Charmonia production and gluon distribution in the proton

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xFitter
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Outline

1. Exclusive production of J/ψ meson
   - Photoproduction in γp collisions
   - Photoproduction in pp and $p\bar{p}$ collisions
2. Semiexclusive production of J/ψ meson
   - Diffractive photoproduction with electromagnetic dissociation
   - Diffractive resonance excitation
   - Diffractive partonic excitation
3. Inclusive production
   - J/Ψ production
   - J/Ψ production from radiative decay of χ_c mesons
4. Conclusions

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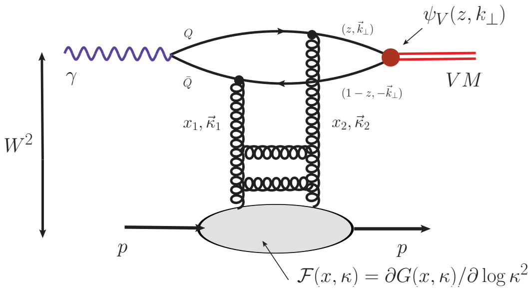
Diagram for exclusive photoproduction $\gamma p \to J/\Psi p$



- $\psi_v(z, k^2)$ → wave function of the vector meson
- $\mathcal{F}(x, \kappa^2)$ → unintegrated gluon distribution function
- $x \sim (Q^2 + M^2_{J/\Psi})/W^2$

The production amplitude for $\gamma p \to J/\Psi p$

The full amplitude:

$$\mathcal{M}_T(W,\Delta^2) = (i + \rho_T)\, \Im m \mathcal{M}_T(W,\Delta^2=0,Q^2=0)\, \exp\left(-\frac{B(W)\Delta^2}{2}\right)$$

The imaginary part of the amplitude can be written as:

$$\Im m\, \mathcal{M}_T(W,\Delta^2=0,Q^2=0) = W^2 \frac{c_v\sqrt{4\pi\alpha_{em}}}{4\pi^2} \int_0^1 \frac{dz}{z(1-z)} \int_0^\infty \pi dk^2 \psi_V(z,k^2)$$
$$\int_0^\infty \frac{\pi d\kappa^2}{\kappa^4} \alpha_S(q^2) \mathcal{F}(x_{eff},\kappa^2) \Big(A_0(z,k^2)\, W_0(k^2,\kappa^2) + A_1(z,k^2)\, W_1(k^2,\kappa^2) \Big)$$

Real part

$$\rho_T = \frac{\Re e \mathcal{M}_T}{\Im m \mathcal{M}_T} = \frac{\pi}{2}\Delta_{\mathbb{P}}$$

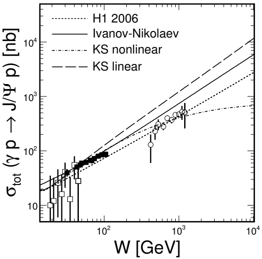
Slope parameter

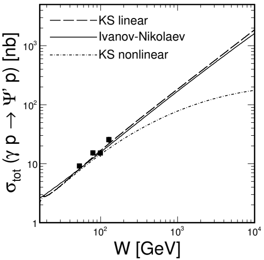
$$B(W) = B_0 + 2\alpha'_{eff} \log\left(\frac{W^2}{W_0^2}\right)$$

Total cross section for $\gamma p \to J/\Psi(\Psi') \, p$

Total cross section can be written as:

$$\sigma_T(\gamma p \to J/\Psi p) = \frac{1+\rho_T{}^2}{16\pi B(W)} \left| \frac{\Im m \mathcal{M}_T(W, \Delta^2 = 0, Q^2 = 0)}{W^2} \right|^2$$

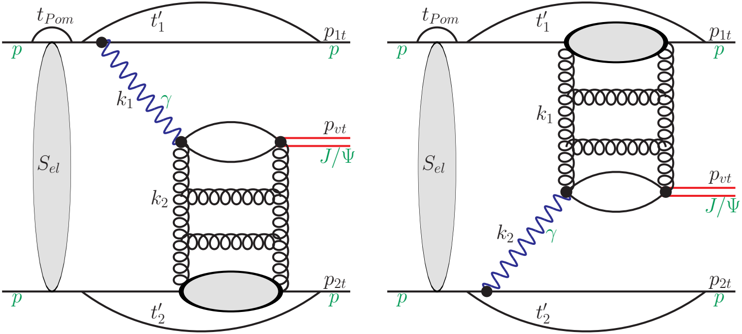




HERA data and extracted LHCb data

- H1 Collaboration, Phys. Lett. B541 (2002) 251
- H1 Collaboration, Eur. Phys. J. C46 (2006) 585
- H1 Collaboration, Eur. Phys. J. C73 (2013) 2466

