Measures of naturalness for EW fine-tuning in SUSY: UV vs. IR?

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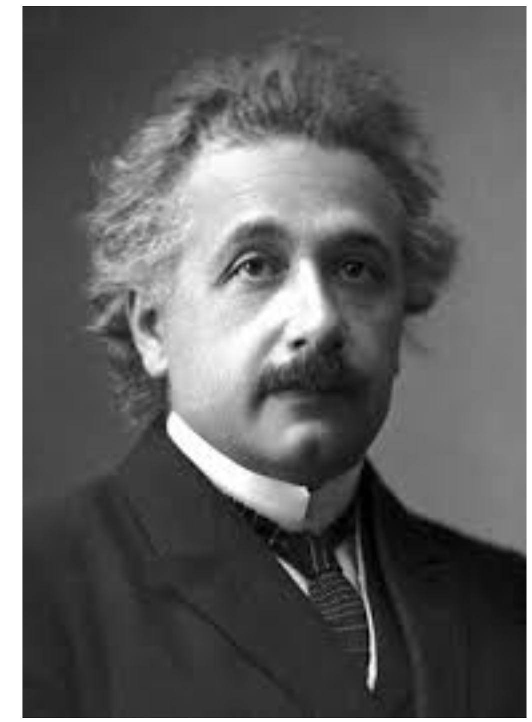
DESY naturalness forum
April 25,2018

twin pillars of guidance:
naturalness & simplicity



``The appearance of fine-tuning in a scientific theory is like a cry of distress from nature, complaining that something needs to be better explained"

S. Weinberg

``Everything should be made as simple as possible, but not simpler"

A. Einstein

Nature sure looks like SUSY

- stabilize Higgs mass

  Witten, Kaul

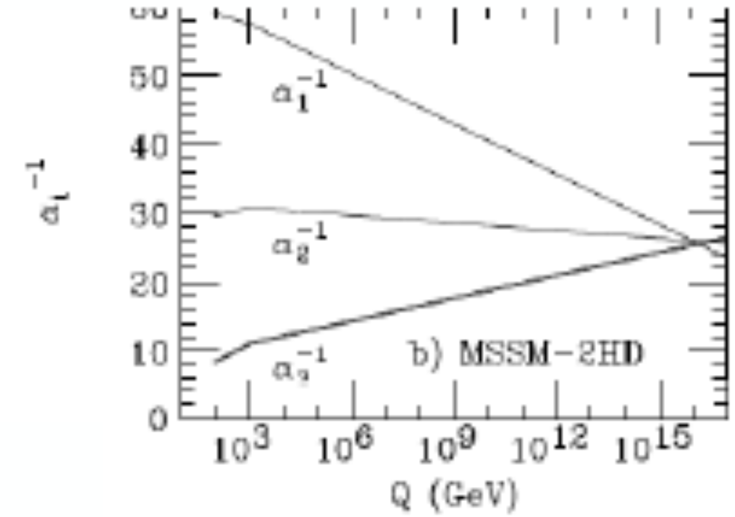
- measured gauge couplings unify

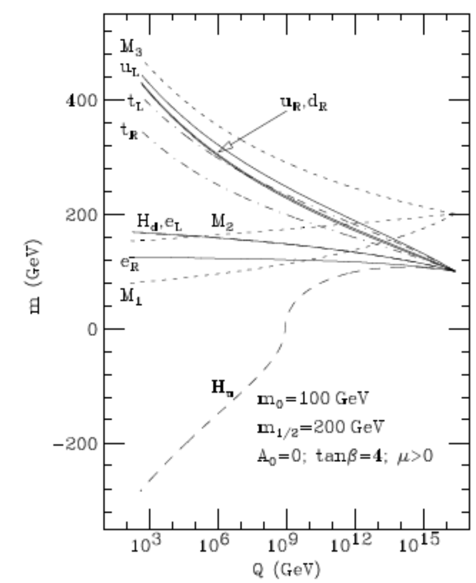
  Dimopoulos, Raby, Wilczek

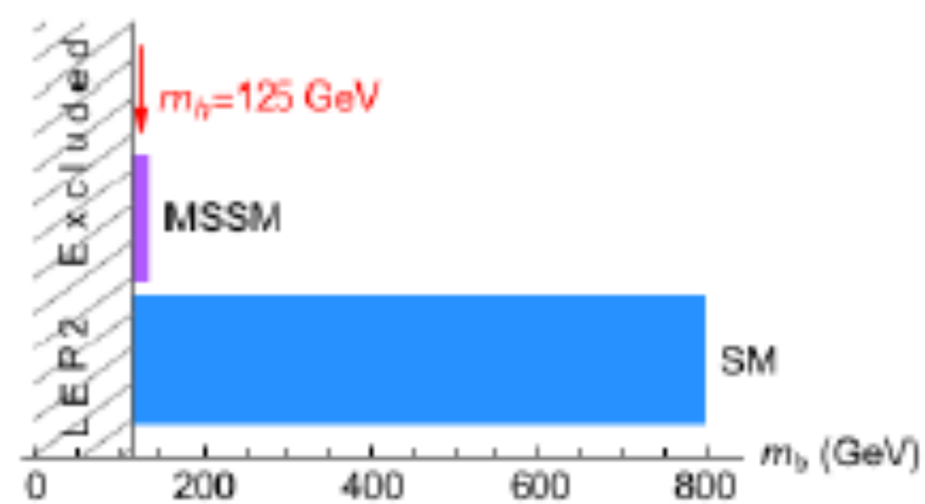
- m(t)~173 GeV for REWSB Ibanez, Ross

- mh(125): squarely within SUSY window

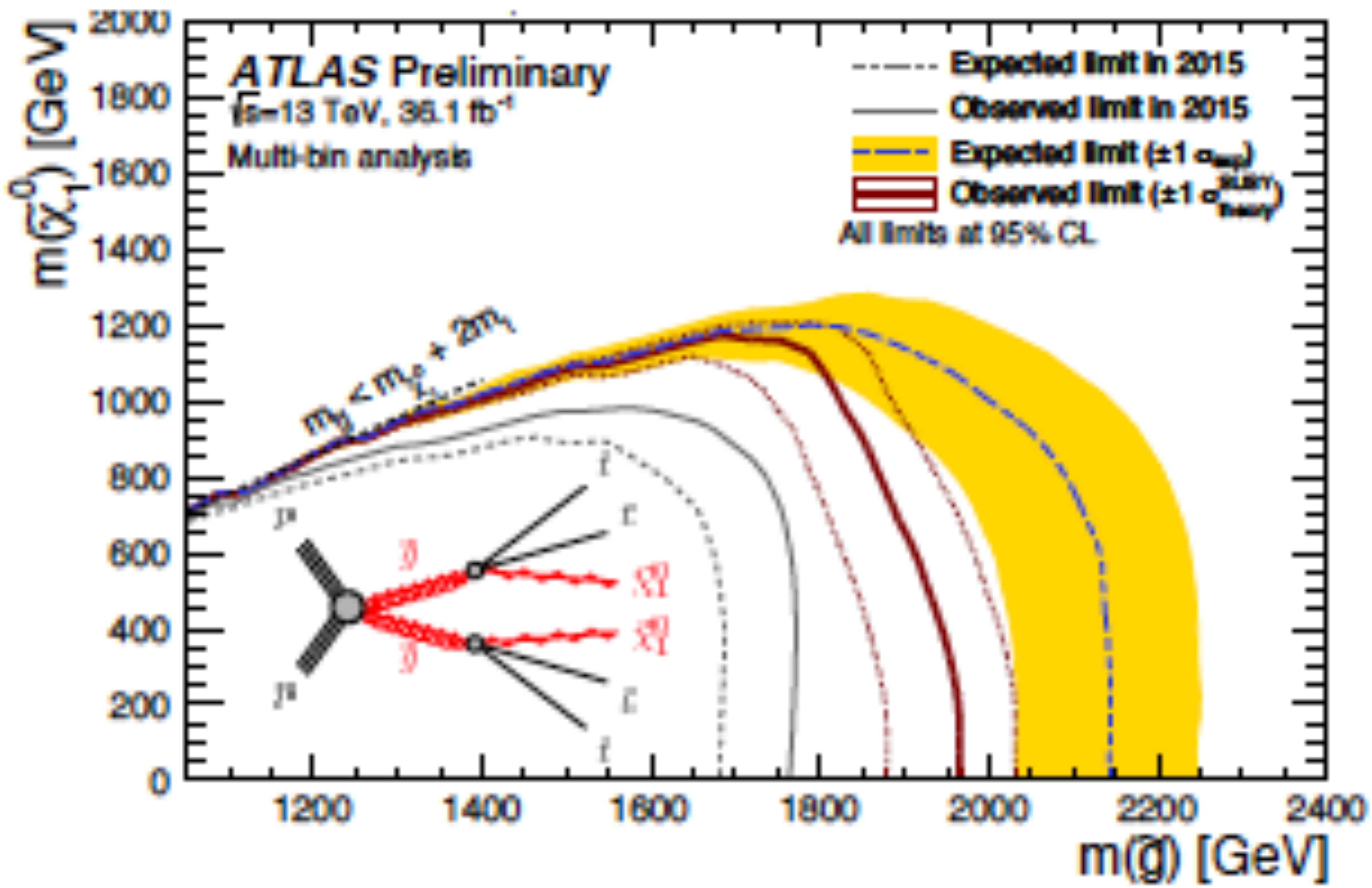
  Haber, Hempfling;
  Okada, Yamaguchi, Yanagida;
  Brignole, Ellis, Zwirner;
  Barbieri, Frigeni;
  Chankowski, Pokorski, Rosiek







recent search results from Atlas run 2 @ 13 TeV:



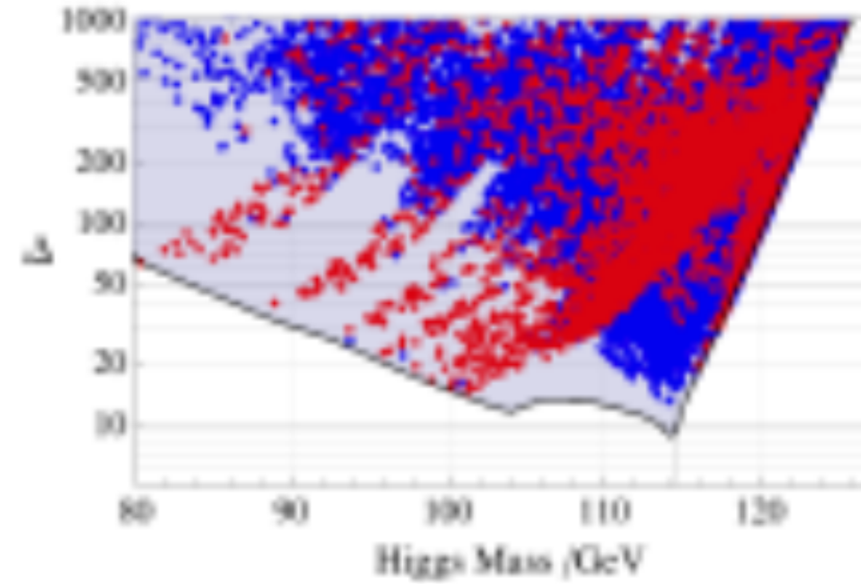
evidently $m_{\tilde{g}} > 1.9$ TeV

compare: BG naturalness (1987): $m_{\tilde{g}} < 0.35$ TeV

These bounds appear in sharp conflict with EW ``naturalness''

	mass
gluino	400 GeV
uR	400 GeV
eR	350 GeV
chargino	100 GeV
neutralino	50 GeV

Barbieri-Giudice 10% bounds, 1987

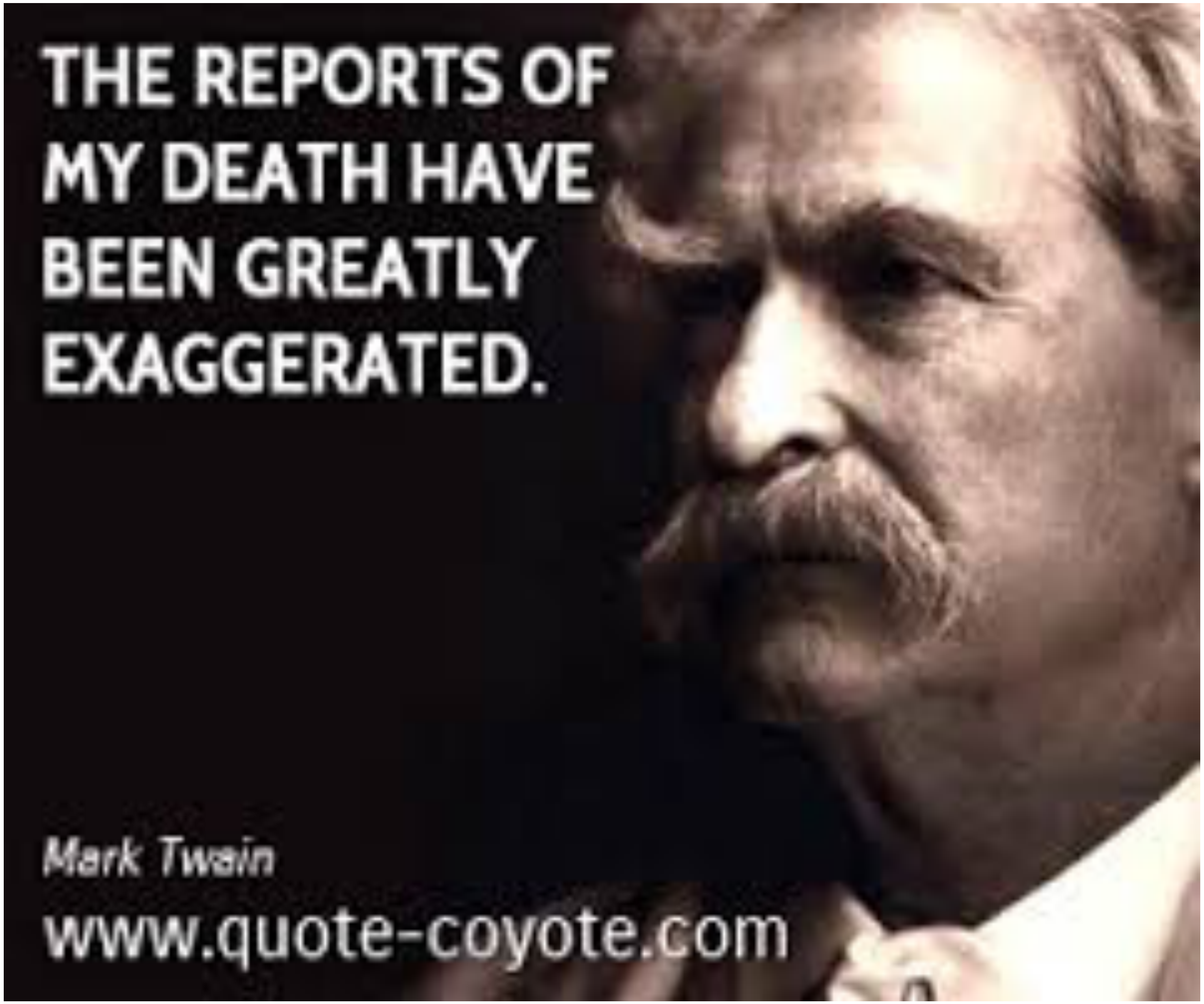


Cassel, Ghilencea, Ross,2009

$\Delta \to 1000$
as $m_h \to 125$ GeV
0.1% tuning!?

or is SUSY dead?
how to disprove SUSY?
when it becomes ``unnatural''?
this brings up naturalness issue

Mark Twain, 1835-1910 (or SUSY)



1897

``...settling the ultimate fate of naturalness is perhaps the most profound theoretical question of our time''

Arkani-Hamed et al.,
arXiv:1511.06495

``Given the magnitude of the stakes involved,
it is vital to get a clear verdict
on naturalness from experiment''

This should be matched by theoretical scrutiny
of what we mean by naturalness

Let us attempt a working definition of naturalness:

- An observable O is natural if all independent contributions to O are comparable to or less than O

$\mathcal{O} = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$ for $i = 1 - n$ parameters p_i with coefficients a_i

- Because if one contribution, say $|a_1 p_1| \gg \mathcal{O}$ then some other contribution will have to be large opposite sign such that there is near perfect cancellation
- This is considered highly implausible, hence unnatural
- Something is lacking in the theory- the theory *as is* is likely wrong- needs some added feature or should discard
- Nature is natural!

Most claims against SUSY stem from overestimates of EW fine-tuning.
These arise from violations of the

Prime directive on fine-tuning:

``Thou shalt not claim fine-tuning of dependent quantities one against another!"

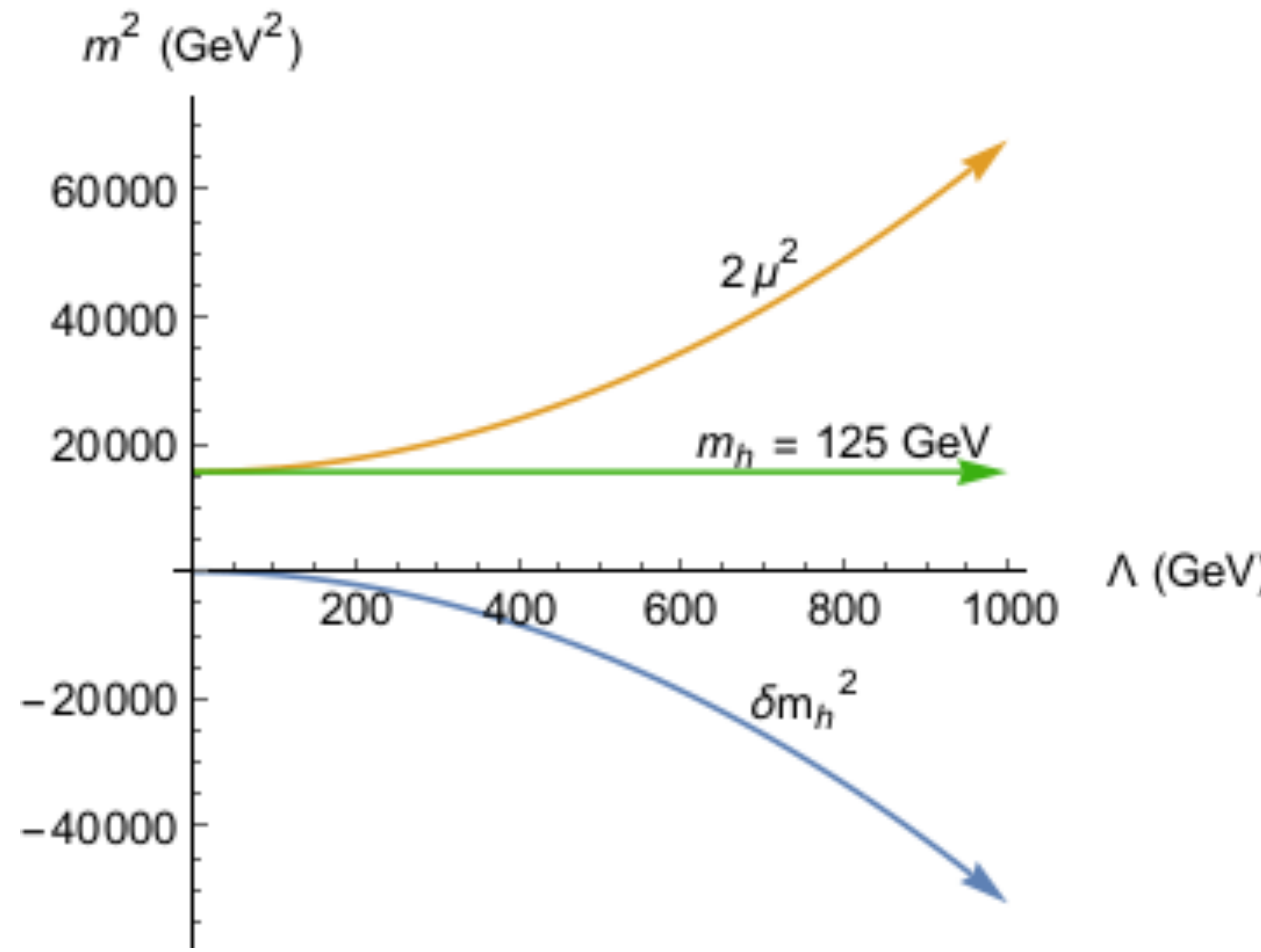
HB, Barger, Mickelson, Padeffke-Kirkland, arXiv:1404.2277

Is observable $\mathcal{O} = \mathcal{O} + a + b - f(b) + c$ fine-tuned for $b > \mathcal{O}$?

Reminder: naturalness in the SM

Higgs sector of SM is ``natural'' only up to cutoff

$$
\begin{aligned}
V &= -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \\
m_h^2 &\simeq 2\mu^2 + \delta m_h^2 \\
\delta m_h^2 &\simeq \frac{3}{4\pi^2}\left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda\right)\Lambda^2
\end{aligned}
$$

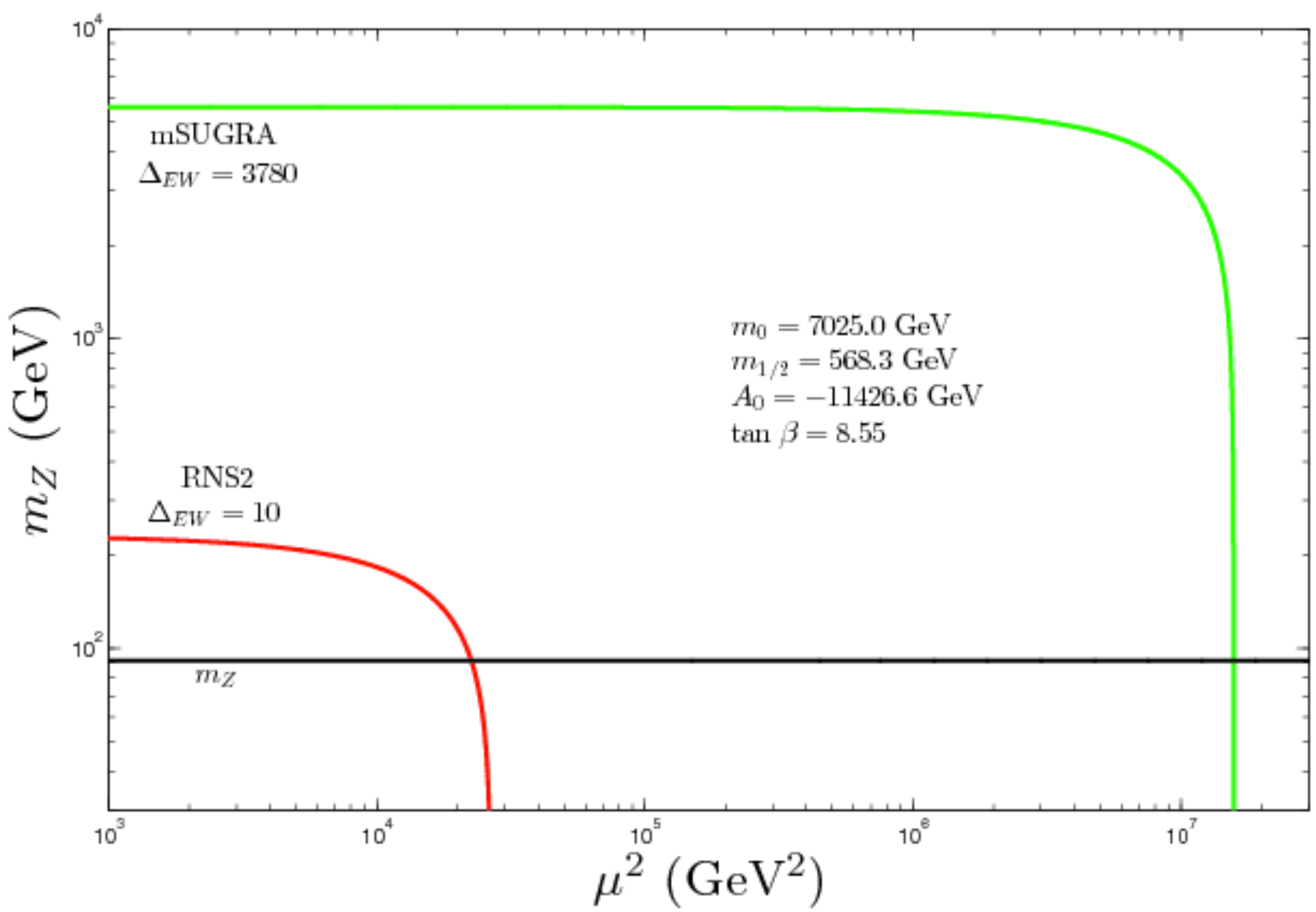


Since δm_h^2 is *independent* of μ^2,
can freely dial (fine-tune) μ^2 to maintain $m_h = 125$ GeV

Naturalness: $\delta m_h^2 < m_h^2 \Rightarrow \Lambda < 1$ TeV!
New physics at or around the TeV scale!

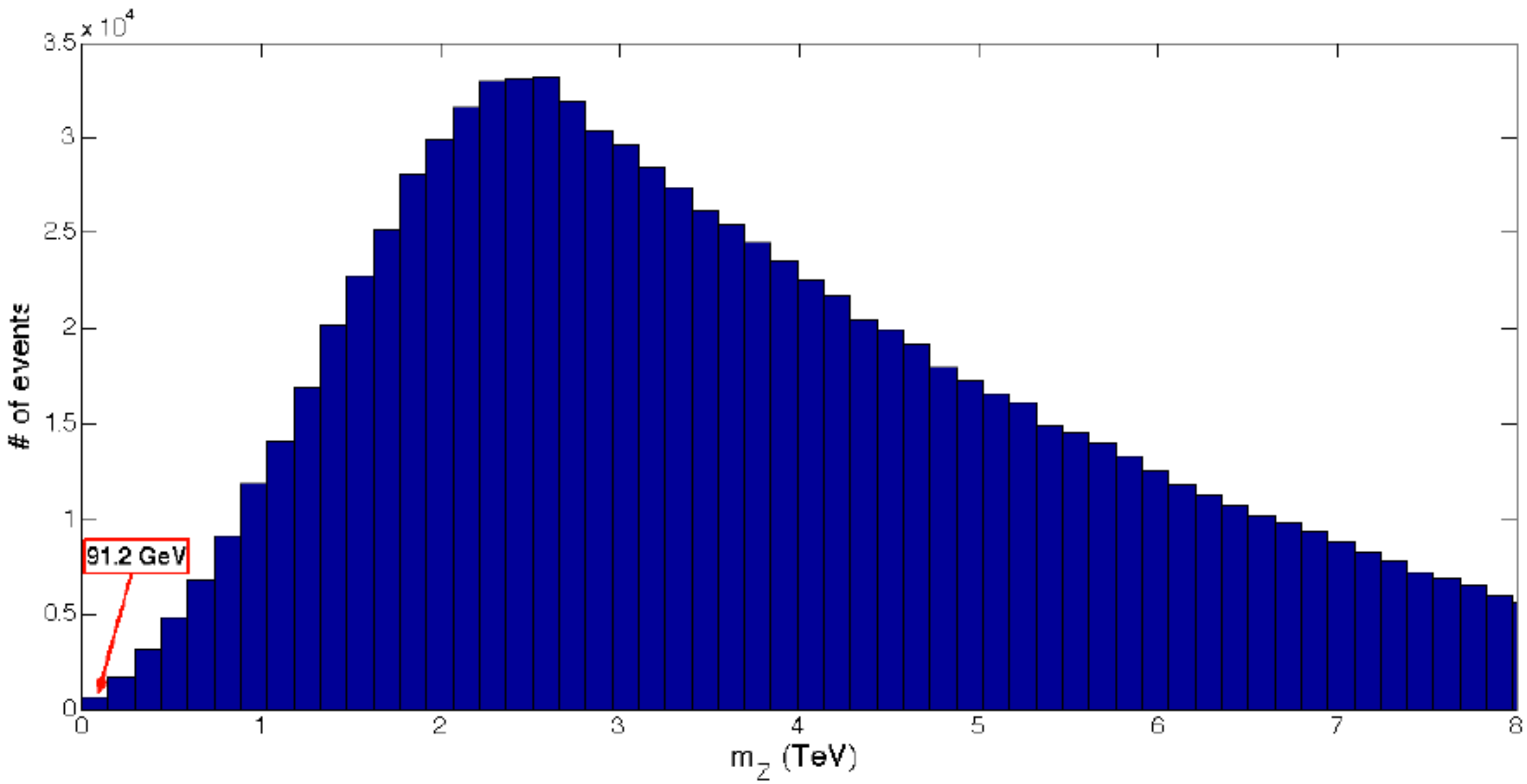
Next: simple electroweak fine-tuning in SUSY:
dial value of mu so that Z mass comes out right:
everybody does it but it is hidden inside spectra codes (Isajet, SuSpect, SoftSUSY, Spheno, SSARD)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2$$



e.g. in CMSSM/ mSUGRA: one then concludes nature gives this:

If you didn't fine-tuned, then here is m(Z)



The 20 dimensional pMSSM parameter space then includes

$$M_1,\ M_2,\ M_3,$$
$$m_{Q_1},\ m_{U_1},\ m_{D_1},\ m_{L_1},\ m_{E_1},$$
$$m_{Q_3},\ m_{U_3},\ m_{D_3},\ m_{L_3},\ m_{E_3},$$
$$A_t,\ A_b,\ A_\tau,$$
$$m^2_{H_u},\ m^2_{H_d},\ \mu,\ B.$$

scan over parameters

Natural value of m(Z) from pMSSM is ~2-4 TeV

Three measures of fine-tuning:

#1: Simplest, most conservative SUSY measure: Δ_{EW}

Working only at the weak scale, minimize scalar potential: calculate m(Z) or m(h)

No large uncorrelated cancellations in m(Z) or m(h)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \quad \sim -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

$\Delta_{EW} \equiv max_i \left|C_i\right| / (m_Z^2/2)$ with $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$ etc.

simple, direct, unambiguous interpretation:

- $|\mu| \sim m_Z \sim 100 - 200$ GeV (Chan, Chatto…,Nath; HB, Barger, Huang)
- $m_{H_u}^2$ should be driven to small negative values such that $-m_{H_u}^2 \sim 100 - 200$ GeV at the weak scale and
- that the radiative corrections are not too large: $\Sigma_u^u \lesssim 100 - 200$ GeV

CETUP*-12/002, FTPI-MINN-12/22, UMN-TH-3109/12, UH-511-1195-12

Radiative natural SUSY with a 125 GeV Higgs boson

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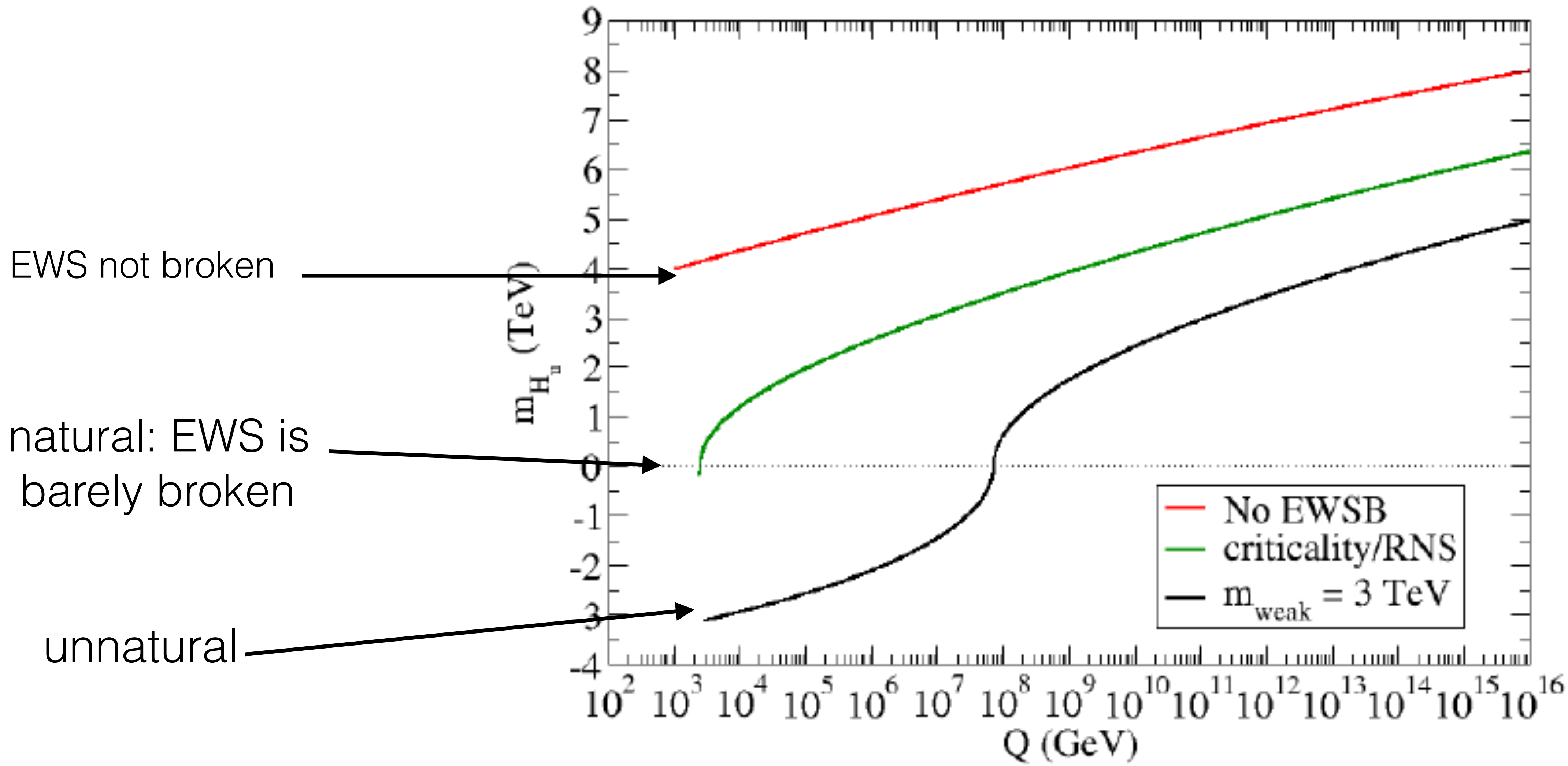
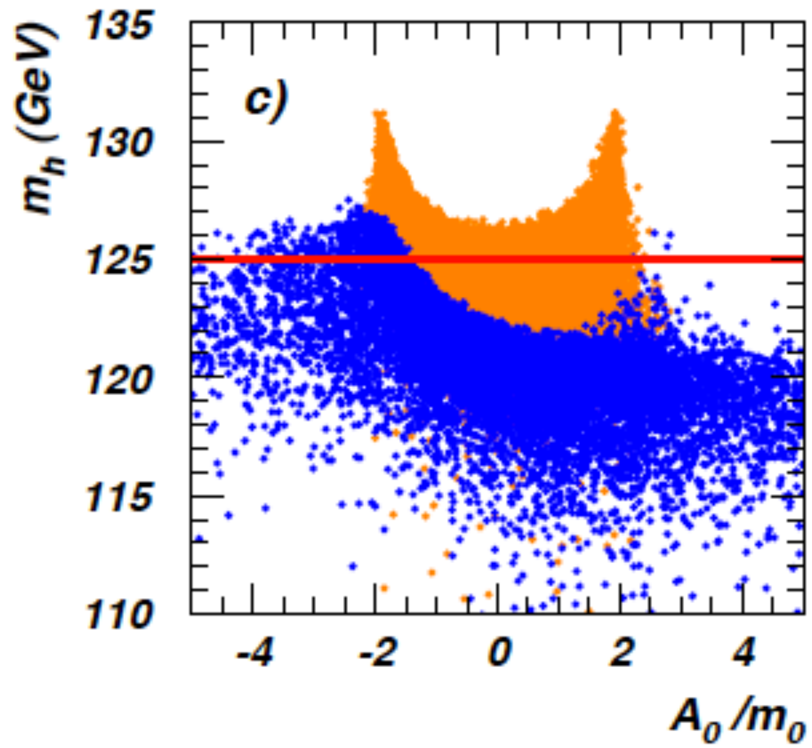
PRL109 (2012) 161802

