Propagation of systematics

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Why do we need systematics?

Example Measurement of the Higgs boson mass

Measurement 1

m=121.4 GeV

Measurement 2

m=126.0 GeV

Combined measurement

m=123.7 GeV

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Why do we need systematics?

Example Measurement of the Higgs boson mass

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Adding uncertainties

Measurement 1

m=121.4±0.8 GeV

Measurement 2

m=126.0±0.4 GeV

Combined measurement

m=123.7±0.45 GeV

$$(\delta \overline{x})^2 = \sum_{i=1}^{N}\left(\frac{\partial \overline{x}}{\partial x_i}\delta x_i\right)^2 = \sum_{i=1}^{N}\left(\frac{\delta x_i}{N}\right)^2$$

Why do we need systematics?

Example Measurement of the Higgs boson mass

$$\bar{x} = \sum_{i=1} \frac{x_i}{\sigma_i^2} \Big/ \sum_i \frac{1}{\sigma_i^2}$$

Weighted average!

Measurement 1

m=121.4±0.8 GeV

Measurement 2

m=126.0±0.4 GeV

$$(\sigma_{\bar{x}})^2 = \frac{\sum_{i=1} (\sigma_i^{-2}\sigma_i)^2}{(\sum_i \sigma_i^{-2})^2} \implies \sigma_{\bar{x}} = \sqrt{\frac{1}{\sum_i \sigma_i^{-2}}}$$

Combined measurement

m=125.1±0.36 GeV

Why do we need systematics?

Example Measurement of the Higgs boson mass

$$\bar{x} = \sum_{i=1} \frac{x_i}{\sigma_i^2} \Big/ \sum_i \frac{1}{\sigma_i^2}$$

Weighted average!

Measurement 1

m=121.4±0.8 GeV

Measurement 2

m=126.0±0.4 GeV

$$(\sigma_{\bar{x}})^2 = \frac{\sum_{i=1} (\sigma_i^{-2}\,\sigma_i)^2}{(\sum_i \sigma_i^{-2})^2} \Longrightarrow \sigma_{\bar{x}} = \sqrt{\frac{1}{\sum_i \sigma_i^{-2}}}$$

Combined measurement

m=125.1±0.36 GeV

Compare to: m=123.7±0.45 GeV

Why do we need systematics?

Example Measurement of the Higgs boson mass

What if the errors are not independent? As is the case for (some) systematics

Measurement 1

m=121.4±0.8 GeV

Measurement 2

m=126.0±0.4 GeV

Combined measurement

m=? GeV

Incorporate systematics

Question How can we incorporate the systematic error due to using an incorrect model?

Incorporate systematics

Question How can we incorporate the systematic error due to using an incorrect model?

Answer Improve the model - introduce more adjustable parameters into the model

Difficult to decide how to introduce the additional parameters.

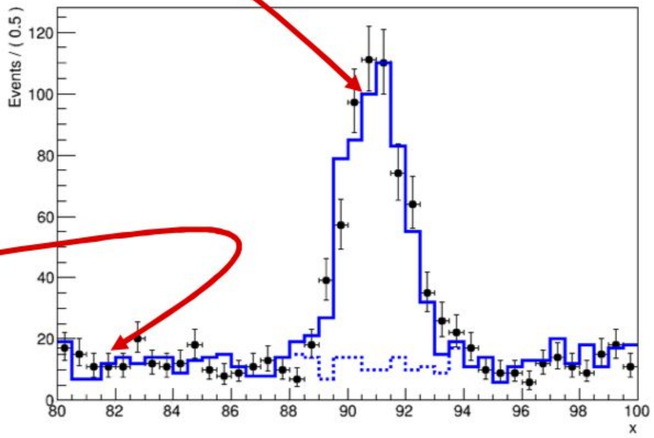
Definition of systematic uncertainty

"Systematic uncertainties are all uncertainties that are not directly due to the statistics of the data."