Diagram for exclusive production of $J/\Psi(\Psi')$ meson in proton-proton collisions



photon-Pomeron Pomeron-photon

Amplitude for process $pp \to p\, J/\Psi\, p$

Full apmlitude for $pp \longrightarrow pVp$

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{p}_1,\boldsymbol{p}_2) &= \int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\, S_{el}(\boldsymbol{k})\, \boldsymbol{M}^{(0)}(\boldsymbol{p}_1-\boldsymbol{k},\boldsymbol{p}_2+\boldsymbol{k}) \\ &= \boldsymbol{M}^{(0)}(\boldsymbol{p}_1,\boldsymbol{p}_2) - \delta\boldsymbol{M}(\boldsymbol{p}_1,\boldsymbol{p}_2) \end{aligned}$$

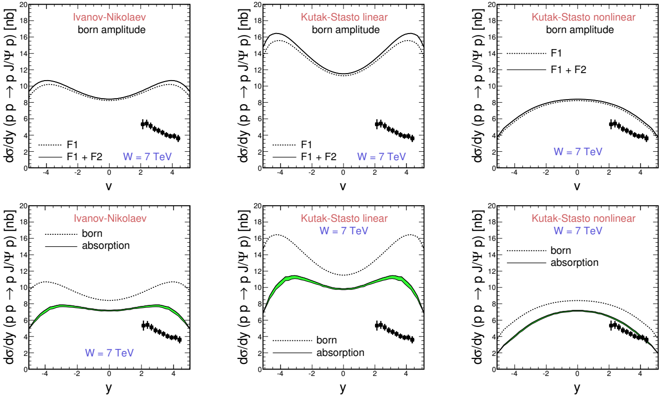
Amplitude without absorption

$$\begin{aligned} \boldsymbol{M}^{(0)}(\boldsymbol{p}_1,\boldsymbol{p}_2) &= e_1 \frac{2}{z_1}\frac{\boldsymbol{p}_1}{t_1}\mathcal{F}_{\lambda_1'\lambda_1}(\boldsymbol{p}_1,t_1)\mathcal{M}_{\gamma h_2\to Vh_2}(s_2,t_2,Q_1^2) \\ &+ e_2 \frac{2}{z_2}\frac{\boldsymbol{p}_2}{t_2}\mathcal{F}_{\lambda_2'\lambda_2}(\boldsymbol{p}_2,t_2)\mathcal{M}_{\gamma h_1\to Vh_1}(s_1,t_1,Q_2^2) \end{aligned}$$

Absorptive corrections for the amplitude

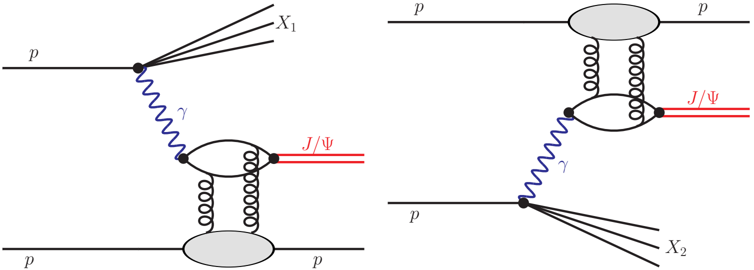
$$\delta\boldsymbol{M}(\boldsymbol{p}_1,\boldsymbol{p}_2) = \int \frac{d^2\boldsymbol{k}}{2(2\pi)^2}\, T(\boldsymbol{k})\, \boldsymbol{M}^{(0)}(\boldsymbol{p}_1-\boldsymbol{k},\boldsymbol{p}_2+\boldsymbol{k})$$

Rapidity distribution



- R. Aaij et al. (LHCb collaboration), J. Phys. **G40** (2013) 045001
- R. Aaij et al. (LHCb collaboration), arXiv:1401.3288 [hep-ex]
- At large p_t we get an enhancement factor of the cross section of order of 10
- Absorption must be included

Diagrams representation of the electromagnetic excitation



- The schematic diagrams representation of the electromagnetic excitation of one (left panel) or second (right panel) photon
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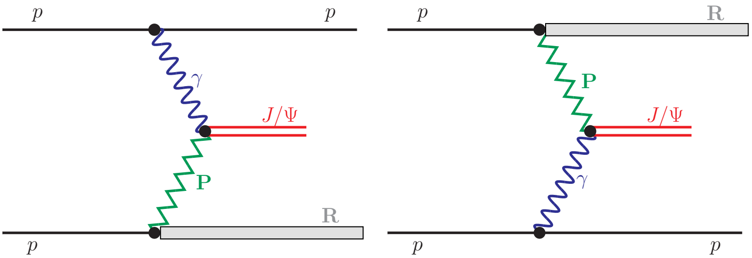
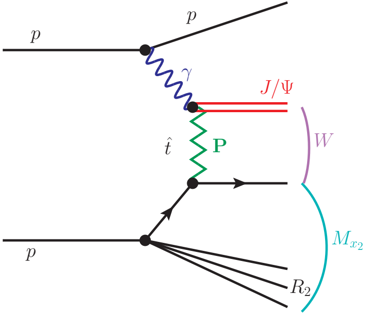
Difractive resonance with strong disociation



