Exact results in QFT

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Contents of the talk

1. Some information about myself
2. Exact results in QFT

A brief summary of my studies

- I was born in Milan (Italy)

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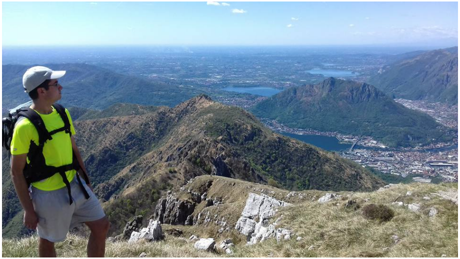
- I did my bachelor and my master in Milan (Italy)
- I did my PhD in Oviedo (Spain)
- This is my first postdoc

Free time activities

Hiking



Running

Free time activities

I'm also learning how to play the classic guitar (but I'm still at the beginning).

Contents of the talk

A list of very nice results

- Seiberg-Witten theory.
  [N. Seiberg, E. Witten (1995)]
- The AdS/CFT correspondence.
  [J. Maldacena (1997)]
- Supersymmetric localization.
  [V.Pestun (2012)]

The Seiberg-Witten theory in a nutshell

[N. Seiberg, E. Witten (1995)]

We want to solve $\mathcal{N} = 0$ ☹

(UV) High energy perurbative QCD $\Longrightarrow$ (IR) Low-energy confinement

$\mathcal{N} = 1 \quad B \leftrightarrow F$

$\mathcal{N} = 2 \quad B \leftrightarrow F \leftrightarrow B$

Let's consider $\mathcal{N} = 2$ ☺

(UV) $4d$ $\mathcal{N} = 2$ QFT $\Longrightarrow$ (IR) Exact low energy effective action

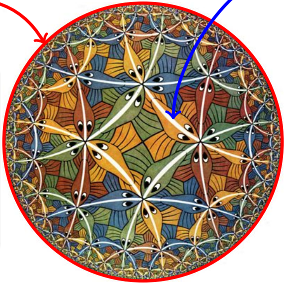
The AdS/CFT correspondence-holography

[J.M. Maldacena (1997)]

gauge theory ⇔ Holographic gravity

boundary

bulk



CFT_d

- gauge theory
- gauge invariant operators $O(x^\mu)$...
- global symmetries

AdS_{d+1}

- gravity theory
- fields: $g_{\mu\nu}, \dots$
- gauge symmetries

- Dictionary

$$\text{Observable in QFT} \quad \Leftrightarrow \quad \text{Observable in the gravity theory}$$

- energy scale in QFT ⇔ radial coordinate in the gravity dual

Supersymmetric localization

[V.Pestun (2012)]

Supersymmetric QFT on a compact manifold $\mathcal{M}$ (e.g. $\mathbb{S}^d$) with a supercharge $\mathcal{Q}$

Partition function

$$Z = \int \mathcal{D}[\Phi] e^{-S[\Phi]}$$

Infinite dimensional path integral over all the fields configuration

$\Rightarrow$

$$Z = \int da\ f(a)$$

normal finite dimensional integral

$\Rightarrow$ exact observables computation in a supersymmetric QFTs

PhD research activity

- $5d$ $\mathcal{N} = 1$ QFT are not renormalizable $\mapsto$ **fixed point** theory, for a proper choice of the gauge group and of the matter content.

  [N. Seiberg (1995)],[K.A. Intriligator, D.R. Morrison and N. Seiberg (1997)]

- In some cases: enhancement of the global symmetry group at UV fixed point.

  [N. Seiberg (1995)]

$$\boxed{\text{(UV)} \quad G_{UV}} \quad \rightarrow \text{ RG flow } \rightarrow \quad \boxed{\text{(IR)} \quad G_{IR}} \qquad G_{UV} > G_{IR}$$

- Gravity dual for $5d$ $\mathcal{N} = 1$ QFT $\Rightarrow$ *study of holographic RG flows.*

  [A. Brandhuber and Y. Oz (1999)],[O. Bergman and D. Rodríguez-Gómez (2012)]