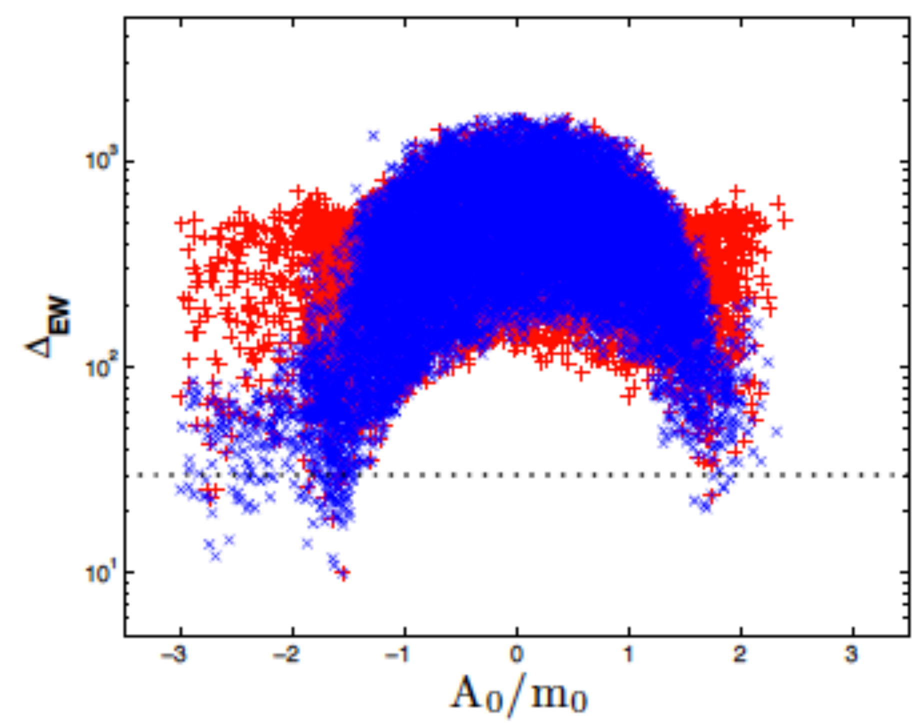
radiative corrections drive $m_{H_u}^2$ from unnatural GUT scale values to naturalness at weak scale: radiatively-driven naturalness



Evolution of the soft SUSY breaking mass squared term $sign(m_{H_u}^2)\sqrt{|m_{H_u}^2|}$ vs. Q

Large value of A_t reduces $\Sigma_u^u(\tilde{t}_{1,2})$ contributions to Δ_{EW} while uplifting m_h to ~ 125 GeV





$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8 g_Z^2 (\frac{1}{4} - \frac{2}{3} x_W) \Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right]$$

$$\Delta_t = (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)/2 + M_Z^2 \cos 2\beta (\frac{1}{4} - \frac{2}{3} x_W)$$

$$F(m^2) = m^2 \left(\log \frac{m^2}{Q^2} - 1 \right) \qquad Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$$

#2: Higgs mass or large-log fine-tuning Δ_{HS}

It is tempting to pick out one-by-one
quantum fluctuations but must combine log divergences
before taking any limit

$$m_h^2 \simeq \mu^2 + m_{H_u}^2(weak) \simeq \mu^2 + m_{H_u}^2(\Lambda) + \delta m_{H_u}^2$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t\right) \qquad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, mHu and running;
then we can integrate from m(SUSY) to Lambda

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{U_3}^2 + A_t^2\right)\ln(\Lambda/m_{SUSY})$$

$$\Delta_{HS} \sim \delta m_h^2/(m_h^2/2) < 10$$

$$m_{\tilde{t}_{1,2},\tilde{b}_1} < 500 \text{ GeV}$$

$$m_{\tilde{g}} < 1.5 \text{ TeV}$$

old natural SUSY

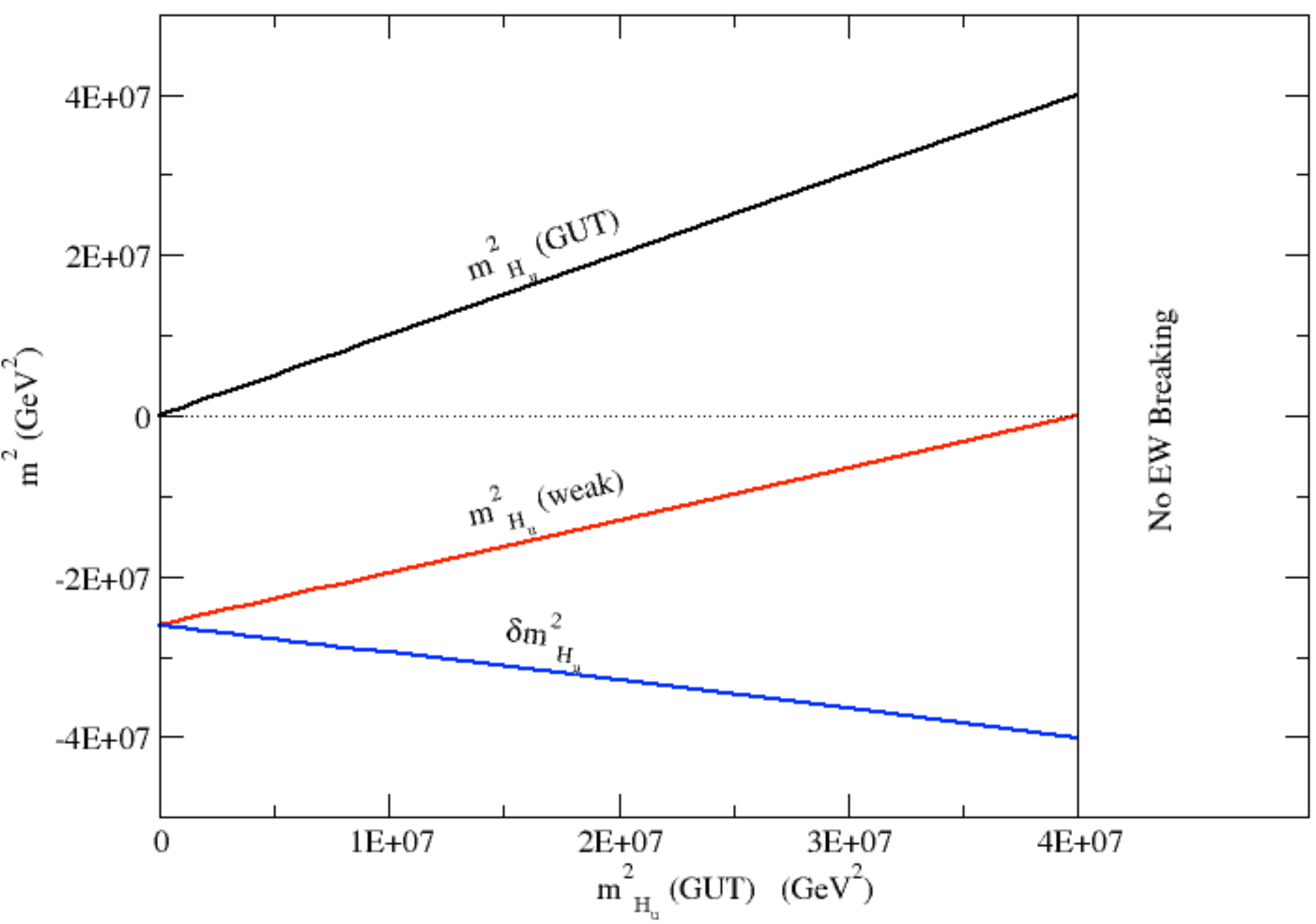
then A_t can't be too big

What's wrong with this argument?

In zeal for simplicity, have made several simplifications: most egregious is that one sets m(Hu)^2=0 at beginning to simplify

$m^2_{H_u}(\Lambda)$ and $\delta m^2_{H_u}$ are *not* independent!

violates prime directive!



The larger $m^2_{H_u}(\Lambda)$ becomes, then the larger becomes the cancelling correction!

HB, Barger, Savoy, arXiv:1502.04127

To fix: combine dependent terms:

$m_h^2 \simeq \mu^2 + \left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ where now both μ^2 and $\left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ are $\sim m_Z^2$

After re-grouping: $\Delta_{HS} \simeq \Delta_{EW}$

Instead of: the radiative correction $\delta m_{H_u}^2 \sim m_Z^2$
we now have: the radiatively-corrected $m_{H_u}^2 \sim m_Z^2$

#3. What about EENZ/BG measure?

$$\Delta_{BG} = max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right| = max_i \left| \frac{p_i}{m_Z^2} \frac{\partial m_Z^2}{\partial p_i} \right|$$

p_i are the theory parameters

applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

now apply to high (e.g. GUT) scale parameters

$$\begin{aligned} m_Z^2 \simeq\ & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & -0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ & -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned}$$

applied to most parameters,

Δ_{BG} large, looks fine-tuned for *e.g.* $m_{\tilde{g}} \simeq M_3 > 1.8$ TeV

$$\Delta_{BG}(M_3^2) = 3.84\frac{M_3^2}{m_z^2} \simeq 1500$$

#3. What about EENZ/BG measure?

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applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

What if we apply to high (e.g. GUT) scale parameters ?

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For correlated scalar masses $\equiv m_0$,
scalar contribution collapses:
what looks fine-tuned isn't: *focus point SUSY*
multi-TeV scalars are *natural*

Feng, Matchev, Moroi

But wait! in more complete models,
soft terms not independent

violates prime directive!

e.g. in SUGRA, for well-specified hidden sector,
each soft term calculated as multiple of m(3/2);
soft terms must be combined!

e.g. dilaton-dominated SUSY breaking: $m_0^2 = m_{3/2}^2$ with $m_{1/2} = -A_0 = \sqrt{3} m_{3/2}$

$$\begin{aligned} m_{H_u}^2 &= a_{H_u} \cdot m_{3/2}^2, \\ m_{Q_3}^2 &= a_{Q_3} \cdot m_{3/2}^2, \\ A_t &= a_{A_t} \cdot m_{3/2}, \\ M_i &= a_i \cdot m_{3/2}, \\ & \cdots . \end{aligned}$$

since μ hardly runs, then

$$\begin{aligned} m_Z^2 &\simeq -2\mu^2 + a \cdot m_{3/2}^2 \\ &\simeq -2\mu^2 - 2m_{H_u}^2(weak) \end{aligned}$$

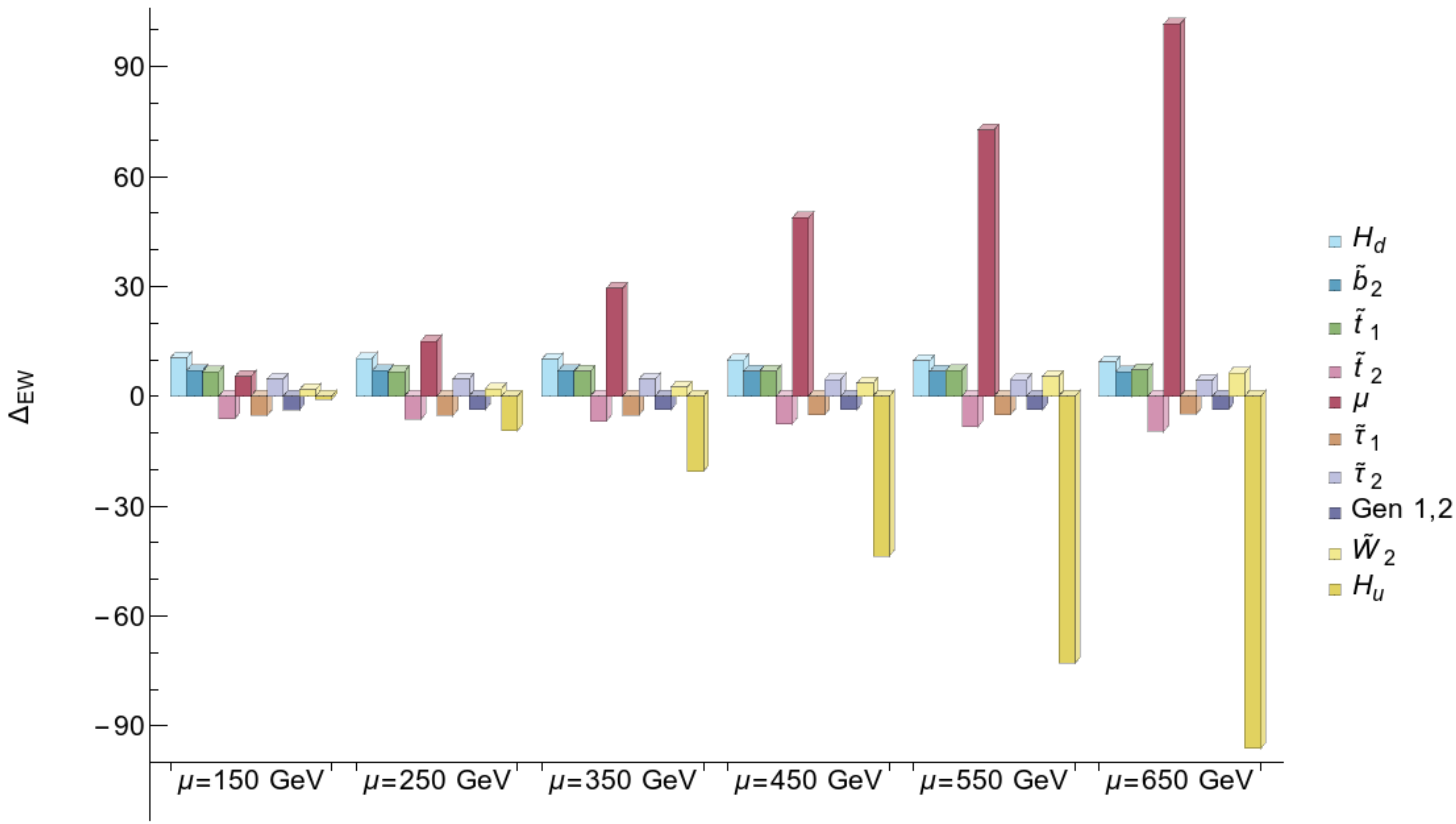
$$m_{H_u}^2(weak) \sim -(100-200)^2 \ \mathrm{GeV}^2 \sim -a \cdot m_{3/2}^2/2$$

using μ^2 and $m_{3/2}^2$ as fundamental,
then $\Delta_{BG} \simeq \Delta_{EW}$ even using high scale parameters!

On SUSY parameters

- parameters are introduced by theorists to parametrize our ignorance of SUSY breaking
- in any more fundamental theory, soft terms are calculated in terms of single soft breaking parameter
- e.g. $m_{3/2}$ in SUGRA or AMSB, Λ in GMSB
- we think Δ_{EW} is a better measure of whether nature is fine-tuned, rather than our effective theories with artificially-introduced parameters

How much is too much fine-tuning?



HB, Barger, Savoy

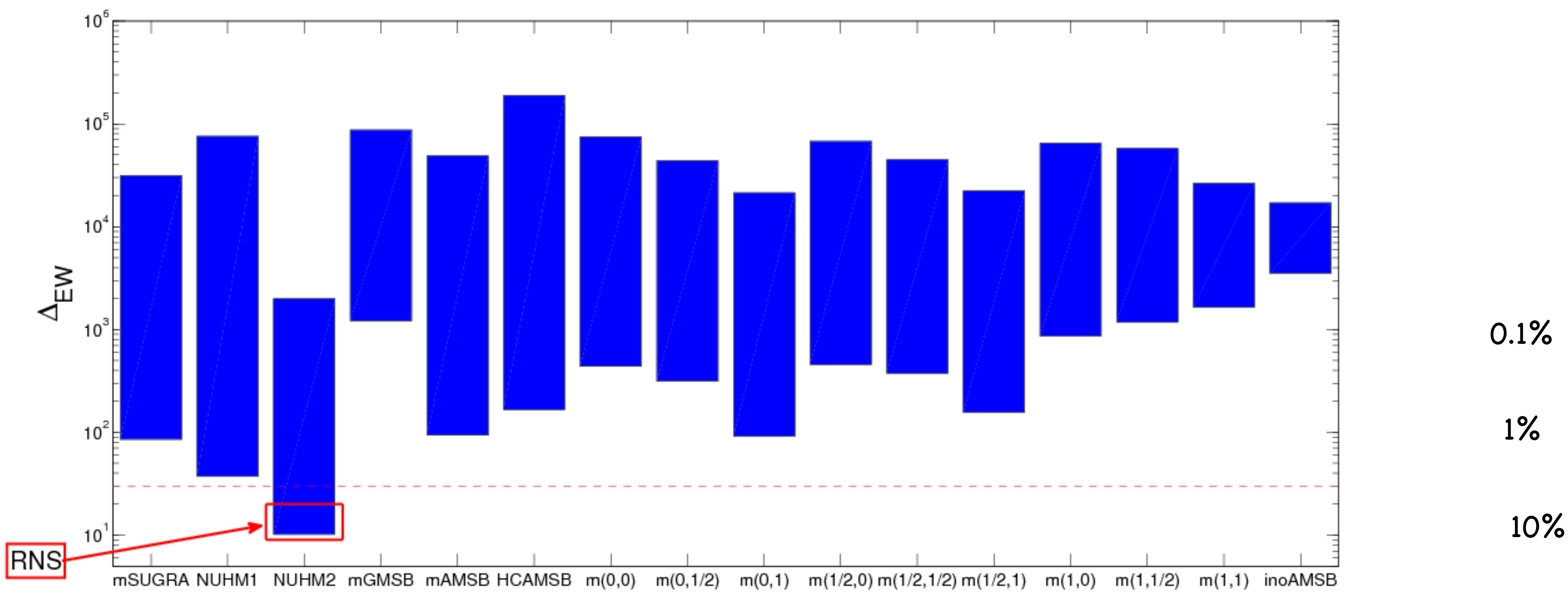
Visually, large fine-tuning has already developed by $\mu \sim 350$ or $\Delta_{EW} \sim 30$

Nature is natural $\Rightarrow \Delta_{EW} < 20 - 30$ (take 30 as conservative)

Δ_{EW} is highly selective:
most constrained models are ruled out
except NUHM2 and its generalizations:

D. Matalliotakis and H. P. Nilles, Nucl. Phys. B **435** (1995) 115; P. Nath and R. L. Arnowitt, Phys. Rev. D **56** (1997) 2820; J. Ellis, K. Olive and Y. Santoso, *Phys. Lett.* **B 539** (2002) 107; J. Ellis, T. Falk, K. Olive and Y. Santoso, *Nucl. Phys.* **B 652** (2003) 259; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, *J. High Energy Phys.* **0507** (2005) 065.

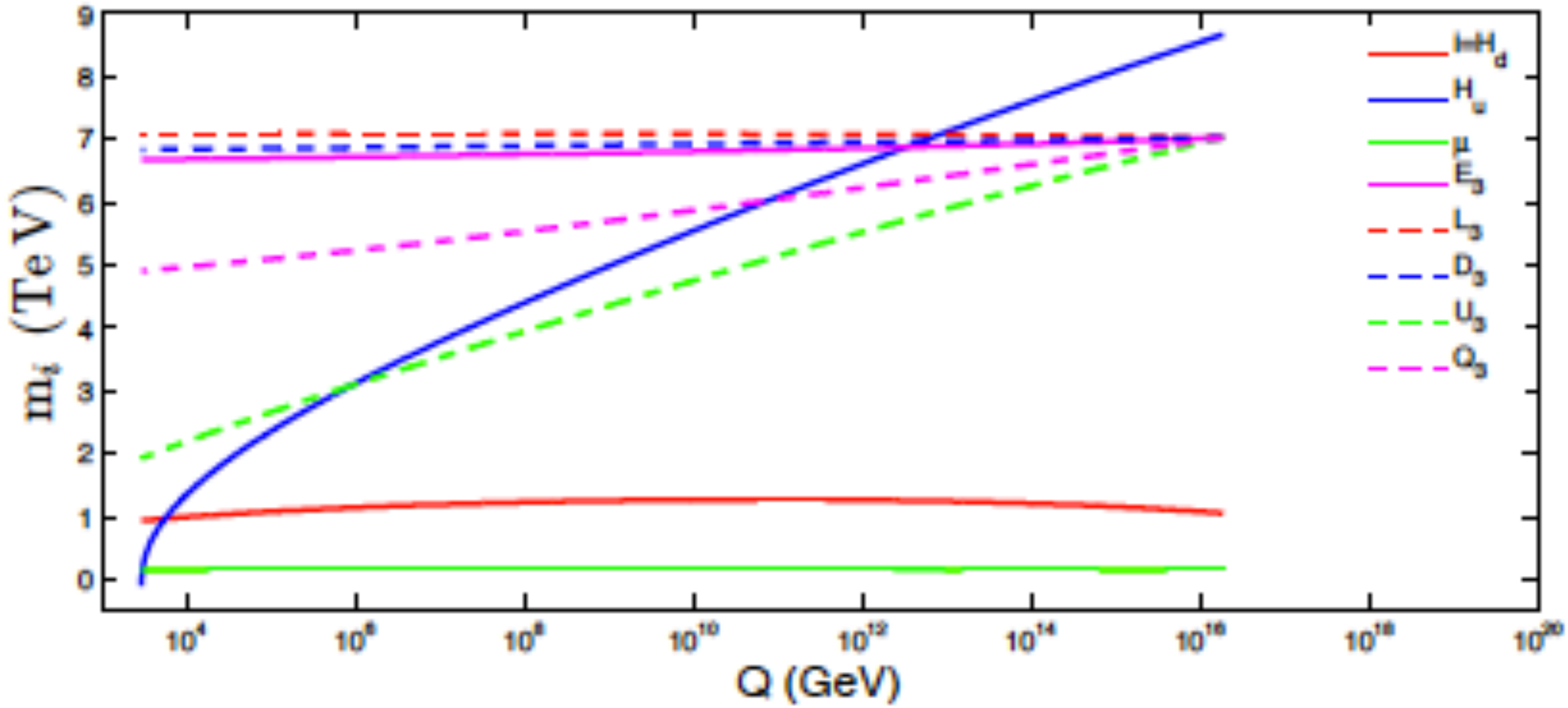
scan over p-space with m(h)=125.5+-2.5 GeV:



HB, Barger, Mickelson,Padeffke-Kirkland, PRD89 (2014) 115019

Applied properly, all three measures agree:

naturalness is unambiguous and highly predictive!



Radiatively-driven natural SUSY, or RNS:

(typically need mHu~25-50% higher than m0)

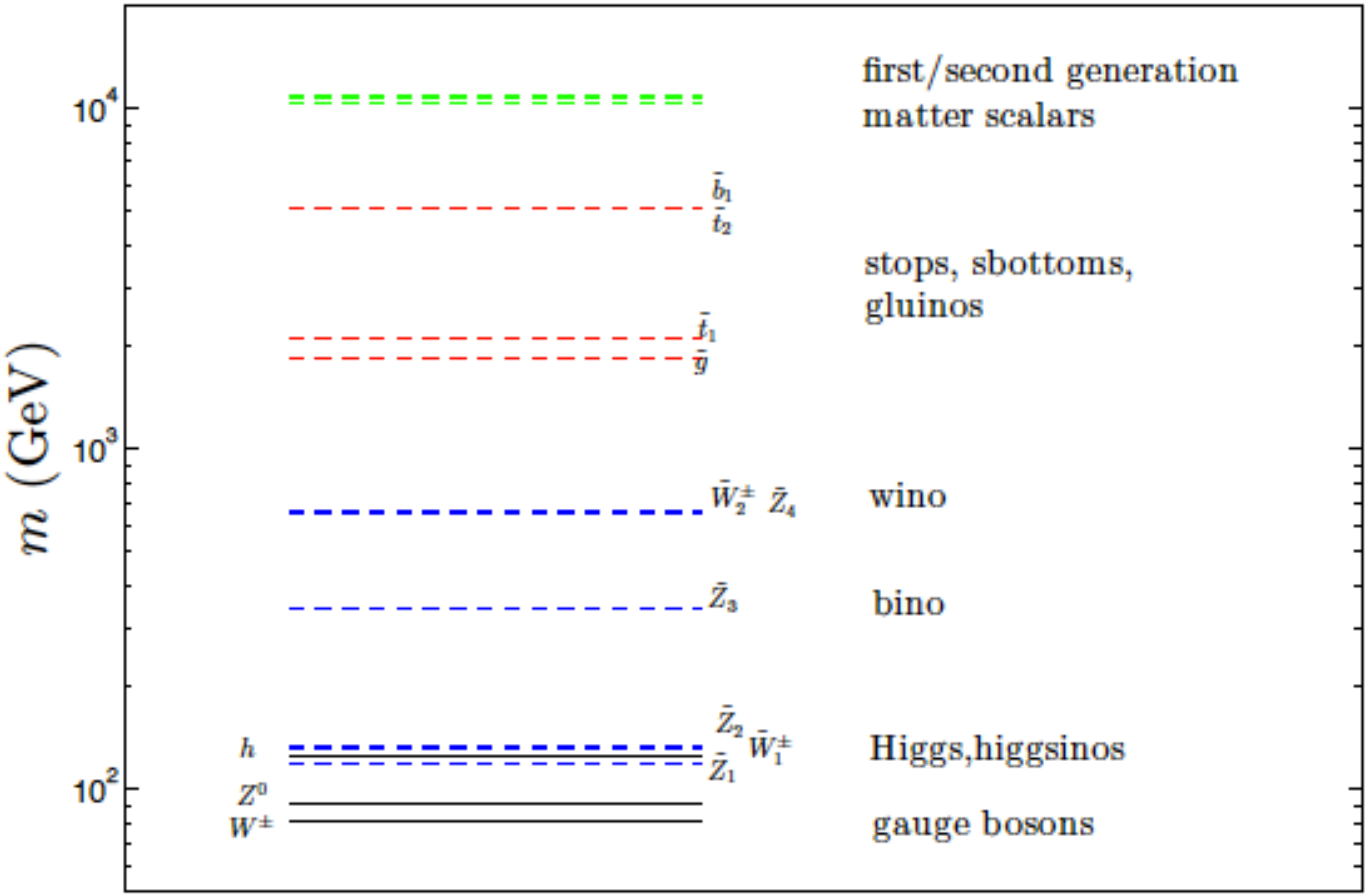
H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, *Phys. Rev. Lett.* **109** (2012) 161802.

H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, *Phys. Rev.* **D 87** (2013) 115028 [arXiv:1212.2655 [hep-ph]].

bounds from naturalness (3%)	BG/DG	Delta_EW
mu	350 GeV	0.35 TeV
gluino	400-600 GeV	5-6 TeV
t1	450 GeV	3 TeV
sq/sl	550-700 GeV	10-30 TeV

h(125) and LHC limits are perfectly compatible with 3-10% naturalness: no crisis!

Typical spectrum for low Δ_{EW} models



There is a Little Hierarchy, but it is no problem

$$\mu \ll m_{3/2}$$

Some topics for discussion

- What is correct measure for EW naturalness (in SUSY/other models)? Δ_{HS}?, Δ_{BG} (what are right p_i?) Δ_{EW}? or is naturalness all *subjective*?
- Is naturalness/fine-tuning a path to falsifiability of weak scale SUSY?
- How should contributions to Δ be organized? Factors of 2 etc. (Ross, Schmidt-Hoberg, Staub)
- What about non-holonomic soft terms (NHSTs) $\mu' \tilde{H}_u \tilde{H}_d$:heavy higgsinos while low fine-tuning?
- Are NHSTs large $\sim m_{weak}$ or highly suppressed (Martin, 1999)?
- How much is too much fine-tuning? Important for sparticle mass upper limits/falsifiability
- What about fine-tuning in QCD sector- strong CP and $\bar{\theta} F \tilde{F}$: axions or other solutions?
- Should one insist on naturalness in both EW and QCD sectors? Interplay between axions and SUSY?
- μ problem: $\mu \sim m_{3/2}$ or $\mu \sim m_{weak} \ll m_{soft}$?
- Is there a mechanism behind barely broken EW symmetry in SUSY?
- What does naturalness imply for future accelerators? LHC, HL-LHC, HE-LHC, ILC250, ILC500-600?, FCC, CepC, CppC?
- What does naturalness imply for dark matter? WIMPs? axions? both? other?