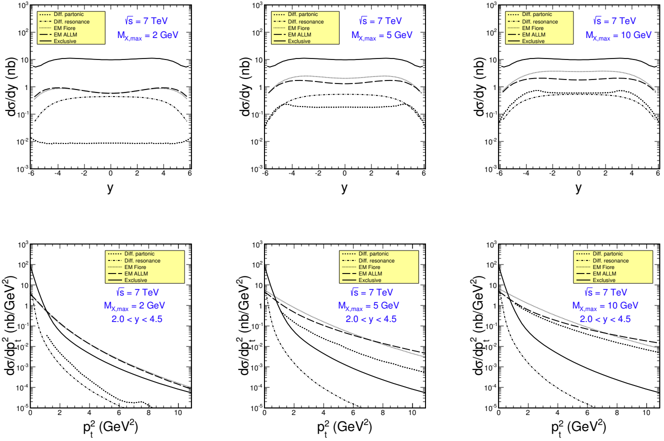
- low $p_T \to$ Dissociation into nucleon resonances/low mass continuum states. Dominated by $N^*(1680)$, $J^P = \frac{5}{2}^+$, $N^*(2220)$, $J^P = \frac{9}{2}^+$, $N^*(2700)$, $J^P = \frac{13}{2}^+$.
  A model by L.L. Jenkovszky, O.E. Kuprash, J.W. Lämsa, V.K. Magas and R. Orava (2011).
- large $p_T \to$ Incoherent diffractive photoproduction of J/ψ off partons. Large diffractive masses are possible here.

Difractive partonic with strong disociation



- dissociative production of vector mesons at large p_T probes the perturbative QCD Pomeron. (Ryskin, Forshaw et al.). An alternative to the “jet - gap - jet” type of processes.

Results



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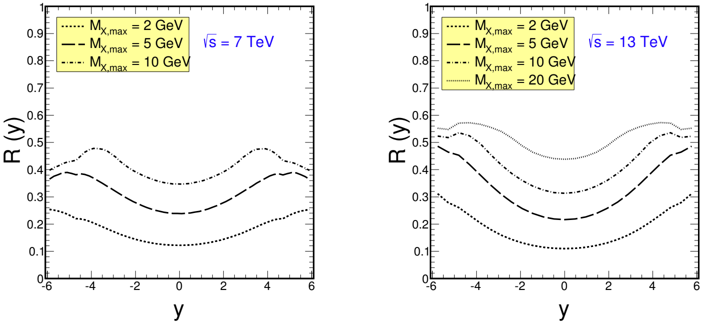
Ratio of dissociative to exclusive cross section



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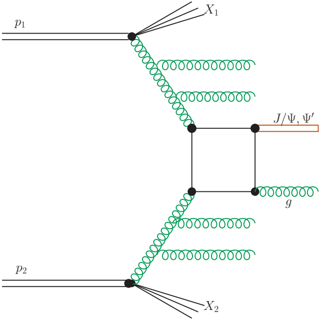
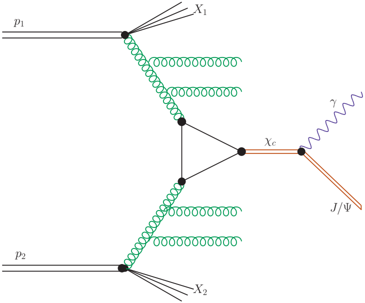
The main color-singlet mechanism of production of J/Ψ meson





- We restrict to gluon-gluon fusion mechanism (high energy)
- We use unintegrated gluon distribution from the KMR (Durham group) and KS (Kutak-Staśto)

Differential cross section for J/Ψ

- The differential cross section in the k_t factorization can be writen as:

$$\frac{d\sigma(pp \to J/\psi g X)}{dy_{J/\psi} dy_g d^2p_{J/\psi,t} d^2p_{g,t}} = \frac{1}{16\pi^2 \hat{s}^2} \int \frac{d^2q_{1t}}{\pi} \frac{d^2q_{2t}}{\pi} \overline{|\mathcal{M}_{\mathbf{g^* g^* \to Vg}}|^2} \times \times \delta^2 \left(\mathbf{q_{1t}} + \mathbf{q_{2t}} - \mathbf{p_{V,t}} - \mathbf{p_{g,t}}\right) \mathcal{F}_{\mathbf{g}}(\mathbf{x_1}, \mathbf{q_{1t}^2}, \mu_{\mathbf{F}}^2) \mathcal{F}_{\mathbf{g}}(\mathbf{x_2}, \mathbf{q_{2t}^2}, \mu_{\mathbf{F}}^2)$$

