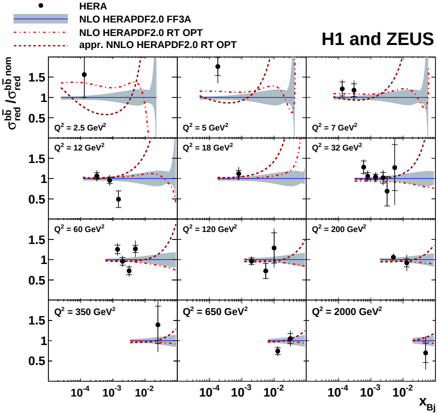
Plots proposed for the paper



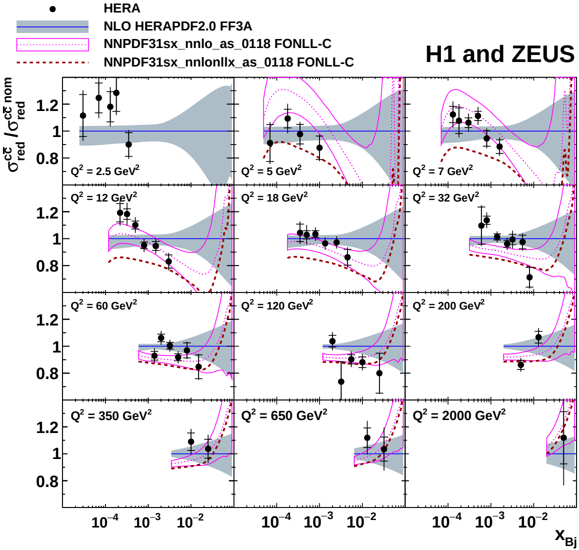


Plots proposed for the paper

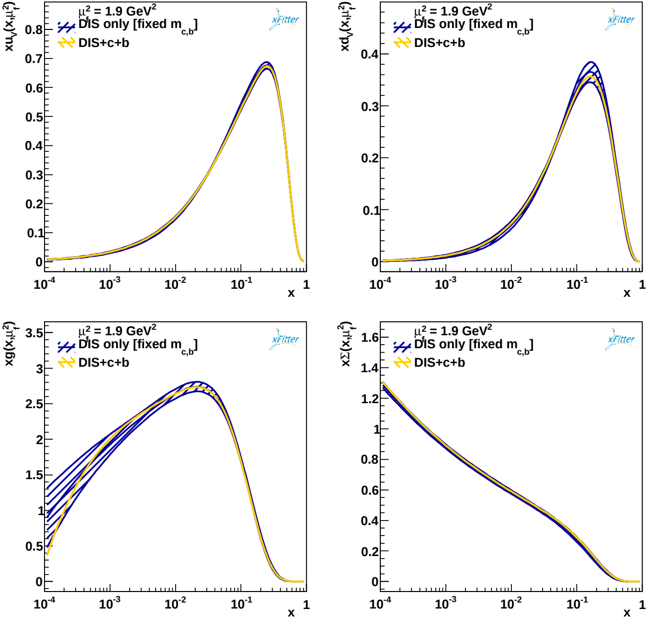




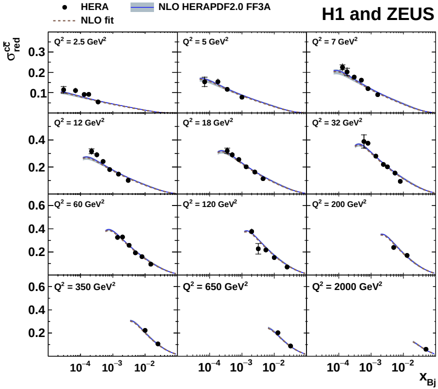
Plots proposed for the paper

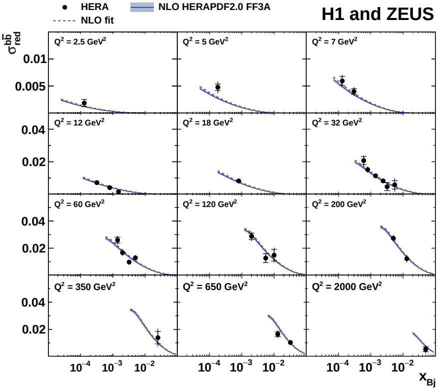


Plots proposed for the paper

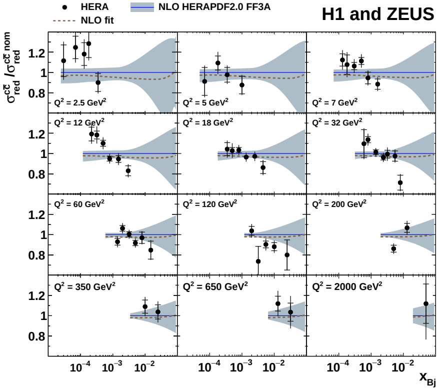


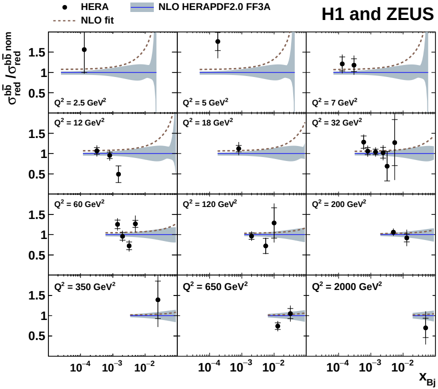
Plots proposed for the paper



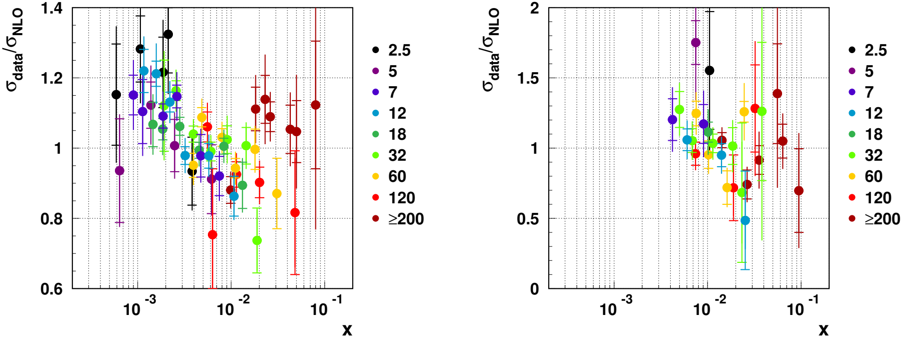


Plots proposed for the paper

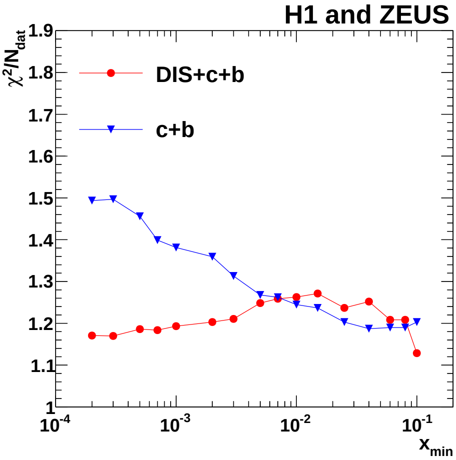




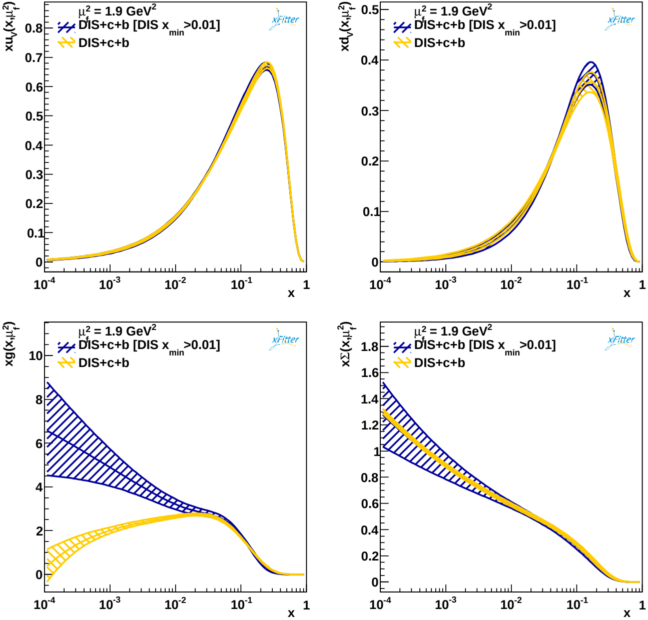
Plots proposed for the paper



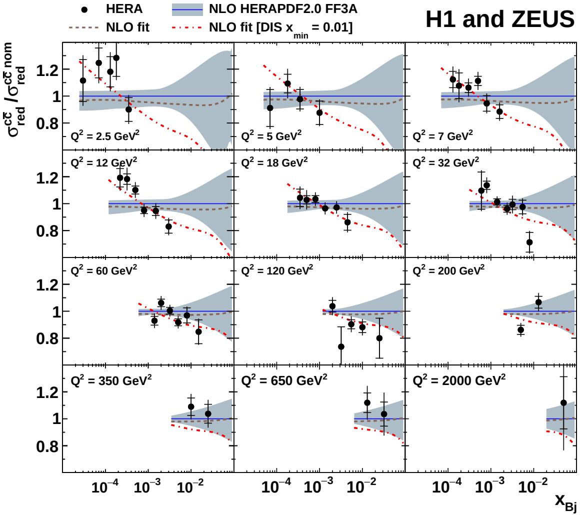
Plots proposed for the paper



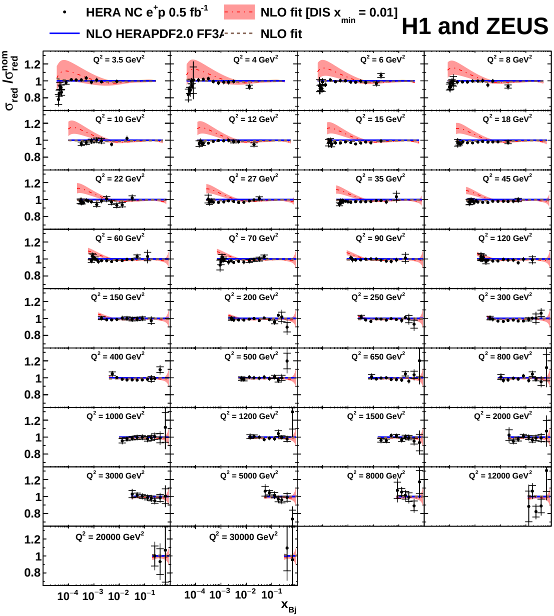
Plots proposed for the paper



Plots proposed for the paper



Plots proposed for the paper



BACKUP

ZEUS D^+ HERA-II [DESY-13-028]