Mirage mediation:
comparable moduli- & anomaly-mediation

Choi, Falkowski, Nilles, Olechowski, Pokorski

Generalized mirage mediation model:

HB, Barger, Serce, Tata: arXiv:1610.06205

$$M_a = (\alpha + b_a g_a^2)\, m_{3/2}/16\pi^2, \tag{10}$$
$$A_\tau = (-a_3\alpha + \gamma_{L_3} + \gamma_{H_d} + \gamma_{E_3})\, m_{3/2}/16\pi^2, \tag{11}$$
$$A_b = (-a_3\alpha + \gamma_{Q_3} + \gamma_{H_d} + \gamma_{D_3})\, m_{3/2}/16\pi^2, \tag{12}$$
$$A_t = (-a_3\alpha + \gamma_{Q_3} + \gamma_{H_u} + \gamma_{U_3})\, m_{3/2}/16\pi^2, \tag{13}$$
$$m_i^2(1,2) = (c_m\alpha^2 + 4\alpha\xi_i - \dot{\gamma}_i)\,(m_{3/2}/16\pi^2)^2, \tag{14}$$
$$m_j^2(3) = (c_{m3}\alpha^2 + 4\alpha\xi_j - \dot{\gamma}_j)\,(m_{3/2}/16\pi^2)^2, \tag{15}$$
$$m_{H_u}^2 = (c_{H_u}\alpha^2 + 4\alpha\xi_{H_u} - \dot{\gamma}_{H_u})\,(m_{3/2}/16\pi^2)^2, \tag{16}$$
$$m_{H_d}^2 = (c_{H_d}\alpha^2 + 4\alpha\xi_{H_d} - \dot{\gamma}_{H_d})\,(m_{3/2}/16\pi^2)^2, \tag{17}$$

elevate a_3, c_m, c_{m3}, c_{H_u}, c_{H_d} from discrete to continuous:
soft terms depend on location of fields in compactified manifold!

p-space: $\alpha,\ m_{3/2},\ c_m,\ c_{m3},\ a_3,\ c_{H_u},\ c_{H_d},\ \tan\beta \quad (GMM)$

$\alpha,\ m_{3/2},\ c_m,\ c_{m3},\ a_3,\ \tan\beta, \mu,\ m_A \quad (GMM').$ <=

allows for natural mirage mediation

Allows to generate mini-landscape spectra

Buchmuller, Hamaguchi, Lebedev, Ratz

Lebedev, Nilles, Raby, Ramos-Sanches, Ratz, Vaudrevange

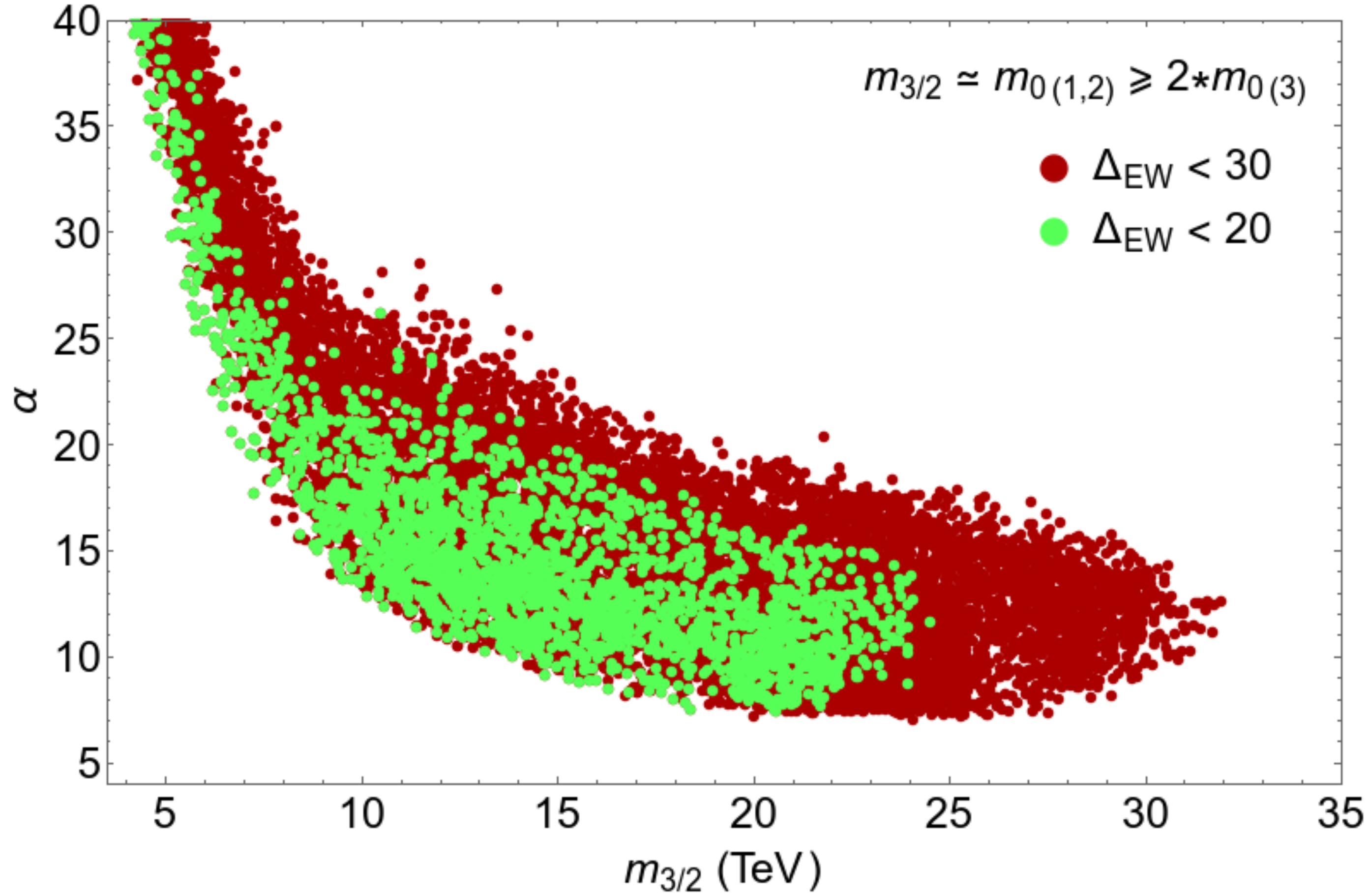
but with radiatively-driven naturalness

HB, Barger, Savoy, Serce, Tata, arXiv:1705.01578

- Begin with heterotic string with orbifold compactification
- Look for fertile patch of landscape giving MSSM
- 1,2 gen lives on orbifold fixed points/tori: in 16 of SO(10)
- 3rd gen, Higgs, gauge live more in bulk: split multiplets
- m(1,2)~m(3/2)~10-30 TeV
- m(3)~m(H)~A's~m(inos)~1-3 TeV
- soft terms that of mirage mediation
- programmed Isajet 7.86

To generate minilandscape, take:

$$c_m = (16\pi^2/\alpha)^2 \text{ so that } m_0(1,2) \simeq m_{3/2}$$

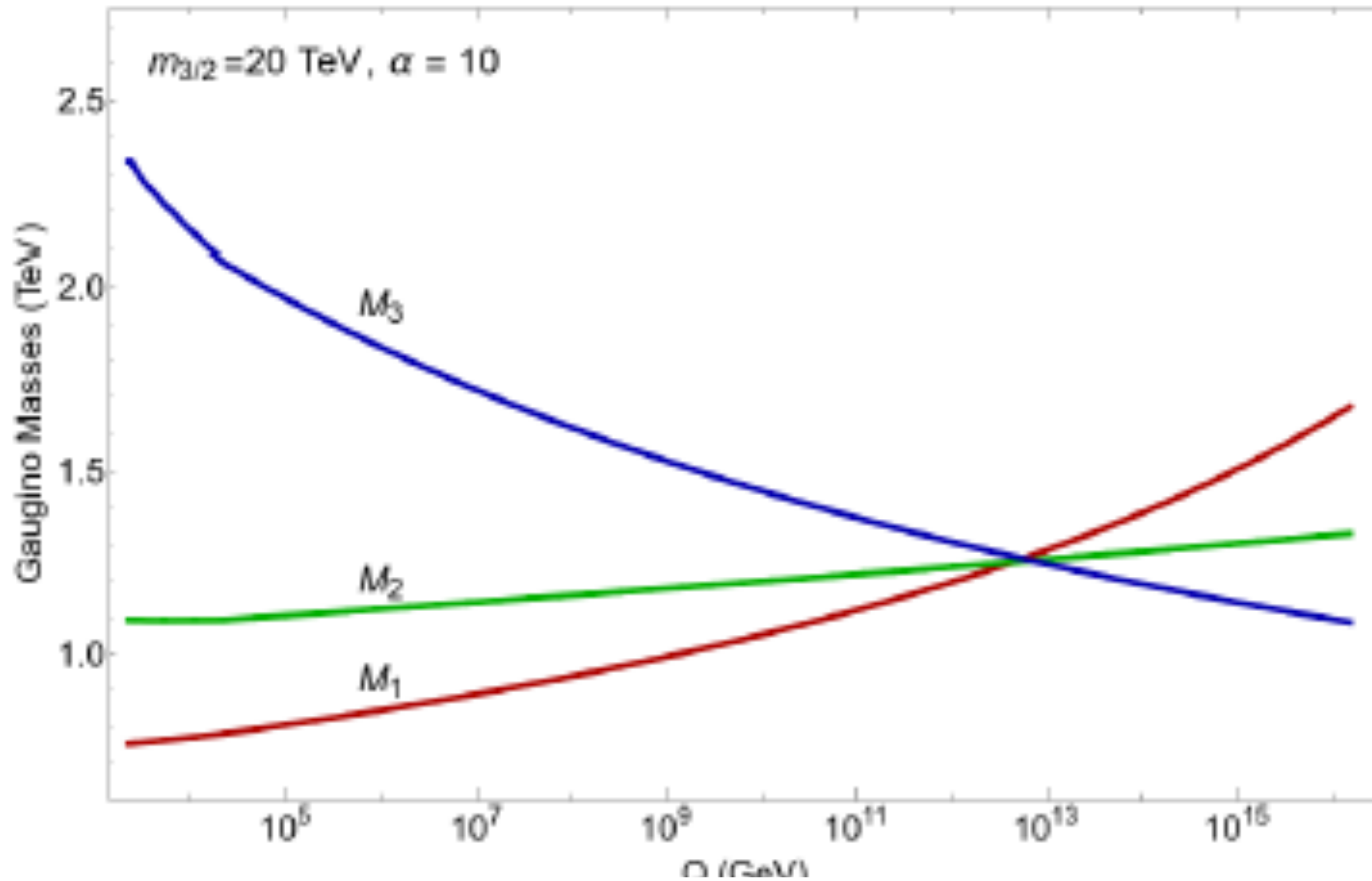


Then get upper bound $m_{3/2} < 25 - 30$ TeV and $\alpha > 7$ else too large $m_0(1,2)$ drives 3rd generation tachyonic

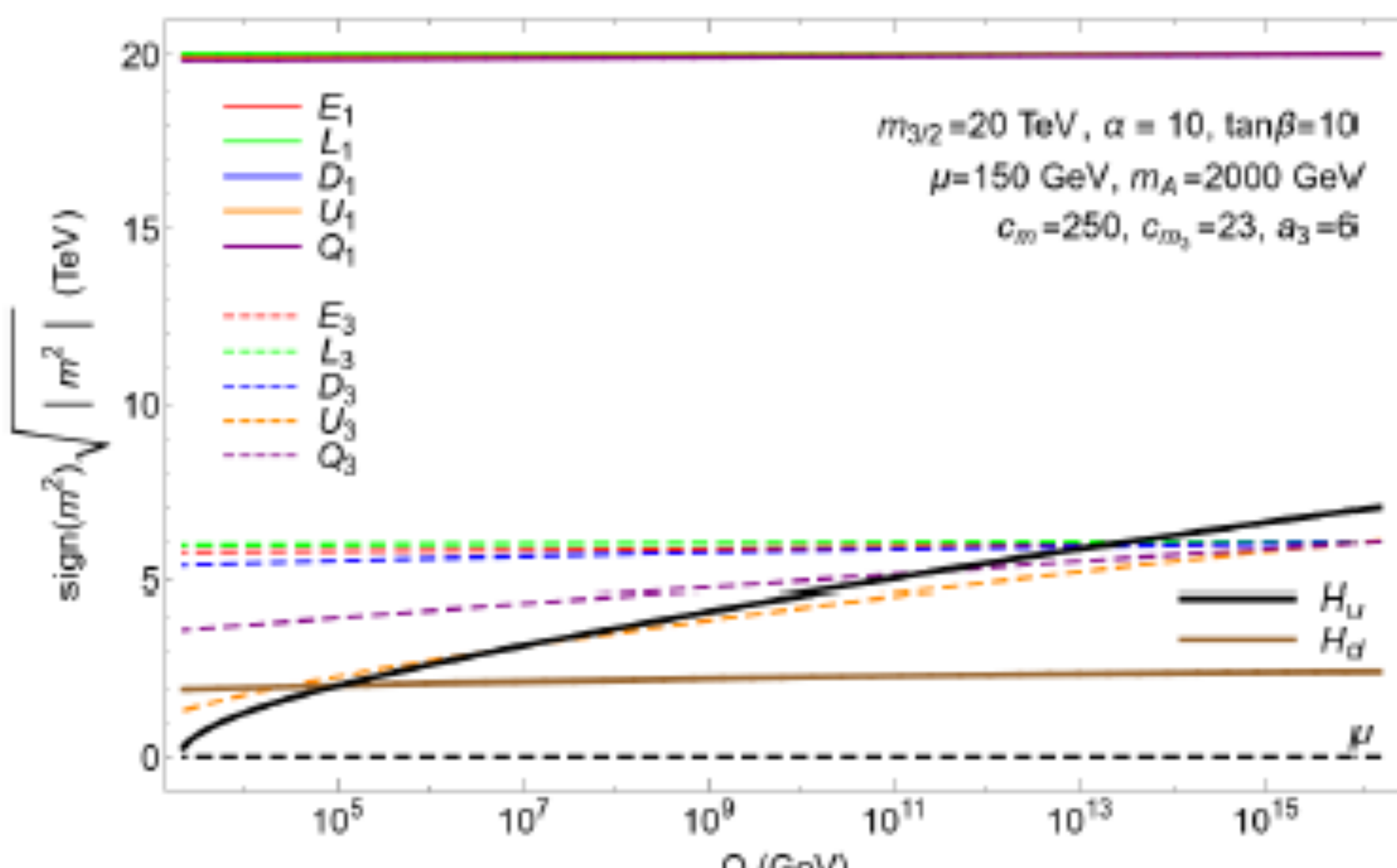
Martin, Vaughn, 2-loop RGEs

Increased upper bound on m(gluino)<6 TeV

Alpha bound => mirage unif scale >10^11 GeV
(not too much compression of inos)



$m_{3/2}$=20 TeV, α = 10
Gaugino Masses (TeV)
M_3
M_2
M_1
Q (GeV)



E_1
L_1
D_1
U_1
Q_1
E_3
L_3
D_3
U_3
Q_3
$m_{3/2}$=20 TeV, α = 10, tanβ=10
μ=150 GeV, m_A=2000 GeV
c_m=250, c_{m_3}=23, a_3=6
$\mathrm{sign}(m^2)\sqrt{|m^2|}$ (TeV)
H_u
H_d
μ
Q (GeV)

$$\Delta_{EW} = 17.6$$

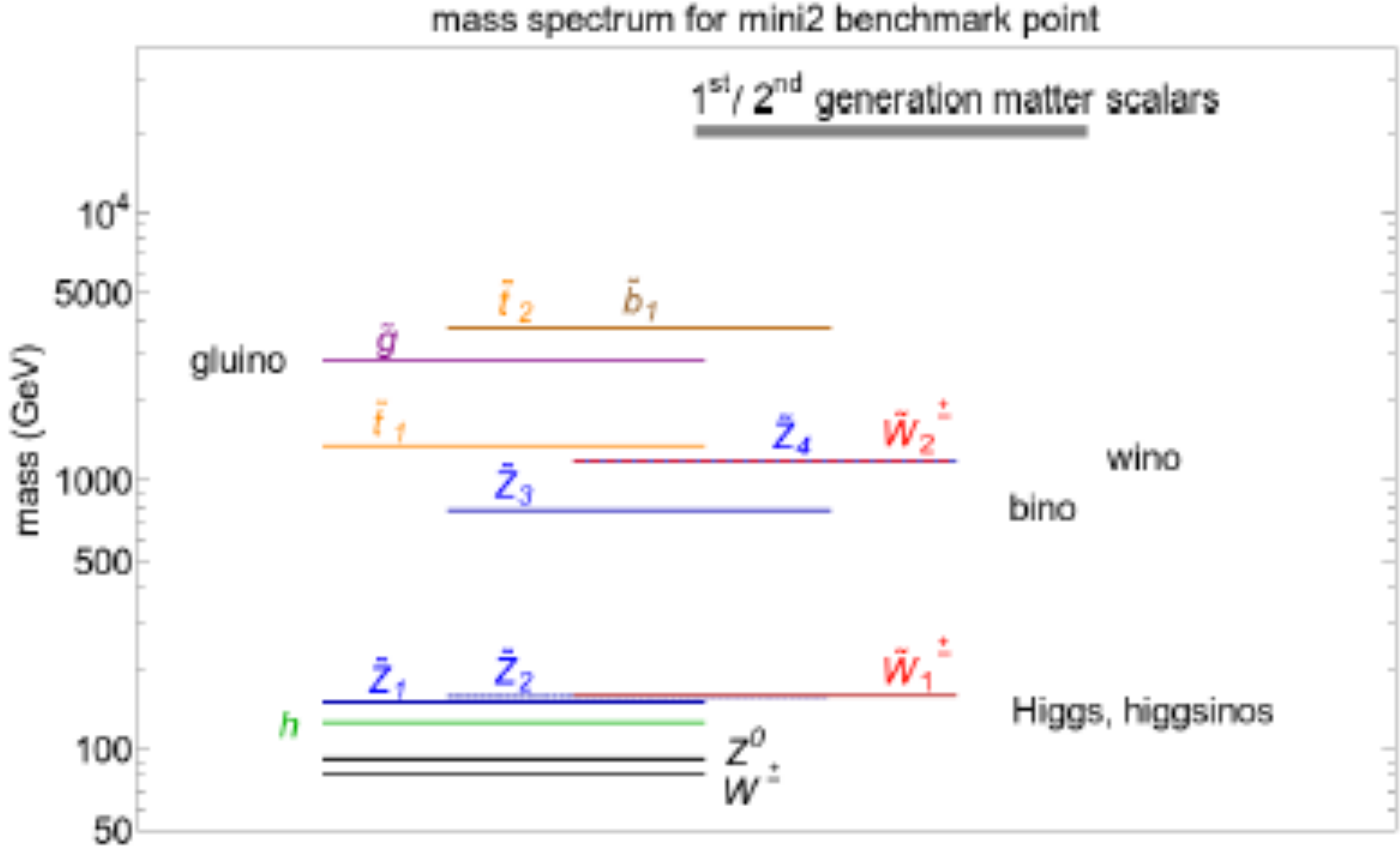


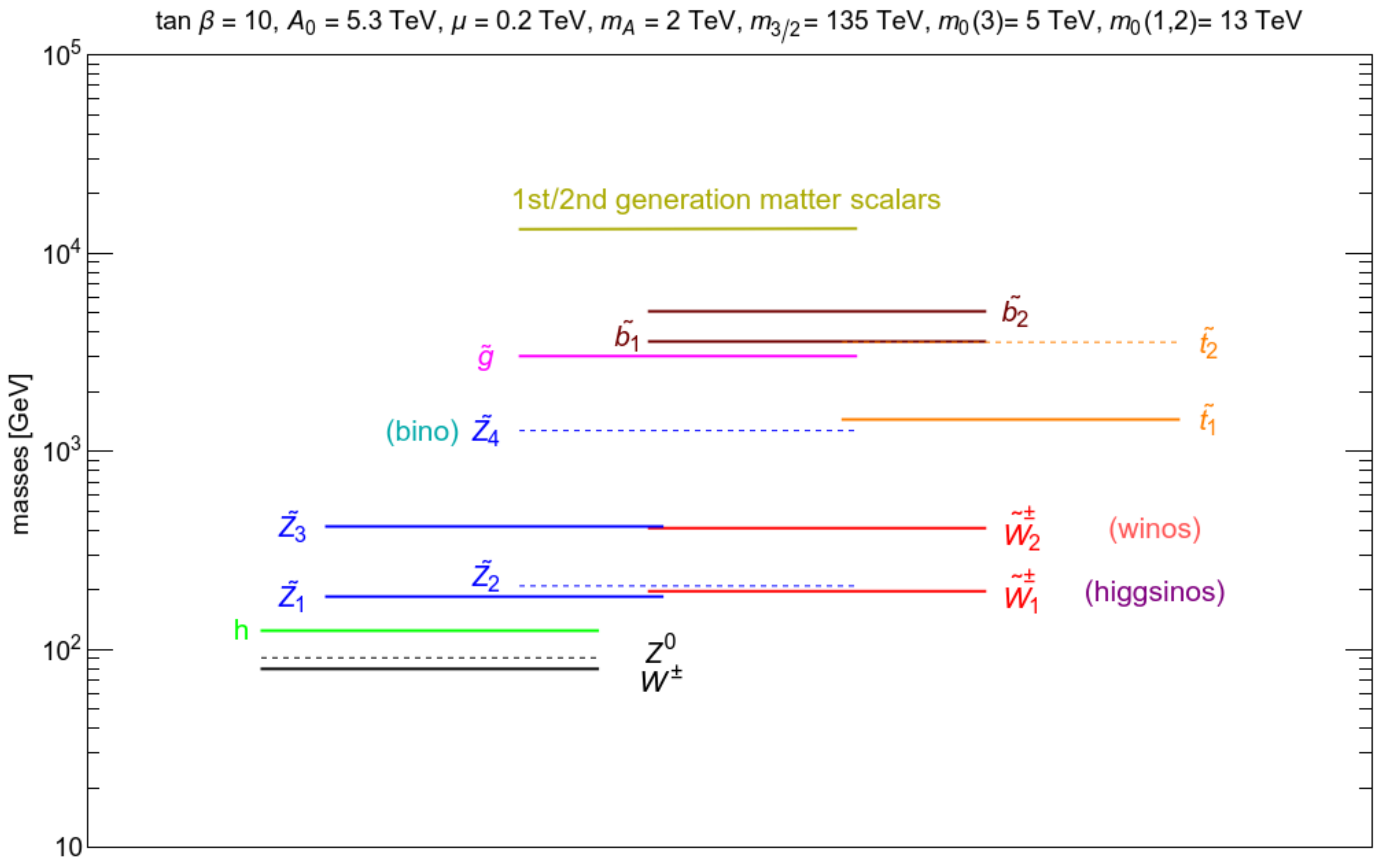
Figure 7: The superparticle mass spectra from the natural mini-landscape point mini2 of Table 1.

Can also construct natural AMSB models

- Begin with usual mAMSB: $m_0(bulk), m_{3/2}, \tan\beta$
- Allow Higgs fields to develop independent bulk soft terms: why shouldn't they? They live in different multiplets (RS)
- Allow small (compared to $m_{3/2}$) bulk A_0 terms (RS)
- Added freedom allows for light higgsinos, highly mixed stops
- Natural AMSB with $m_h \sim 125$ GeV!
- gauginos still ordered as usual but: $\mu < M_2 < M_1 < M_3$
- May need ILC with $\sqrt{s} > 2m(higgsino) \sim 500 - 600$ GeV to sort out gaugino hierarchy (unified, mirage or AMSB?) via *Higgsino code*!

HB, Barger, Sengupta

natural AMSB with m(h)~125 GeV



Summary so far:

First order question:
why is the weak scale m(W,Z,h)~100 GeV?
Because mu(weak), mHu(weak)~100-200 GeV
and top squarks ~few TeV but highly mixed

Second order question:
Why might mu<< m(SUSY)
and why are soft terms such that
mHu(weak)~100-200 GeV?

Some answers: see tomorrow talk!

SUSY mu problem: mu term is SUSY, not SUSY breaking:
expect mu~M(Pl) but phenomenology requires mu~m(Z)

- NMSSM: mu~m(3/2); but beware singlets!
- Giudice-Masiero: mu forbidden by some symmetry: generate via Higgs coupling to hidden sector
- Kim-Nilles: invoke SUSY version of DFSZ axion solution to strong CP: $W \ni \lambda_\mu S^2 H_u H_d / m_P$

KN: PQ symmetry forbids mu term,
but then it is generated via PQ breaking $\mu \sim \lambda_\mu f_a^2 / m_P$

Little Hierarchy due to mismatch between
PQ breaking and SUSY breaking scales?

$m_{3/2} \sim m_{hid}^2 / M_P$

$f_a \ll m_{hid}$

Higgs mass tells us where
to look for axion!

$$m_a \sim 6.2 \mu\text{eV} \left(\frac{10^{12}\ \text{GeV}}{f_a} \right)$$

Little Hierarchy from radiative PQ breaking? exhibited within context of MSY/CCK model

Murayama, Suzuki, Yanagida (1992);
Gherghetta, Kane (1995)

Choi, Chun, Kim (1996)

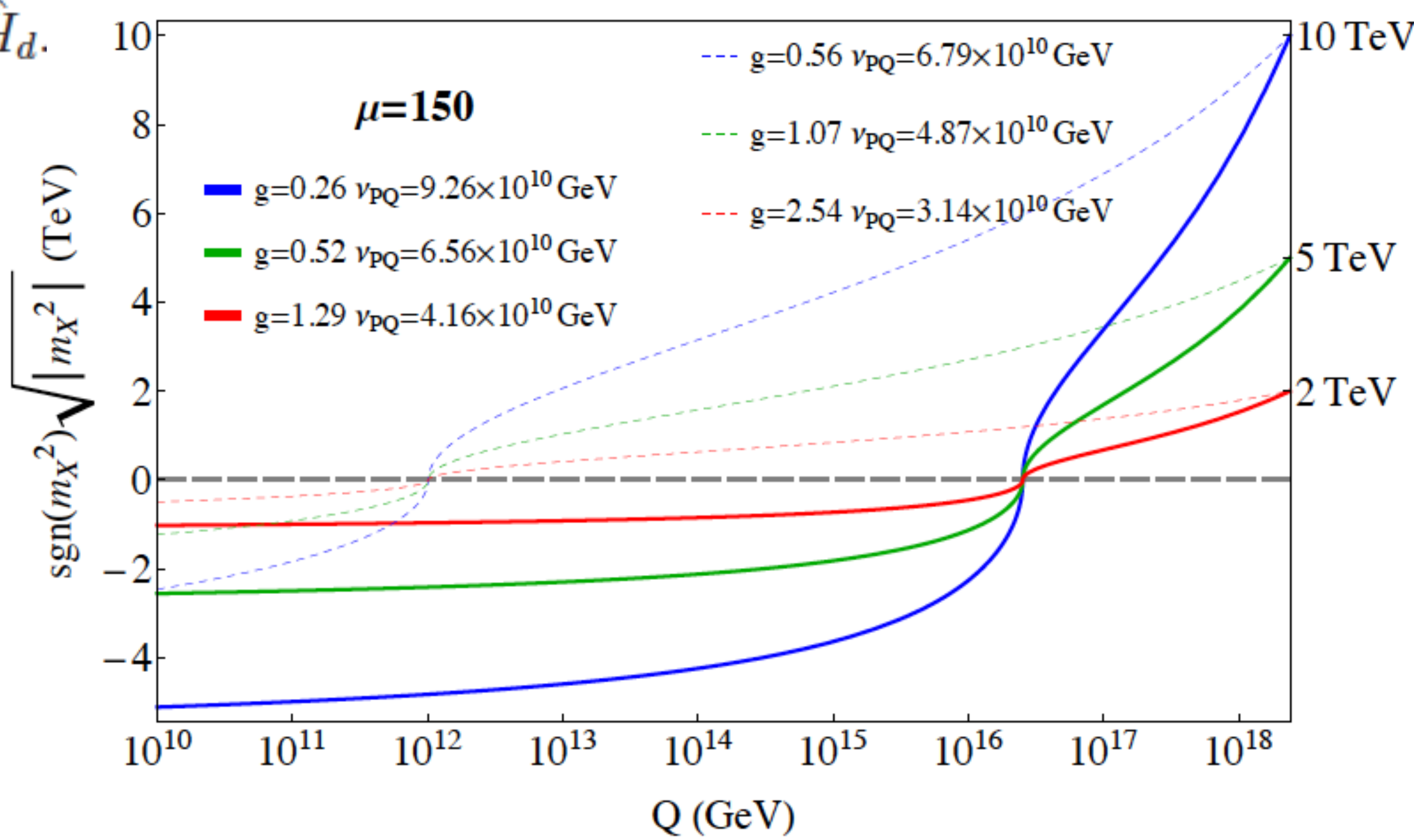
Bae, HB, Serce, PRD91 (2015) 015003

augment MSSM with PQ charges/fields:

$$\hat{f}' = \frac{1}{2}h_{ij}\hat{X}\hat{N}_i^c\hat{N}_j^c + \frac{f}{M_P}\hat{X}^3\hat{Y} + \frac{g}{M_P}\hat{X}\hat{Y}\hat{H}_u\hat{H}_d.$$

$$M_{N_i^c} = v_X\, h_i|_{Q=v_X}$$

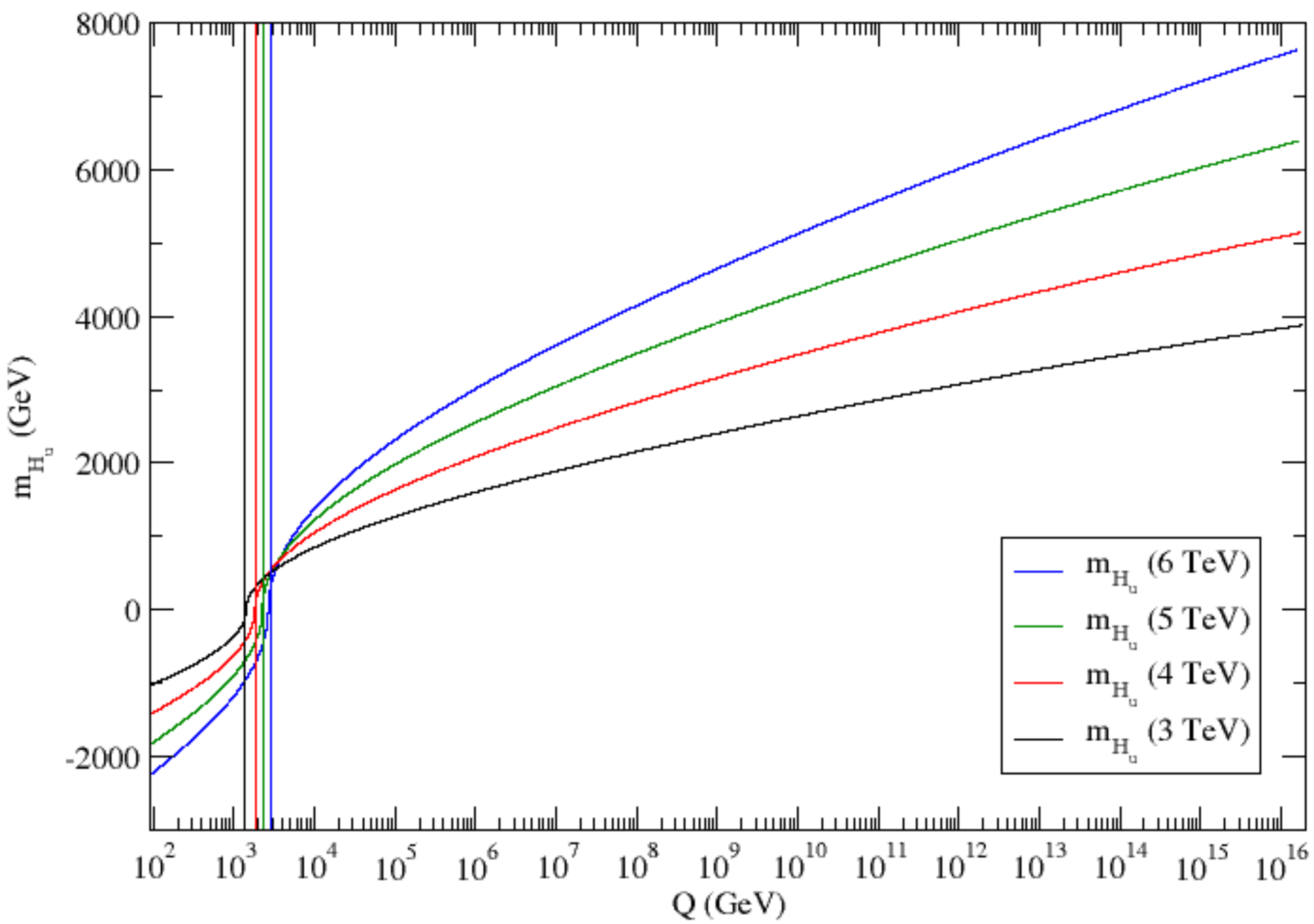
$$\mu = g\frac{v_X v_Y}{M_P}\ .$$



Large $m_{3/2}$ generates small $\mu \sim 100 - 200$ GeV!

Why might mHu have the value needed to give naturalness at weak scale?

1. For right correlations amongst soft terms, get "generalized focus point"



HB, Barger, Savoy

e.g.

For $\mu = 150$ GeV, $\tan\beta = 10$ and

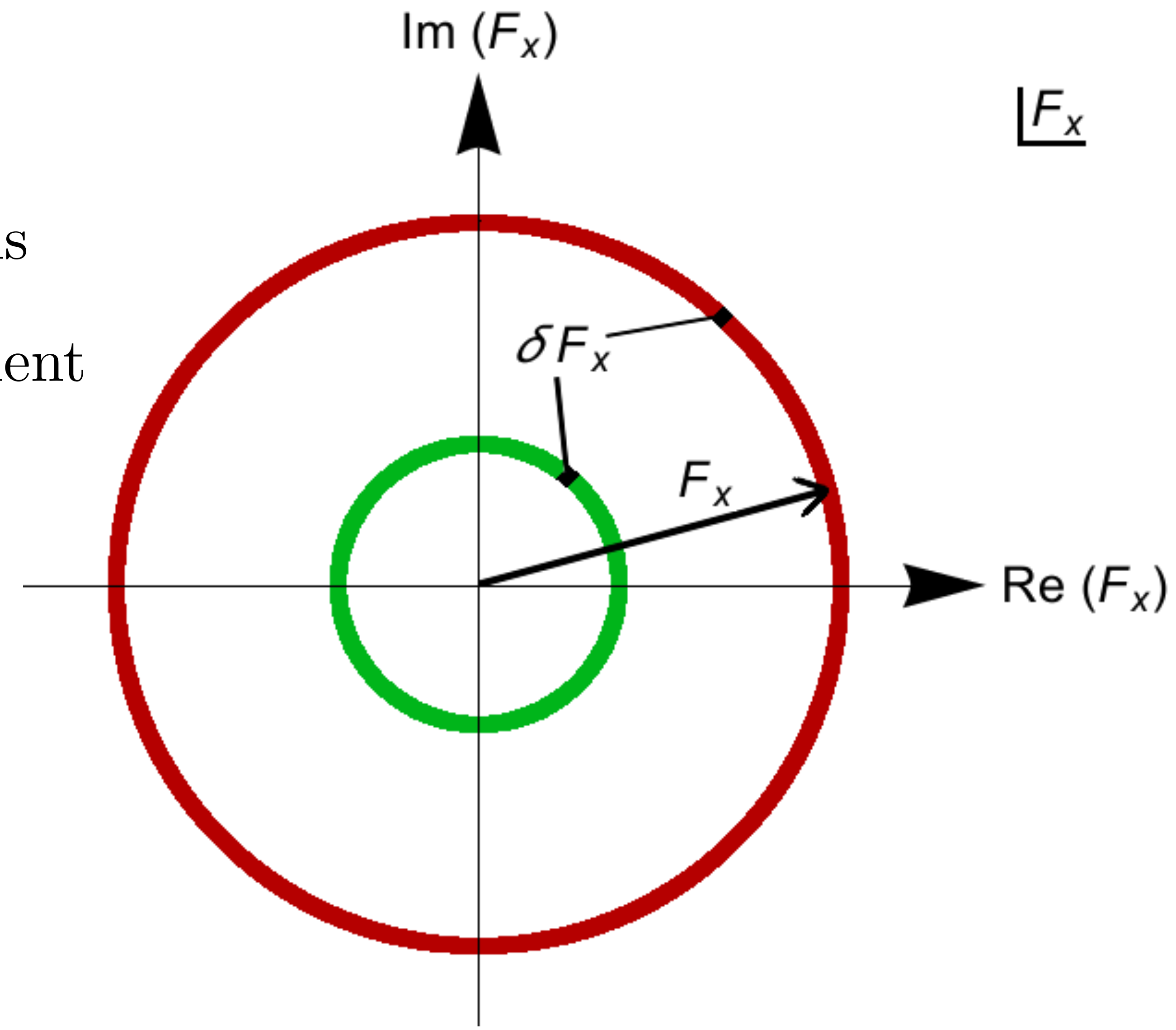
$$
\begin{aligned}
m_0^2 &= m_{3/2}^2 \\
A_0 &= -1.6 m_{3/2} \\
m_{1/2} &= m_{3/2}/5 \\
m_{H_d}^2 &= m_{3/2}^2/2.
\end{aligned}
$$

$$m_{H_u}^2(GUT) = 1.8 m_{3/2}^2 - (212.52\ \text{GeV})^2.$$

Why do soft terms take on values needed for natural (barely-broken) EWSB?