- **We calculate the dominant color-single $gg \to Vg$ contribution taking into account transverse momenta of initial gluons**
- The corresponding matrix element squared for the $gg \to Vg$ is

$$|\mathcal{M}_{\mathbf{gg \to Vg}}|^2 \propto \alpha_s^3 |\mathbf{R(0)}|^2$$

- Anna Cisek and Antoni Szczurek - Phys. Rev. **D97** (2018) 034035

χ_c production

* In the k_t-factorization approach the leading-order **cross section for the χ_c meson production** can be written as:

$$\sigma_{\mathbf{pp}\to\chi_\mathbf{c}} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\frac{d^2q_{1t}}{\pi}\frac{d^2q_{2t}}{\pi}\,\delta\left((q_1+q_2)^2 - M^2_{\chi_c}\right)\sigma_{gg\to\chi_c}(x_1,x_2,q_1,q_2)$$
$$\times\mathcal{F}_\mathbf{g}(\mathbf{x_1},\mathbf{q^2_{1t}},\mu^2_\mathbf{F})\mathcal{F}_\mathbf{g}(\mathbf{x_2},\mathbf{q^2_{2t}},\mu^2_\mathbf{F})$$

* The matrix element squared for the $gg\to\chi_c$ subprocess is

$$|\mathcal{M}_{\mathbf{gg}\to\chi_\mathbf{c}}|^\mathbf{2} \propto \alpha^\mathbf{2}_\mathbf{s}|\mathbf{R}'(\mathbf{0})|^\mathbf{2}$$

* For running coupling constants we choose:

$$\alpha_s^2 \to \alpha_s(\mu_\mathbf{1}^\mathbf{2})\alpha_s(\mu_\mathbf{2}^\mathbf{2})$$

where $\mu_\mathbf{1}^\mathbf{2} = \mathbf{max}(\mathbf{q^2_{1t}},\mathbf{m^2_t})$ and $\mu_\mathbf{2}^\mathbf{2} = \mathbf{max}(\mathbf{q^2_{2t}},\mathbf{m^2_t})$

Cross section for χ_c

- After some manipulation:

$$\sigma_{pp\to\chi_c} = \int dy d^2p_t d^2q_t \frac{1}{s\mathbf{x_1}\mathbf{x_2}} \frac{1}{m^2_{t,\chi_c}} \overline{|\mathcal{M}_{g^*g^*\to\chi_c}|^2} \mathcal{F}_g(\mathbf{x_1}, \mathbf{q^2_{1t}}, \mu^2_F) \mathcal{F}_g(\mathbf{x_2}, \mathbf{q^2_{2t}}, \mu^2_F)/4$$

- Which can be also used to calculate rapidity and transverse momentum distribution of the χ_c mesons
- In the last equation:

$$\mathbf{p_t} = \mathbf{q_{1t}} + \mathbf{q_{2t}} \qquad \mathbf{q_t} = \mathbf{q_{1t}} - \mathbf{q_{2t}}$$

$$\mathbf{x_1} = \frac{\mathbf{m_{t,\chi_c}}}{\sqrt{\mathbf{s}}} \exp(\mathbf{y}) \qquad \mathbf{x_2} = \frac{\mathbf{m_{t,\chi_c}}}{\sqrt{\mathbf{s}}} \exp(-\mathbf{y})$$

- The factor $\frac{1}{4}$ is the jacobian of transformation from $(\mathbf{q_{1t}}, \mathbf{q_{2t}})$ to $(\mathbf{p_t}, \mathbf{q_t})$ variables

α_s - scale

For running coupling constants we choose different scale:

- χ_c

$\alpha_s^2 \to \alpha_s(\mu_1^2)\alpha_s(\mu_2^2)$

1. prescription 1
   - $\mu_1^2 = \mathbf{q_{1t}^2}$
   - $\mu_2^2 = \mathbf{q_{2t}^2}$
2. prescription 2
   - $\mu_1^2 = \mathbf{max(q_{1t}^2, m_t^2)}$
   - $\mu_2^2 = \mathbf{max(q_{2t}^2, m_t^2)}$

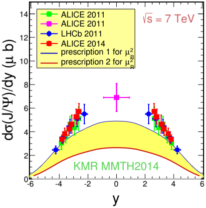
- J/Ψ

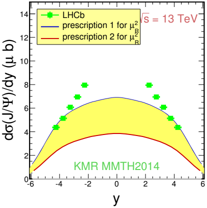
$\alpha_s^3 \to \alpha_s(\mu_1^2)\alpha_s(\mu_2^2)\alpha_s(\mu_3^2)$

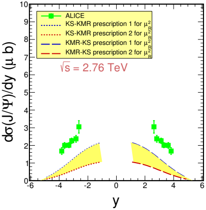
1. prescription 1
   - $\mu_1^2 = \mathbf{q_{1t}^2}$
   - $\mu_2^2 = \mathbf{q_{2t}^2}$
   - $\mu_3^2 = \mathbf{m_t^2}$
2. prescription 2
   - $\mu_1^2 = \mathbf{max(q_{1t}^2, m_t^2)}$
   - $\mu_2^2 = \mathbf{max(q_{2t}^2, m_t^2)}$
   - $\mu_3^2 = \mathbf{m_t^2}$

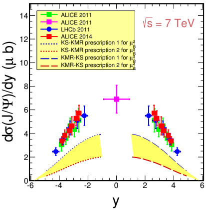
rapidity dependence for J/Ψ meson (direct)