Simple (ideal) example

Create a likelihood function for a measurement where signal and background have perfectly known properties

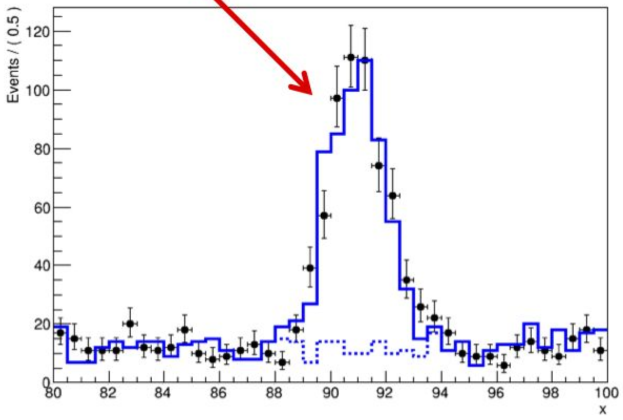
$$L(\vec{N} \mid \mu) = \prod_{bins} Poisson(N_i \mid \mu \cdot \tilde{s}_i + \tilde{b}_i)$$

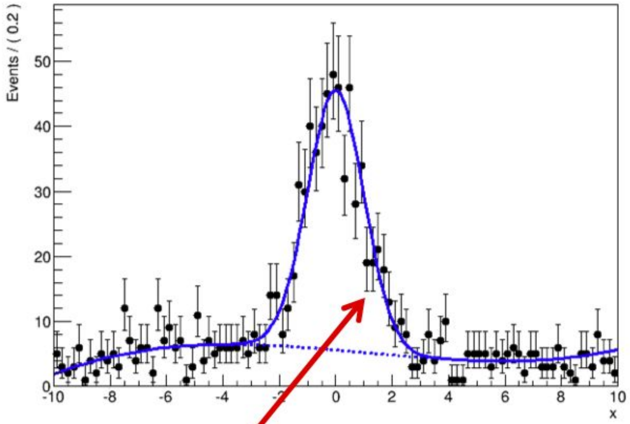


Modelling uncertainties

Additional model parameters $\longrightarrow$ Nuisance Parameters

$$L(\vec{N} \mid \mu) = \prod_{bins} Poisson(N_i \mid \mu \cdot \tilde{s}_i + \tilde{b}_i)$$





$$L(x \mid f, m, \sigma, a_0, a_1, a_2) = fG(x, m, \sigma) + (1 - f)Poly(x, a_0, a_1, a_2)$$

Modelling uncertainties

Additional model parameters ⟶ Nuisance Parameters

- Is this model correct?
- Is it flexible enough?
- How do model parameters connect to known detector/theory uncertainties?

Better to add parameters based on known systematic uncertainties on the analysis.

$$L(x \mid f, m, \sigma, a_0, a_1, a_2) = fG(x, m, \sigma) + (1 - f)Poly(x, a_0, a_1, a_2)$$

Typical systematic uncertainties

Uncertainty from the statics of the MC simulation

- Effect of uncertainties on MC sample size

Theory related

- Difference between simulators (for example: Sherpa vs MG5)
- Uncertainty on the cross-section

Detector simulation related

- Jet energy scale
- B-tagging efficiency

Naive approach to systematics

Vary setting ⟶ rerun analysis ⟶ observe the difference

- Make a nominal measurement
- Change a setting ‘1 sigma’ up and down ⟶ redo measurement
- Systematic uncertainty assumed to be propagated effect of shifted setting

$$\mu = \underbrace{\mu_{nom} \pm \sigma_{stat}} \pm \underbrace{(\mu_{syst}^{up} - \mu_{syst}^{down})/2} \pm \ldots$$