2. string theory landscape?

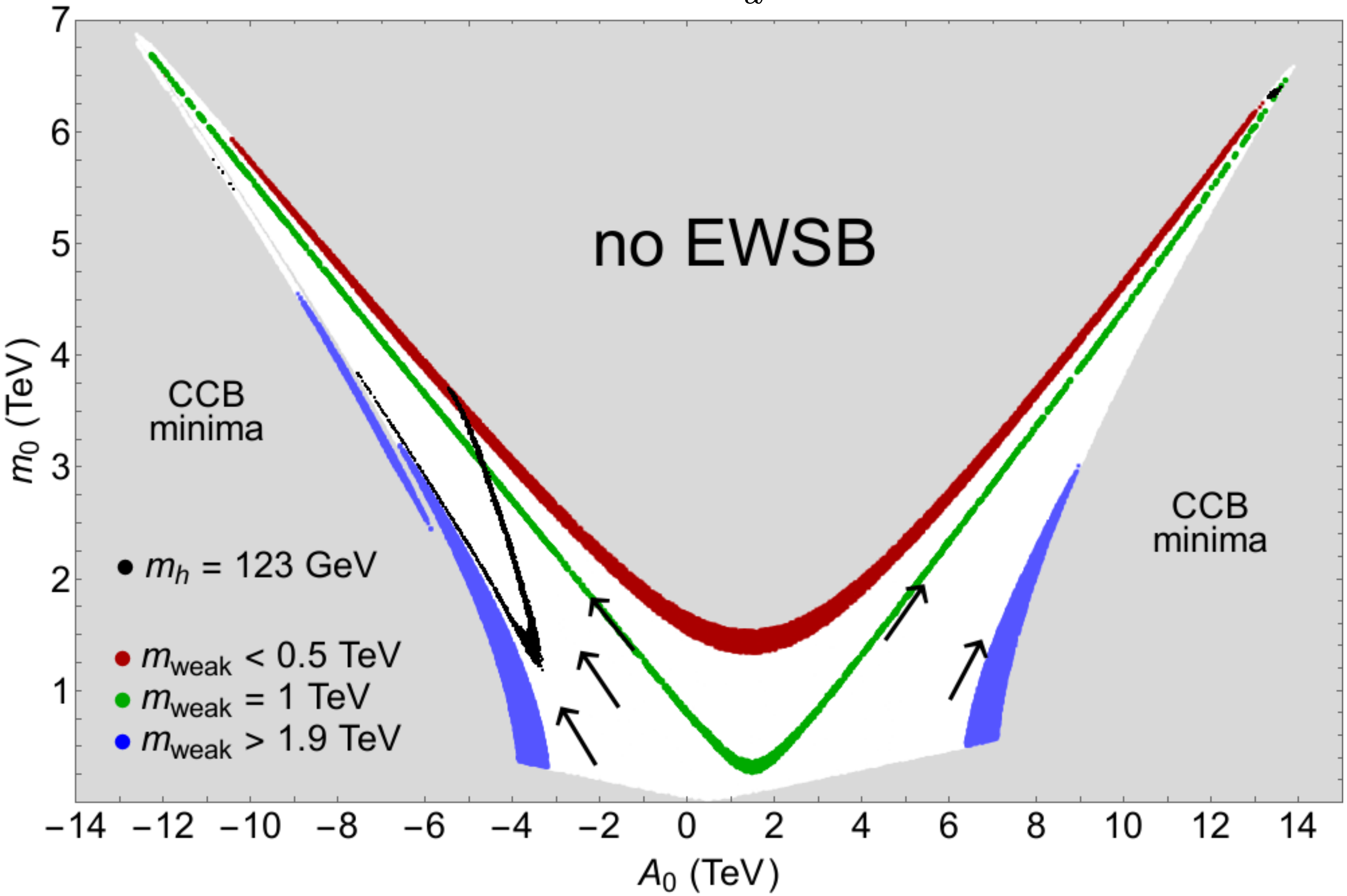
- assume model like MSY/CCK where $\mu \sim 100$ GeV
- then $m(weak)^2 \sim |m^2_{H_u}|$
- If all values of SUSY breaking field $\langle F_X \rangle$ equally likely, then mild (linear) statistical draw towards large soft terms
- This is balanced by anthropic requirement of weak scale $m_{weak} \sim 100$ GEV



$m(weak) <\sim 400$ GeV (Agrawal et al.)

Anthropic selection of $m_{weak} \sim 100$ *GeV*:
If m_W too large, then weak interactions $\sim (1/m_W^4)$ too weak
weak decays, fusion reactions suppressed
elements not as we know them

$$m_{H_u} = 1.3 m_0$$



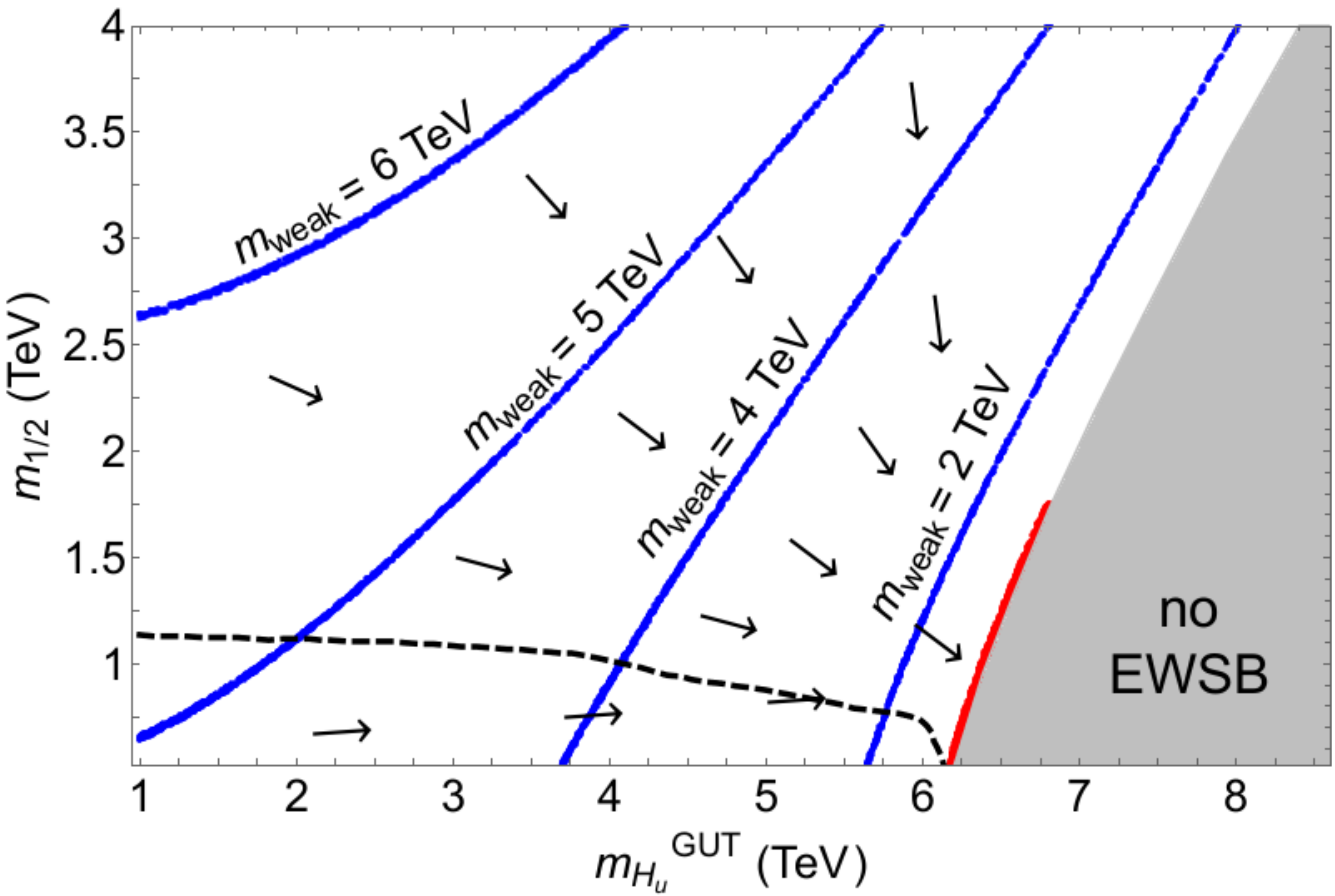
statistical draw to large soft terms balanced by anthropic draw toward red (m(weak)~100 GeV): then m(Higgs)~125 GeV and natural SUSY spectrum!

Denef, Douglas, JHEP0405 (2004) 072

Giudice, Rattazzi, NPB757 (2006) 19;

HB, Barger, Savoy, Serce, PLB758 (2016) 113

$m_0 = 5\ \mathrm{TeV}$



statistical/anthropic draw toward FP-like region

Statistical analysis of SUSY breaking scale:
M. Douglas, hep-th/0405279

some reasonable assumptions

- string theory landscape contains vast ensemble of N=1, d=4 SUGRA EFTs at high scales
- the EFTs contain the SM as weak scale EFT
- the EFTs contain visible sector +potentially large hidden sector
- visible sector contains MSSM plus extra gauge singlets (e.g. a PQ sector, RN neutrinos,...)
- SUGRA is broken spontaneously via superHiggs mechanism via either F- or D- terms or in general a combination

Scalar potential is given by usual SUGRA form:

$$
\begin{aligned}
V &= e^{K/m_P^2}\left(g^{i\bar{j}} D_i W D_{\bar{j}} W^* - \frac{3}{m_P^2}|W|^2\right) + \frac{1}{2}\sum_\alpha D_\alpha^2 \\
&= e^{K/m_P^2}\left(\sum_i |F_i|^2 - 3\frac{|W|^2}{m_P^2}\right) + \frac{1}{2}\sum_\alpha D_\alpha^2
\end{aligned}
$$

- W = holomorphic superpotential
- K = real Kähler function
- $F_i = D_i W = DW/D\phi^i \equiv \partial W/\partial\phi^i + (1/m_P^2)(\partial K/\partial\phi^i)W$ are F-terms
- $D_\alpha \sim \sum \phi^\dagger g t_\alpha \phi$ are D-terms
- ϕ^i are chiral superfields

minimize V:

- $\partial V/\partial\phi^i = 0$
- $\partial^2 V/\partial\phi^i\partial\phi^j > 0$
- $\Lambda_{cc} = m_{hidden}^4 - 3e^{K/m_P^2}|W|^2/m_P^2$ with
- $m_{hidden}^4 = \sum_i |F_i|^2 + \frac{1}{2}\sum_\alpha D_\alpha^2$ is hidden sector mass scale

gravitino mass $m_{3/2} = e^{K/2m_P^2} m_P \sim m_{hidden}^2/m_P$ with $m_{hidden} \sim 10^{12}$ GeV

Denef&Douglas: statistics of SUSY breaking in landscape

DD observation: W_0 distributed uniformly as complex variable allows dynamical neutralization of Λ while not influencing SUSY breaking

Then, number of flux vacua containing spontaneously broken SUGRA with SUSY breaking scale m^2_{hidden} is:

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} dm^2_{hidden}$$

- $f_{cc} \sim \Lambda/m^4$ where DD maintain $m \sim m_{string}$ and not m_{hidden}
- $f_{SUSY}(m^2_{hidden}) \sim (m^2_{hidden})^{2n_F+n_D-1}$ for uniformly distributed values of F and D breaking fields
- $f_{EWFT} \sim m^2_{weak}/m^2_{soft}$ (?) where $m_{soft} \sim m_{3/2} \sim m^2_{hidden}/m_P$

$$n = 2n_F + n_D - 1$$

$$f_{SUSY} \sim m^n_{soft}$$

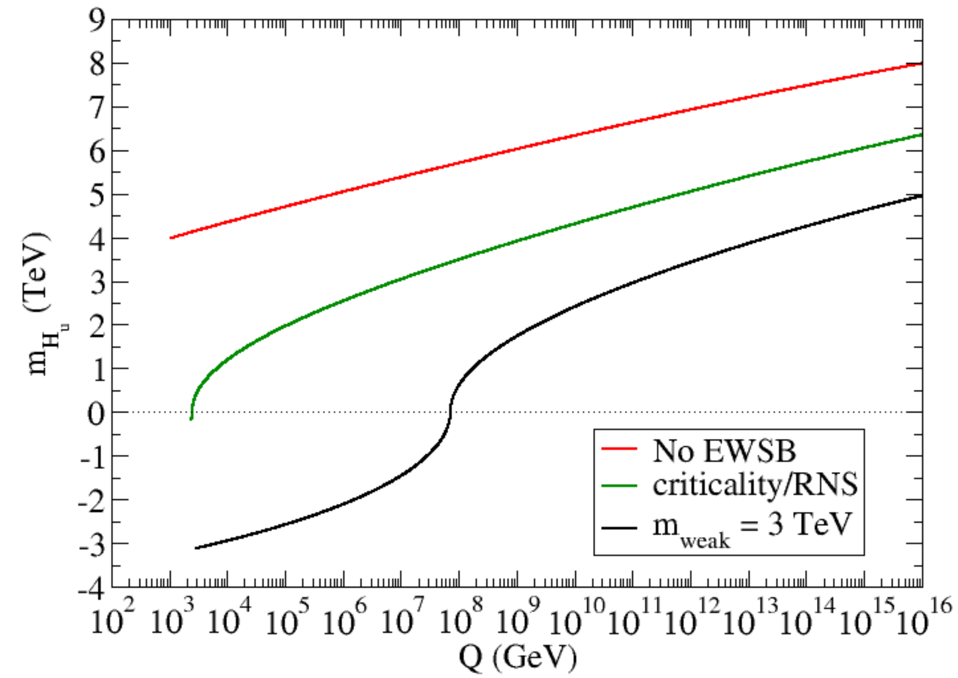
n_F	n_D	n
0	1	0
1	0	1
0	2	1
1	1	2
0	3	2
2	0	3
2	1	4

landscape favors high scale SUSY breaking
tempered by f(EWFT) anthropic penalty!

What about DD/AD anthropic penalty $f_{EWFT} \sim m^2_{weak}/m^2_{soft}$?

This fails in a variety of *practical* cases:

- A-terms get large: $\Rightarrow CCB$ minima
- $m^2_{H_u}$ too large: fail to break EW symmetry



Must require proper EWSB!

Even if EWS properly broken, then

- large A_t *reduces* EWFT in the $\Sigma^u_u(\tilde{t}_{1,2})$
- large $m^2_{H_u}(m_{GUT})$ needed to radiatively drive $m^2_{H_u}$ to natural value at weak scale

Better proposal: $f_{EWFT} \Rightarrow \Theta(30 - \Delta_{EW})$
keeps calculated weak scale within factor ~ 4 of measured weak scale
$m_{weak} \equiv m_{W,Z,h} \sim 100$ GeV

Assume $\mu \sim 100-200$ GeV via *e.g.* rad PW breaking: then m_Z variable and may be large depending on soft terms $m^2_{H_{u,d}}$ and $\Sigma^{u,d}_{u,d}(i)$

$$\frac{m_Z^2}{2} = \frac{m^2_{H_d} + \Sigma^d_d - (m^2_{H_u} + \Sigma^u_u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

For practical calculations, adopt NUHM3 SUGRA model:

- $m_0(1,2)$ = gen(1,2) common soft mass
- $m_0(3)$ = gen(3) common soft mass
- $m_{H_u}^2$ up-Higgs soft mass
- $m_{H_d}^2$ down-Higgs soft mass
- $m_{1/2}$ = unified gaugino mass
- A_0 = unified trilinear soft term
- $\tan\beta$

Trade $m_{H_u}^2,\ m_{H_d}^2 \Leftrightarrow \mu,\ m_A$

$$m_0(1,2),\ m_0(3),\ m_{1/2},\ A_0,\ \tan\beta,\ \mu,\ m_A \quad (NUHM3)$$

Recent work: place on more quantitative footing: scan soft SUSY breaking parameters as m(soft)^n along with f(EWFT) penalty

We scan according to m_{soft}^n over:

- $m_0(1,2):\ 0.1-40$ TeV,
- $m_0(3):\ 0.1-20$ TeV,
- $m_{1/2}:\ 0.5-10$ TeV,
- $A_0:\ 0-\ -60$ TeV,
- $m_A:\ 0.3-10$ TeV,

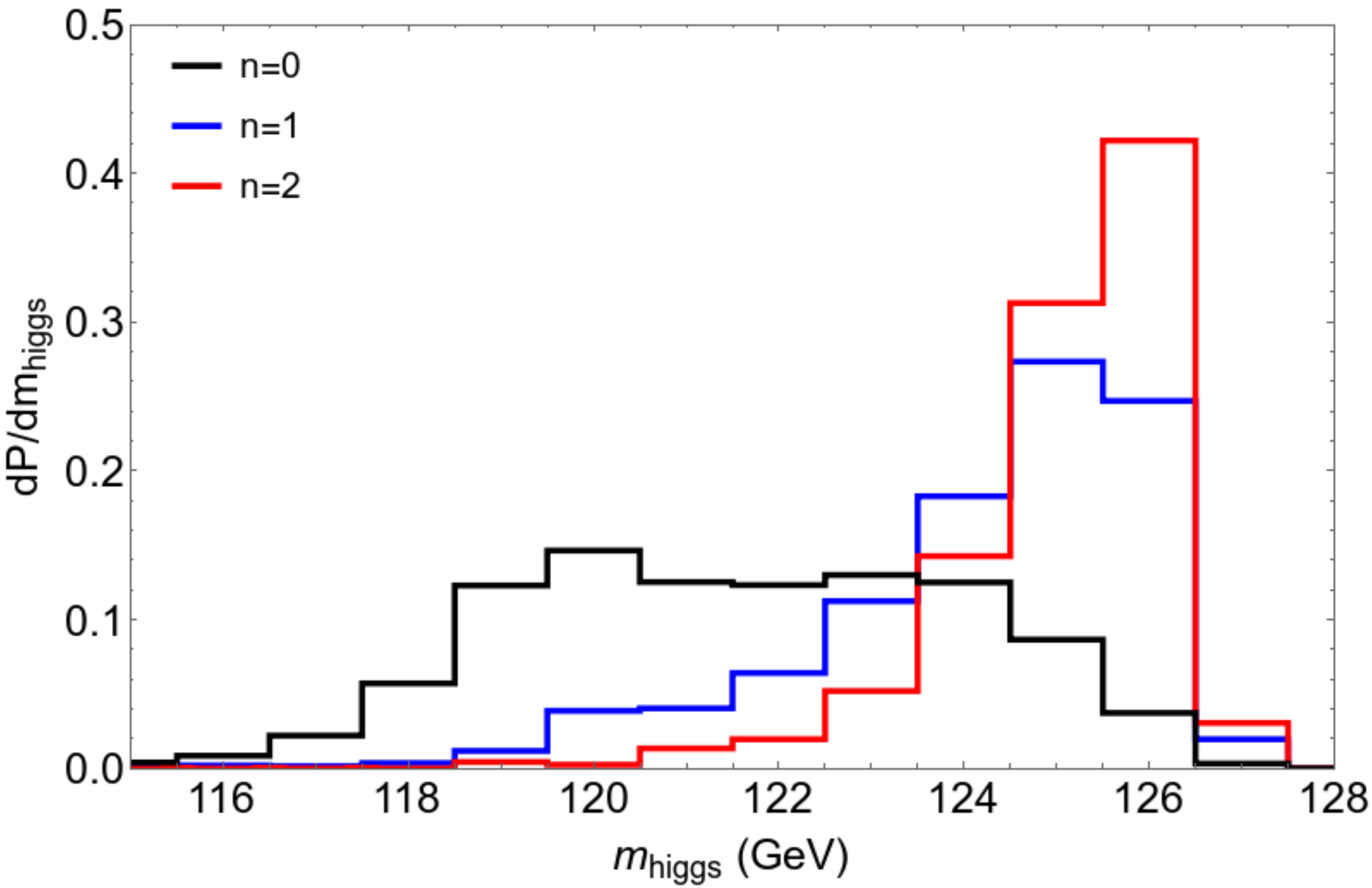
$\tan\beta:3-60$ (flat)

mu=150 GeV (fixed)

HB, Barger, Serce, Sinha (arXiv:1712.01399)

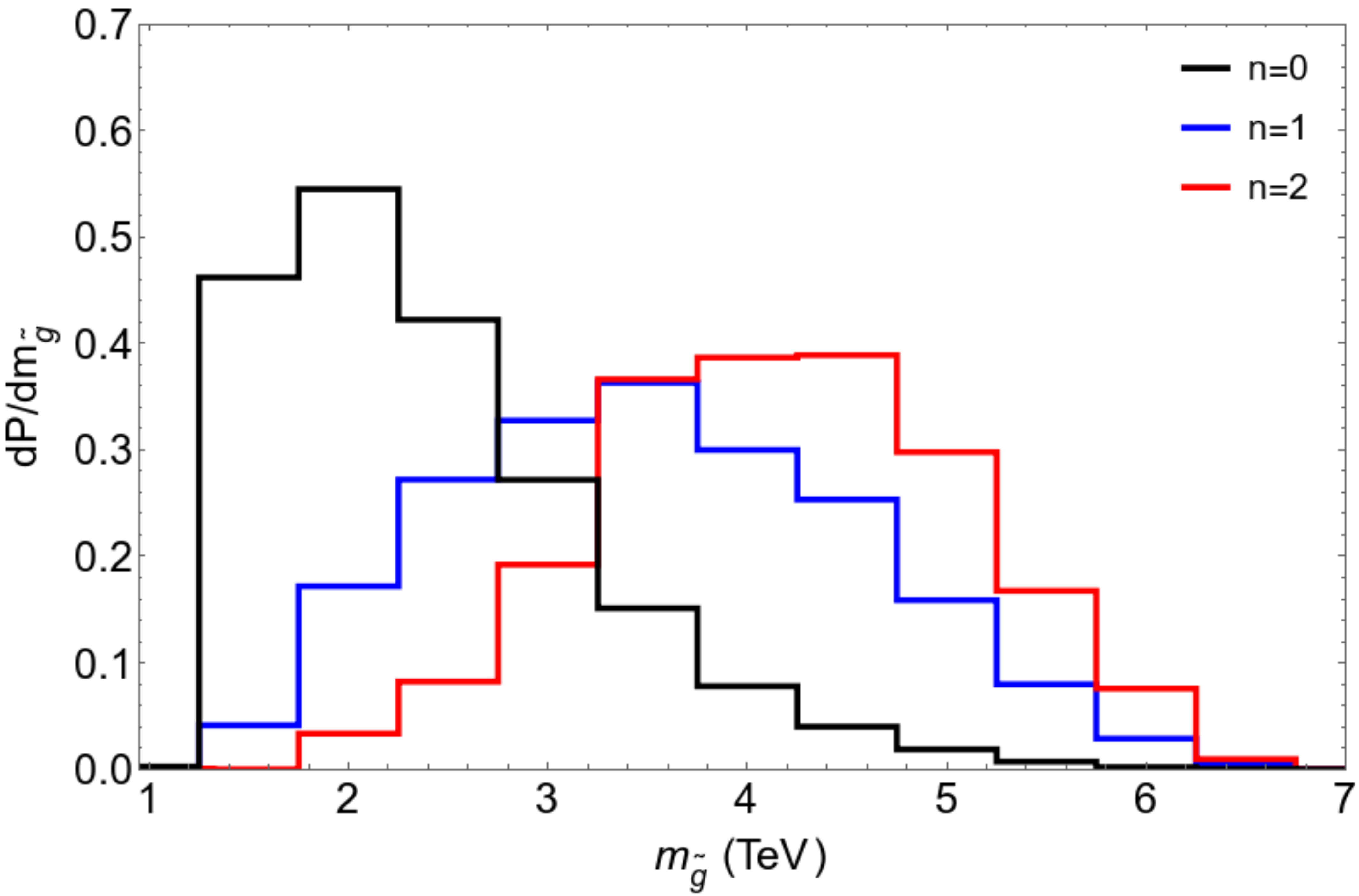
Making the picture more quantitative:

$$ac[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} dm^2_{hidden}$$



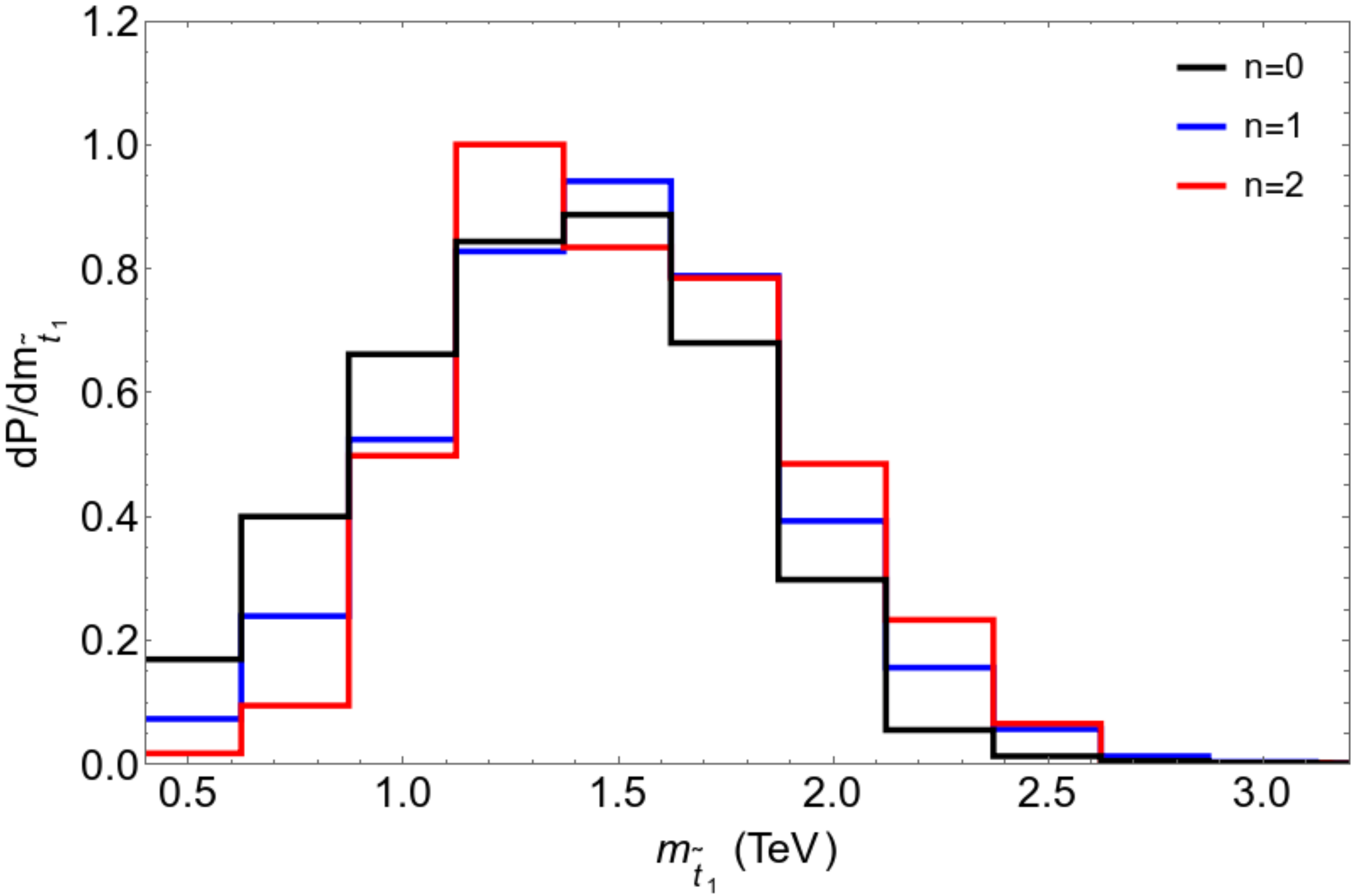
m(h)~125 most favored for n=1,2

What is corresponding distribution for gluino mass?



typically beyond LHC 14 reach (may need HE-LHC)

and m(t1)?



n=0
n=1
n=2
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$dP/dm_{\tilde{t}_1}$
$m_{\tilde{t}_1}$ (TeV)

Conclusion: SUSY IS alive and well!