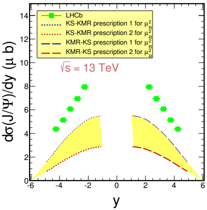
- **2,76 TeV** - B. Abelev et al.; Phys. Let. B. **718** (2012) 295-306
- **7 TeV** - B. Abelev et al.; Eur.Phys. J. C. **74** (2014) 2974
  7 TeV - K. Aamodt et al.; Phys. Let. B. **704** (2011) 442
  7 TeV - R. Aaij et al.; Eur.Phys. J. C. **71** (2011) 1645
- **13 TeV** - R. Aaij et al.; JHEP 1510 (2015) 172



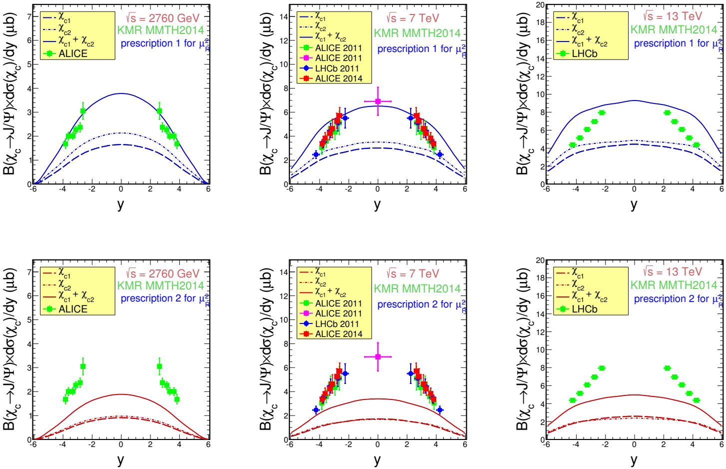




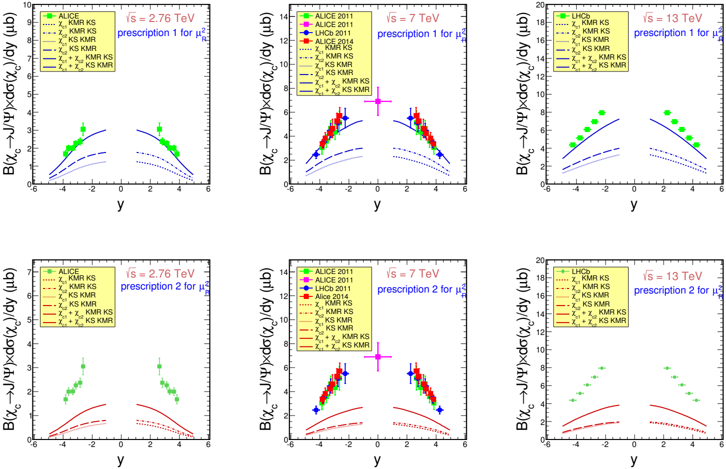




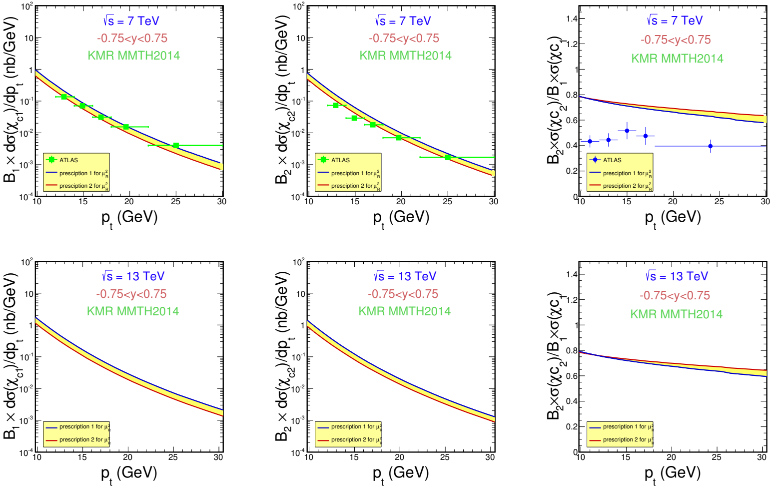
rapidity distribution J/Ψ from χ_c decays



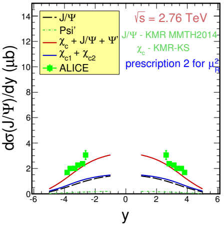
rapidity distribution J/Ψ from χ_c decays

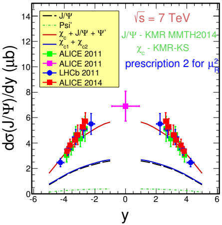


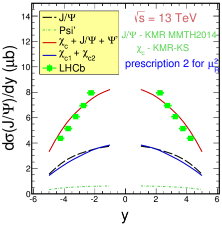
p_t distribution for χ_c meson



rapidity dependence







- Better solution is to take prescription 2 for α_s scale
- The best solution is to take KMR UGDF for J/ψ and ψ' mesons and mixed UGDfs for χ_c mesons
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Conclusions

- We have compared our results with **HERA** ($\gamma p \longrightarrow J/\Psi(\Psi')\, p$) and **LHCb** ($pp \longrightarrow p\, J/\Psi\, p$) data.
- $d\sigma/dp_t$ is interesting (**spin flip**, **Pomeron-Odderon** fusion) but difficult to measure.
- **Absorptive corrections** have been included.
- In γ-Pomeron fusion reactions in proton-proton scattering, **electromagnetic dissociation is of the same size as strong, diffractive dissociation**. It even dominates in some regions of the phase space.
- Electromagnetic dissociation is calculable from F_2 data. **Resonance excitation is important at low excited masses** .