From statistical analysis

Systematic uncertainty from error propagation

Naive approach to systematics

Vary setting ⟶ rerun analysis ⟶ observe the difference

Pro Easy to do

Con loss of information

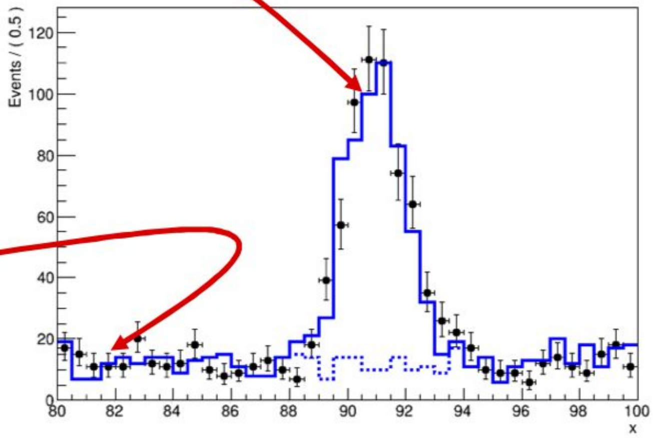
Solution ‘Profiling’ Incorporate a description of systematic uncertainties in the likelihood function. How?

Back to simple (ideal) example

Model with fixed b

Likelihood is defined as P(observed data|theory)

$$L(\vec{N} \mid \mu) = \prod_{bins} Poisson(N_i \mid \mu \cdot \tilde{s}_i + \tilde{b}_i)$$



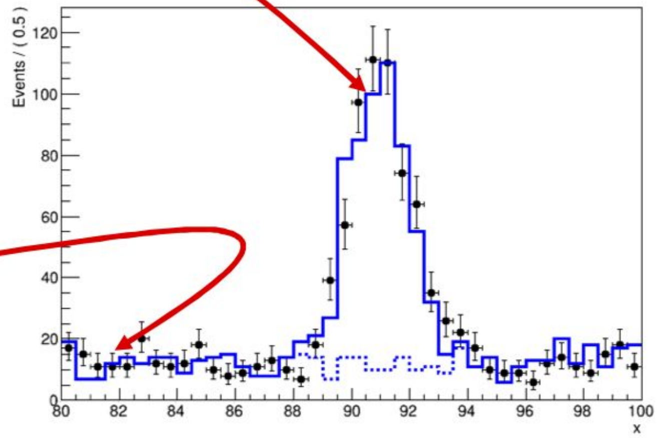
Back to simple (ideal) example

Now consider *b* to be uncertain

L(N | s) ⟶ L(N | s,b)

Experimental data is insufficient to constrain both *s* and *b* ⟶ Need to add additional measurement to constrain *b*

$$L(\vec{N} \mid \mu) = \prod_{bins} Poisson(N_i \mid \mu \cdot \tilde{s}_i + \tilde{b}_i)$$



Sideband measurement

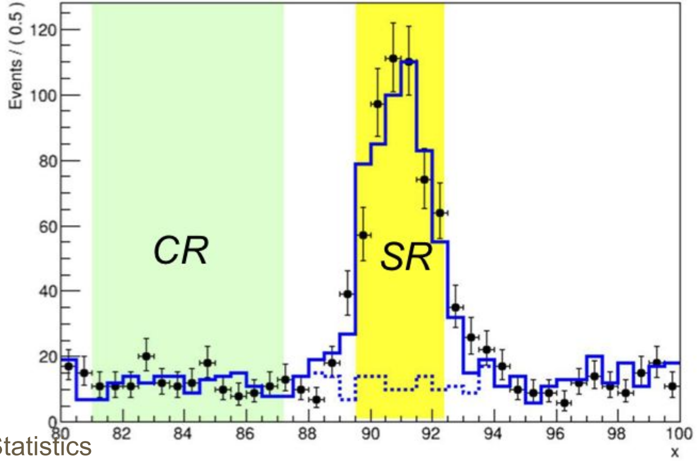
Use a Control Region (CR) to estimate level of background in a Signal Region (SR)