Uncorrelated

δ_1 Boxcut variation $(\pm 1\%)$

δ_5, δ_6 Signal extraction procedure $(< \pm 1\%,\ ^{+0.7\%}_{-1.5\%})$

Point-to-point correlated

δ_2 Electron energy scale $(< \pm 1\%)$

δ_3 Hadronic energy scale $(< \pm 1\%)$

δ_4 FLT trigger efficiency $(< \pm 1\%)$

δ_7 Decay length smearing $(\pm 1\%)$

δ_8 Beauty contribution to the cross sections $(\pm 2\%)$

δ_9, δ_{10} MC shape $(\pm 2\%, \pm 4\%)$

δ_{11} Tracking efficiency $(\pm 1.5\%)$

δ_{12} MVD hit efficiency $(\pm 0.9\%)$

δ_{13} $\chi^2_{sec.\ vtx.}$ smearing $(+2\%)$

δ_{14} $D^+ \to K^-\pi^+\pi^+$ branching ratio $(\pm 2.1\%)$

δ_{15} Luminosity $(\pm 1.9\%)$

More details in H1ZEUS HF meeting 19.04.2013

ZEUS D^* HERA-II [DESY-13-054]

Uncorrelated

- δ_3, δ_4 Boxcut variations ($\pm1\%$, $\pm1\%$)
- δ_5, δ_6 Background shape ($< \pm1\%$, $< \pm1\%$)
- δ_{11} Acceptance statistical uncertainty ($\pm1\%$)
- **$\boldsymbol{\delta_{15}}$ Resolved photon contribution to the acceptance ($+2\%$)**

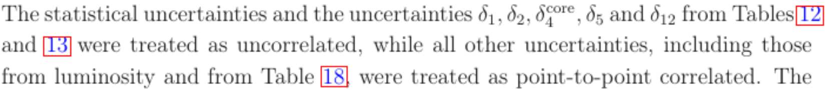
Correlated

- δ_1 Hadronic energy scale ($\pm1\%$)
- δ_2 Electron energy scale ($\pm1\%$)
- δ_7 $p_T(\pi_s)$ correction ($\pm1.5\%$)
- **$\boldsymbol{\delta_8}$ Signal outside the $M_{K\pi}$ window ($+2\%$)**
- **$\boldsymbol{\delta_9}$ Tracking efficiecny ($\pm2\%$)**
- δ_{12} Beauty contribution to the acceptance ($< \pm1\%$)
- δ_{13}, δ_{14} PHP and diffractive MC normalisations ($< \pm1\%$, $< \pm1\%$)
- **$\boldsymbol{\delta_{16}}$, $\boldsymbol{\delta_{17}}$ MC shape ($\pm2\%$, $^{+3\%}_{-2\%}$)**
- δ_{18} Luminosity ($\pm1.9\%$)
- δ_{19} $D^* \to D^0\pi^+, D^0 \to K^-\pi^+$ ($\pm1.9\%$)

More details in H1ZEUS HF meeting 19.04.2013

ZEUS vertex HERA-II DESY-14-083

- Treatment of systematics: as in the original paper for $m_b(m_b)$ extraction



The statistical uncertainties and the uncertainties $\delta_1, \delta_2, \delta_4^{\mathrm{core}}, \delta_5$ and δ_{12} from Tables 12 and 13 were treated as uncorrelated, while all other uncertainties, including those from luminosity and from Table 18, were treated as point-to-point correlated. The

Treated as correlated between ZEUS D^+, D^* and VTX HERA-II:

- **tracking**
- **e/h energy scales**
- **lumi (also with beauty ZEUS el)**

H1 b vertex DESY-09-096, ZEUS μ DESY-09-056, ZEUS el DESY-11-005, ZEUS μ HERA-I DESY-10-047

ZEUS $b \to \mu$ DESY-09-056, H1 b vertex DESY-09-096

- Charm data were included in previous H1ZEUS combination: treat same sources as correlated for beauty

ZEUS $b \to e$ DESY-11-005:

- Only lumi treated as correlated

ZEUS $b \to \mu$ HERA-I DESY-10-047:

- muon detection (7%) and lumi treated as correlated

Statistical correlation between charm and beauty data

$\delta^2_{\mathrm{stat}} = \delta^2_{\mathrm{uncor}} + \delta^2_{\mathrm{cor}} \quad \rightarrow \quad \delta^2_{\mathrm{cor}} = \delta^2_{\mathrm{stat}} \times |\rho|,\ \delta^2_{\mathrm{uncor}} = \delta^2_{\mathrm{stat}} - \delta^2_{\mathrm{cor}}$

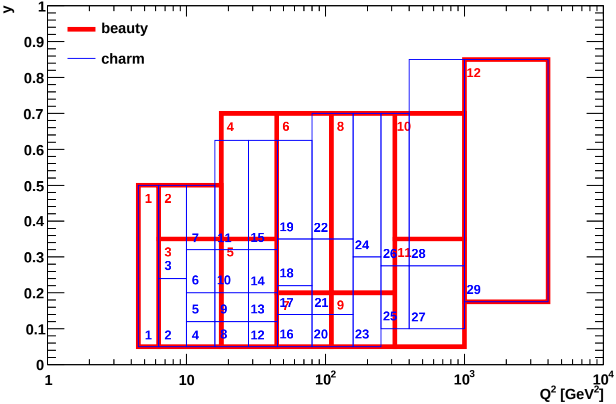
$\rho < 0 \Rightarrow$ correlated uncertainties are assigned with different ± signs for c and b

- ZEUS μ 2005: correlation factors ρ are used directly
- $\delta^2_{\mathrm{stat}} = \delta^2_{\mathrm{uncor}} + \delta^2_{\mathrm{cor}} \quad \rightarrow \quad \delta^2_{\mathrm{cor}} = \delta^2_{\mathrm{stat}} \times |\rho|,\ \delta^2_{\mathrm{uncor}} = \delta^2_{\mathrm{stat}} - \delta^2_{\mathrm{cor}}$ $\rho < 0 \Rightarrow$ correlated uncertainties are assigned with different ± signs for c and b
- ZEUS vertex: to account for different c and b phase space, charm correlation factors are scaled by NLO predictions (see next slides)
- H1 vertex: to account for different c and b binning, split correlations according to NLO predictions (see next slides)

ZEUS vertex, statistical correlations

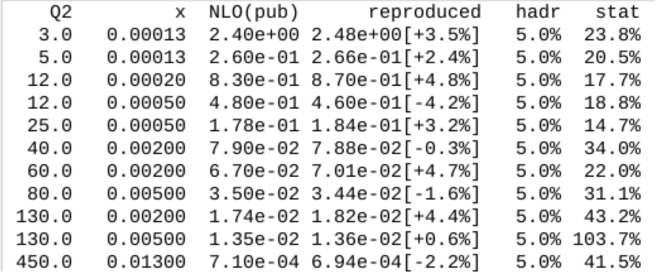
- Charm: $E_T(\mathrm{jet} > 4.2\mathrm{GeV})$, beauty $E_T(\mathrm{jet} > 5\mathrm{GeV})$
- Charm correlation factors are scaled by theory predictions: $\sigma(E_T(\mathrm{jet} > 4.2\mathrm{GeV}))/\sigma(E_T(\mathrm{jet} > 5\mathrm{GeV}))$
- This fraction ranges from 0.65 at low Q^2, high x to 0.95 at high Q^2

H1 vertex, statistical correlations



- 29 c bins, 12 b bins, original Q^2-y bin boundaries provided by H1
- Found 54 overlays $\Rightarrow$ need 54 correlated uncertainty sources
- Use NLO predictions to split statistical uncertainty into uncorrelated and needed number of correlated parts (number of overlays σ_i):
  $\delta^2_{\rm stat} = \delta^2_{\rm uncor} + \sum_i \delta^2_{{\rm cor},i}$, $\quad \delta^2_{{\rm cor},i} = \delta^2_{\rm stat} \times |\rho| \times \frac{\sigma_i}{\sigma}$ small impact: backup