Conclusions

- Diffractive dissociation requires modelling, there is only little data to constrain it. **The resonance contribution is concentrated at very small t**, similar to the coherent elastic contribution
- **We have calculated the color-singlet contribution** in the NRQCD k_t-factorization
- **We have compared our results with ALICE and LHCb data for J/Ψ and ATLAS for χ_{c1} and χ_{c2}**
- Our results in rapidity are consistent with experimental data for KMR UGDF and better when nonlinear effects are included
- **Data at 13 TeV may require saturation effects in the small-x gluon**

Backup

Helicity conserving and helicity flip amplitudes

The full amplitude for the $pp \to pVp$ process can be written as

$$\mathcal{M}^{\lambda_1\lambda_2\to\lambda_1'\lambda_2'\lambda_V}_{h_1h_2\to h_1h_2V}(s,s_1,s_2,t_1,t_2) = \mathcal{M}_{\gamma\mathbf{P}} + \mathcal{M}_{\mathbf{P}\gamma}$$

$$= \langle p_1',\lambda_1'|J_\mu|p_1,\lambda_1\rangle\epsilon^*_\mu(q_1,\lambda_V)\frac{\sqrt{4\pi\alpha_{em}}}{t_1}\mathcal{M}^{\lambda_{\gamma^*}\lambda_2\to\lambda_V\lambda_2}_{\gamma^*h_2\to Vh_2}(s_2,t_2,Q_1^2)$$

$$+\langle p_2',\lambda_2'|J_\mu|p_2,\lambda_2\rangle\epsilon^*_\mu(q_2,\lambda_V)\frac{\sqrt{4\pi\alpha_{em}}}{t_2}\mathcal{M}^{\lambda_{\gamma^*}\lambda_1\to\lambda_V\lambda_1}_{\gamma^*h_1\to Vh_1}(s_1,t_1,Q_2^2)$$

Simple structure:

$$\langle p_1',\lambda_1'|J_\mu|p_1,\lambda_1\rangle\epsilon^*_\mu(q_1,\lambda_V) = \frac{(\boldsymbol{e}^{*(\lambda_V)}\boldsymbol{q}_1)}{\sqrt{1-z_1}}\,\frac{2}{z_1}\cdot$$

$$\cdot\chi^\dagger_{\lambda'}\left\{F_1(Q_1^2) - \frac{i\kappa_p F_2(Q_1^2)}{2m_p}(\boldsymbol{\sigma}_1\cdot[\boldsymbol{q}_1,\boldsymbol{n}])\right\}\chi_\lambda$$

- The coupling with F_1 - proton helicity conserving, F_2 - proton helicity flip

Diffractive production with electromagnetic dissociation

The cross section for such proces can be written as:

$$\frac{d\sigma(pp \to XVp; s)}{dy d^2\boldsymbol{p}} = \int \frac{d^2\boldsymbol{q}}{\pi \boldsymbol{q}^2} \mathcal{F}^{(\mathrm{in})}_{\gamma/p}(z_+, \boldsymbol{q}^2) \frac{1}{\pi} \frac{d\sigma^{\gamma^* p \to Vp}}{dt}(z_+ s, t = -(\boldsymbol{q} - \boldsymbol{p})^2) + (z_+ \leftrightarrow z_-)$$

$$z_\pm = e^{\pm y} \sqrt{\boldsymbol{p}^2 + m_V^2} / \sqrt{s}$$

Structure function of proton

$$\mathcal{F}^{(\mathrm{inel})}_{\gamma/p}(z, \boldsymbol{q}^2, M_X^2) = \frac{\alpha_{\mathrm{em}}}{\pi}(1 - z)\theta(M_X^2 - M_{\mathrm{thr}}^2) \frac{F_2(x_{Bj}, Q^2)}{M_X^2 + Q^2 - m_p^2} \cdot \left[\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + z(M_X^2 - m_p^2) + z^2 m_p^2} \right]^2$$

$$Q^2 = \frac{1}{1 - z} \left[\boldsymbol{q}^2 + z(M_X^2 - m_p^2) + z^2 m_p^2 \right], x_{Bj} = \frac{Q^2}{Q^2 + M_X^2 - m_p^2}$$