Define *b* as the amount of background in the SR

Define τ as a scale factor for the difference in size between SR and CR

$$L_{SR}(s,b) = Poisson(N_{SR} \mid s+b)$$

$$L_{CR}(b) = Poisson(N_{CR} \mid \tilde{\tau} \cdot b)$$



Simultaneous fit to the full likelihood

$$L_{full}(s,b) = Poisson(N_{SR} \mid s+b) \cdot Poisson(N_{CR} \mid \tilde{\tau} \cdot b)$$

From sideband to systematic uncertainty

Sideband is not a systematic uncertainty

$$L_{full}(s,b) = Poisson(N_{SR} \mid s+b) \cdot Poisson(N_{CR} \mid \tilde{\tau} \cdot b)$$

If *b* is taken from MC simulation

- For example: 8% cross-section uncertainty (systematic)

'Measured background rate by MC simulation'

$$L_{full}(s,b) = Poisson(N_{SR} \mid s+b) \cdot \underbrace{Gauss(\tilde{b} \mid b, 0.08)}$$

'Subsidiary measurement' of background rate

→ Subsidiary measurement

Definition of systematic uncertainty

“Systematic uncertainties are all uncertainties that are not directly due to the statistics of the data.”

What is included in “the statistics of the data”?

- Subsidiary measurements?
- Control measurements?
- Calibrations?

→ Can model systematics like sidebands in the likelihoods

Definition of systematic uncertainty

Systematic uncertainty includes

- Parameter(s) of which the true value is unknown
- A model that describes the effect of those parameters on the measurement
- A subsidiary measurement

⟶ Implies a specific distribution: Gaussian, Poisson or other

Slang

Profiled likelihood

$$L(N,0 \mid s,\alpha) = Poisson(N \mid s + b(\alpha)) \cdot Gauss(0 \mid \alpha, 1)$$

Where the nuisance parameters are “profiled”

Constraint term

$$Gauss(0 \mid \alpha, 1)$$

It “constrains” the parameter α

Shape systematics

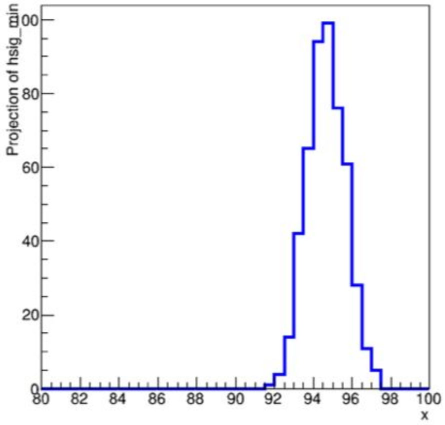
So far we looked at counting measurements.

Same technique for shape fits.

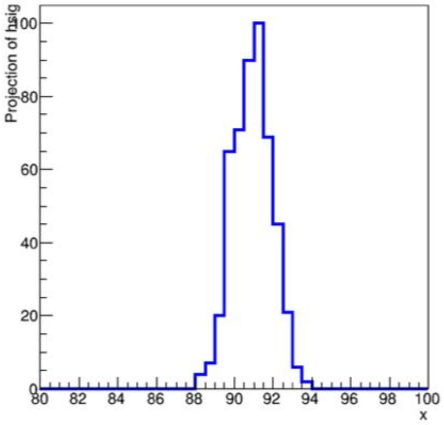
Need to define a Morphing algorithm to define s(x) for every value of a.

More on this tomorrow!

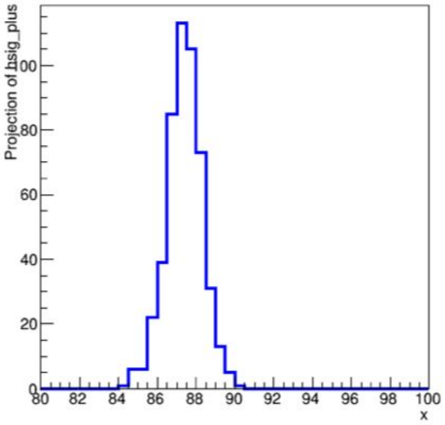
s(x)|a=-1



s(x)|a=0



s(x)|a=1



Overconstrained

All systematic uncertainties are calculated under the assumption that your model is correct.