- old calculations of naturalness over-estimate fine-tuning
- naturalness: Little Hierarchy mu<< m(SUSY) allowed
- radiatively-driven naturalness: mu~100-200 GeV, m(t1)<3 TeV, m(gluino)<5-6 TeV
- SUSY DFSZ axion: solve strong CP, solve SUSY mu problem; generate mu<< m(SUSY)
- landscape pull on soft terms towards RNS, m(h)~125 GeV
- natural mirage-mediation/mini-landscape
- natural NUHM2: HL-LHC can cover via SSdB+Z1Z2j channels
- natural mirage/mini-landscape may escape detection at HL-LHC; need LHC33!
- expect ILC as higgsino factory
- DM= axion+higgsino-like WIMP admixture: detect both?
- higgsino-like WIMP detection likely; axion more difficult

Backup

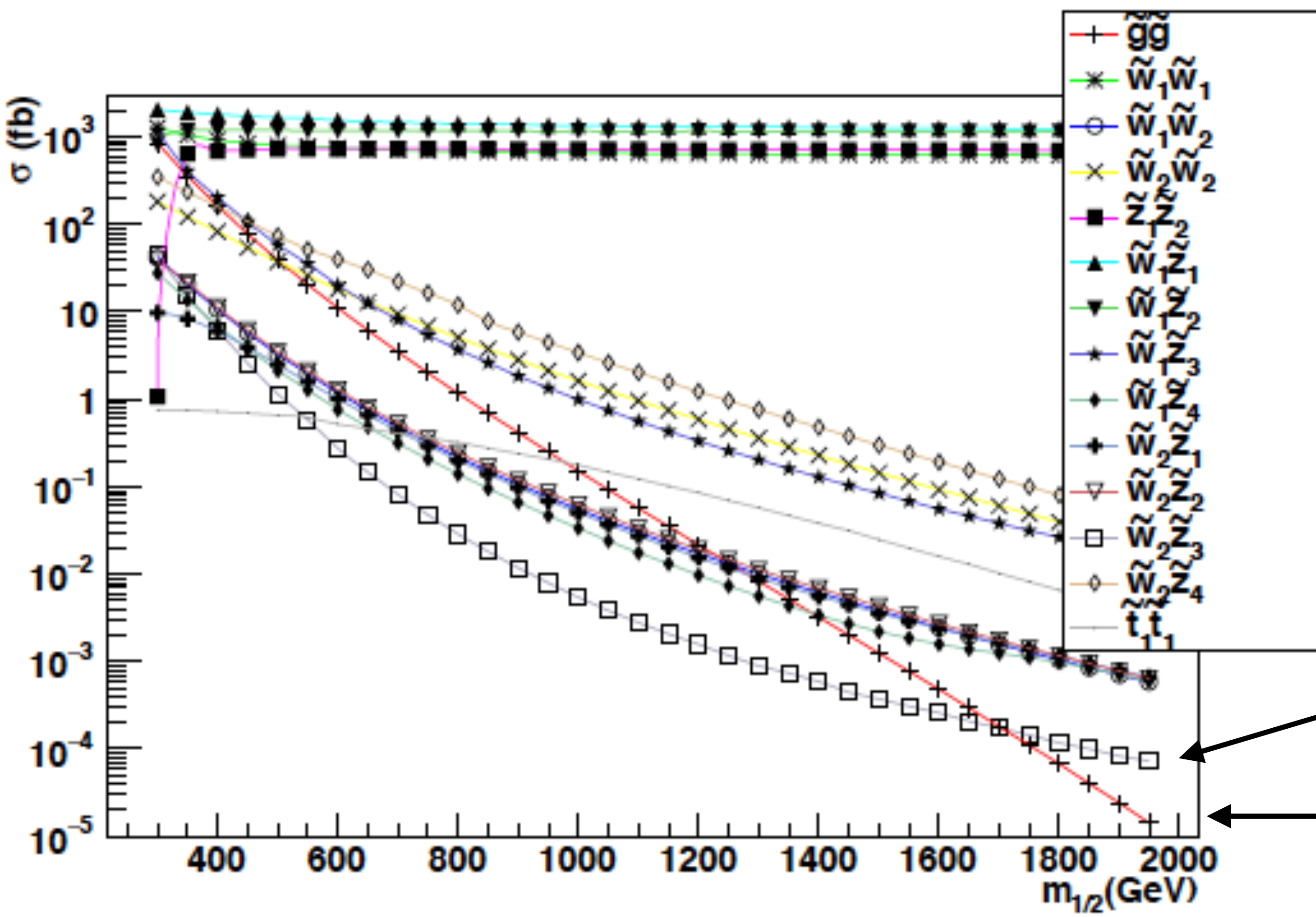
Prospects for SUSY at LHC:

signature list for radiatively-driven natural SUSY:

- $\tilde{g}\tilde{g}$
- $\tilde{t}_1\tilde{t}_1^*$
- $\tilde{Z}_1\tilde{Z}_2$ (higgsino pair production)
- $\tilde{W}_2^{\pm}\tilde{Z}_4$ (wino pair production)

Sparticle prod'n along RNS model-line at LHC14:

mu=150 GeV



higgsinos

gauginos

stops

gluinos

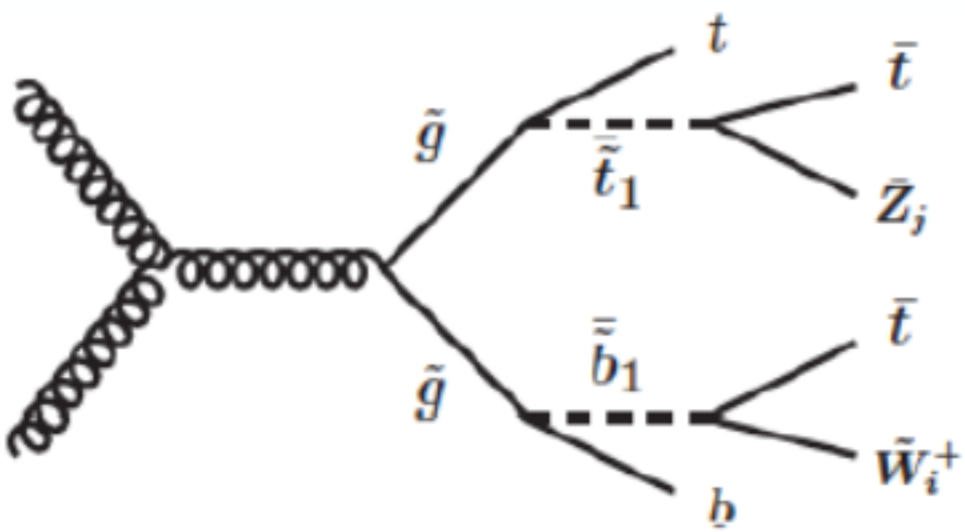
higgsino pair production dominant-but only soft visible energy release from higgsino decays

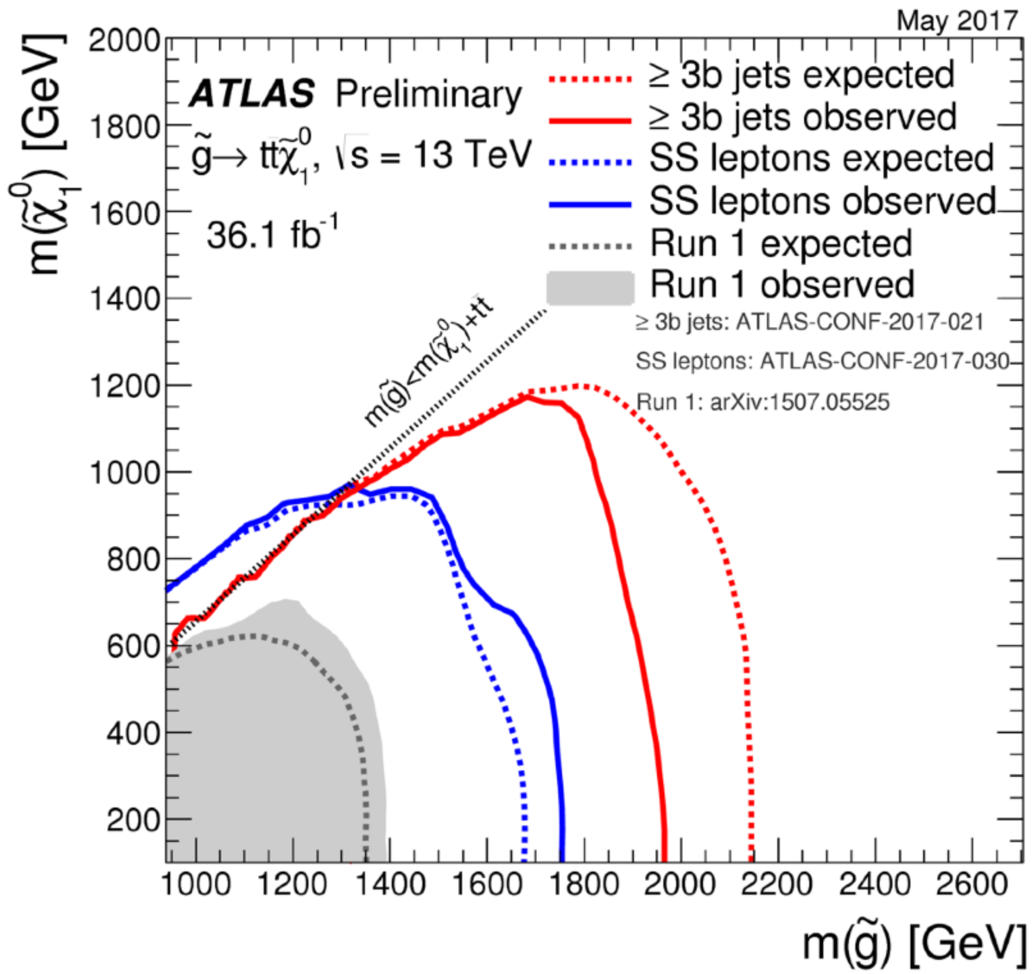
largest visible cross section: wino pairs

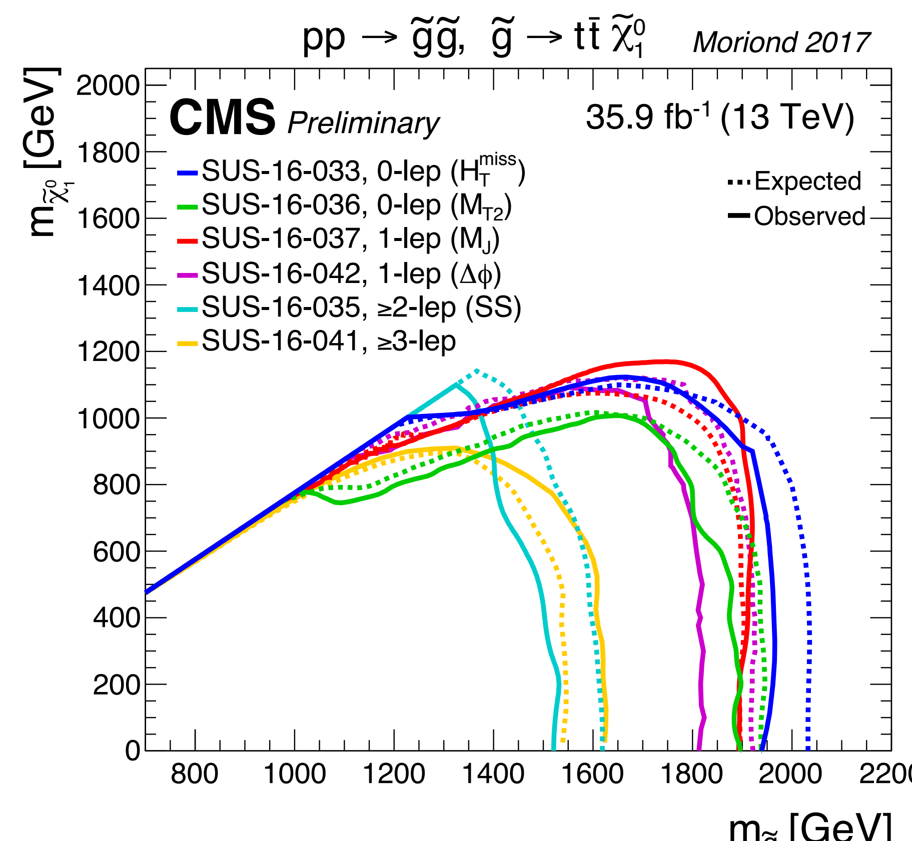
gluino pairs sharply dropping

stops at bottom

gluino pair cascade decay signatures

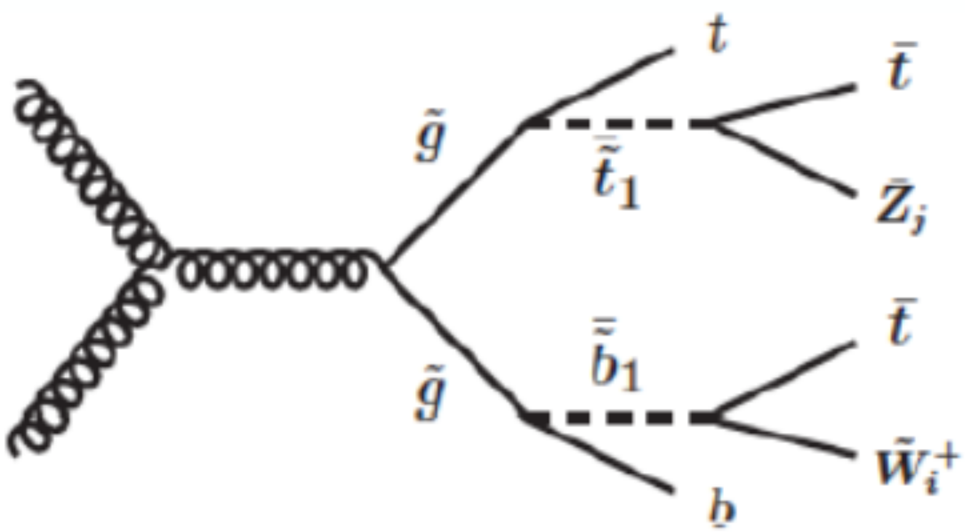






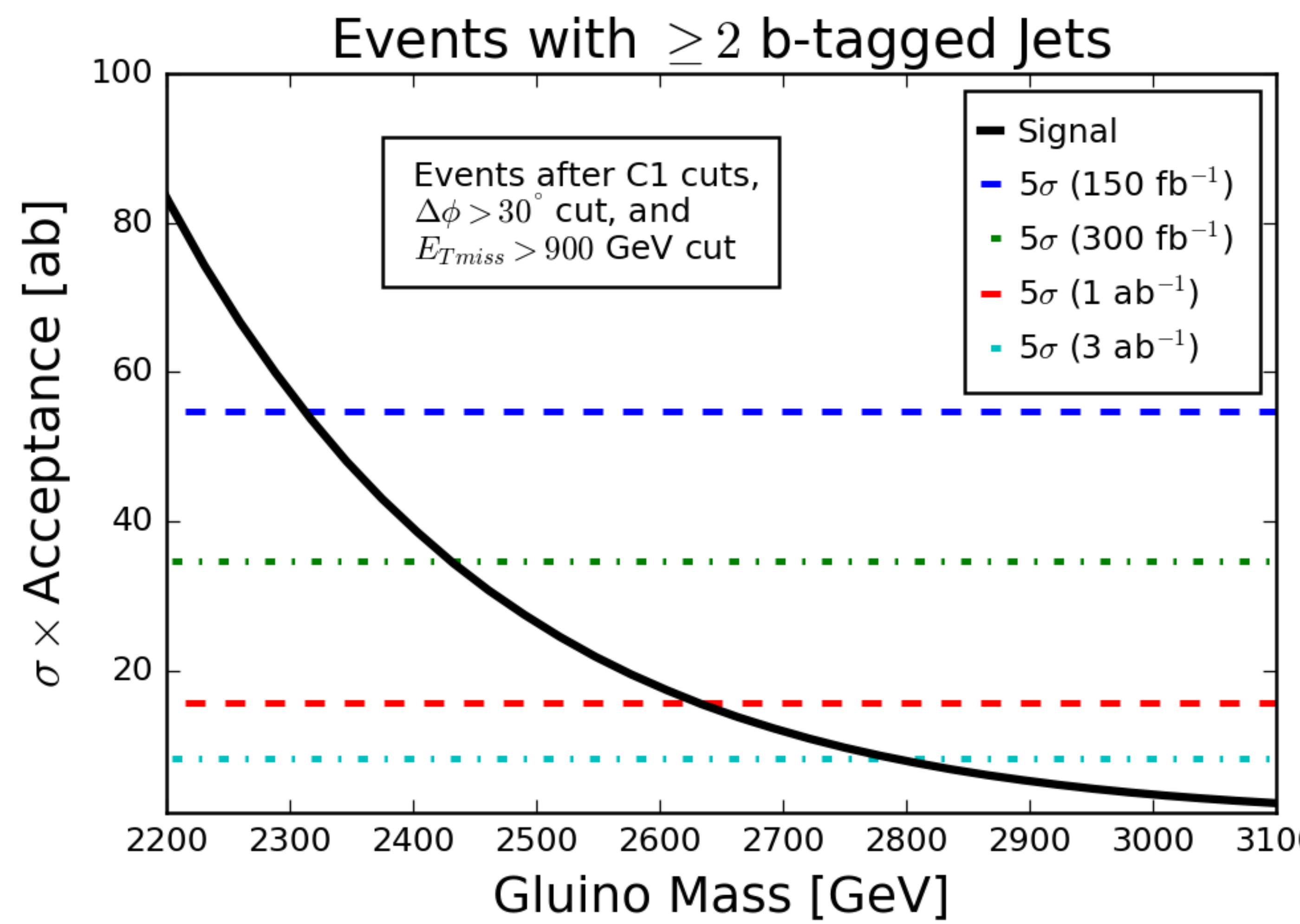
Current limits for m(Z1)~150 GeV:
m(glno)>~2 TeV

gluino pair cascade decay signatures



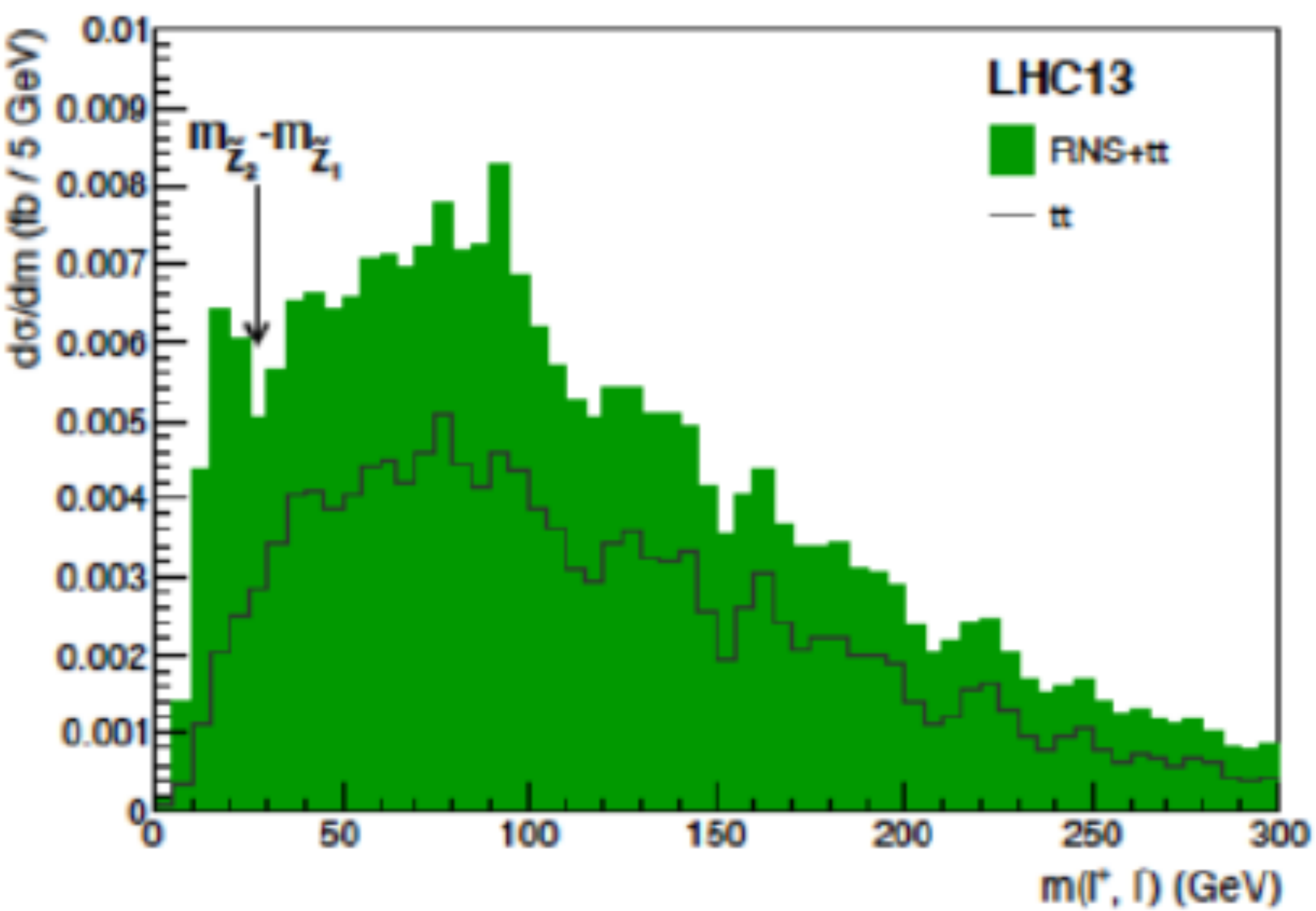
Estimated HL-LHC reach for gluinos

HL-LHC reach to
m(glno)~2.8 TeV;
RNS: m(glno)<~5 TeV



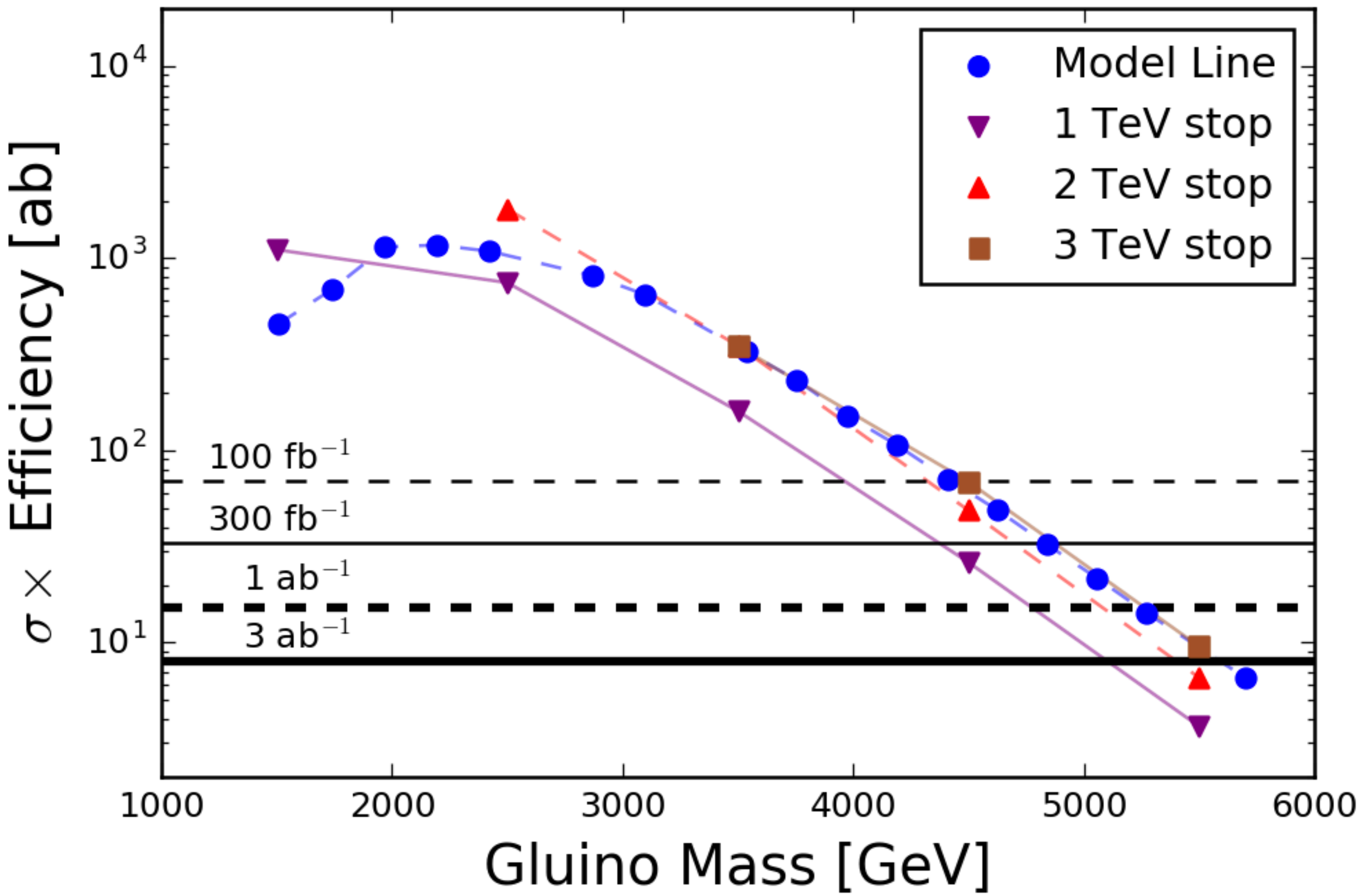
HB, Barger, Huang, Gainer, Savoy, Sengupta, Tata

LHC14 has some reach for gluino pair production in RNS; if a signal is seen, should be distinctive



OS/SF dilepton mass edge apparent from cascade decays with z2->z1+l+lbar

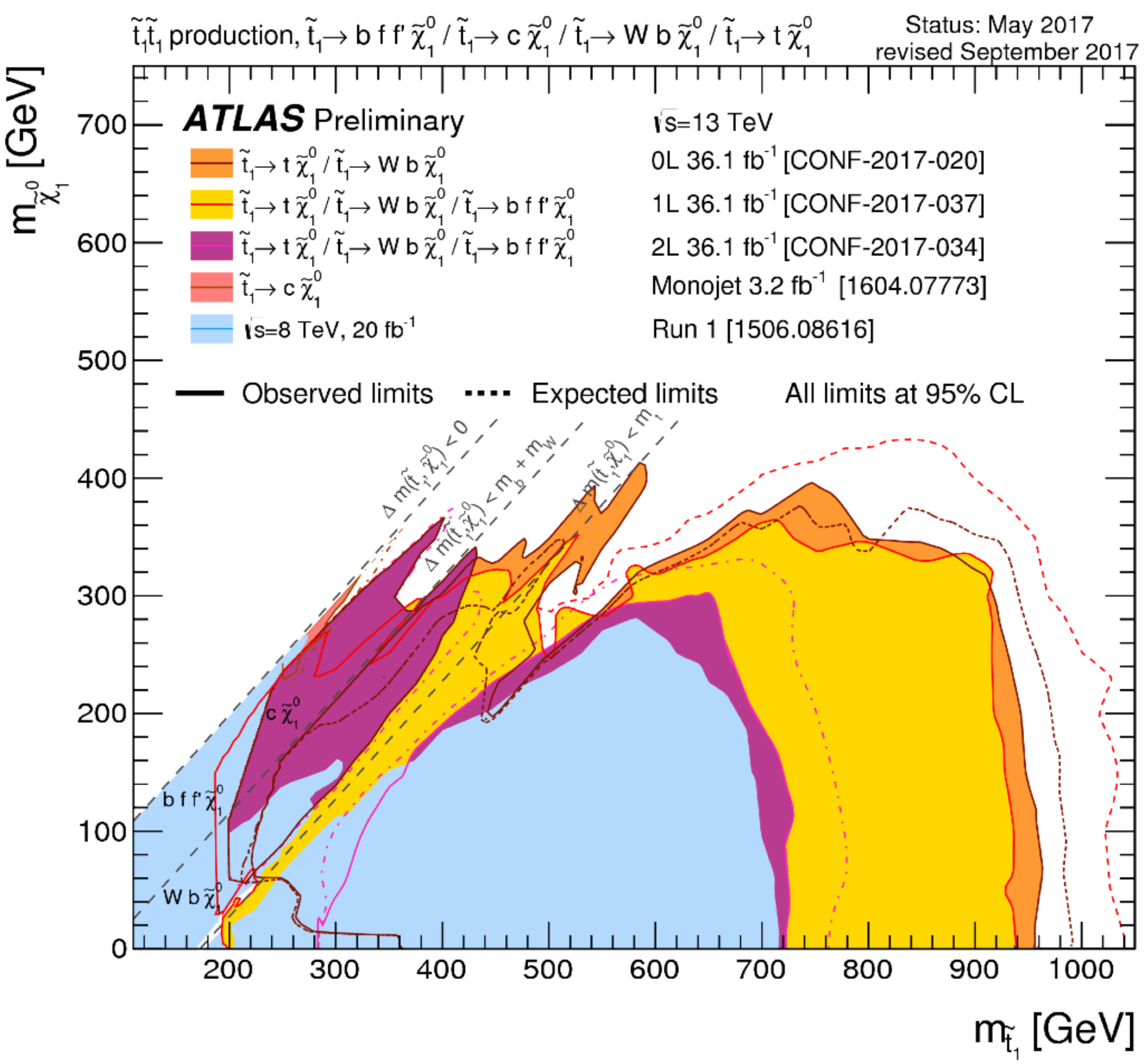
Gluino 5-sigma reach at LHC33: to about m(glno)~5-5.5 TeV

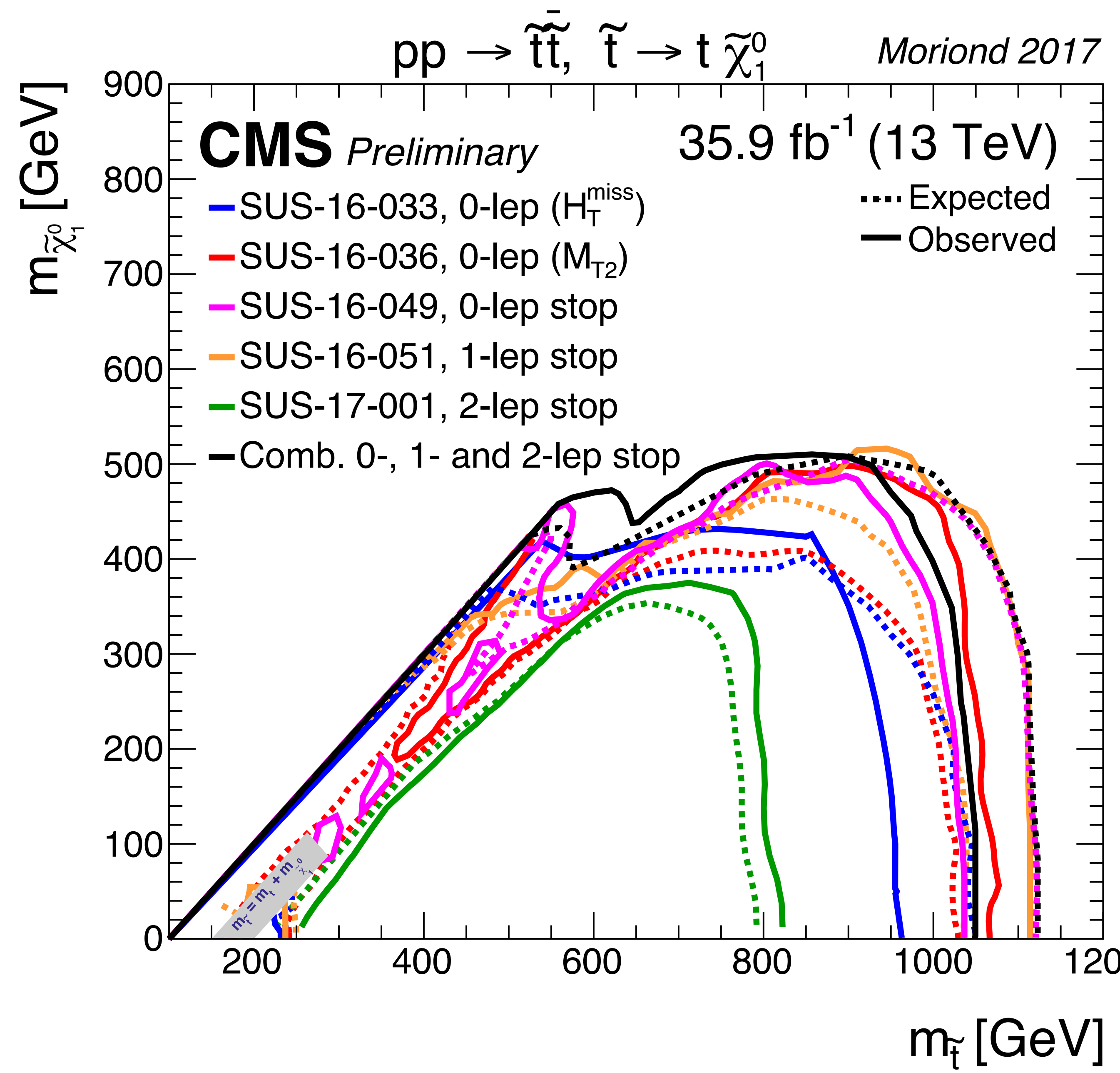


>=4 jets; >=2-b-jets;MET>1500 GeV

HB,Barger, Gainer,Huang, Savoy, Serce, Tata

Present limits on top squarks from LHC

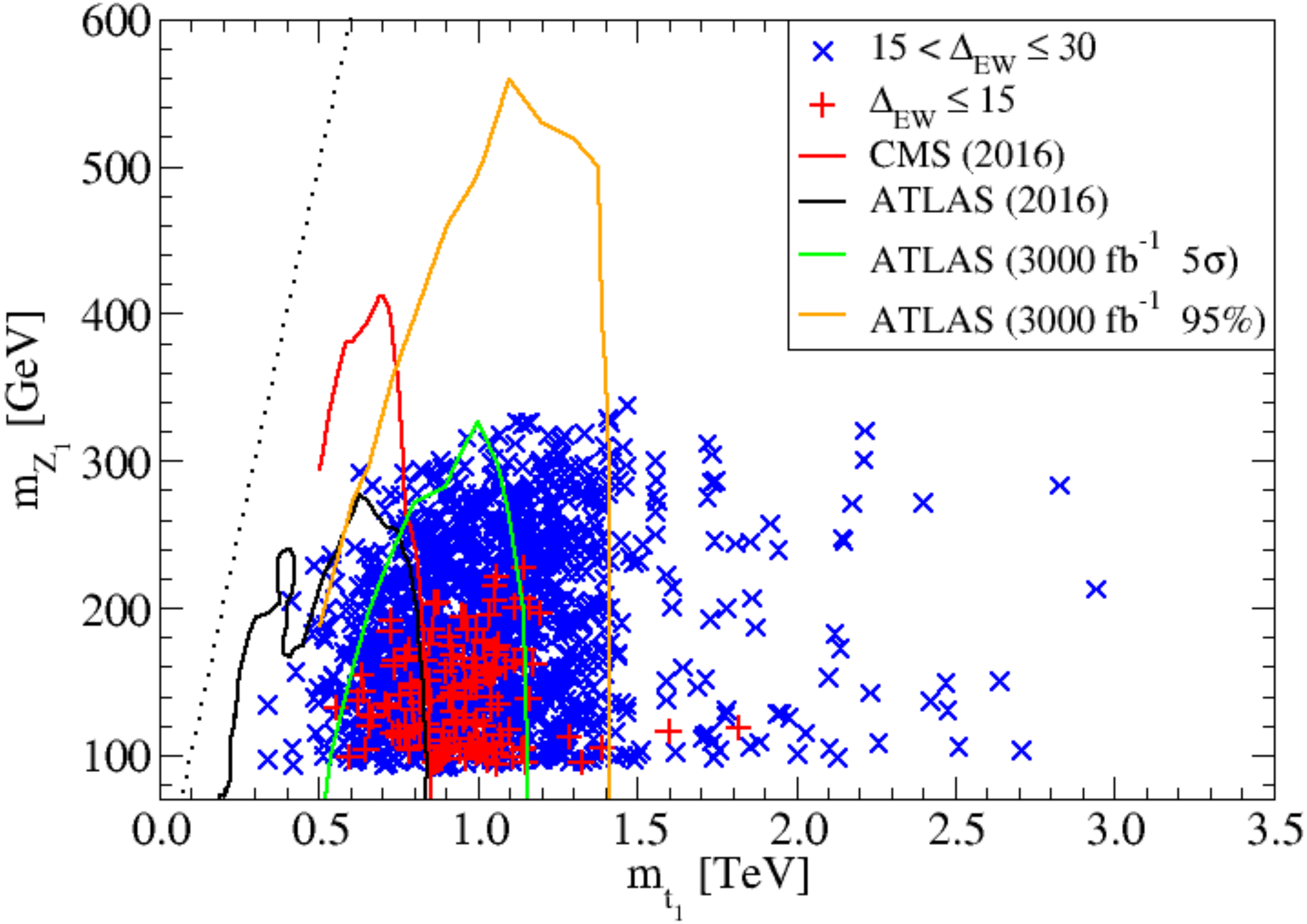




Evidently m(t1)>~1 TeV for m(LSP)~150 GeV

- * TeV-scale top squark needed for m(h)~125 GeV
- * Also needed for b-> s gamma

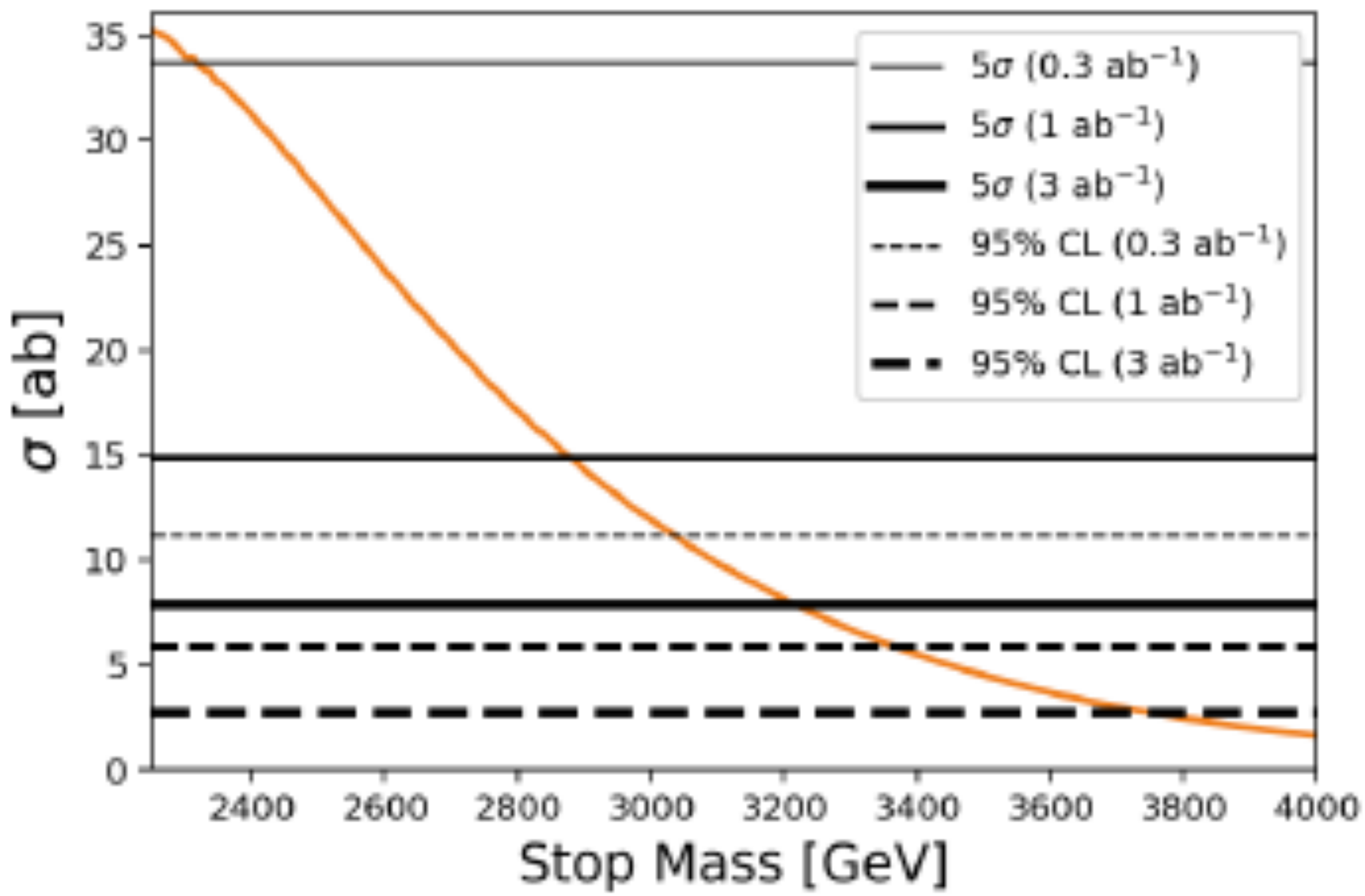
Prospects for top squarks in natural SUSY



m(t1) can range up to 3 TeV with little cost to naturalness;
the hunt for stops has only begun!