PhD research activity

- The study of gauge theories on curved background $\mapsto$ reaction to curvature measures different things: partition functions, index ...
  [V.Pestun (2012)]...
- Procedure that allows to define a SQFT on a curved background.
  [G.Festuccia and N. Seiberg (2011)]
  $\Rightarrow$ *study of* $5d$ *SQFT on a Riemann manifold*.
- A particularly useful background: $\mathbb{S}^1 \times \mathbb{S}^4 \ \mapsto$ computation of the Superconformal Index (SCI) of the theory.
  [J.Kinney, J.M.Maldacena, S.Minwalla and S.Raju (2007)],[J.Bhattacharya, S.Bhattacharyya, S.Minwalla and S.Raju (2008)],[H.-C. Kim,S.-S.Kim, K. Lee (2012)]
- Limits of the SCI $\leftrightarrow$ subset of operators of the theory
  [A.Gadde, L. Rastelli and collaborators (2012)]
  $\Rightarrow$ particular limit limit of the $5d$ SCI.

PhD research activity

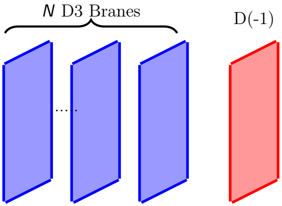
- The partition function takes the form

$$Z = \int Z_{pert} Z_{inst}$$

in the case of pure gauge theories (with 8 supercharges) Z_{inst} coincides with the Hilbert series of the instanton moduli space.

[D. Rodríguez-Gómez and G.Zafrir (2014)],[C.A.Keller,N. Mekareeya, J.Song and Y.Tachikawa (2012)]

- We can use string theory to study instantons on $\mathbb{C}^2$. Higgs branch of a system of D_p and D_{p+4} branes (e.g. $p = 3$)



Research activity during the post-doc

For the moment I will mostly focused on $4d$ QFTs.

- $4d\ \mathcal{N} = 2 \rightarrow 4\text{d}\ \mathcal{N} = 1$
  [D.Gaiotto, S. Razamat (2015)]
- Discovery of $4d$ QFTs with $\mathcal{N} = 3$ (Non Lagrangian theories!).
  []I. Garcia Etxabarria, D. Regalado (2015)
  $\rightarrow$ a lot of things to explore
  - Study of the SCI.
  - Seiberg-Witten curve.

THANKS FOR YOUR ATTENTION

The prepotential $\mathcal{F}$

$$\mathcal{F}(A) = \frac{1}{2}\tau_0 A^2 + \frac{i}{\pi} A^2 log(\frac{A^2}{\Lambda^2}) + \frac{1}{2\pi i} A^2 \sum_{l=1}^{+\infty} c_l (\frac{\Lambda}{A})^{4l}$$

The low energy effective action

$$\frac{1}{4\pi} \mathrm{Imm} \left[\int d^4\theta \frac{\partial \mathcal{F}}{\partial A} \overline{A} + \int d^2\theta \frac{1}{2} \frac{\partial^2 \mathcal{F}}{\partial A^2} W_\alpha W^\alpha \right]$$

Supersymmetric localization 1

- QFT with a fermionic odd charge $\mathcal{Q}$, such that $\mathcal{Q}^2 = B$
- BPS observable $\mathcal{Q}\mathcal{O}_{BPS} = 0$
- we want to evaluate

$$\langle \mathcal{O}_{BPS} \rangle = \int_F \mathcal{D}[X] \mathcal{O}_{BPS} e^{-S[X]}$$

- it holds that

$$\langle \mathcal{O}_{BPS} \rangle = \langle \mathcal{O}_{BPS} + \mathcal{Q}O \rangle$$

Supersymmetric localization 2

Let's consider the deformed path-integral

$$\langle \mathcal{O}_{BPS} \rangle = \int_F \mathcal{D}[X] \mathcal{O}_{BPS} e^{-S[X] - t\mathcal{Q}P_F[X]}$$

It holds that

$$\frac{d}{dt}\langle \mathcal{O}_{BPS} \rangle = 0 \quad \Rightarrow \quad t \to +\infty$$

the integral is dominated by the saddle points of the localising action

$$X = X_0 + \frac{1}{\sqrt{t}}\delta X \quad \Rightarrow \quad S[X_0] + \frac{1}{2}\int\int \frac{\delta^2 S_{loc}[X_0]}{\delta X^2}\,|_{X=X_0}\,(\delta X)^2$$