BACKUP. ZEUS μ HERA-I: extrapolation



```
   Q2          x  NLO(pub)      reproduced    hadr   stat
  3.0    0.00013  2.40e+00 2.48e+00[+3.5%]    5.0%  23.8%
  5.0    0.00013  2.60e-01 2.66e-01[+2.4%]    5.0%  20.5%
 12.0    0.00020  8.30e-01 8.70e-01[+4.8%]    5.0%  17.7%
 12.0    0.00050  4.80e-01 4.60e-01[-4.2%]    5.0%  18.8%
 25.0    0.00050  1.78e-01 1.84e-01[+3.2%]    5.0%  14.7%
 40.0    0.00200  7.90e-02 7.88e-02[-0.3%]    5.0%  34.0%
 60.0    0.00200  6.70e-02 7.01e-02[+4.7%]    5.0%  22.0%
 80.0    0.00500  3.50e-02 3.44e-02[-1.6%]    5.0%  31.1%
130.0    0.00200  1.74e-02 1.82e-02[+4.4%]    5.0%  43.2%
130.0    0.00500  1.35e-02 1.36e-02[+0.6%]    5.0% 103.7%
450.0    0.01300  7.10e-04 6.94e-04[-2.2%]    5.0%  41.5%
```

- visible cross section are reported for μ (from b) + jet final state
- hadronisation corrections are needed for NLO: "...change the NLO QCD predictions by typically 5% or less." [DESY-10-047]
- $\Rightarrow$ apply 5% as uncorrelated uncertainty
- Experimental: treat 7% muon detection and 2% lumi as correlated, all other as uncorrelated (statistical and remaining uncorrelated systematics are much larger)

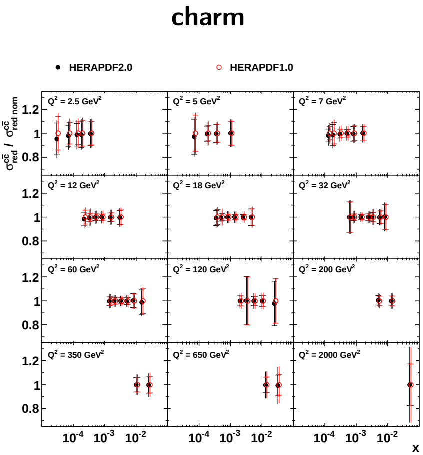
BACKUP. Impact of using HERAPDF2.0 vs HERAPDF1.0 for extrapolation

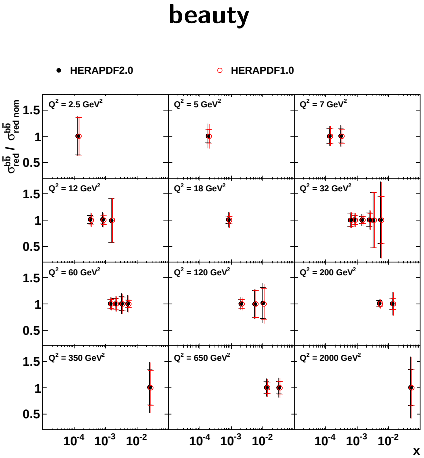
$$\sigma_{\mathrm{red}} = \sigma_{\mathrm{vis}} \frac{\sigma_{\mathrm{red}}^{\mathrm{NLO}}}{\sigma_{\mathrm{vis}}^{\mathrm{NLO}}}$$

Reminder:

- Need FFNS variant with consistent masses, scales and α_s settings: exists for HERAPDF1.0, does not exist for HERAPDF2.0
- Decision: check HERAPDF2.0 impact on central values

BACKUP. HERAPDF2.0 vs 1.0

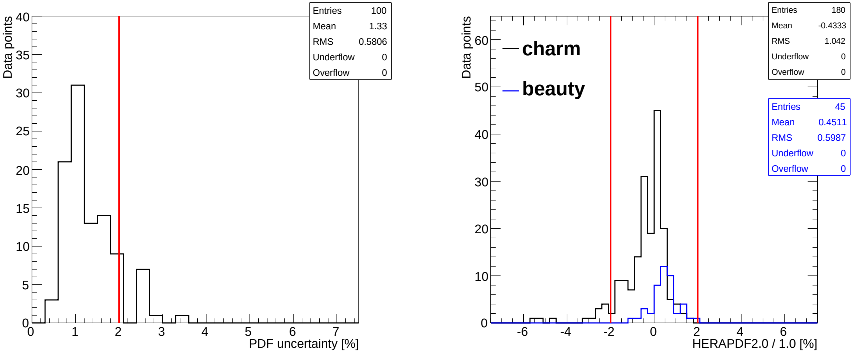




Only small impact on charm at low Q^2, x, well within uncertainties

BACKUP. HERAPDF2.0 vs 1.0

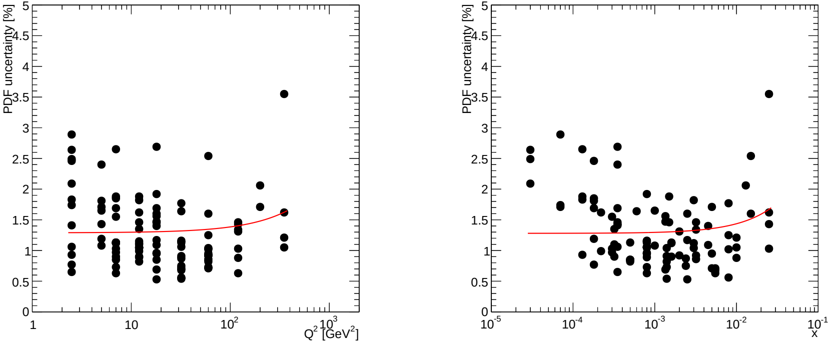
Which uncertainty assign to PDFs in extrapolation:



- $2\% \gtrsim$ RMS(HERAPDF1.0 unc.) $\oplus$ Mean(HERAPDF2.0/1.0)
- treat as bin-to-bin uncorrelated (effective sum of many variations)

BACKUP. HERAPDF2.0 vs 1.0

No substantial Q^2, x dependence:



- large Q^2, x points seem to be just more affected by numerical precision
- **Proceed with using HERAPDF1.0 for central values, assign uncorrelated 2% uncertainty for all points: consistent with HERAPDF2.0**

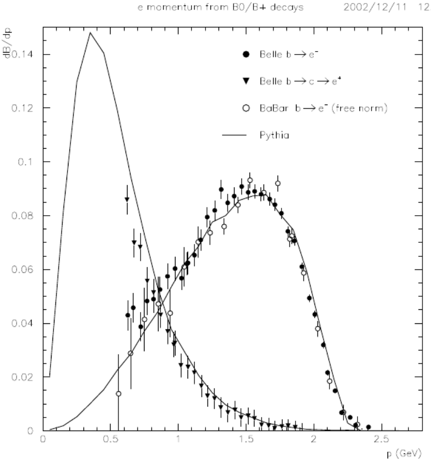
Beauty semileptonic decay spectrum used for extrapolation

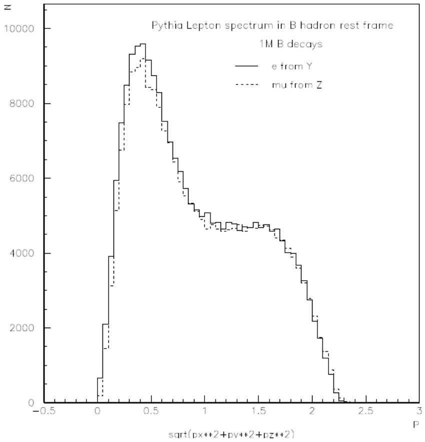
Massimo Corradi, 14 May 2008

HVQDIS for Beauty

The $B \to \mu X$ spectrum used in FMNR has been plugged in HVQDIS.

It was taken from Pythia and checked with BaBar/Belle data

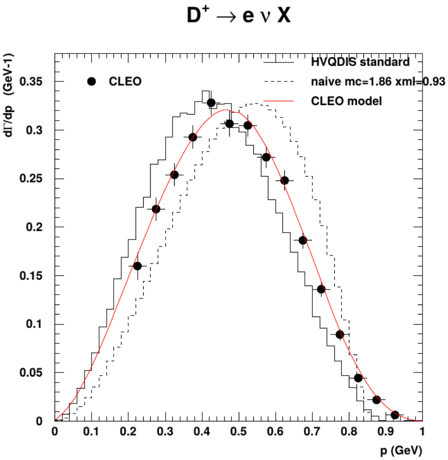




Charm semileptonic decay spectrum used for extrapolation

Massimo Corradi, 19 June 2008

Charm decay, HVQDIS and CLEO data



electron momentum in D^+ decay
CLEO data at $\psi(3770)$
$\simeq D$ rest frame ($E_{\text{kin}} = 16$MeV)