- Some systematic uncertainties are not described well by one parameter.
- Theory uncertainties: Difficulties are not in modelling procedure but in what we know.

Try to think about how many degrees of freedom your systematic uncertainty has: Is the true point covered by your NP?

Summary

Construct a likelihood function that describes your measurement

Nuisance parameters can be incorporated by Profiling

- Not dependent on method (Frequentist/Bayesian)

Important to check your profiled likelihood model

- Overconstrained

Back-up

Slide from Wouter

Modeling a detector calibration uncertainty

$$L_{full}(s,b) = Poisson(N_{SR} \mid s+b) \cdot Gauss(\tilde{b} \mid b, 0.08)$$

- Now consider a detector uncertainty, e.g. jet energy scale calibration, which can affect the analysis acceptance in a non-trivial way (unlike the cross-section example)

Nominal calibration

Signal rate (our parameter of interest)

Assumed calibration

$$L(N,\tilde{\alpha} \mid s,\alpha) = Poisson(N \mid s + \tilde{b}(\alpha/\tilde{\alpha})\cdot 2)) \cdot Gauss(\tilde{\alpha} \mid \alpha, \sigma_{\alpha})$$

Observed event count

Nominal background expectation from MC (a constant), obtained with a=a˜

Response function for JES uncertainty (a 1% JES change results in a 2% acceptance change)

Uncertainty on nominal calibration (here 5%)

“Subsidiary measurement” Encodes ‘external knowledge’ on JES calibration

Modeling a detector calibration uncertainty

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- Simplify expression by renormalizing “subsidiary measurement”

$$Gauss(\tilde{\alpha}\,|\,\alpha,\sigma_{\alpha})$$

Signal rate (our parameter of interest)

$$L(N\,|\,s,\alpha) = Poisson(N\,|\,s+\tilde{b}(1+0.1\alpha))\cdot Gauss(0\,|\,\alpha,1)$$

Observed event count

Nominal background expectation from MC (a constant)

Response function for normalized JES parameter
[a unit change in α – a 5% JES change – still results in a 10% acceptance change]

“Normalized subsidiary measurement”

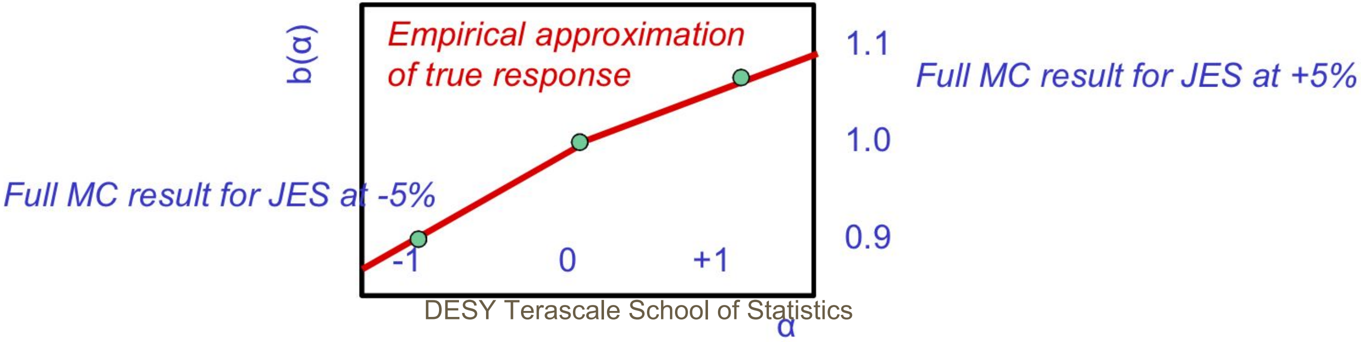
The scale of parameter α is now chosen such that values ±1 corresponds to the nominal uncertainty (in this example 5%)