Difractive resonance with strong disociation

The large gap is proided by te Pomeron exchange, and we write the cross secion in such way:

$$\frac{d\sigma(\gamma p \to VX)}{dt dM_X^2} = \left(\frac{s_{\gamma p}}{M_X^2}\right)^{2\alpha_{\mathbf{P}}^{\mathrm{eff}}(t)-2} \cdot A_0 f_{\gamma\to V}^2(t) \cdot F(M_X^2, t)$$

The function $f_{\gamma\to V}(t) = \exp[B_{\gamma\to V} t/2]$ is a formfactor of the $\gamma \to V$ transition, while $F(M_X^2, t)$ contains the information on the dynamics of the diffractive dissociation.

$$F(M_X^2, t) = \frac{x(1-x)^2}{(M_X^2 - m_p^2)(1+\tau)^{3/2}} \left(\Im m A(M_X^2, t) + A_{\mathrm{Roper}}(M_X^2, t)\right)$$

$$x = \frac{|t|}{M_X^2 + |t|}, \; \tau = \frac{4 m_p^2 x^2}{|t|}$$

Difractive resonance with strong disociation

Explicitly, they contribute to the $p\mathbb{P} \to X$ amplitude as:

$$\Im m A(M_X^2, t) = \sum_{n=1,3} [f(t)]^{2(n+1)} \cdot \frac{\Im m\, \alpha(M_X^2)}{(J_n - \Re e\, \alpha(M_X^2))^2 + (\Im m\, \alpha(M_X^2))^2}$$

We can now compute the contribution from diffractive excitation of small masses from the formula

$$\frac{d\sigma(pp \to XVp; s)}{dy d^2\boldsymbol{p} dM_X^2} = \int \frac{d^2\boldsymbol{q}}{\pi \boldsymbol{q}^2} \mathcal{F}^{(\mathrm{el})}_{\gamma/p}(z_+, \boldsymbol{q}^2) \frac{1}{\pi} \frac{d\sigma(\gamma p \to VX)}{dt dM_X^2}(z_+ s) + (z_+ \leftrightarrow z_-)$$

$$\mathcal{F}^{(\mathrm{el})}_{\gamma/p}(z, \boldsymbol{q}^2) = \frac{\alpha_{\mathrm{em}}}{\pi}(1-z)\left[\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + z^2 m_p^2}\right]^2 \frac{4m_p^2 G_E^2(Q^2) + Q^2 G_M^2(Q^2)}{4m_p^2 + Q^2}$$

$$Q^2 = \frac{\boldsymbol{q}^2 + z^2 m_p^2}{1-z}$$

Difractive partonic with strong disociation

Cross section

$$\frac{d\sigma^{diff,partonic}_{pp\to Vj}}{dy_V dy_j d^2p_t} = \frac{1}{\pi} x_1 q_{\text{eff}}(x_1, \mu_F^2) x_2 \gamma_{el}(x_2) \frac{d\sigma(\gamma q \to Vq)}{d\hat{t}} + (x_1 \leftrightarrow x_2)$$

$$q_{\text{eff}}(x, \mu_F^2) = \frac{81}{16} g(x, \mu_F^2) + \sum_f \left[q_f(x, \mu_F^2) + \bar{q}_f(x, \mu_F^2) \right]$$

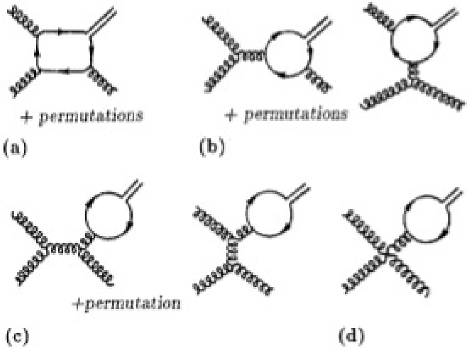
Factorization scale: $\mu_F^2 = m_V^2 + |\hat{t}|$

Simple formula for Pomeron-exchange

$$\frac{d\sigma_{\gamma q \to Vq}}{d\hat{t}} \propto \alpha_s^2(\bar{Q}_t^2) \alpha_s^2(|\hat{t}|) \frac{m_V^3 \Gamma(V \to l^+ l^-)}{(\bar{Q}_t^2)^4}$$

$\bar{Q}_t^2 = m_V^2 + |\hat{t}|$

Matrix elements for J/Ψ (Ψ')



$$\mathcal{M}_{\mathbf{a}}(\mathbf{gg} \to \mathbf{J}/\psi\mathbf{g}) = tr\{\epsilon_{\mathbf{1}}(\mathbf{p_c} - \mathbf{k_1} + m_c)\epsilon_{\mathbf{2}} \times (-\mathbf{p_c} - \mathbf{k_3} + m_c)\epsilon_{\mathbf{3}} J(S,L)\} C_\Psi$$
$$\times tr\{T^a T^b T^c T^d\}[k_1^2 - 2(p_c k_1)]^{-1} \times [k_3^2 - 2(p_{\bar{c}} k_3)]^{-1} + 5\ permutations$$