HVQDIS spectrum from Rapgap (Jetset) softer than CLEO data

naive free-quark model not so good

CLEO model based on ISGW parametrisation OK

Data and uncertainties[%]: charm

bin	Q^2 [GeV^2]	x	$\sigma_{\rm red}$	stat	uncor	cor	tot
1	2.5	0.00003	0.11423	8.9	10.7	9.4	16.9
2	2.5	0.00007	0.11054	5.8	6.7	8.2	12.1
3	2.5	0.00013	0.09111	7.1	6.2	7.9	12.3
4	2.5	0.00018	0.09170	4.8	9.6	7.2	12.9
5	2.5	0.00035	0.05437	5.3	8.2	6.9	12.0
6	5.0	0.00007	0.15321	11.6	9.6	8.2	17.1
7	5.0	0.00018	0.15385	5.3	3.4	7.8	10.0
8	5.0	0.00035	0.11642	5.2	5.3	5.7	9.3
9	5.0	0.00100	0.07763	4.8	8.7	5.6	11.4
10	7.0	0.00013	0.22486	4.3	3.3	6.7	8.6
11	7.0	0.00018	0.20231	6.8	5.7	7.2	11.4
12	7.0	0.00030	0.17669	2.3	2.4	5.4	6.4
13	7.0	0.00050	0.16158	2.5	1.8	5.2	6.0
14	7.0	0.00080	0.11994	4.6	4.0	4.9	7.8
15	7.0	0.00160	0.09023	4.1	3.9	5.2	7.7
16	12.0	0.00022	0.31613	4.9	2.9	5.7	8.0
17	12.0	0.00032	0.29041	2.9	1.5	6.3	7.1
18	12.0	0.00050	0.24098	2.4	1.3	4.6	5.3
19	12.0	0.00080	0.18134	2.1	1.4	4.5	5.1
20	12.0	0.00150	0.14761	3.2	1.5	5.1	6.2
21	12.0	0.00300	0.10103	4.4	4.0	5.1	7.8
22	18.0	0.00035	0.31977	5.2	3.3	5.2	8.1
23	18.0	0.00050	0.29049	2.6	1.4	6.4	7.0
24	18.0	0.00080	0.25539	2.2	1.2	4.2	4.9
25	18.0	0.00135	0.20163	2.0	1.1	4.1	4.7
26	18.0	0.00250	0.16300	1.9	1.3	4.2	4.7
27	18.0	0.00450	0.11367	5.5	4.1	5.4	8.7
28	32.0	0.00060	0.38846	8.5	9.3	5.8	13.9
29	32.0	0.00080	0.37557	2.3	1.4	4.4	5.2
30	32.0	0.00140	0.28070	2.0	1.1	3.4	4.1
31	32.0	0.00240	0.21897	2.3	1.4	3.9	4.7
32	32.0	0.00320	0.20149	3.6	1.6	5.4	6.6
33	32.0	0.00550	0.15534	4.2	3.0	4.1	6.6
34	32.0	0.00800	0.09403	8.7	5.4	6.0	11.9
35	60.0	0.00140	0.32542	3.2	1.4	4.8	5.9
36	60.0	0.00200	0.32893	2.3	1.2	4.1	4.9
37	60.0	0.00320	0.25762	2.2	1.2	3.6	4.4
38	60.0	0.00500	0.19250	2.3	1.6	4.1	5.0
39	60.0	0.00800	0.15960	4.8	3.1	3.4	6.7
40	60.0	0.01500	0.09458	8.1	6.5	4.9	11.5
41	120.0	0.00200	0.37661	3.3	2.6	5.0	6.5
42	120.0	0.00320	0.22743	14.6	13.7	2.7	20.2
43	120.0	0.00550	0.21729	3.3	1.6	5.4	6.5
44	120.0	0.01000	0.15186	3.9	2.3	5.2	6.9
45	120.0	0.02500	0.07022	13.6	12.6	4.4	19.1
46	200.0	0.00500	0.23889	3.1	2.4	4.5	6.0
47	200.0	0.01300	0.17035	3.4	2.3	5.0	6.5
48	350.0	0.01000	0.22300	5.1	3.0	6.4	8.7
49	350.0	0.02500	0.10646	6.1	2.9	7.4	10.0
50	650.0	0.01300	0.20260	5.4	3.7	9.1	11.2
51	650.0	0.03200	0.08846	7.8	3.8	12.8	15.4
52	2000.0	0.05000	0.06026	16.0	6.7	26.4	31.6

Data and uncertainties[%]: beauty

bin	Q^2 [GeV^2]	x	$\sigma_{\rm red}$	stat	uncor	cor	tot
1	2.5	0.00013	0.00184	28.4	22.4	11.4	37.9
2	5.0	0.00018	0.00476	10.5	7.1	19.8	23.5
3	7.0	0.00013	0.00593	8.8	11.2	12.7	19.1
4	7.0	0.00030	0.00398	8.5	10.3	15.2	20.2
5	12.0	0.00032	0.00715	4.9	5.8	10.5	13.0
6	12.0	0.00080	0.00409	4.6	6.9	11.1	13.9
7	12.0	0.00150	0.00145	32.2	26.9	3.6	42.1
8	18.0	0.00080	0.00817	4.8	5.0	12.8	14.5
9	32.0	0.00060	0.02074	8.9	7.8	8.9	14.8
10	32.0	0.00080	0.01516	5.8	6.1	10.0	13.1
11	32.0	0.00140	0.01135	3.9	5.3	9.0	11.2
12	32.0	0.00240	0.00824	9.0	9.5	12.9	18.4
13	32.0	0.00320	0.00464	32.2	41.9	3.0	52.9
14	32.0	0.00550	0.00579	39.8	20.4	57.4	72.8
15	60.0	0.00140	0.02599	4.8	6.9	8.8	12.2
16	60.0	0.00200	0.01672	7.5	6.5	10.5	14.4
17	60.0	0.00320	0.00975	10.7	7.7	14.4	19.5
18	60.0	0.00500	0.01287	5.4	4.2	14.7	16.2
19	120.0	0.00200	0.02876	6.3	5.4	9.0	12.2
20	120.0	0.00550	0.01268	21.2	14.9	10.9	28.1
21	120.0	0.01000	0.01485	20.5	20.6	23.6	37.5
22	200.0	0.00500	0.02737	3.8	3.7	6.9	8.7
23	200.0	0.01300	0.01231	9.5	4.8	19.5	22.2
24	350.0	0.02500	0.01381	20.4	26.2	35.0	48.2
25	650.0	0.01300	0.01641	8.1	7.5	13.1	17.1
26	650.0	0.03200	0.01027	8.1	8.7	14.6	18.8
27	2000.0	0.05000	0.00522	30.6	15.2	47.6	58.6