The response function as empirical model of full simulation

Slide from Wouter

$$L(N,0|s,\alpha) = Poisson(N|s+b(\alpha)) \cdot Gauss(0|\alpha,1)$$

- Note that the response function is generally not linear, but can in principle *always be determined by your full simulation chain*
  - But you cannot run your full simulation chain for any arbitrary 'systematic uncertainty variation' ⇨ Too much time consuming
  - Typically, run full MC chain for nominal and ±1σ variation of systematic uncertainty, and approximate response for other values of NP with interpolation
  - For example run at nominal JES and with JES shifted up and down by ±5%



Piecewise linear interpolation

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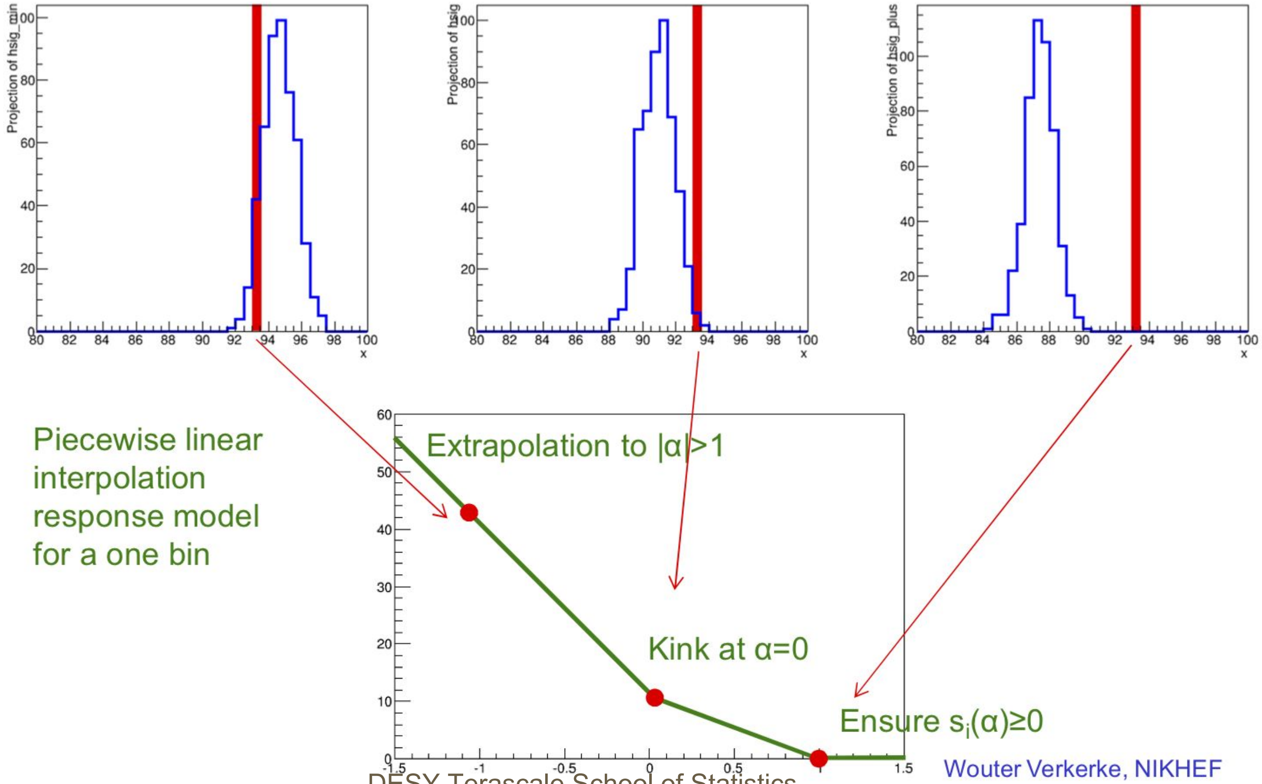
- Simplest solution is piece-wise linear interpolation for each bin