HL-LHC reach extends to m(t1)~1.2-1.4 TeV

Reach of LHC33 for top squarks



- $\tilde{t}_1 \to b\tilde{W}_1; \sim 50\%$
- $\tilde{t}_1 \to t\tilde{Z}_1; \sim 25\%$
- $\tilde{t}_1 \to t\tilde{Z}_2; \sim 25\%$

- A. $\tilde{t}_1\tilde{t}_1^* \to b\bar{b} + E_T^{\rm miss} \quad \sim 25\%$,
- B. $\tilde{t}_1\tilde{t}_1^* \to b\bar{t},\ \bar{b}t + E_T^{\rm miss} \quad \sim 50\%$,
- C. $\tilde{t}_1\tilde{t}_1^* \to t\bar{t} + E_T^{\rm miss} \quad \sim 25\%$.

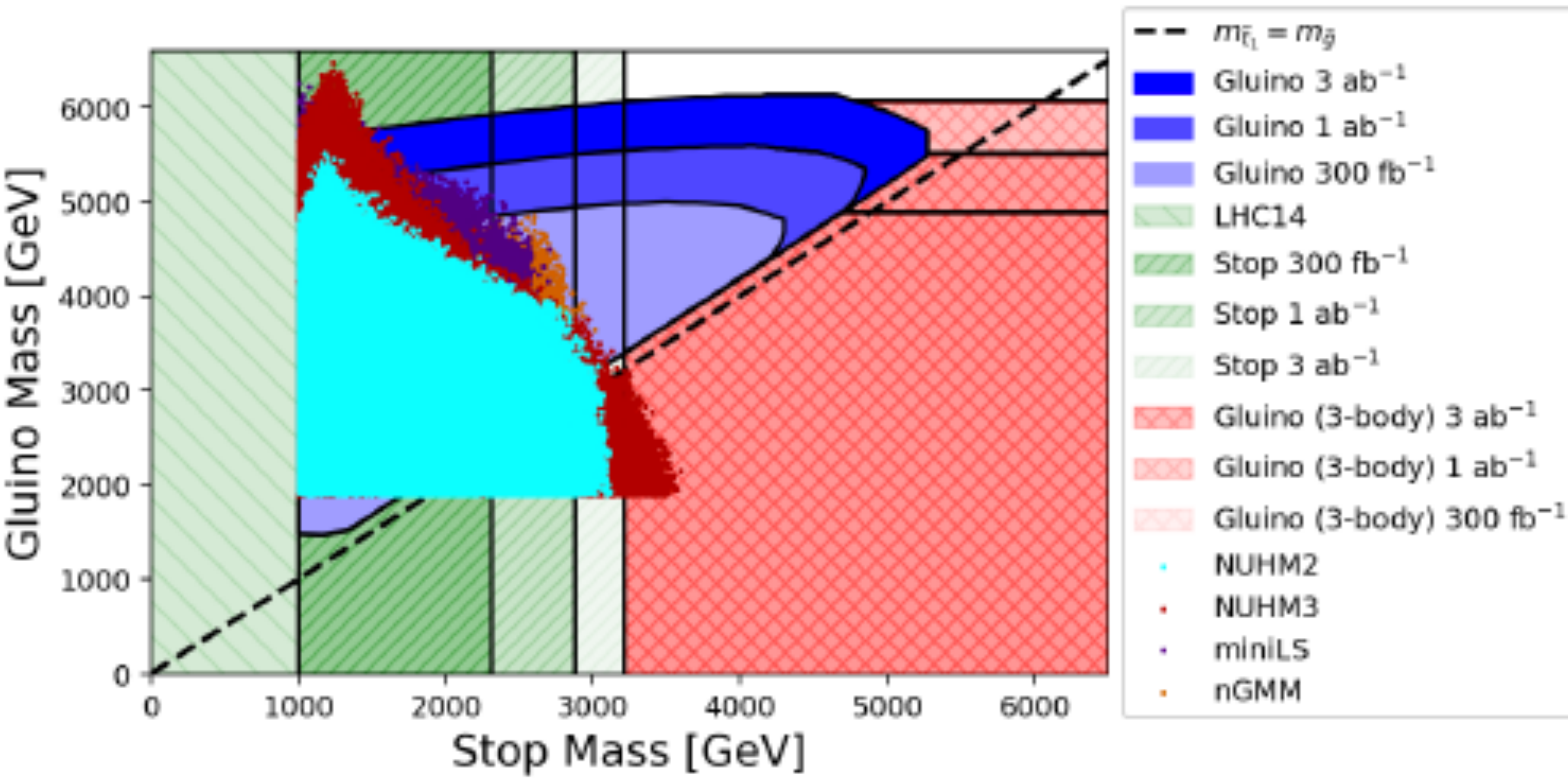
LHC33 reach extends to m(t1)~3-3.8 TeV

n(b-jets)>=2; MET>750 GeV

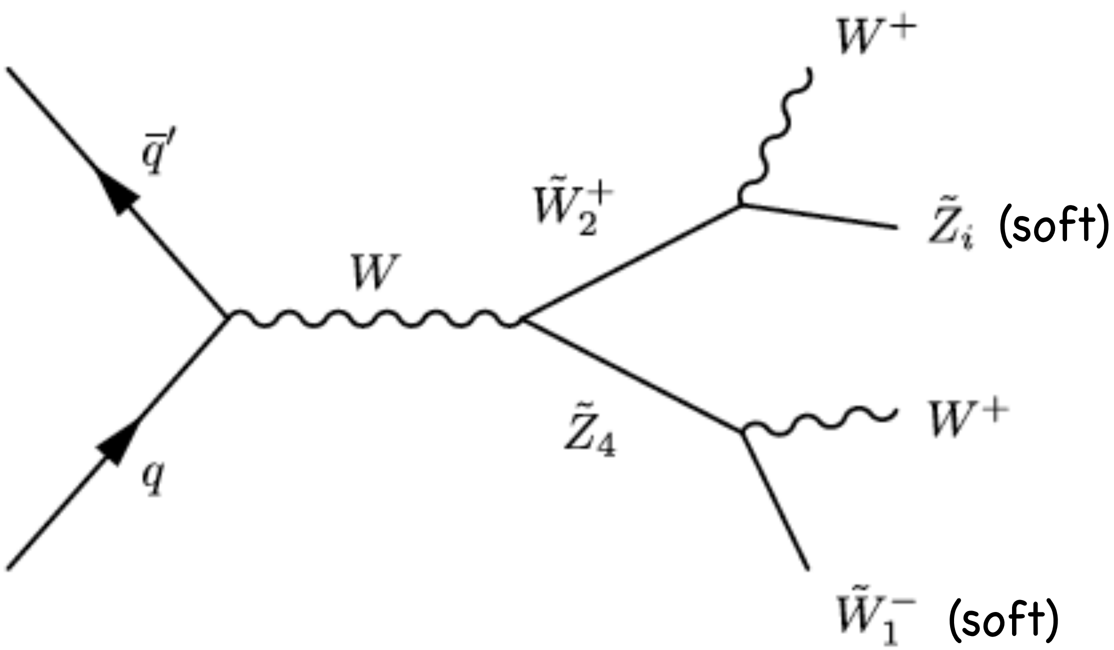
HB, Barger, Gainer, Serce, Tata

Combined LHC33 reach for t1 and glno covers all natural SUSY p-space!

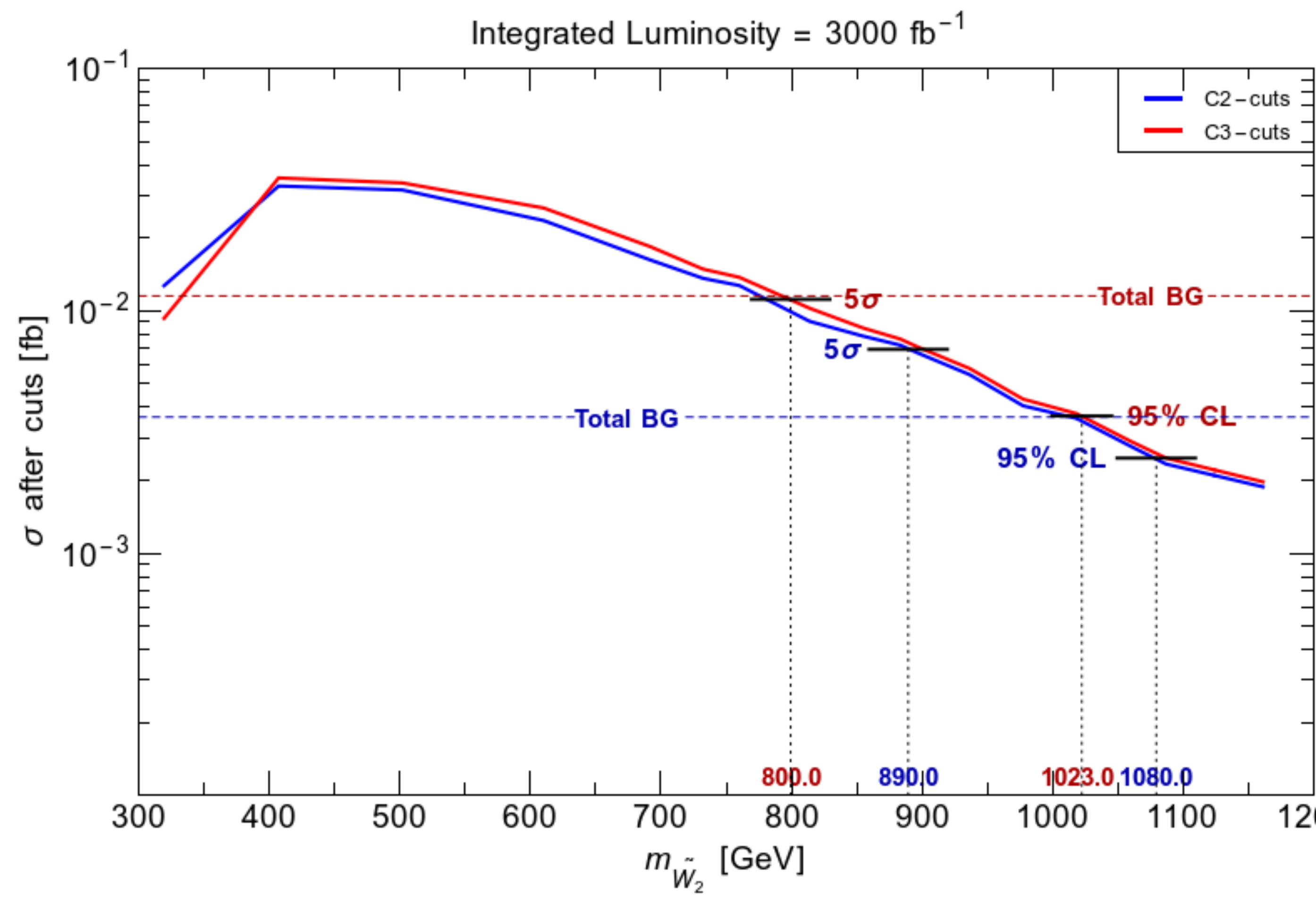
(need to re-do for LHC27)



Distinctive same-sign diboson (SSdB) signature from SUSY models with light higgsinos!



wino pair production

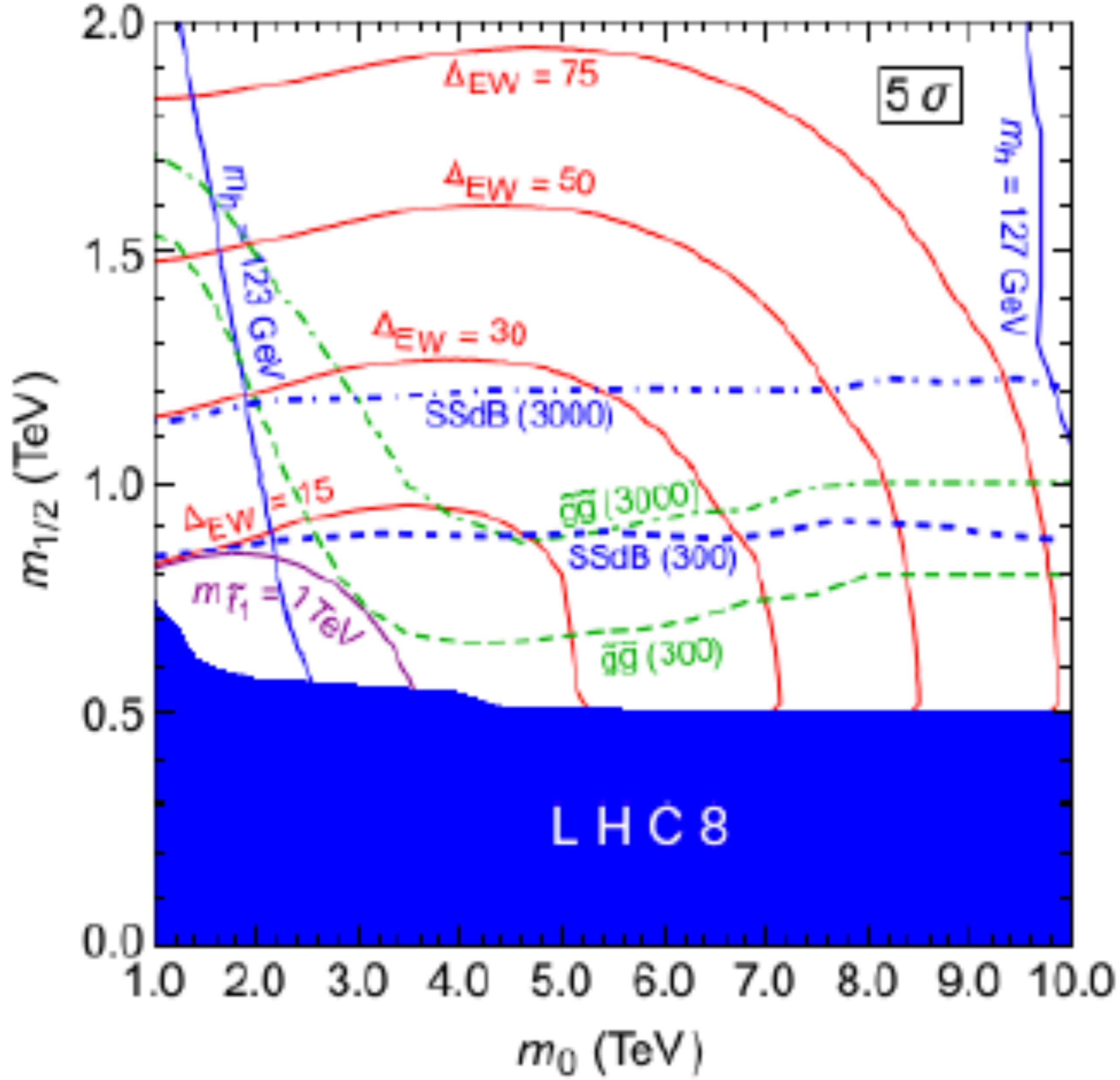


This channel offers good reach of LHC14 for RNS; it is also indicative of wino-pair prod'n followed by decay to higgsinos

H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, *Phys. Rev. Lett.* **110** (2013) 151801.

HB, Barger, Gainer, Sengupta, Tata

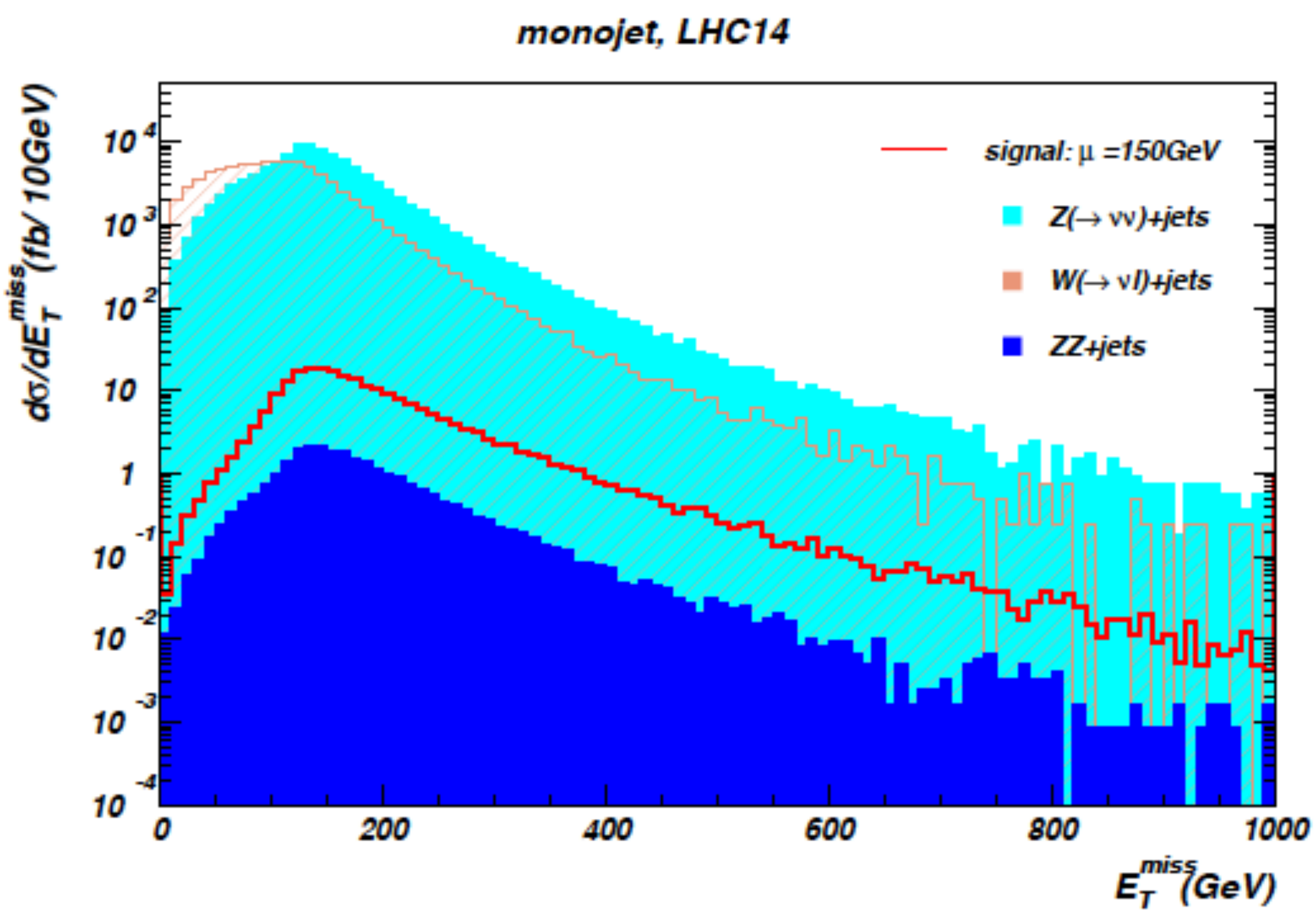
Good old m0 vs. mhf plane still viable, but needs mu~100-200 GeV as possible in NUHM2 instead of CMSSM/mSUGRA



For models with ino mass unif'n, reach via SSdB may exceed glno pairs for high luminosity

HB,Barger,Savoy, Tata; arXiv:1604.07438

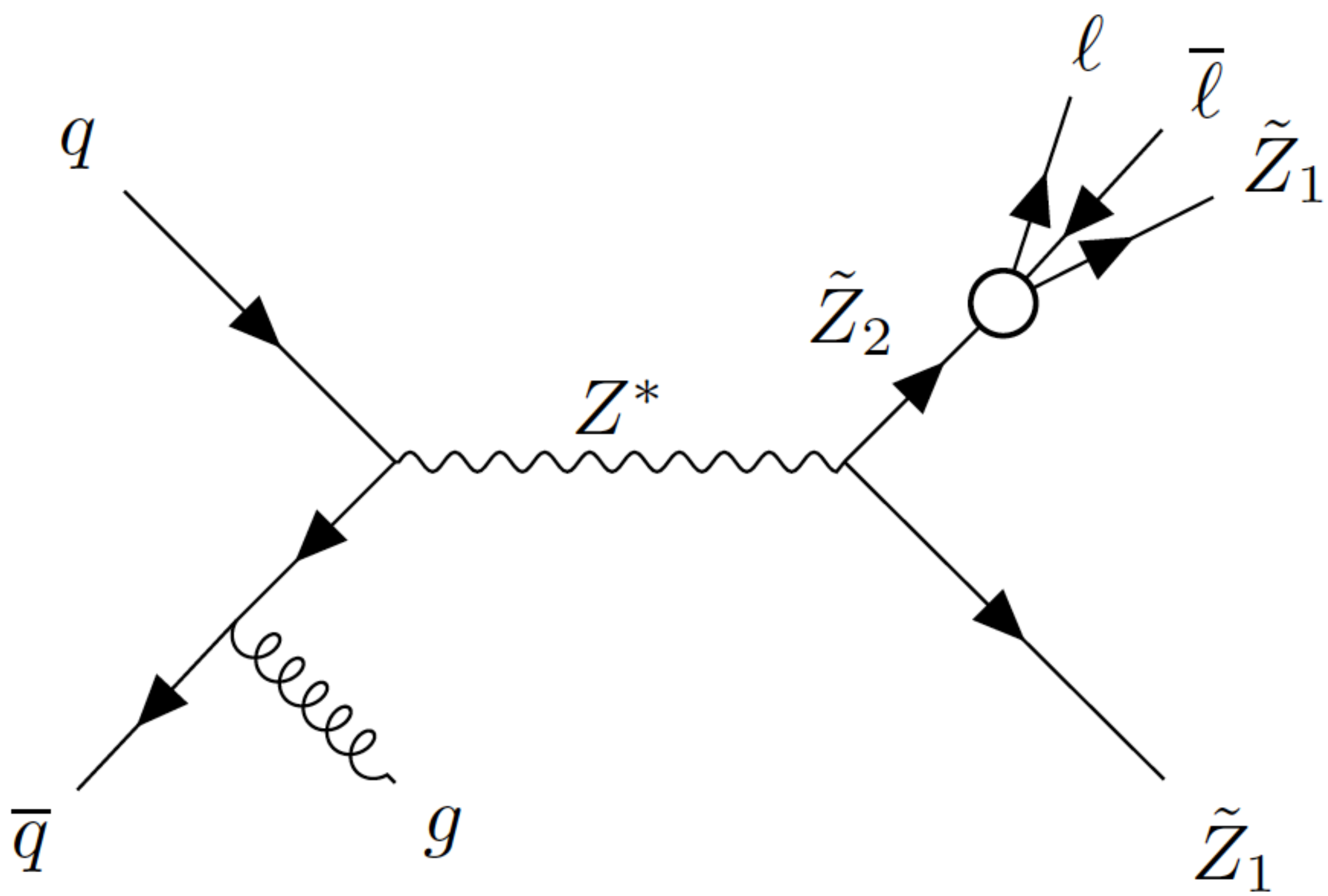
See direct higgsino pair production recoiling from ISR (monojet signal)?



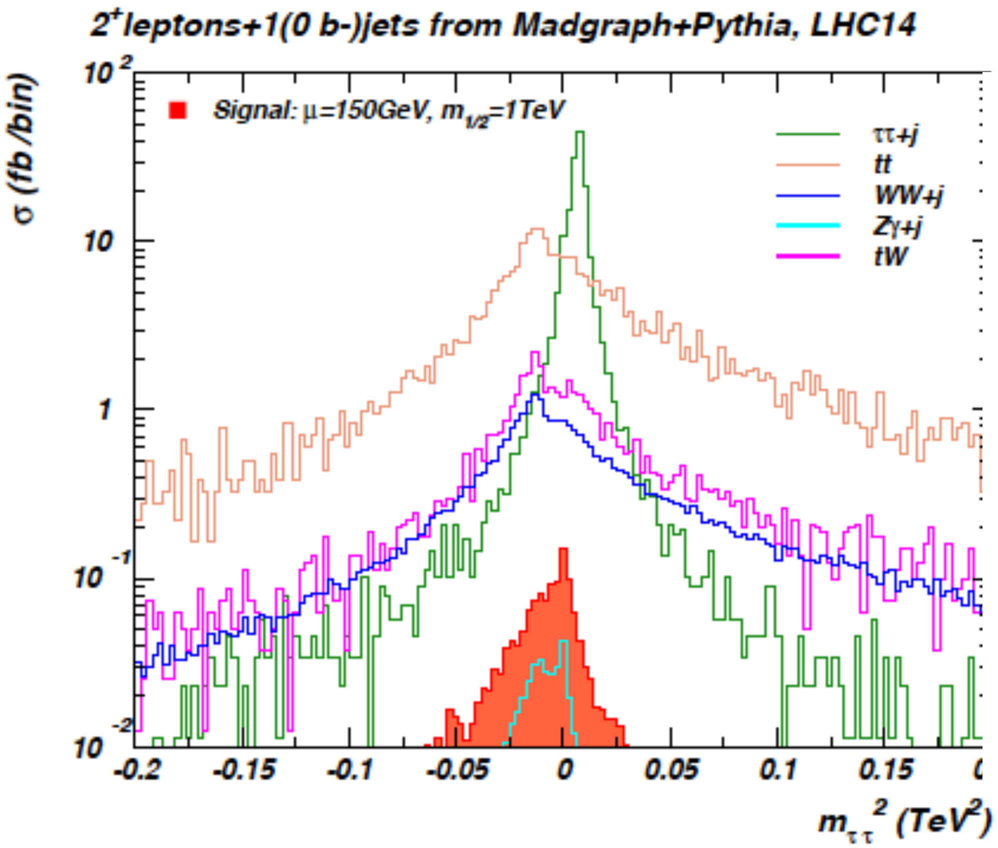
typically 1% S/BG after cuts:
very tough to do!

What about $pp \to \tilde{Z}_1 \tilde{Z}_2 j$ with $\tilde{Z}_2 \to \tilde{Z}_1 \ell^+ \ell^-$?

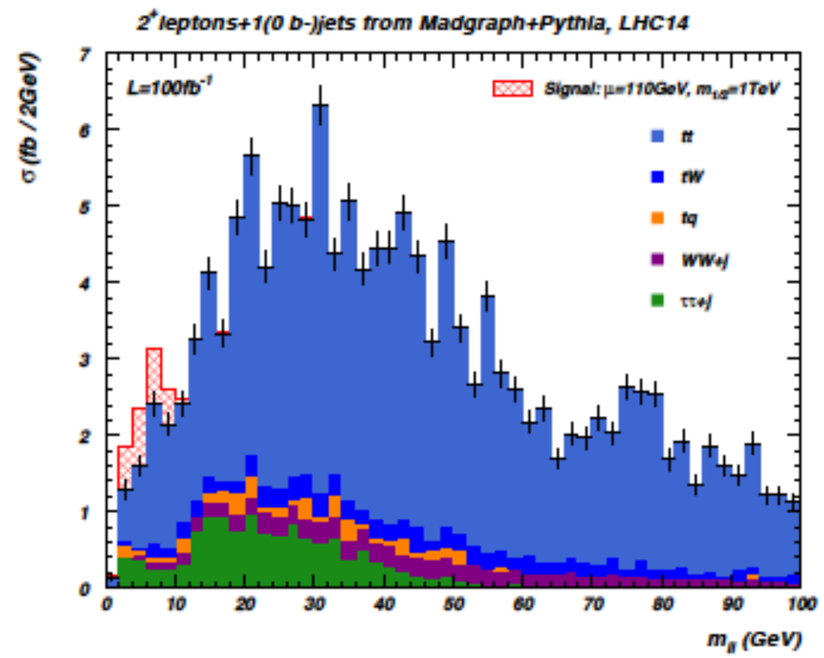
Han, Kribs, Martin, Menon, PRD89 (2014) 075007;
HB, Mustafayev, Tata, PRD90 (2014) 115007;

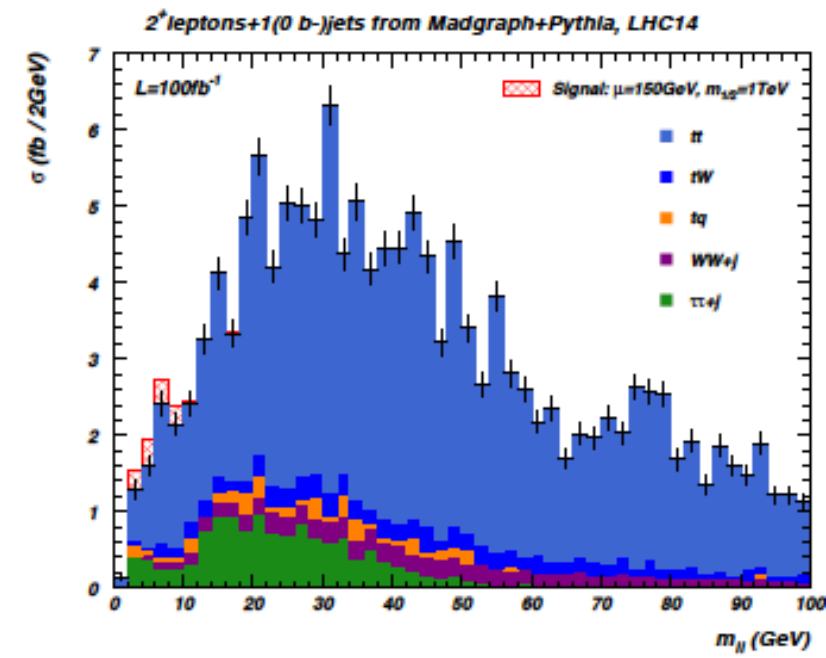


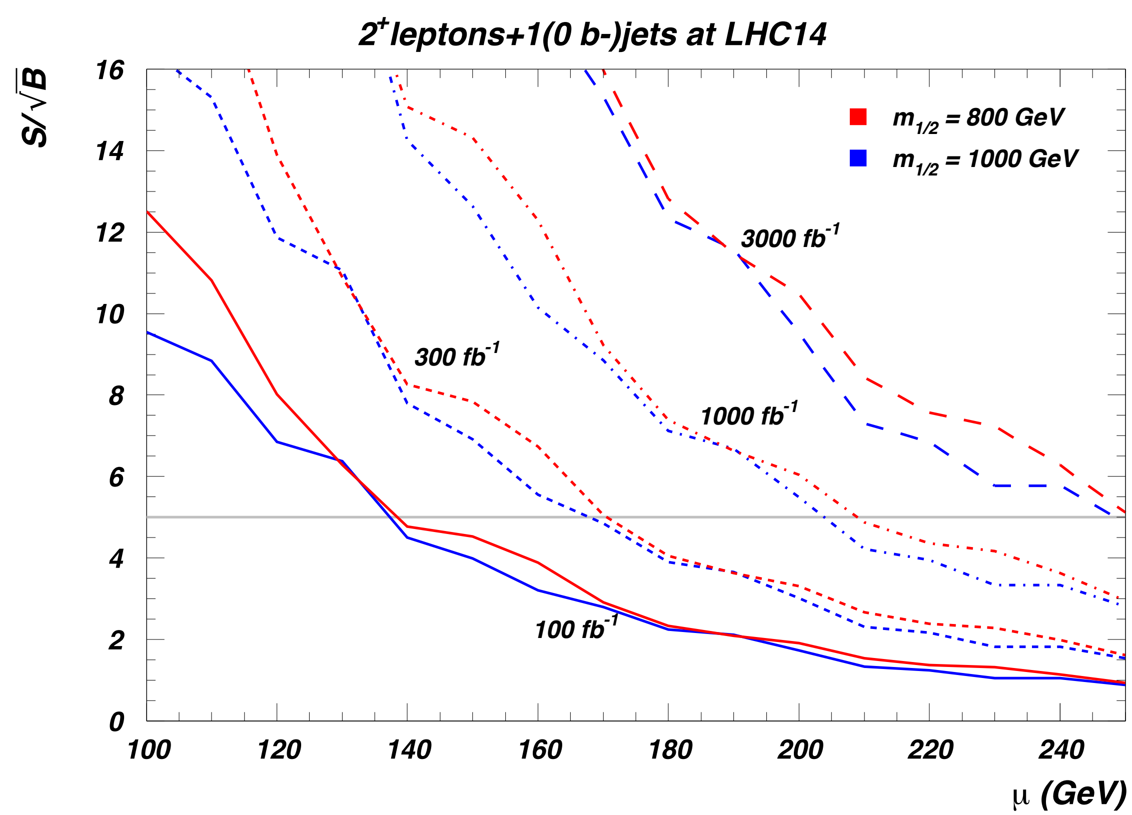
use MET to construct m^2(tau-tau)



cut m(ditau)^2<0







HL-LHC 5-sigma reach
to mu~250 GeV!