Data correlations[%]: charm vs charm

bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
1	100	35	30	25	23	26	21	25	11	39	30	32	23	17	8	34	31	26	23	10	−0	28	29	22	19	7	−2	16	28	17	10	5	0	−1	17	10	7	4	2	−3	11	2	3	5	−4	7	4	3	1	3	−1	1
2	35	100	41	34	33	31	29	35	18	49	39	46	36	27	16	42	40	38	35	18	8	36	39	32	31	17	3	21	40	28	19	9	5	1	25	17	16	12	6	2	17	4	8	13	−2	13	9	7	4	6	3	2
3	30	41	100	31	32	26	28	33	18	42	37	45	37	28	19	37	38	38	36	20	11	33	38	33	33	20	7	17	38	29	22	10	8	4	25	18	19	16	8	5	18	5	9	15	1	14	11	8	6	7	5	3
4	25	34	31	100	29	23	25	31	19	35	25	38	34	27	19	32	33	34	32	22	12	29	33	30	31	22	7	15	34	27	21	11	11	2	23	17	18	15	9	6	16	4	9	11	1	13	10	8	5	6	6	3
5	23	33	32	29	100	21	25	32	24	32	27	39	37	27	28	29	32	35	34	26	21	27	33	32	34	28	15	13	33	29	23	14	14	9	22	18	19	19	12	10	16	5	11	14	3	13	12	8	8	7	8	3
6	26	31	26	23	21	100	19	24	12	34	25	30	23	16	9	30	27	24	23	11	3	25	26	21	19	9	1	15	26	17	11	6	3	0	16	10	9	6	3	0	11	3	4	6	−2	7	5	4	2	3	1	1
7	21	29	28	25	25	19	100	29	17	33	26	43	40	26	20	31	39	41	36	25	14	29	39	37	37	26	10	15	38	34	30	13	16	4	25	21	26	23	14	8	20	5	14	17	4	17	15	14	8	13	11	9
8	25	35	33	31	32	24	29	100	25	36	29	44	42	30	27	33	37	40	40	29	20	31	37	37	40	30	14	16	39	35	29	15	18	7	26	22	24	21	14	12	19	6	13	17	5	16	14	10	8	9	10	4
9	11	18	18	19	24	12	17	25	100	16	15	27	30	24	28	15	22	26	28	27	25	16	23	26	30	29	21	7	23	25	22	15	18	13	17	15	18	19	15	14	13	4	11	14	6	12	12	7	8	6	10	3
10	39	49	42	35	32	34	33	36	16	100	41	54	41	31	14	53	46	46	41	21	5	46	45	40	35	18	2	24	48	34	23	11	6	2	33	20	20	14	8	0	23	4	9	14	−2	17	11	10	5	8	4	3
11	30	39	37	25	27	25	26	29	15	41	100	41	32	21	14	36	36	34	32	16	6	31	36	30	29	14	5	17	36	26	18	8	5	4	23	16	16	12	6	2	16	4	8	13	−1	13	9	7	5	7	3	3
12	32	46	45	38	39	30	43	44	27	54	41	100	58	43	32	48	56	59	56	38	22	45	58	54	54	38	18	23	57	50	40	21	22	10	40	31	36	30	20	12	30	7	18	25	5	26	20	16	12	13	13	6
13	23	36	37	34	37	23	40	42	30	41	32	58	100	43	40	38	47	57	57	47	30	39	49	55	61	50	24	18	50	54	48	25	31	13	37	34	41	39	25	19	29	8	22	26	9	26	23	17	15	13	18	7
14	17	27	28	27	27	16	26	30	24	31	21	43	43	100	29	28	34	40	41	35	24	27	35	37	41	37	21	13	35	36	33	19	21	14	28	22	27	27	19	14	21	5	15	20	6	19	17	11	11	9	13	5
15	8	16	19	19	28	9	20	27	28	14	14	32	40	29	100	14	23	32	36	39	40	20	26	34	42	44	34	6	28	35	32	24	29	19	22	22	29	30	22	21	17	5	17	21	9	18	20	13	15	9	16	5
16	34	42	37	32	29	30	31	33	15	53	36	48	38	28	14	100	42	44	38	21	5	43	41	39	34	19	3	21	45	34	24	11	8	2	32	21	21	15	9	0	23	4	10	14	−1	18	11	11	6	9	5	4
17	31	40	38	33	32	27	39	37	22	46	36	56	47	34	23	42	100	51	46	30	14	39	57	46	43	29	11	20	51	41	31	18	16	6	36	27	28	21	14	7	28	6	14	18	2	22	16	14	9	11	7	4
18	26	38	38	34	35	24	41	40	26	46	34	59	57	40	32	44	51	100	55	40	24	42	52	57	55	42	19	19	54	53	44	22	27	11	41	35	39	34	23	14	31	7	20	25	6	27	22	19	13	15	14	7
19	23	35	36	32	34	23	36	40	28	41	32	56	57	41	36	38	46	55	100	49	31	38	48	54	59	50	24	19	52	53	47	30	31	14	41	36	39	37	26	18	30	7	24	29	8	27	25	16	17	16	20	9
20	10	18	20	22	26	11	25	29	27	21	16	38	47	35	39	21	30	40	49	100	36	25	32	42	52	54	30	9	35	44	43	30	34	18	31	29	36	39	27	21	23	6	22	25	10	22	24	14	17	12	22	8
21	−0	8	11	12	21	3	14	20	25	5	6	22	30	24	40	5	14	24	31	36	100	11	17	27	35	42	36	3	20	30	29	22	29	23	18	19	25	29	22	23	14	4	16	21	9	15	19	11	15	9	16	5
22	28	36	33	29	27	25	29	31	16	46	31	45	39	27	20	43	39	42	38	25	11	100	39	39	36	23	7	19	44	36	26	13	13	1	32	22	25	18	11	5	23	5	12	15	1	19	14	13	9	10	8	5
23	29	39	38	33	33	26	39	37	23	45	36	58	49	35	26	41	57	52	48	32	17	39	100	47	45	31	13	20	53	43	33	16	17	6	39	29	29	23	15	8	29	6	15	20	3	24	16	15	9	12	9	5
24	22	32	33	30	32	21	37	37	26	40	30	54	55	37	34	39	46	57	54	42	27	39	47	100	55	45	20	17	51	53	44	23	30	11	39	35	40	36	24	16	31	7	22	25	8	27	23	19	14	15	16	7
25	19	31	33	31	34	19	37	40	30	35	29	54	61	41	42	34	43	55	59	52	35	36	45	55	100	58	30	16	50	56	51	31	36	18	39	37	47	44	30	22	29	8	26	30	11	28	27	19	18	16	22	9
26	7	17	20	22	28	9	26	30	29	18	14	38	50	37	44	19	29	42	50	54	42	23	31	45	58	100	36	9	36	49	46	37	38	24	31	31	43	46	32	25	24	7	25	28	11	25	28	15	20	13	25	7
27	−2	3	7	7	15	1	10	14	21	2	5	18	24	21	34	3	11	19	24	30	36	7	13	20	30	36	100	0	15	23	24	20	26	26	14	15	22	25	20	20	11	3	14	19	7	13	17	9	13	7	14	4
28	16	21	17	15	13	15	15	16	7	24	17	23	18	13	6	21	20	19	19	9	3	19	20	17	16	9	0	100	21	16	11	5	3	−0	14	10	10	7	4	1	10	2	5	6	−0	9	5	5	3	4	3	2
29	28	40	38	34	33	26	38	39	23	48	36	57	50	35	28	45	51	54	52	35	20	44	53	51	50	36	15	21	100	49	38	19	21	8	42	32	36	28	19	11	32	7	20	23	5	28	20	19	13	17	14	8
30	17	28	29	27	29	17	34	35	25	34	26	50	54	36	35	34	41	53	53	44	30	36	43	53	56	49	23	16	49	100	47	28	33	13	40	36	45	40	27	19	32	8	26	28	10	31	26	22	18	18	21	10
31	10	19	22	21	23	11	30	29	22	23	18	40	48	33	32	24	31	44	47	43	29	26	33	44	51	46	24	11	38	47	100	23	34	14	32	33	41	41	27	19	25	7	25	28	10	26	24	20	17	18	22	11
32	5	9	10	11	14	6	13	15	15	11	8	21	25	19	24	11	18	22	30	30	22	13	16	23	31	37	20	5	19	28	23	100	19	16	19	19	26	26	19	13	14	3	17	19	5	17	22	9	15	9	17	5
33	0	5	8	11	14	3	16	18	18	6	5	22	31	21	29	8	16	27	31	34	29	13	17	30	36	38	26	3	21	33	34	19	100	14	20	22	30	30	24	21	17	5	19	22	10	17	19	15	14	13	17	7
34	−1	1	4	2	9	0	4	7	13	2	4	10	13	14	19	2	6	11	14	18	23	1	6	11	18	24	26	−0	8	13	14	16	14	100	8	8	14	17	13	12	7	2	8	13	4	8	11	5	8	5	9	3
35	17	25	25	23	22	16	25	26	17	33	23	40	37	28	22	32	36	41	41	31	18	32	39	39	39	31	14	14	42	40	32	19	20	8	100	28	32	25	18	10	27	5	19	21	5	26	19	17	13	15	14	8
36	10	17	18	17	18	10	21	22	15	20	16	31	34	22	22	21	27	35	36	29	19	22	29	35	37	31	15	10	32	36	33	19	22	8	28	100	30	28	18	12	25	5	22	21	6	24	20	19	14	19	18	10
37	7	16	19	18	19	9	26	24	18	20	16	36	41	27	29	21	28	39	39	36	25	25	29	40	47	43	22	10	36	45	41	26	30	14	32	30	100	38	26	18	27	6	25	27	9	29	26	22	18	20	23	11
38	4	12	16	15	19	6	23	21	19	14	12	30	39	27	30	15	21	34	37	39	29	18	23	36	44	46	25	7	28	40	41	26	30	17	25	28	38	100	25	19	22	6	24	26	9	25	25	19	19	17	24	10
39	2	6	8	9	12	3	14	14	15	8	6	20	25	19	22	9	14	23	26	27	22	11	15	24	30	32	20	4	19	27	27	19	24	13	18	18	26	25	100	15	14	4	15	17	7	17	16	11	11	10	14	6
40	−3	2	5	6	10	0	8	12	14	0	2	12	19	14	21	0	7	14	18	21	23	5	8	16	22	25	20	1	11	19	19	13	21	12	10	12	18	19	15	100	8	4	12	13	8	10	12	7	9	7	11	4
41	11	17	18	16	16	11	20	19	13	23	16	30	29	21	17	23	28	31	30	23	14	23	29	31	29	24	11	10	32	32	25	14	17	7	27	25	27	22	14	8	100	4	19	19	4	25	18	18	12	18	17	9
42	2	4	5	4	5	3	5	6	4	4	4	7	8	5	5	4	6	7	7	6	4	5	6	7	8	7	3	2	7	8	7	3	5	2	5	5	6	6	4	4	4	100	4	4	2	4	3	3	3	2	3	1
43	3	8	9	9	11	4	14	13	11	9	8	18	22	15	17	10	14	20	24	22	16	12	15	22	26	25	14	5	20	26	25	17	19	8	19	22	25	24	15	12	19	4	100	21	6	20	19	16	14	19	23	13
44	5	13	15	11	14	6	17	17	14	14	13	25	26	20	21	14	18	25	29	25	21	15	20	25	30	28	19	6	23	28	28	19	22	13	21	21	27	26	17	13	19	4	21	100	6	20	22	16	15	15	20	10
45	−4	−2	1	1	3	−2	4	5	6	−2	−1	5	9	6	9	−1	2	6	8	10	9	1	3	8	11	11	7	−0	5	10	10	5	10	4	5	6	9	9	7	8	4	2	6	6	100	5	5	4	4	4	6	2
46	7	13	14	13	13	7	17	16	12	17	13	26	26	19	18	18	22	27	27	22	15	19	24	27	28	25	13	9	28	31	26	17	17	8	26	24	29	25	17	10	25	4	20	20	5	100	20	19	15	19	17	10
47	4	9	11	10	12	5	15	14	12	11	9	20	23	17	20	11	16	22	25	24	19	14	16	23	27	28	17	5	20	26	24	22	19	11	19	20	26	25	16	12	18	3	19	22	5	20	100	17	16	17	19	12
48	3	7	8	8	8	4	14	10	7	10	7	16	17	11	13	11	14	19	16	14	11	13	15	19	19	15	9	5	19	22	20	9	15	5	17	19	22	19	11	7	18	3	16	16	4	19	17	100	13	18	15	13
49	1	4	6	5	8	2	8	8	8	5	5	12	15	11	15	6	9	13	17	17	15	9	9	14	18	20	13	3	13	18	17	15	14	8	13	14	18	19	11	9	12	3	14	15	4	15	16	13	100	12	15	8
50	3	6	7	6	7	3	13	9	6	8	7	13	13	9	9	9	11	15	16	12	9	10	12	15	16	13	7	4	17	18	18	9	13	5	15	19	20	17	10	7	18	2	19	15	4	19	17	18	12	100	19	18
51	−1	3	5	6	8	1	11	10	10	4	3	13	18	13	16	5	7	14	20	22	16	8	9	16	22	25	14	3	14	21	22	17	17	9	14	18	23	24	14	11	17	3	23	20	6	17	19	15	15	19	100	15
52	1	2	3	3	3	1	9	4	3	3	3	6	7	5	5	4	4	7	9	8	5	5	5	7	9	7	4	2	8	10	11	5	7	3	8	10	11	10	6	4	9	1	13	10	2	10	12	13	8	18	15	100