Visualization of bin-by-bin linear interpolation of distribution

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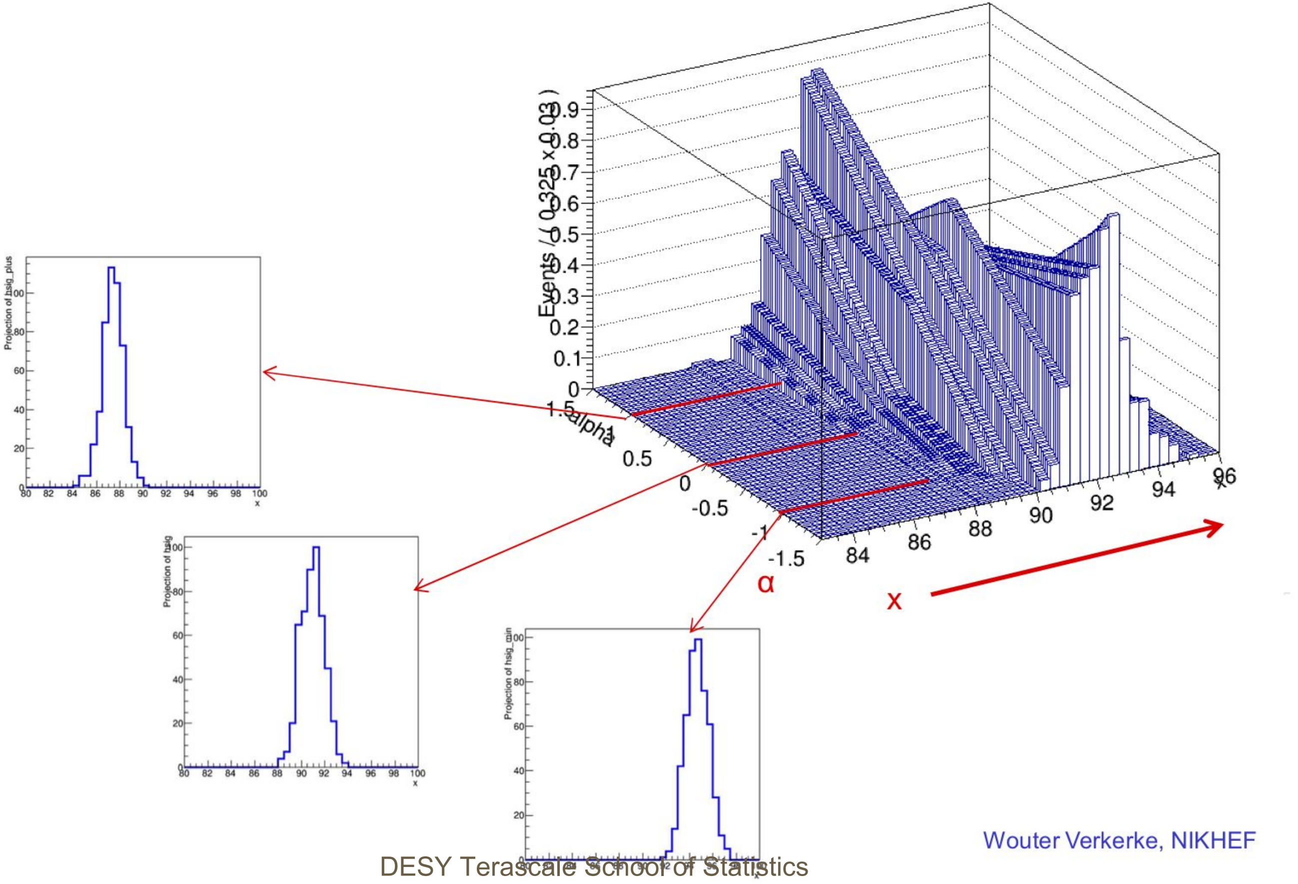

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