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Matrix elements for J/Ψ (Ψ')

$$\mathcal{M}_{\mathbf{b}}(\mathbf{gg} \to \mathbf{J}/\psi\mathbf{g}) = tr\{\gamma_\mu(p_{\bar{c}} - k_3 + m_c)\epsilon_{\mathbf{3}} J(S,L)\} \times G^3(k_1, \epsilon_{\mathbf{1}}, k_2, \epsilon_{\mathbf{2}}, -k, \mu) C_\Psi f^{abe} \times tr\{T^e T^c T^d\}[k^2]^{-1} \times [k_3^2 - 2(p_{\bar{c}} k_3)]^{-1} + 5\ permutations$$

$$\mathcal{M}_{\mathbf{c}}(\mathbf{gg} \to \mathbf{J}/\psi\mathbf{g}) = tr\{\gamma_\mu J(S,L)\} G^3(k_1, \epsilon_{\mathbf{1}}, k_2, \epsilon_{\mathbf{2}}, -k, \mu) \times G^3(-k_3, -\epsilon_{\mathbf{3}}, -p_\Psi, -\epsilon\text{-}, -k, \nu) C_\Psi f^{abe} f^{cfe} \times tr\{T^f T^d\}[k^2]^{-1} \times [m_\Psi^2]^{-1} + 2\ permutations$$

$$\mathcal{M}_{\mathbf{d}}(\mathbf{gg} \to \mathbf{J}/\psi\mathbf{g}) = tr\{\gamma_\nu J(S,L)\} G^{(4)A,B,C}(\epsilon_{\mathbf{1}}, \epsilon_{\mathbf{2}}, \epsilon_{\mathbf{3}}, \nu) C_\Psi \times tr\{T^f T^d\}[k^2]^{-1}[m_\Psi^2]^{-1}$$

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Matrix elements for χ_c

$$\overline{|\mathcal{A}(g^\star + g^\star \to \mathcal{H}[^3P_0^{(1)}]|^2} = \frac{8}{3}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{H}}[^3\mathbf{P}_0^{(1)}]\rangle}{M^5} \mathbf{F}^{[^3\mathbf{P}_0]}(\mathbf{t_1}, \mathbf{t_2}, \varphi)$$

$$\overline{|\mathcal{A}(g^\star + g^\star \to \mathcal{H}[^3P_1^{(1)}]|^2} = \frac{16}{3}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{H}}[^3\mathbf{P}_1^{(1)}]\rangle}{M^5} \mathbf{F}^{[^3\mathbf{P}_1]}(\mathbf{t_1}, \mathbf{t_2}, \varphi)$$

$$\overline{|\mathcal{A}(g^\star + g^\star \to \mathcal{H}[^3P_2^{(1)}]|^2} = \frac{32}{45}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{H}}[^3\mathbf{P}_2^{(1)}]\rangle}{M^5} \mathbf{F}^{[^3\mathbf{P}_2]}(\mathbf{t_1}, \mathbf{t_2}, \varphi)$$

where

$$\langle \mathcal{O}^{\chi_{\mathbf{cJ}}}[^3\mathbf{P}_\mathbf{J}^{(1)}]\rangle = \mathbf{2N_c(2J+1)|R'(0)|^2}$$

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Matrix elements for χ_c

$$\mathbf{F}^{[^3\mathbf{P}_0]}(\mathbf{t_1}, \mathbf{t_2}, \varphi) = \frac{2}{9}\frac{M^2\left(M^2+|\mathbf{p}_t|^2\right)^2\left[(3M^2+t_1+t_2)\cos\varphi+2\sqrt{t_1t_2}\right]^2}{(M^2+t_1+t_2)^4}$$

$$\mathbf{F}^{[^3\mathbf{P}_1]}(\mathbf{t_1}, \mathbf{t_2}, \varphi) = \frac{2}{9}\frac{M^2\left(M^2+|\mathbf{p}_t|^2\right)^2\left[(t_1+t_2)^2\sin^2\varphi+M^2\left(t_1+t_2-2\sqrt{t_1t_2}\cos\varphi\right)\right]}{(M^2+t_1+t_2)^4}$$

$$\mathbf{F}^{[^3\mathbf{P}_2]}(\mathbf{t_1}, \mathbf{t_2}, \varphi) = \frac{1}{3}\frac{M^2}{(M^2+t_1+t_2)^4}(M^2+|\mathbf{p}_t|^2)^2\{3M^4+3M^2(t_1+t_2)+4t_1t_2 \\ +(t_1+t_2)^2\cos^2\varphi+2\sqrt{t_1t_2}[3M^2+2(t_1+t_2)]\cos\varphi\}$$

where $\mathbf{p}_t = \mathbf{q}_{1t} + \mathbf{q}_{2t}$
and $\varphi = \varphi_1 - \varphi_2$ is the angle between $\mathbf{q}_{1t}$ and $\mathbf{q}_{2t}$ so

$$|\mathbf{p}_t|^2 = t_1 + t_2 + 2\sqrt{t_1t_2}\cos\varphi$$

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