Data correlations[%]: beauty vs charm

bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1	0	4	1	3	3	−2	1	1	1	1	−1	−1	1	3	−2	−3	−2	0	−2	0	1	−4	−2	−0	−2	−1
2	1	0	3	1	3	3	−2	1	1	1	2	−1	−1	1	3	−1	−3	−3	0	−2	0	1	−5	−2	−0	−2	−2
3	1	0	2	0	2	1	−1	1	0	0	2	−1	−1	1	2	−0	−3	−3	0	−1	0	−0	−5	−2	−0	−2	−1
4	1	0	2	−0	2	1	−1	1	0	−0	1	−1	−1	1	2	−0	−2	−1	0	−1	0	0	−3	−2	−1	−1	−1
5	0	0	1	−1	2	−0	−0	1	−0	−0	1	−1	−0	1	2	1	−2	0	1	−0	1	1	−2	−1	−1	−1	−1
6	1	0	3	1	3	2	−2	1	1	1	1	−1	−1	1	2	−1	−2	−2	0	−1	0	1	−3	−2	−0	−1	−1
7	−1	−30	1	−0	7	7	−0	4	−0	1	6	−1	−0	1	1	1	−1	−4	1	−1	1	0	−5	−1	−1	1	−2
8	1	0	1	−1	2	1	−0	1	−0	−0	2	−1	−0	1	2	1	−2	−1	1	−0	1	1	−3	−2	−1	−1	−1
9	0	1	−0	−2	1	−2	0	0	−1	−0	0	−1	0	1	1	2	−1	2	1	0	1	1	−0	−0	−1	0	−1
10	1	1	2	2	5	3	−2	1	2	2	2	−3	−1	1	2	−1	−2	−4	0	−2	0	1	−6	−2	−1	−3	−2
11	1	0	3	1	3	3	−1	1	1	1	2	−1	−1	1	3	−1	−3	−4	1	−1	0	0	−5	−2	−0	−2	−1
12	1	2	2	−1	3	1	−1	4	1	1	3	−3	−0	1	3	2	−1	−4	1	−0	1	0	−6	−1	−2	−3	−2
13	−0	1	0	−2	3	−5	0	2	1	1	4	−4	0	1	2	4	1	0	2	1	1	2	−3	0	−3	−1	−1
14	−0	1	0	−1	2	−2	0	1	0	1	1	−4	0	1	0	3	1	0	1	0	1	0	−2	1	−3	−1	−1
15	−0	1	−1	−3	0	−3	1	0	−1	−0	0	−2	1	1	1	4	0	4	2	1	1	1	1	0	−1	0	−0
16	1	0	4	1	4	3	−2	1	1	1	2	−2	−1	1	2	−1	−3	−5	0	−2	0	0	−6	−2	−1	−2	−1
17	1	4	3	0	−1	6	−1	7	1	0	3	−2	−1	1	4	1	−2	−5	0	−1	1	0	−5	−2	−1	−4	−2
18	1	1	2	−1	5	1	−0	1	0	0	4	−2	−0	1	3	2	−2	−5	1	0	1	0	−6	−2	−1	−2	−1
19	0	1	1	−2	4	−6	0	2	0	1	2	−3	0	1	3	3	−1	−0	3	0	1	2	−3	−1	−3	−1	−3
20	−0	1	−1	−3	1	−8	1	2	−1	−1	2	−3	1	1	2	3	−1	6	2	1	1	3	2	−1	−2	1	−1
21	−0	1	−2	−3	−0	−3	1	−0	−1	−1	−1	−1	1	1	1	3	−1	3	2	1	1	1	0	−1	−1	1	−0
22	1	0	3	1	4	2	−1	0	1	1	1	−2	−1	0	1	1	−2	−4	0	−1	0	−0	−5	−2	−2	−2	−1
23	1	3	3	0	6	4	−1	−12	1	1	1	−2	−1	0	1	1	−2	−8	0	−1	1	−2	−9	−1	−3	−4	−3
24	0	1	1	−1	4	4	0	0	0	0	0	−2	0	0	3	3	−2	−2	2	0	1	1	−5	−2	−1	−1	−0
25	0	1	−0	−2	2	2	1	0	0	0	0	−2	1	1	3	4	−2	3	3	1	2	2	−1	−2	−1	−0	−1
26	−0	2	−2	−4	−1	−1	1	−0	−1	−1	−3	−2	1	0	1	4	−2	9	3	1	1	2	4	−1	−2	0	−0
27	−0	1	−2	−3	−1	−3	1	−0	−0	−1	−0	−1	1	1	1	2	−1	4	2	1	1	1	2	−1	−0	1	−0
28	1	0	3	2	3	2	−1	1	−10	2	1	−2	−1	0	1	1	1	−2	1	−1	0	0	−2	−0	−1	−2	−1
29	1	1	2	−0	5	4	−1	−9	1	−3	3	−1	−0	−2	2	2	−3	−6	2	−1	1	−0	−7	−3	−1	−2	−2
30	0	1	0	−2	3	2	1	−1	0	1	−5	−2	0	−0	1	5	−1	−2	2	1	0	1	−3	−1	−2	−1	−1
31	−0	1	−1	−3	1	1	1	1	−1	−0	−1	−9	1	1	2	4	−1	−0	3	1	1	2	−2	−1	−1	1	−1
32	0	3	−0	−2	1	1	1	−0	−0	−0	−8	−1	0	0	1	2	−2	10	2	1	0	0	8	−1	−1	1	−0
33	−1	0	−2	−3	−0	−1	2	0	−1	−3	1	2	1	−1	1	2	−2	0	2	2	2	1	−0	−3	1	1	−0
34	−0	1	−1	−2	−0	−1	0	−0	−1	−1	−0	−1	0	0	1	1	−2	4	2	0	0	1	3	−1	−0	0	−0
35	0	1	1	−2	3	0	−0	0	−0	−1	−0	−2	−0	−0	−12	1	−3	−6	1	0	0	−2	−6	−2	−4	−2	−2
36	−1	−1	−1	−3	1	−0	0	−0	−1	−2	1	−2	0	−0	−10	−10	−2	−4	1	0	0	0	−4	−1	−1	−2	−1
37	−0	−0	−1	−3	1	2	1	−2	−1	−1	3	−1	1	0	−0	3	−14	−10	3	1	1	−1	1	−3	−0	0	−0
38	−1	−0	−2	−4	−2	−1	1	−3	−1	−1	2	−2	1	0	−1	3	−2	−20	3	1	1	−0	−1	−2	−2	−2	−0
39	−0	0	−1	−3	1	−1	1	0	−1	−1	0	−1	1	0	1	3	−1	1	2	1	1	1	0	−1	−1	0	−0
40	−0	0	−2	−2	−1	−1	1	−0	−1	−1	1	0	1	0	1	2	−0	2	1	1	1	0	1	−1	0	1	−0
41	0	1	0	−1	5	1	0	3	1	1	1	−1	0	−2	−2	2	−1	−4	−16	1	0	−3	−2	−0	1	0	−0
42	0	−0	−0	−0	0	0	0	−0	0	−0	0	−0	0	0	0	1	−0	−0	0	0	0	0	−0	−0	−0	−0	−0
43	−1	−0	−2	−2	1	−1	1	0	−1	0	0	−0	1	−1	−4	1	−1	−5	1	−4	2	−5	−6	−2	−1	0	−2
44	−0	1	−1	−2	1	−0	1	−0	−0	0	1	2	0	3	1	2	−2	−8	2	3	−7	−0	−11	−3	1	1	−1
45	−0	−0	−1	−1	−0	−1	1	−0	−0	−0	0	−0	1	0	−0	2	1	0	1	1	0	−0	1	0	−0	0	0
46	−1	0	−1	−4	1	−3	0	−3	−0	−1	−1	−3	0	−1	−3	2	−1	−5	2	0	0	−27	−2	−2	−3	−1	−2
47	−0	0	−0	−1	2	0	1	0	−0	0	1	0	0	0	2	2	−2	2	2	1	1	−1	−13	−3	2	3	−0
48	−0	−0	−0	−1	2	−0	0	1	−0	−0	0	−0	0	−0	1	1	−2	−5	0	1	0	−3	−0	−2	−6	0	0
49	−0	0	−1	−2	−0	−2	1	−1	−0	−0	−0	−1	0	−0	−1	1	−1	2	0	1	−0	−2	−2	−19	−2	−8	−1
50	−1	−2	−1	−3	2	−1	0	−0	−1	−2	−0	0	0	−0	1	0	−3	−7	1	0	0	−0	−0	−3	−30	−2	−2
51	−1	−2	−2	−3	−0	−4	1	−1	−1	2	−1	−2	1	−1	0	2	−1	1	1	1	2	1	3	−1	−1	−25	−2
52	−0	−2	−1	−1	1	−1	0	0	−0	0	−1	0	0	−0	1	1	−1	−5	1	0	0	1	0	−1	1	4	−61