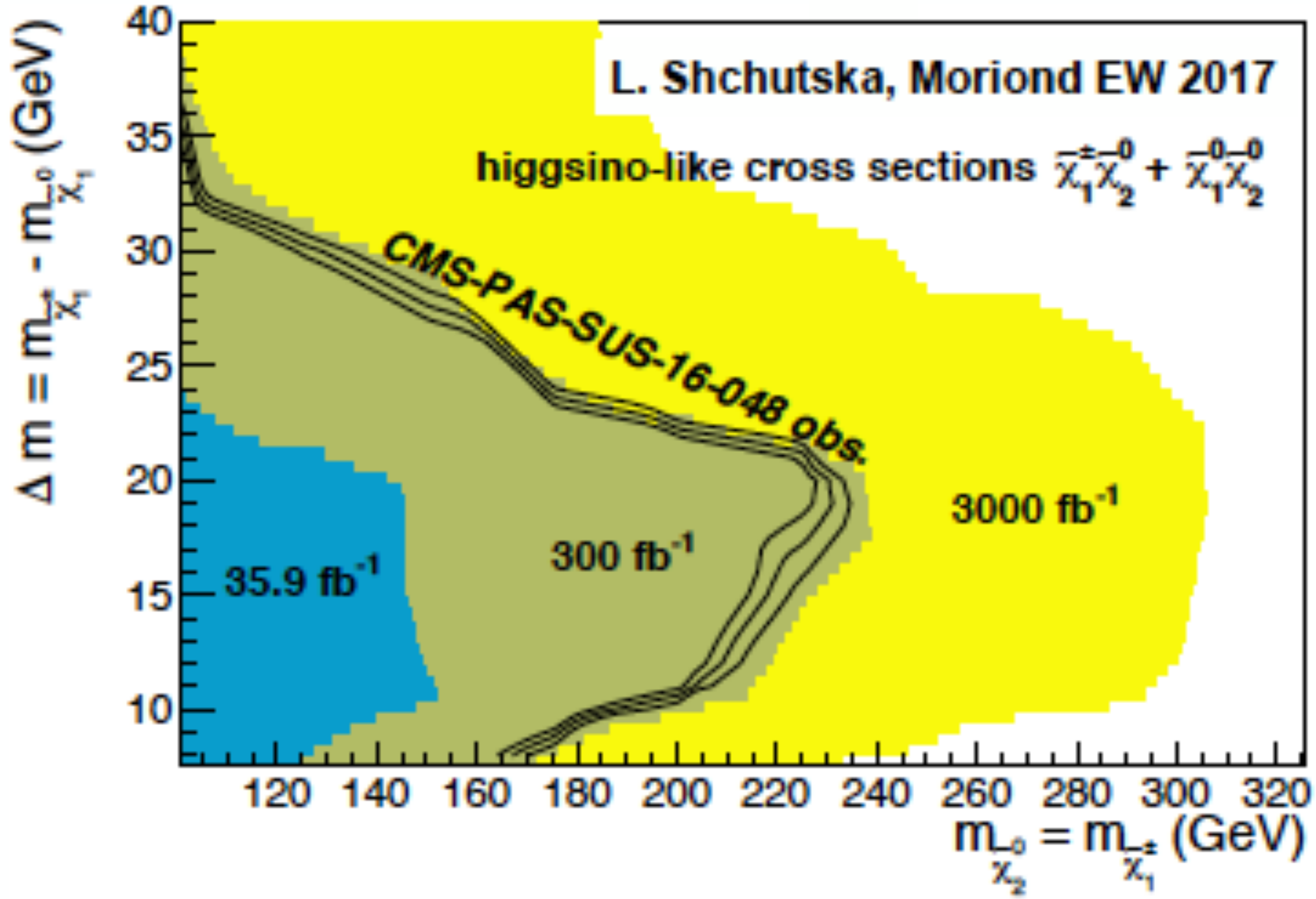
HB, Mustafayev, Tata

CMS analysis: this may be the most important SUSY discovery channel at LHC since it directly probes higgsinos which can't be too far from m(W,Z,h)

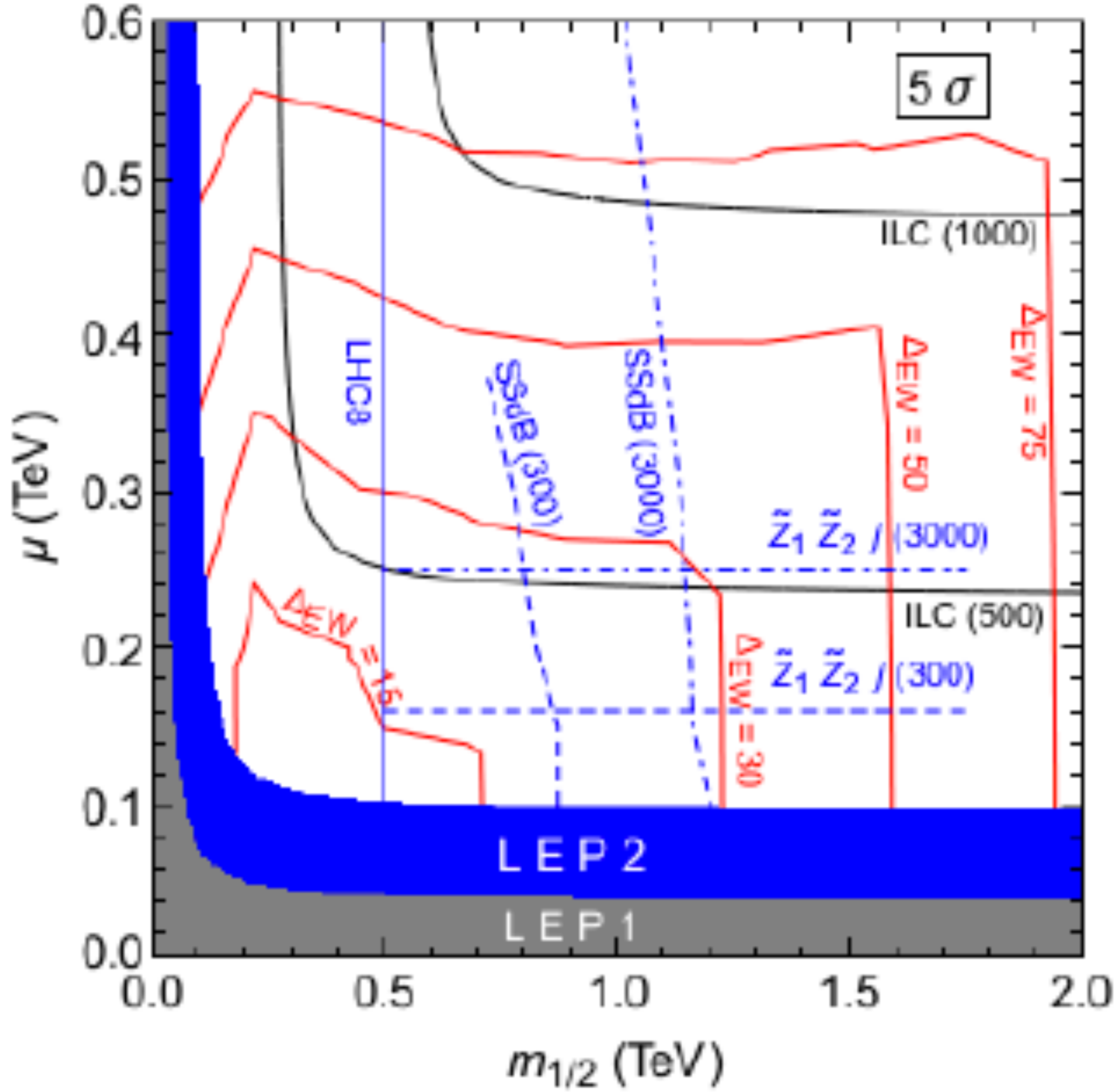
Atlas study underway- results soon?

NatSUSY z2-z1 mass gap may range down to 3 GeV so need to ID very soft, low m(ll) leptons

Higgsino cross section (projection only)



panoramic view of reach of HL-LHC for natural SUSY



Combined SSdB/lljMET searches may cover all Nat SUSY p-space at HL-LHC for models with ino mass unification; in mirage scenario, z2-z1 mass gap can be reduced and M2 can be much higher than in NUHM2

Summary of collider searches

- In light of recent LHC bounds (m(glno)>2 TeV, m(t1)>1 TeV) and m(h) requiring TeV-scale highly mixed top squarks, concern has arisen about an emerging Little Hierarchy problem characterized by m(weak)~100 GeV<< m(SUSY)~multi-TeV rendering perhaps SUSY as ``unnatural”
- We propose an improved naturalness measure based upon scalar potential minimization condition

$$m_Z^2/2 = \frac{m_{H_d}^2+\Sigma_d^d-(m_{H_u}^2+\Sigma_u^u)\tan^2\beta}{\tan^2\beta-1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u(\tilde{t}_{1,2}) - \mu^2$$

This leads to upper bounds from naturalness:

DM=WIMP/axion mix?

- m(higgsinos)~100-300 GeV (the lighter the better)
- m(t1)<~3 TeV
- m(glno)<~6 TeV

process	current	HL-LHC	HE-LHC
glno-glno	m(glno)>2 TeV	~2.8 TeV	5.5 TeV
t1-t1	m(t1)>1 TeV	1.3 TeV	3.5 TeV
SSdB (winos)	x	m(W2)~1 TeV	?
z1z2j->l+lb+j+MET	barely	mu~250 GeV	?

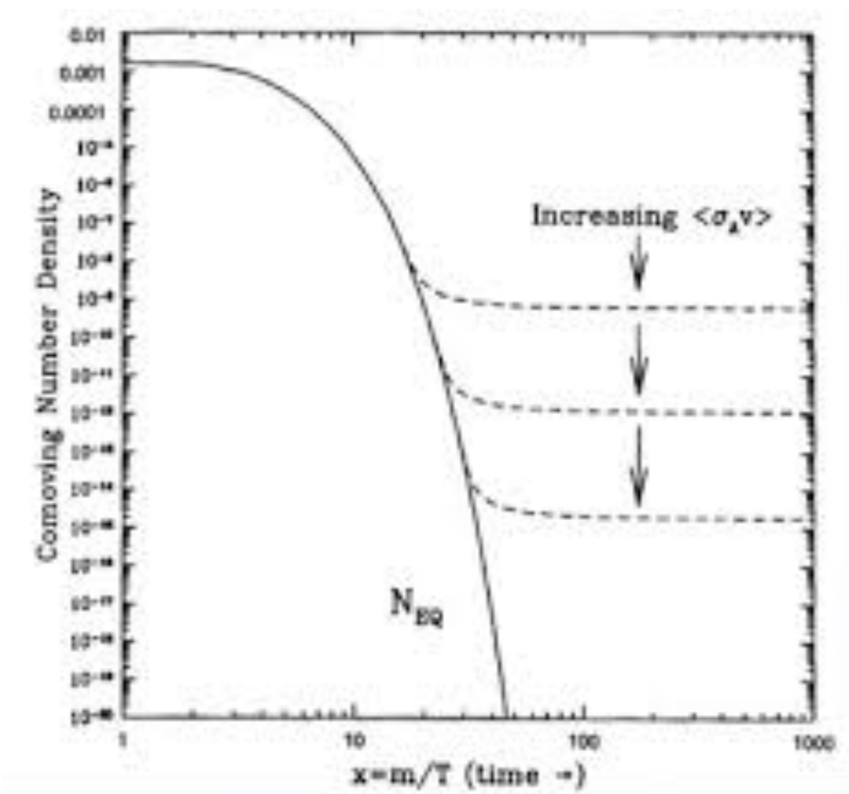
Conclusions:
1. SUSY still natural;
2. hunt for nSUSY has only begun;
3. HL-LHC handle most SUSY with ino-mass unification;
4. other (e.g. mirage) may require HE-LHC to complete search

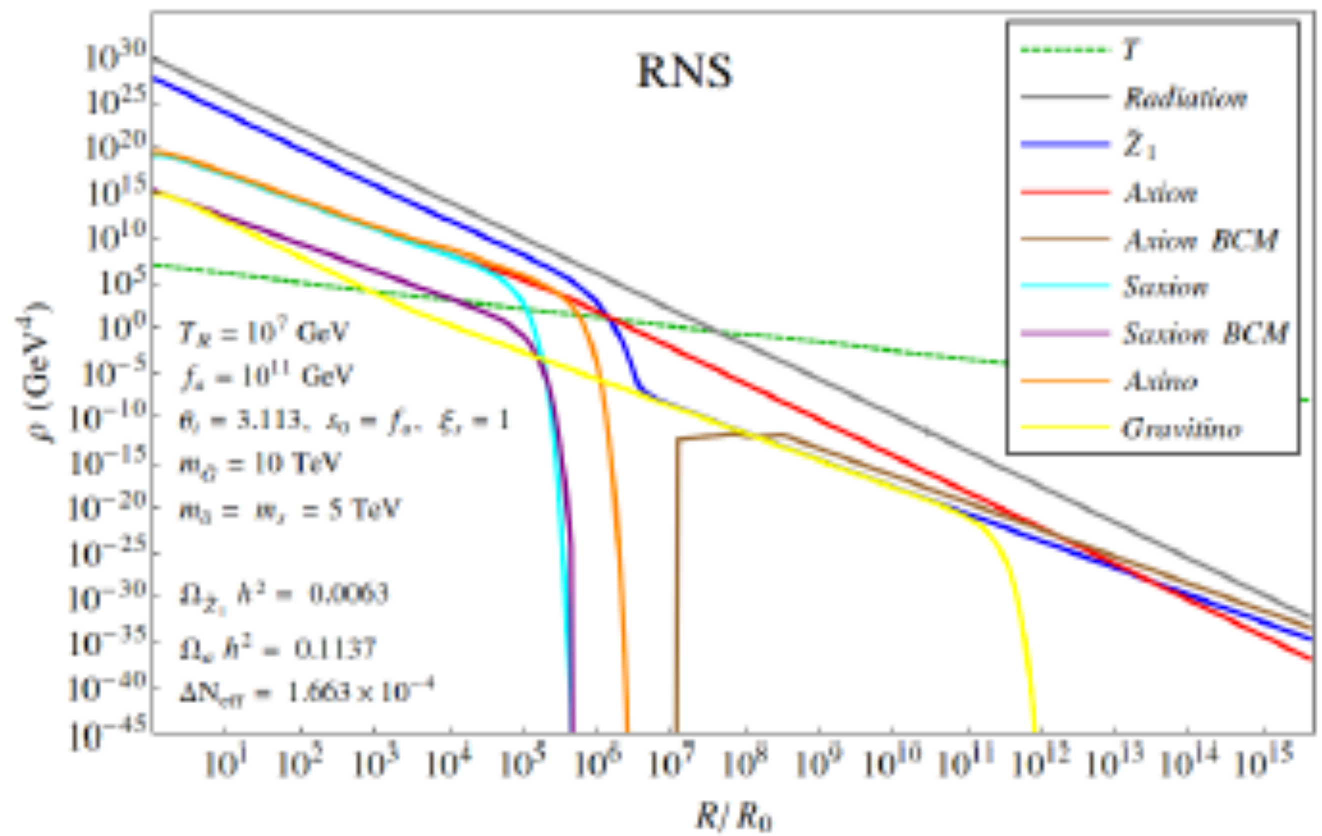
HB, Barger,Gainer, Huang,Tata
Savoy, Mustafayev
Sengupta,Serce

What happens to SUSY WIMP dark matter?

- higgsino-like WIMPs thermally underproduced
- 3 not four light pions => QCD theta vacuum
- F.F~ term should be present but neutron(EDM)=> it is tiny
- strong CP problem => axions: no fine-tuning in QCD sector
- SUSY context: axion superfield, axinos and saxions
- DM= axion+higgsino-like WIMP admixture
- DFSZ SUSY axion: solves mu problem with mu<< m_3/2!
- ultimately detect both WIMP and axion?

usual picture => mixed axion/WIMP

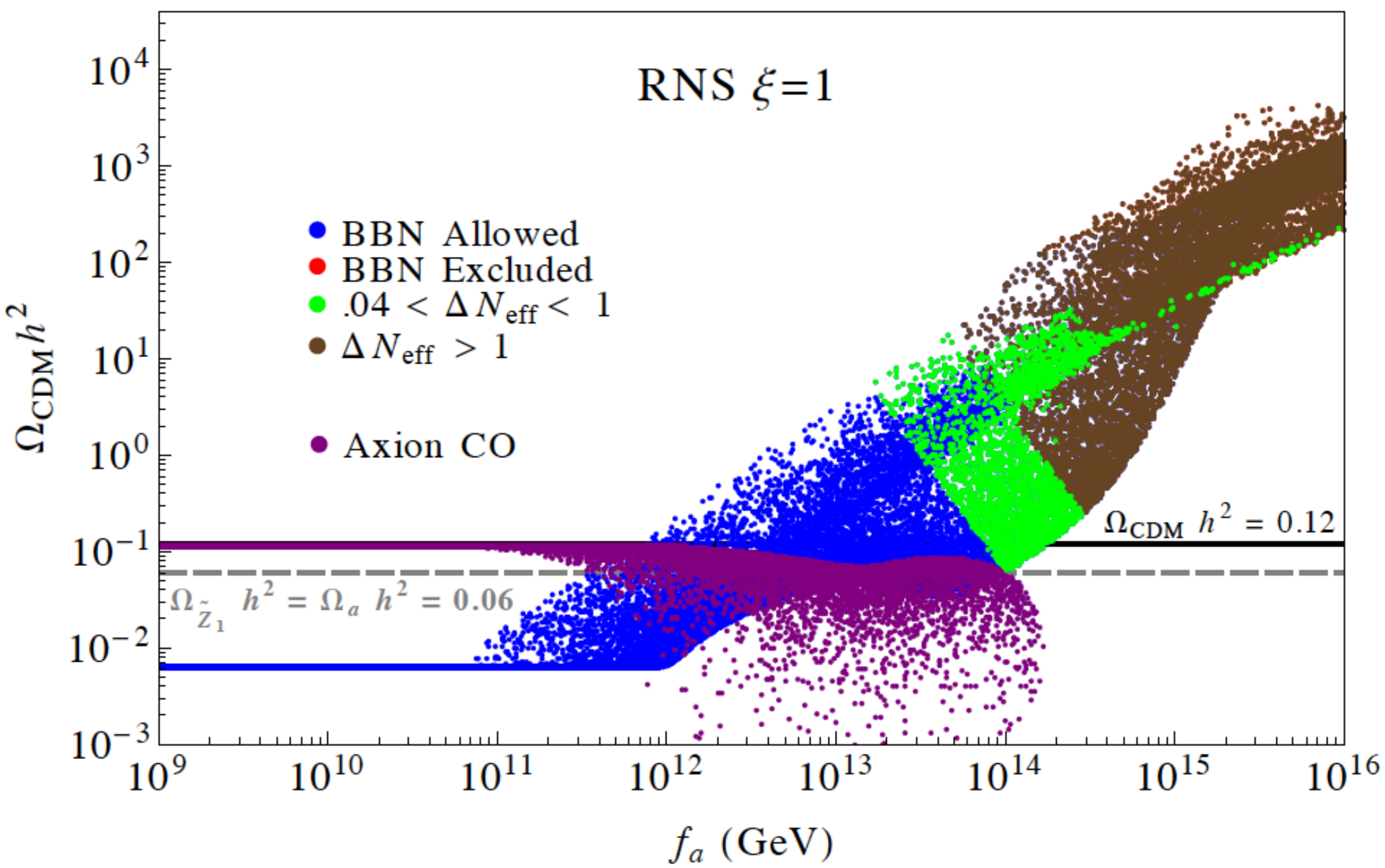




KJ Bae, HB, Lessa, Serce

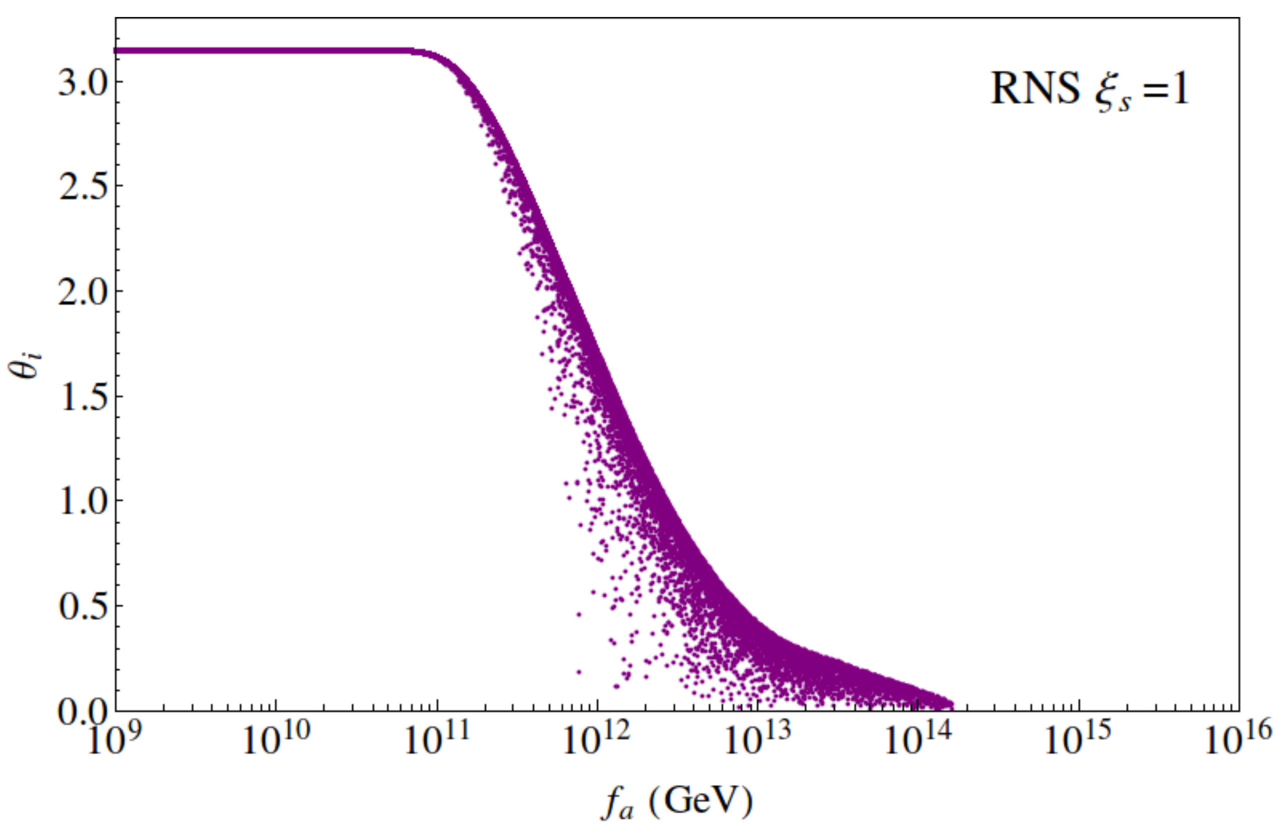
much of parameter space is axion-dominated with 10-15% WIMPs

=>



higgsino abundance

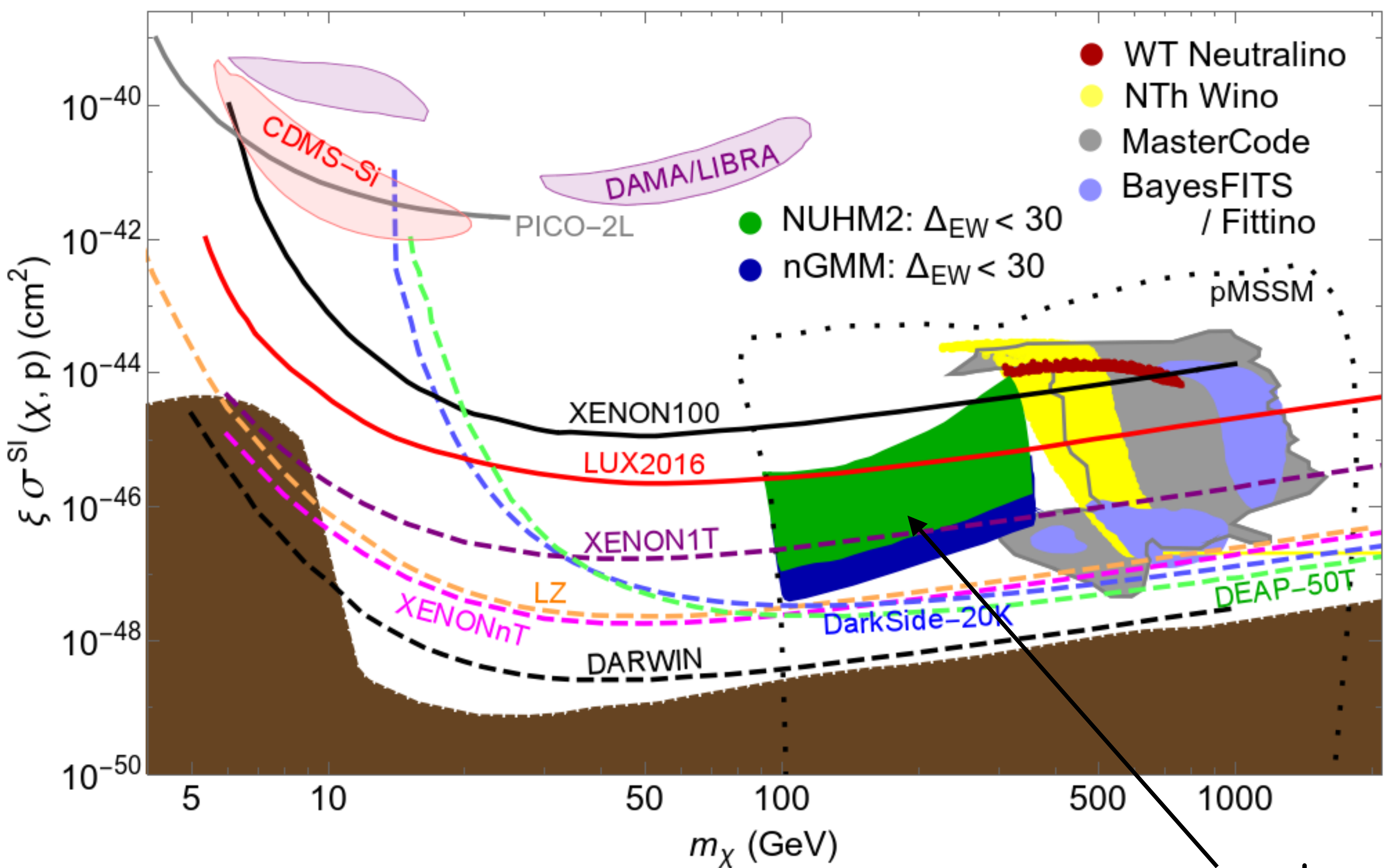
axion abundance



mainly axion CDM for fa<~10^12 GeV; for higher fa, then get increasing wimp abundance

Bae, HB,Lessa,Serce

Direct higgsino detection rescaled for minimal local abundance $\xi \equiv \Omega_\chi^{TP} h^2/0.12$



natural SUSY

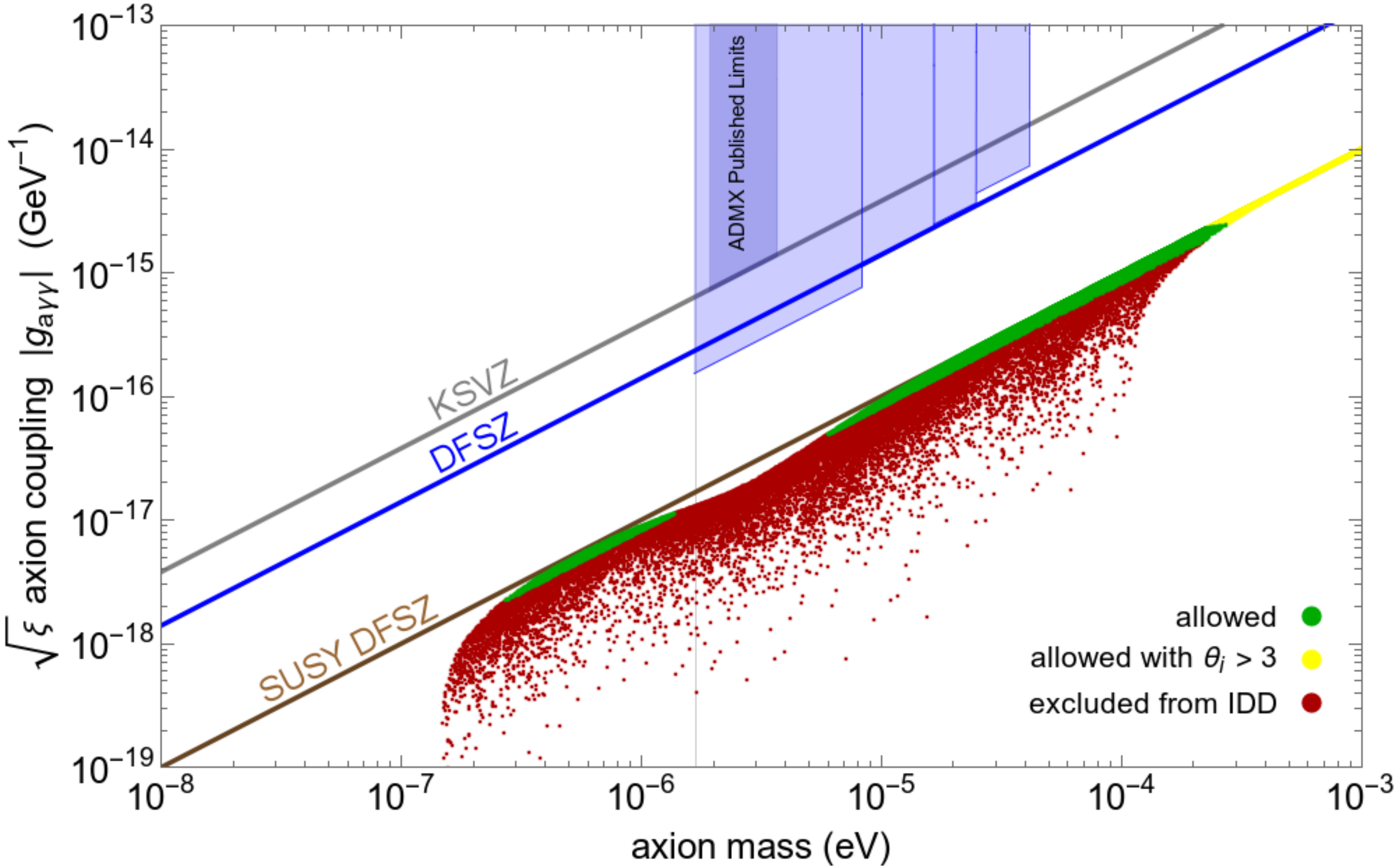
Bae, HB, Barger, Savoy, Serce

$$\mathcal{L} \ni -X_{11}^h \overline{\widetilde{Z}}_1 \widetilde{Z}_1 h$$

$$X_{11}^h = -\frac{1}{2}\left(v_2^{(1)} \sin\alpha - v_1^{(1)} \cos\alpha\right)\left(g v_3^{(1)} - g' v_4^{(1)}\right)$$

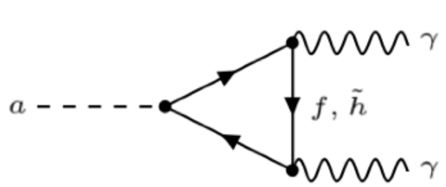
Xe-1-ton now operating!