Data correlations[%]: beauty vs beauty

bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	100	9	10	8	9	10	1	5	7	8	8	6	−0	1	6	7	3	6	4	3	1	4	3	−1	3	2	1
2	9	100	12	11	17	17	1	17	8	11	19	7	−0	1	8	10	5	16	6	3	2	6	9	−0	1	10	1
3	10	12	100	18	19	19	1	12	10	18	17	10	−0	1	14	15	8	9	10	3	2	13	5	−0	3	4	1
4	8	11	18	100	22	21	1	13	10	22	19	13	−0	1	17	16	12	10	11	2	3	14	5	2	7	6	2
5	9	17	19	22	100	26	1	37	11	23	25	12	−0	1	28	18	12	8	13	2	3	22	5	1	9	3	1
6	10	17	19	21	26	100	1	21	11	21	36	16	−0	2	23	16	9	24	13	3	4	19	15	1	11	10	3
7	1	1	1	1	1	1	100	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1	1	−0	0	0	0
8	5	17	12	13	37	21	1	100	7	15	20	9	−0	2	31	11	8	8	8	2	2	26	4	1	9	1	2
9	7	8	10	10	11	11	1	7	100	11	10	6	0	1	8	9	6	6	7	3	2	7	4	2	3	3	1
10	8	11	18	22	23	21	1	15	11	100	21	12	−0	2	18	19	15	11	14	4	7	16	7	4	6	7	2
11	8	19	17	19	25	36	1	20	10	21	100	15	0	3	23	18	10	30	14	3	6	21	18	2	10	16	4
12	6	7	10	13	12	16	1	9	6	12	15	100	0	9	12	9	5	8	9	2	6	12	5	0	9	6	2
13	−0	−0	−0	−0	−0	−0	0	−0	0	−0	0	0	100	0	−0	0	0	0	0	1	0	0	0	0	0	0	0
14	1	1	1	1	1	2	0	2	1	2	3	9	0	100	2	1	1	2	2	3	25	2	1	0	1	3	0
15	6	8	14	17	28	23	1	31	8	18	23	12	−0	2	100	16	9	12	13	2	3	25	8	1	13	6	4
16	7	10	15	16	18	16	1	11	9	19	18	9	0	1	16	100	12	10	13	3	3	15	6	4	4	7	2
17	3	5	8	12	12	9	1	8	6	15	10	5	0	1	9	12	100	8	8	2	2	9	3	5	3	4	1
18	6	16	9	10	8	24	1	8	6	11	30	8	0	2	12	10	8	100	8	3	4	16	32	2	8	25	6
19	4	6	10	11	13	13	1	8	7	14	14	9	0	2	13	13	8	8	100	3	4	13	5	3	4	6	1
20	3	3	3	2	2	3	1	2	3	4	3	2	1	3	2	3	2	3	3	100	4	2	2	0	1	3	0
21	1	2	2	3	3	4	0	2	2	7	6	6	0	25	3	3	2	4	4	4	100	3	4	1	2	5	1
22	4	6	13	14	22	19	1	26	7	16	21	12	0	2	25	15	9	16	13	2	3	100	11	6	12	11	5
23	3	9	5	5	5	15	1	4	4	7	18	5	0	1	8	6	3	32	5	2	4	11	100	2	7	24	5
24	−1	−0	−0	2	1	1	−0	1	2	4	2	0	0	0	1	4	5	2	3	0	1	6	2	100	1	4	1
25	3	1	3	7	9	11	0	9	3	6	10	9	0	1	13	4	3	8	4	1	2	12	7	1	100	7	4
26	2	10	4	6	3	10	0	1	3	7	16	6	0	3	6	7	4	25	6	3	5	11	24	4	7	100	4
27	1	1	1	2	1	3	0	2	1	2	4	2	0	0	4	2	1	6	1	0	1	5	5	1	4	4	100

Parametrisation study

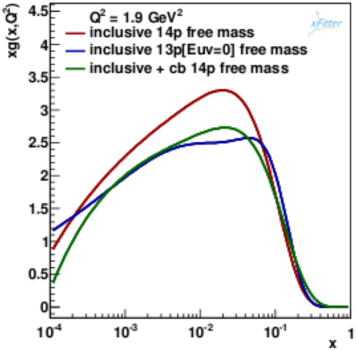
Nominal 14p (HERAPDF2.0):

$$xg(x) = A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}$$
$$xu_v(x) = A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}(1+E_{u_v}x^2)$$
$$xd_v(x) = A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}$$
$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}(1+D_{\bar{U}}x)$$
$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}$$

Additional constrains:

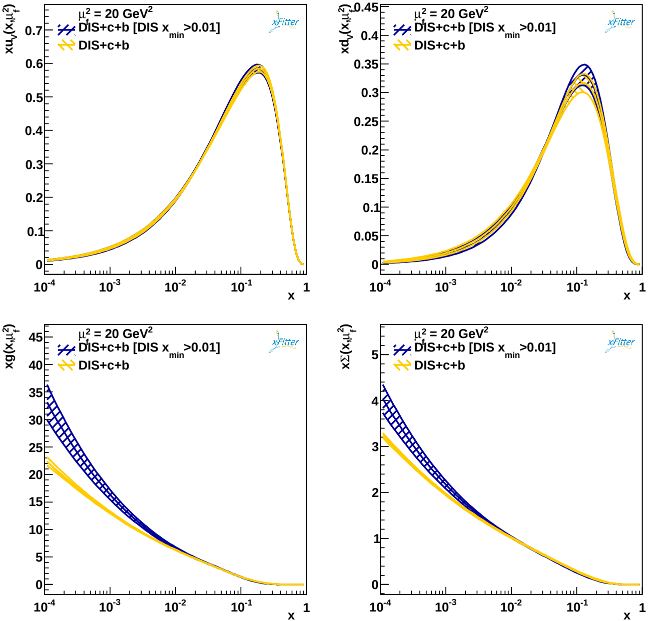
$$A_{\bar{U}} = A_{\bar{D}}(1-f_s),\ B_{\bar{U}} = B_{\bar{D}},\ C'_g = 25$$
$$\int_0^1 [\textstyle\sum_i (q_i(x) + \bar{q}_i(x)) + g(x)]x dx = 1$$
$$\int_0^1 [u(x) - \bar{u}(x)]dx = 2,$$
$$\int_0^1 [d(x) - \bar{d}(x)]dx = 1$$

Parametrisation	DIS $+ c + b$			DIS		
	chi2	mc	mb	chi2	mc	mb
14p nominal	1435	1.29	4.05	1304	1.80	8.45
12p Aprig=0	1439	1.29	4.04	1308	1.78	8.03
13p DUbar=0	1459	1.26	3.98	1330	1.45	3.57
13p Euv=0	1441	1.26	4.02	1319	1.45	4.00
15p Cprig	1435	1.29	4.05	1303	1.80	8.49
15p DDbar	1435	1.29	4.05	1304	1.80	8.44
15p Ddv	1435	1.29	4.05	1304	1.80	8.41
15p Dg	1435	1.29	4.06	1303	1.80	8.53
15p Duv	1418	1.29	4.04	1289	1.80	8.18
15p EDbar	1435	1.29	4.05	1304	1.80	8.49
15p Edv	1435	1.29	4.05	1304	1.80	8.44
15p Eg	1435	1.29	4.05	1303	1.80	8.79
15p EUbar	1435	1.29	4.05	1304	1.80	8.43
15p FDbar	1435	1.29	4.05	1304	1.80	8.52
15p Fdv	1435	1.29	4.05	1304	1.80	8.45
15p Fg	1435	1.29	4.05	1303	1.80	8.74
15p FUbar	1429	1.29	4.04	1297	1.80	8.13
15p Fuv	1429	1.29	4.05	1299	1.80	8.33

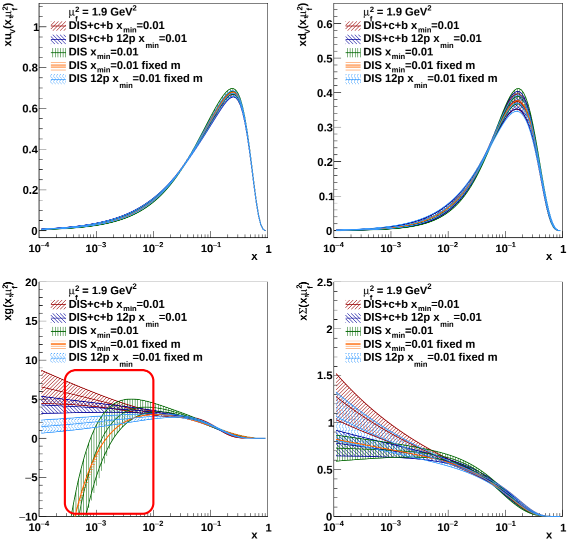


'13p Euv=0' is taken as parametrisation uncertainty

PDFs with $x_{\min} = 0.01$ cut on inclusive data



Comparison of PDFs

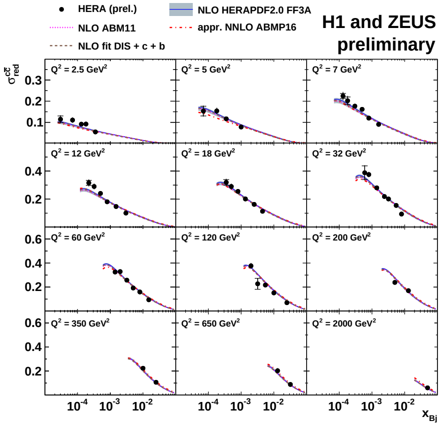


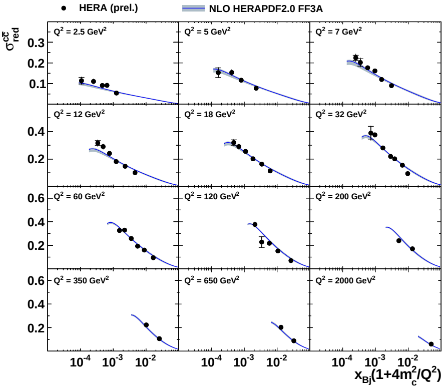
Constraints on low-x gluon are from HF data included in the fit

Cross sections as function of $x(1+4m^2/Q^2)$

x_{Bj}

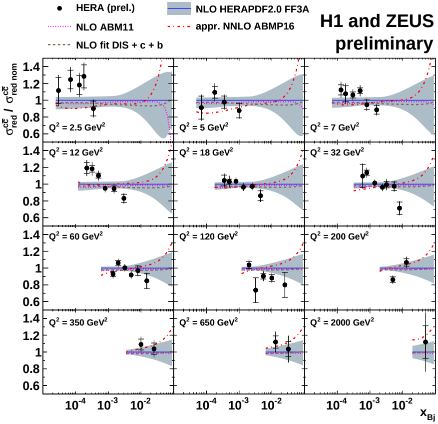
$x_{\mathrm{Bj}}(1+4m^2/Q^2) \approx \xi$



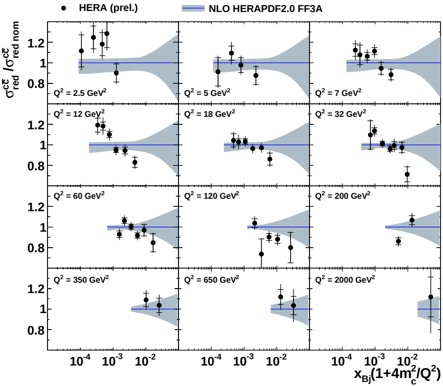


Cross sections as function of $x(1+4m^2/Q^2)$

$\boldsymbol{x_{\mathrm{Bj}}}$

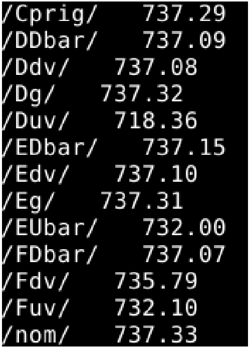


$\boldsymbol{x_{\mathrm{Bj}}(1+4m^2/Q^2) \approx \xi}$



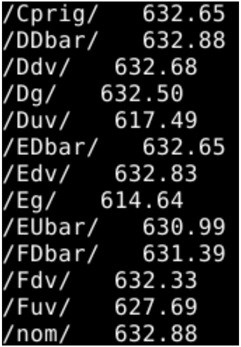
parametrisation scan for $x_{\min} > 0.015$ fit

DIS+c+b



```
/Cprig/    737.29
/DDbar/    737.09
/Ddv/    737.08
/Dg/    737.32
/Duv/    718.36
/EDbar/    737.15
/Edv/    737.10
/Eg/    737.31
/EUbar/    732.00
/FDbar/    737.07
/Fdv/    735.79
/Fuv/    732.10
/nom/    737.33
```

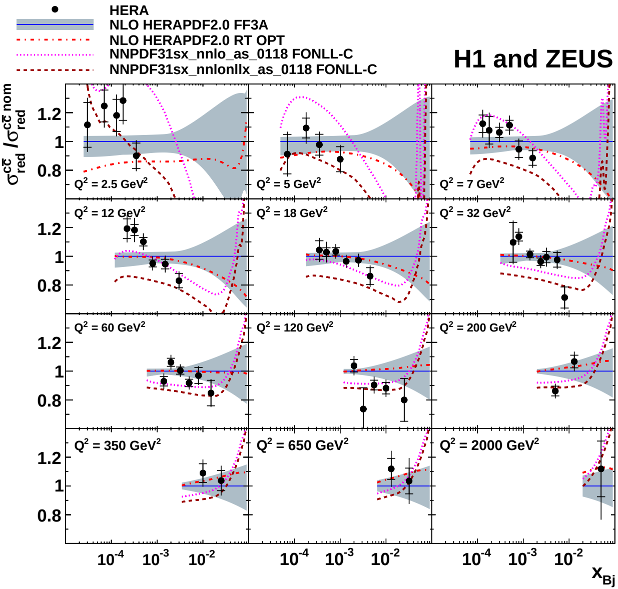
DIS



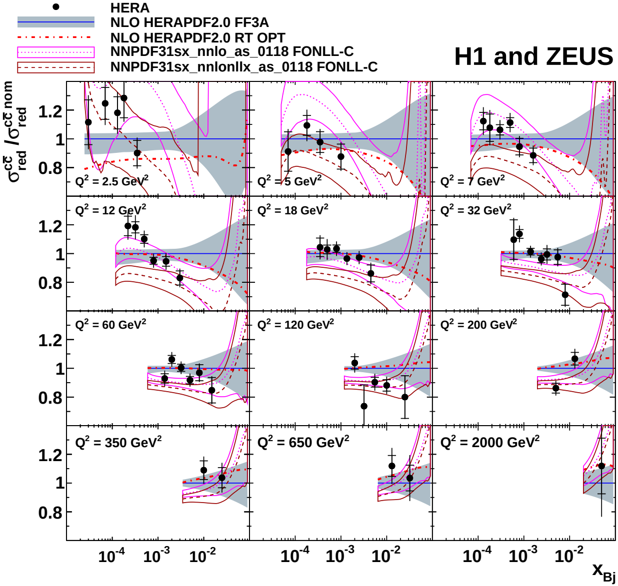
```
/Cprig/    632.65
/DDbar/    632.88
/Ddv/    632.68
/Dg/    632.50
/Duv/    617.49
/EDbar/    632.65
/Edv/    632.83
/Eg/    614.64
/EUbar/    630.99
/FDbar/    631.39
/Fdv/    632.33
/Fuv/    627.69
/nom/    632.88
```

- these are χ^2 values for DIS+c+b and DIS fits with $x_{\min} > 0.015$ cut on inclusive data
- ‘nom’ is nominal 14p parametrisation
- each other entry is when one parameter released (not all fits converged to physcial solutions, e.g. DIS Eg free fir produces negative gluon at $x \sim 1$)
- in general, very similar results to the nominal fit (in particular Duv)

Comparison to NNPDF: lowest Q^2 bin



Comparison to NNPDF: uncertainties



PDF parameters for fits with $x_{\min}$ cuts

Parameter	DIS+c+b x min=0.01	DIS+c+b 12p x min=0.01	DIS x min=0.01	DIS x min=0.01 fixed m	DIS 12p x min=0.01 fixed m
'Bg'	−0.03 ± 0.11	−0.037 ± 0.051	−0.42 ± 0.28	−0.32 ± 0.33	0.16 ± 0.13
'Cg'	11.6 ± 2.8	7.16 ± 0.84	4.3 ± 1.8	4.1 ± 2.1	9.0 ± 1.5
'Aprig'	9 ± 10	-	0.10 ± 0.17	0.09 ± 0.19	-
'Bprig'	0.12 ± 0.19	0.1080	−0.77 ± 0.12	−0.72 ± 0.16	−0.7248
'Cprig'	25.00	25.00	25.00	25.00	25.00
'Buv'	0.693 ± 0.038	0.683 ± 0.042	0.721 ± 0.042	0.708 ± 0.043	0.692 ± 0.042
'Cuv'	4.831 ± 0.085	4.861 ± 0.086	4.844 ± 0.087	4.862 ± 0.086	4.856 ± 0.085
'Euv'	13.0 ± 2.3	13.8 ± 2.7	12.1 ± 2.3	12.9 ± 2.5	13.4 ± 2.5
'Bdv'	0.89 ± 0.10	0.90 ± 0.11	0.95 ± 0.10	0.90 ± 0.11	0.87 ± 0.11
'Cdv'	4.63 ± 0.43	4.67 ± 0.50	4.76 ± 0.47	4.58 ± 0.51	4.51 ± 0.46
'CUbar'	8.00 ± 0.80	7.86 ± 0.99	7.9 ± 1.3	7.8 ± 1.2	8.06 ± 0.88
'DUbar'	12.8 ± 6.7	6.1 ± 4.0	3.3 ± 3.7	4.7 ± 4.6	10.8 ± 6.6
'ADbar'	0.178 ± 0.073	0.321 ± 0.090	0.42 ± 0.13	0.29 ± 0.11	0.200 ± 0.096
'BDbar'	−0.165 ± 0.081	−0.046 ± 0.056	−0.010 ± 0.066	−0.062 ± 0.079	−0.132 ± 0.098
'CDbar'	7.4 ± 2.7	11.6 ± 3.3	14.2 ± 3.9	10.4 ± 3.4	7.5 ± 3.0
'alphas'	0.1060	0.1060	0.1060	0.1060	0.1060
'fs'	0.4000	0.4000	0.4000	0.4000	0.4000
'mc'	1.400 ± 0.014	1.384 ± 0.052	1.924 ± 0.048	1.270	1.270
'mb'	4.10 ± 0.10	4.10 ± 0.11	9.4 ± 3.3	4.170	4.170

$$
\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}\\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}(1+E_{u_v}x^2)\\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}\\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}(1+D_{\bar{U}}x)\\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}
\end{aligned}
$$