Can test completely with ton scale detector or equivalent (subject to minor caveats)



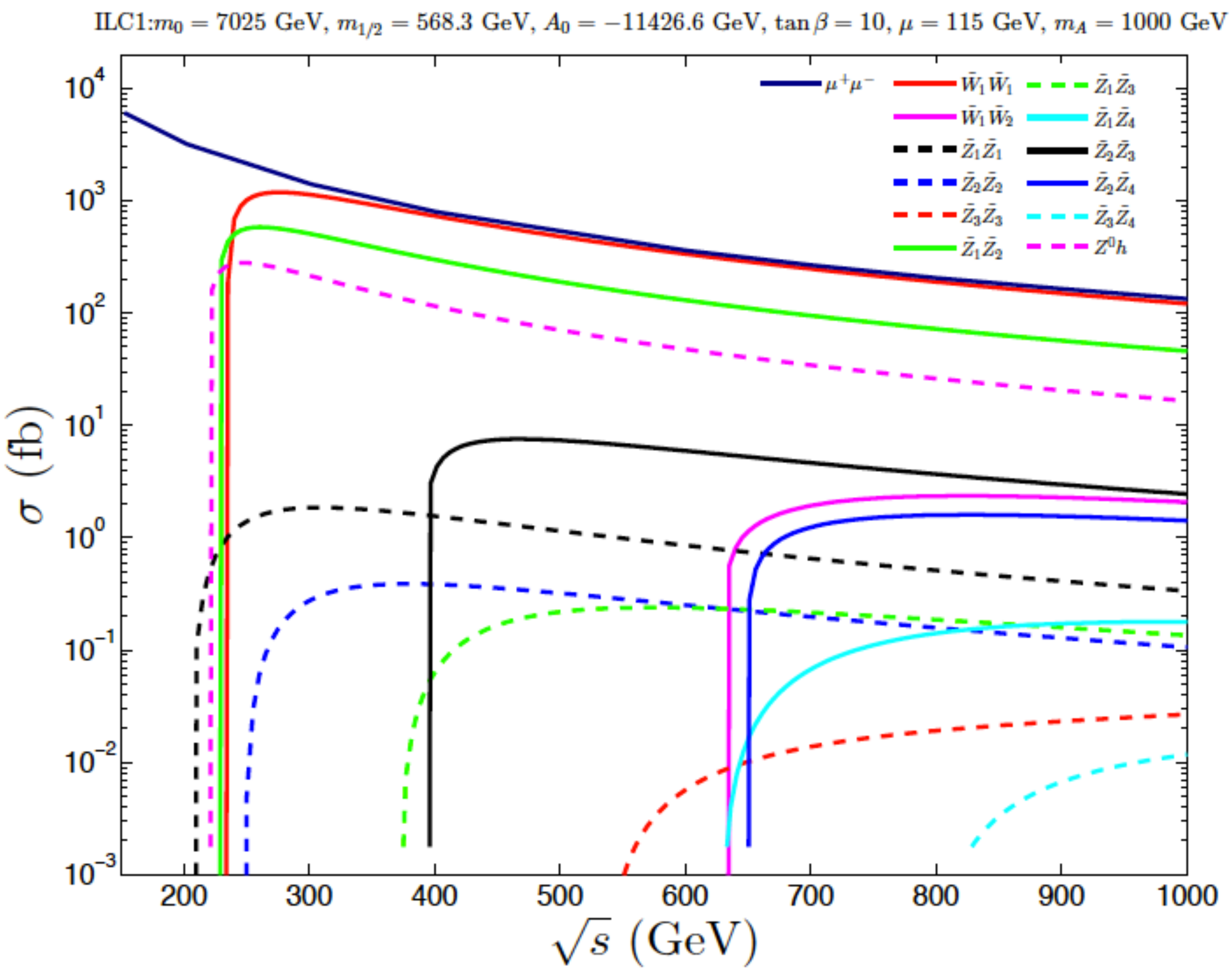
SUSY DFSZ axion: large range in m(a) but coupling reduced

may need to probe broader and deeper!



Smoking gun signature: light higgsinos at ILC:

ILC is Higgs/higgsino factory!



$$\sigma(higgsino) \gg \sigma(Zh)$$

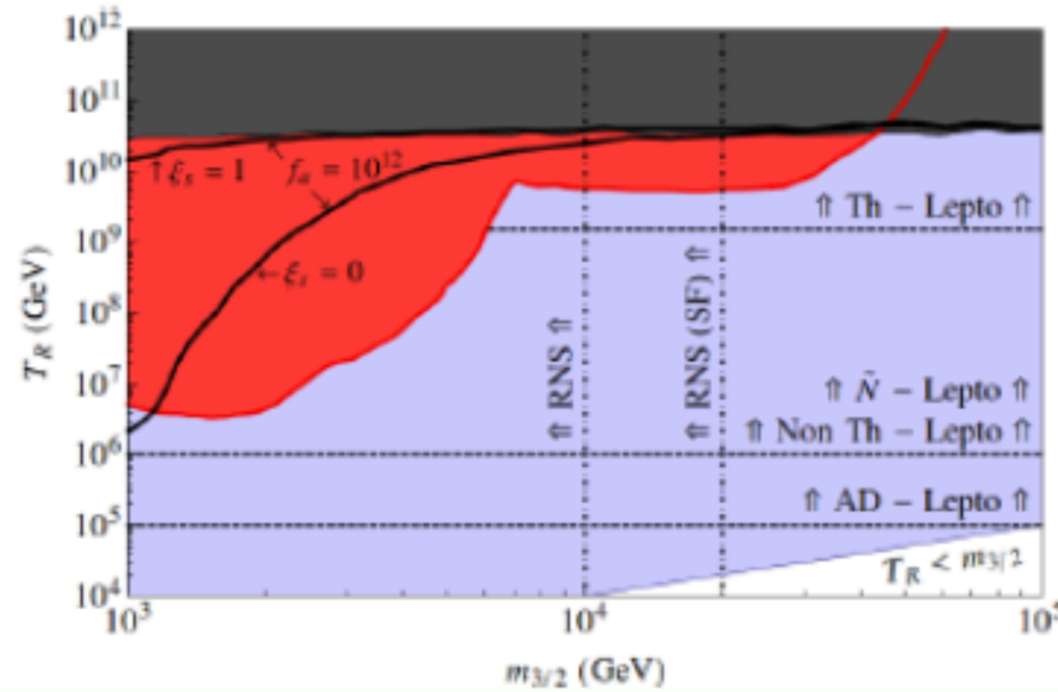
10-15 GeV higgsino mass gaps no problem in clean ILC environment

HB, Barger, Mickelson, Mustafayev, Tata
arXiv:1404:7510

ILC either sees light higgsinos or MSSM dead

Baryogenesis scenarios for radiative natural SUSY

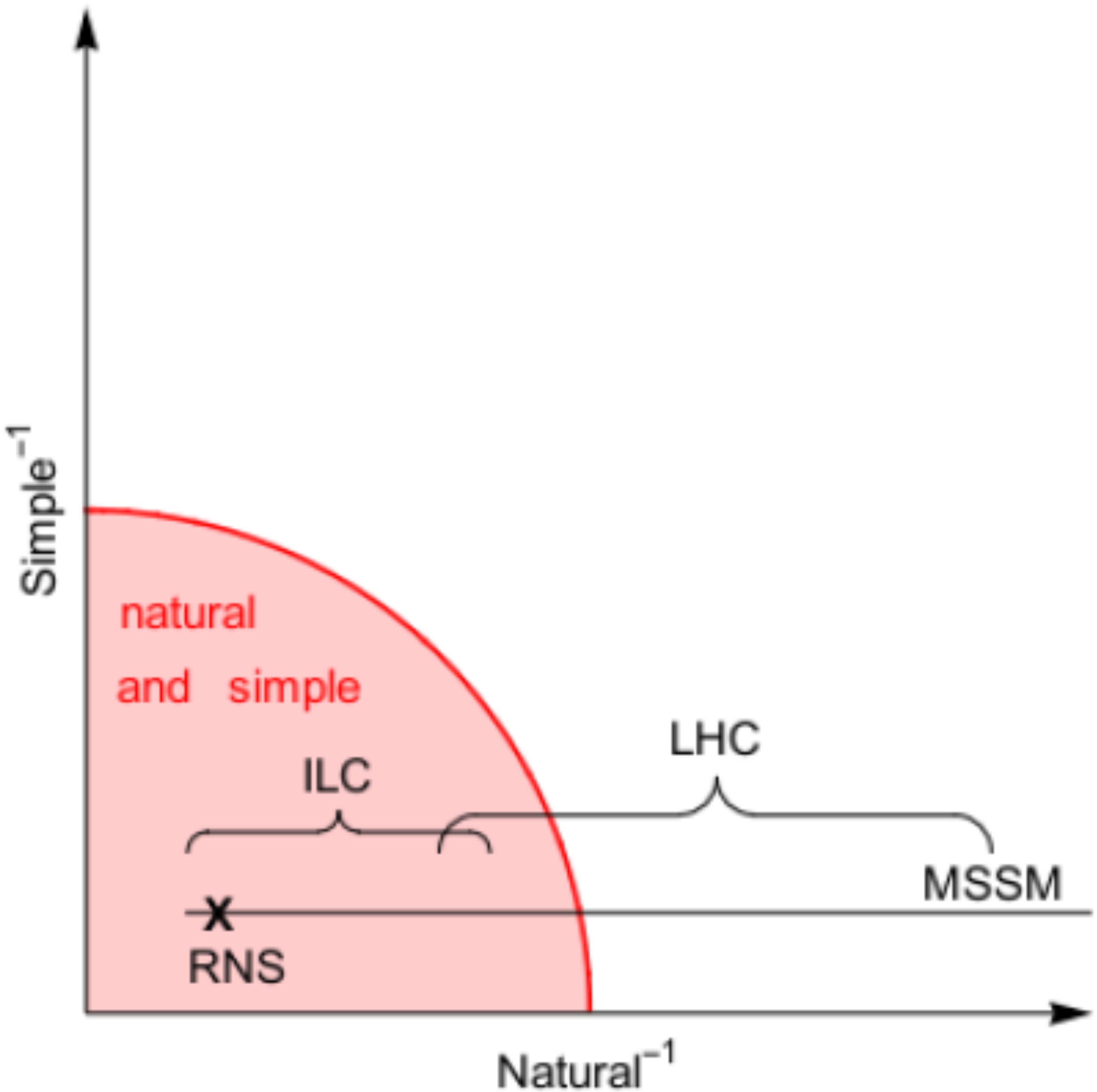
- thermal leptogenesis
- non-thermal (inflaton decay)
- oscillating sneutrino
- Affleck-Dine (AD)



gravitino problem plus axino/saxion problem: still plenty room

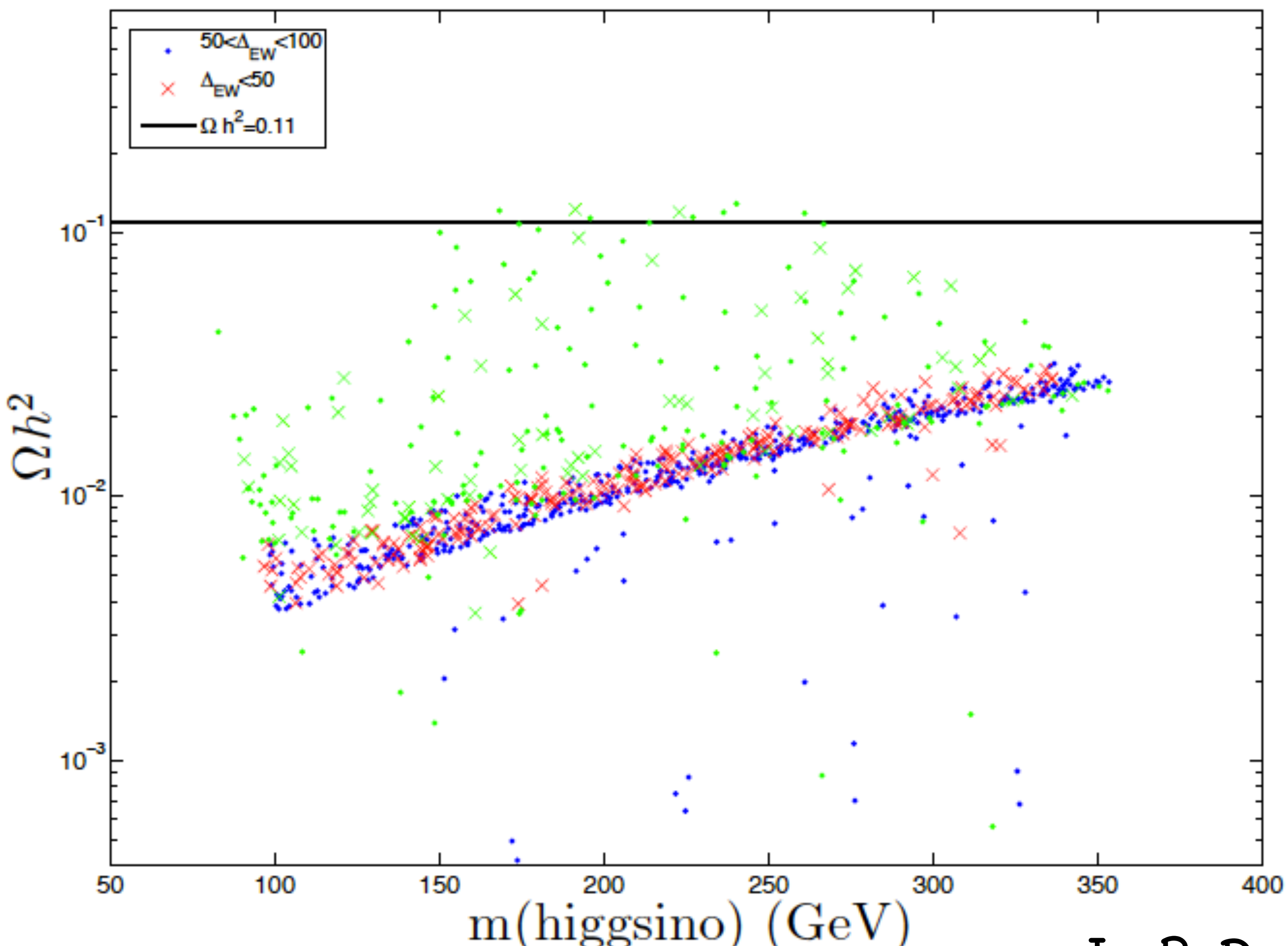
$$f_a = 10^{11},\ 10^{12}\ \text{GeV}$$

Bae,HB, Serce, Zhang, arXiv:1510.00724


Simple^{-1}
natural
and simple
ILC
LHC
MSSM
RNS
Natural^{-1}

Dark matter in RNS

Mainly higgsino-like WIMPs thermally underproduce DM



IsaReD

green: excluded;
red/blue:allowed

HB, Barger, Mickelson

Factor of 10-15 too low

But so far we have addressed only Part 1
of fine-tuning problem:

In QCD sector, the term $\frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu} \tilde{F}_A^{\mu\nu}$ must occur

But neutron EDM says it is not there: strong CP problem

(frequently ignored by SUSY types)

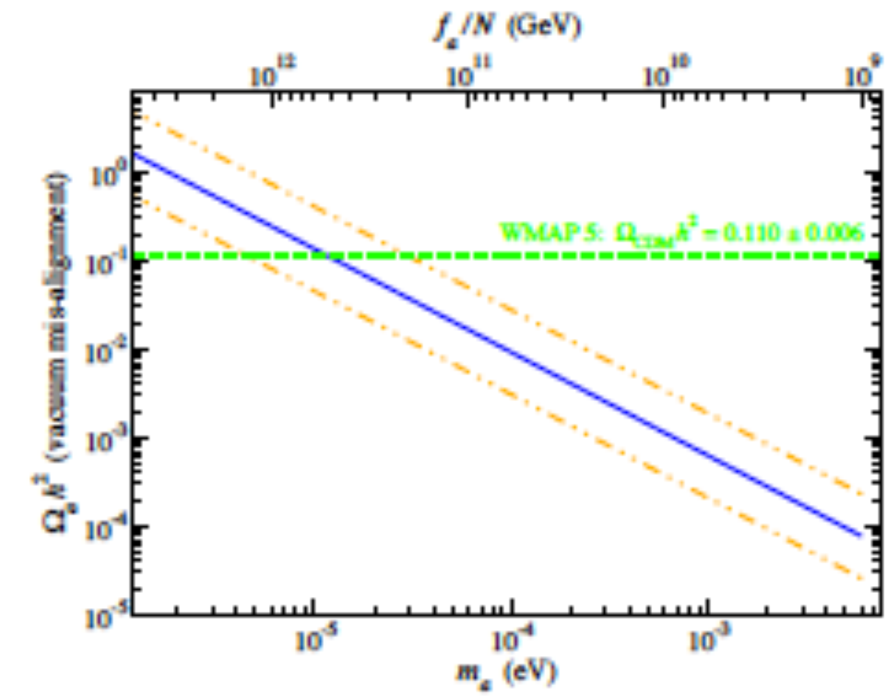
Best solution after 35 years:
PQWW/KSVZ/DFSZ invisible axion

In SUSY, axion accompanied by axino and saxion

Changes DM calculus:
expect mixed WIMP/axion DM (2 particles)

Axion cosmology

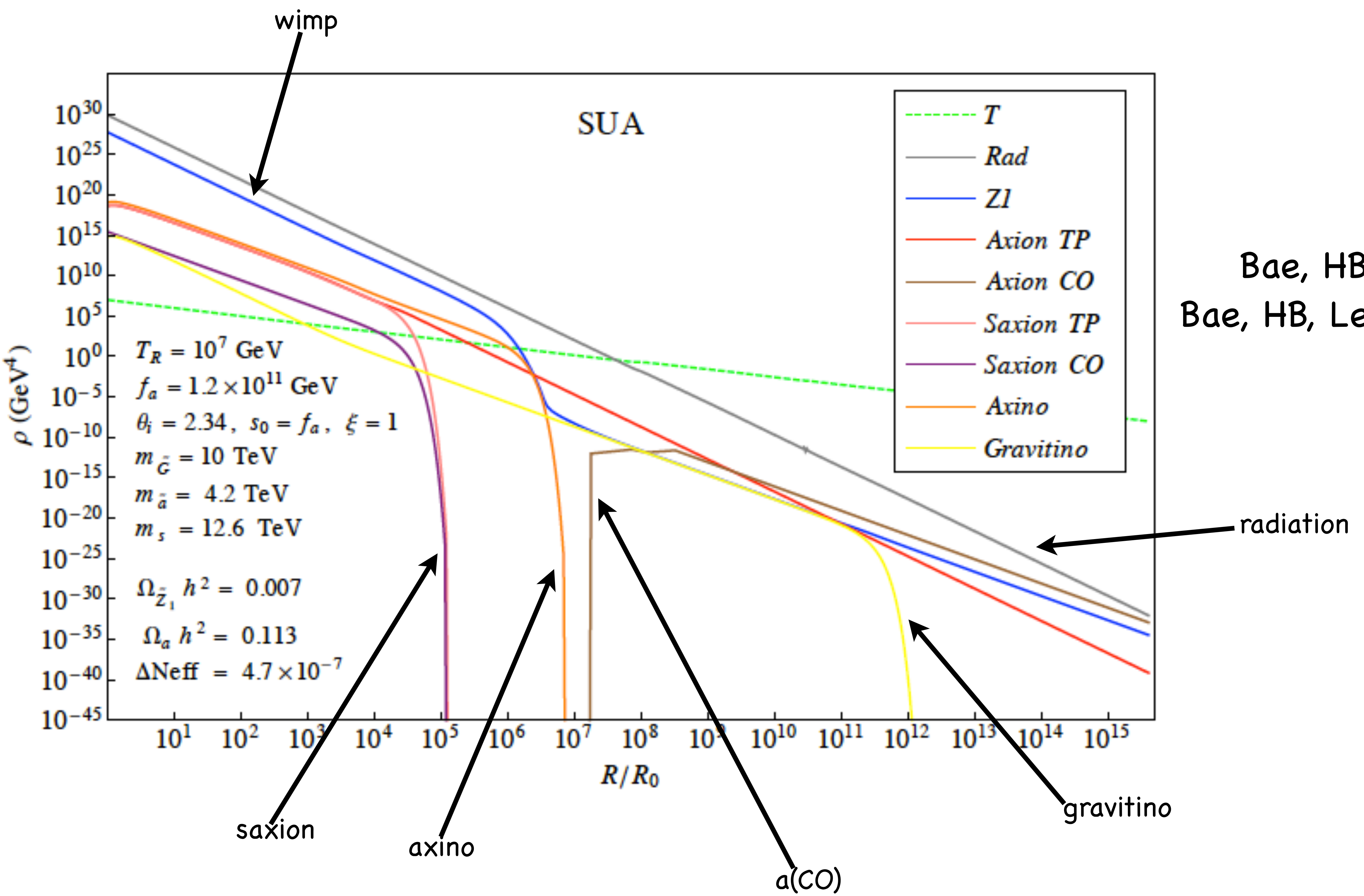
- ⋆ Axion field eq'n of motion: $\theta = a(x)/f_a$
  - $\ddot{\theta} + 3H(T)\dot{\theta} + \frac{1}{f_a^2}\frac{\partial V(\theta)}{\partial \theta} = 0$
  - $V(\theta) = m_a^2(T) f_a^2 (1 - \cos\theta)$
  - Solution for T large, $m_a(T) \sim 0$:
    $\theta = const.$
  - $m_a(T)$ turn-on ~ 1 GeV
- ⋆ a(x) oscillates,
  creates axions with $\vec{p} \sim 0$:
  production via vacuum mis-alignment
- ⋆ $\Omega_a h^2 \sim \frac{1}{2}\left[\frac{6\times 10^{-6} eV}{m_a}\right]^{7/6} \theta_i^2 h^2$
- ⋆ astro bound: stellar cooling $\Rightarrow f_a \gtrsim 10^9 GeV$



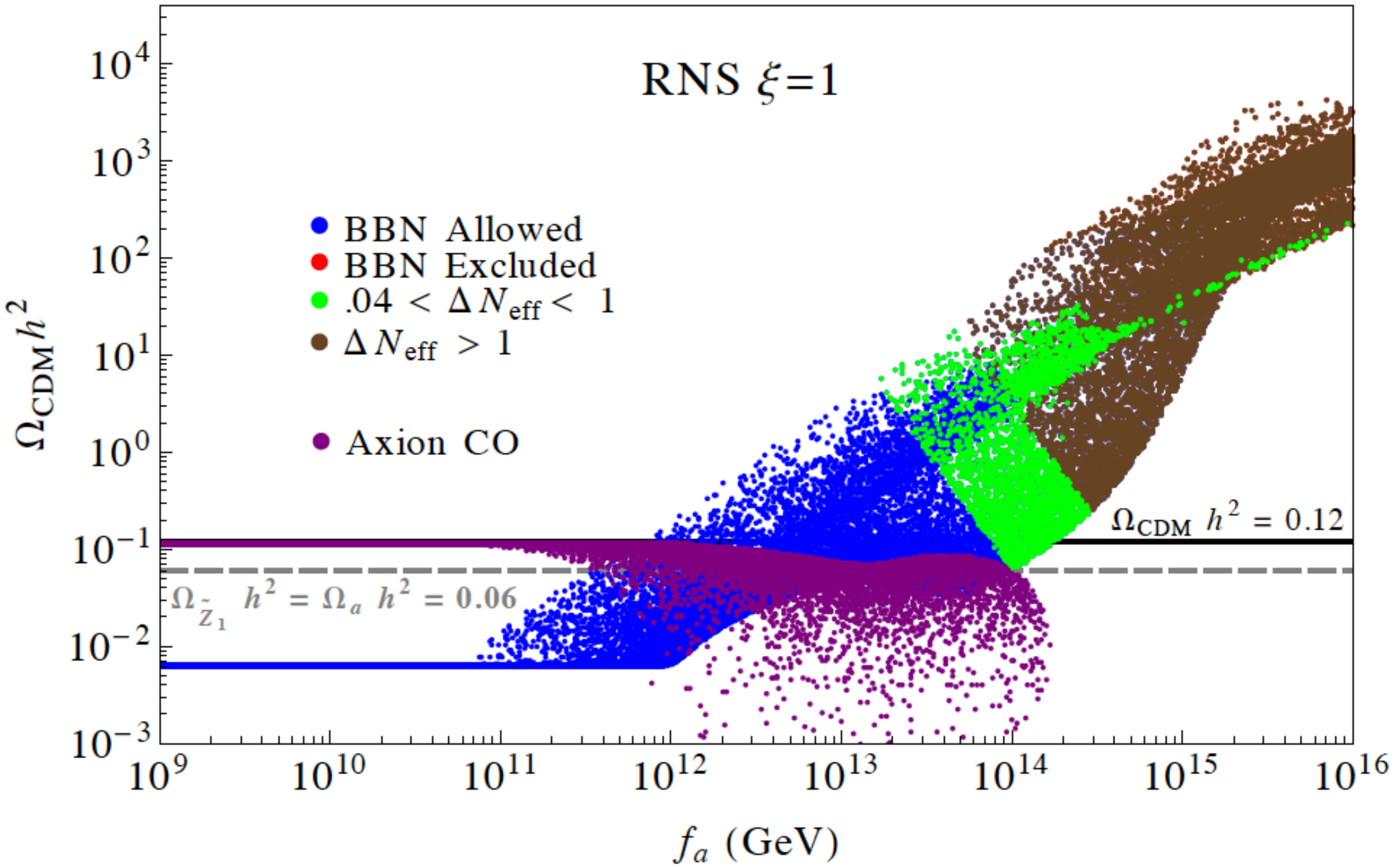
mixed axion-neutralino production in early universe

- neutralinos: thermally produced (TP) or NTP via $\tilde{a}$, s or $\tilde{G}$ decays
  - re-annihilation at $T_D^{s,\tilde{a}}$
- axions: TP, NTP via $s \to aa$, bose coherent motion (BCM)
- saxions: TP or via BCM
  - $s \to gg$: entropy dilution
  - $s \to SUSY$: augment neutralinos
  - $s \to aa$: dark radiation ($\Delta N_{eff} < 1.6$)
- axinos: TP
  - $\tilde{a} \to SUSY$ augments neutralinos
- gravitinos: TP, decay to SUSY

DM production in SUSY DFSZ: solve eight coupled Boltzmann equations

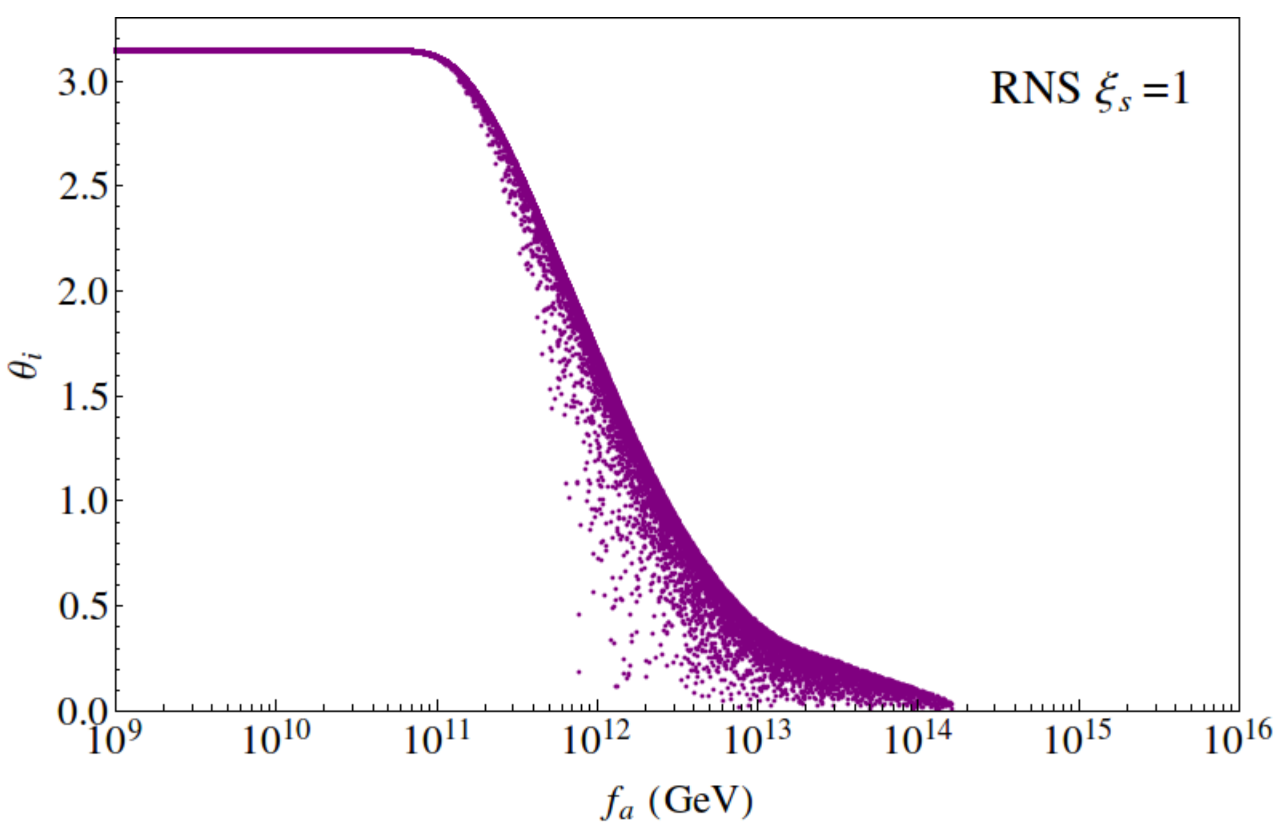


Bae, HB, Chun;
Bae, HB, Lessa, Serce



higgsino abundance

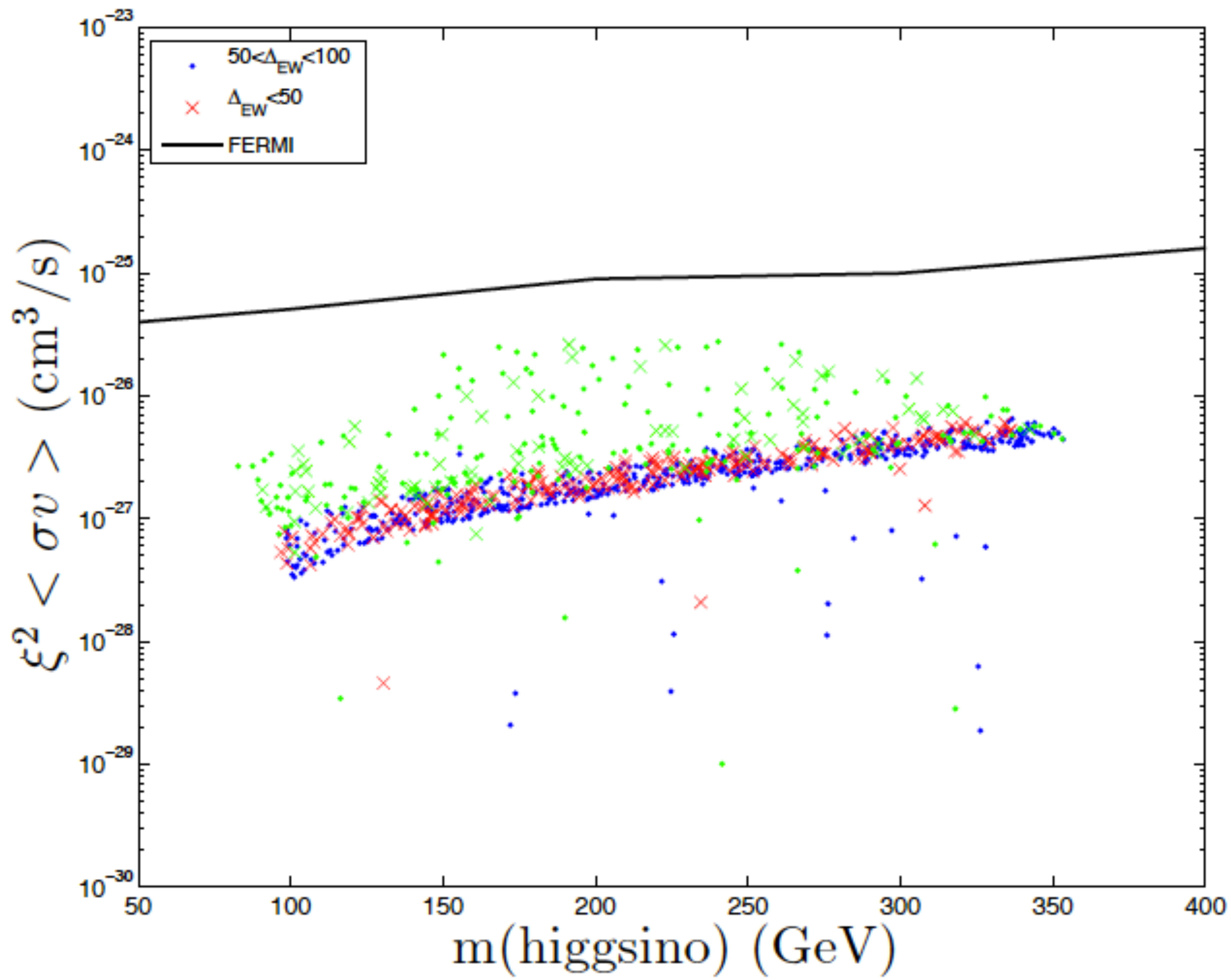
axion abundance



mainly axion CDM for fa<~10^12 GeV; for higher fa, then get increasing wimp abundance

Bae, HB,Lessa,Serce

Higgsino detection via halo annihilations:

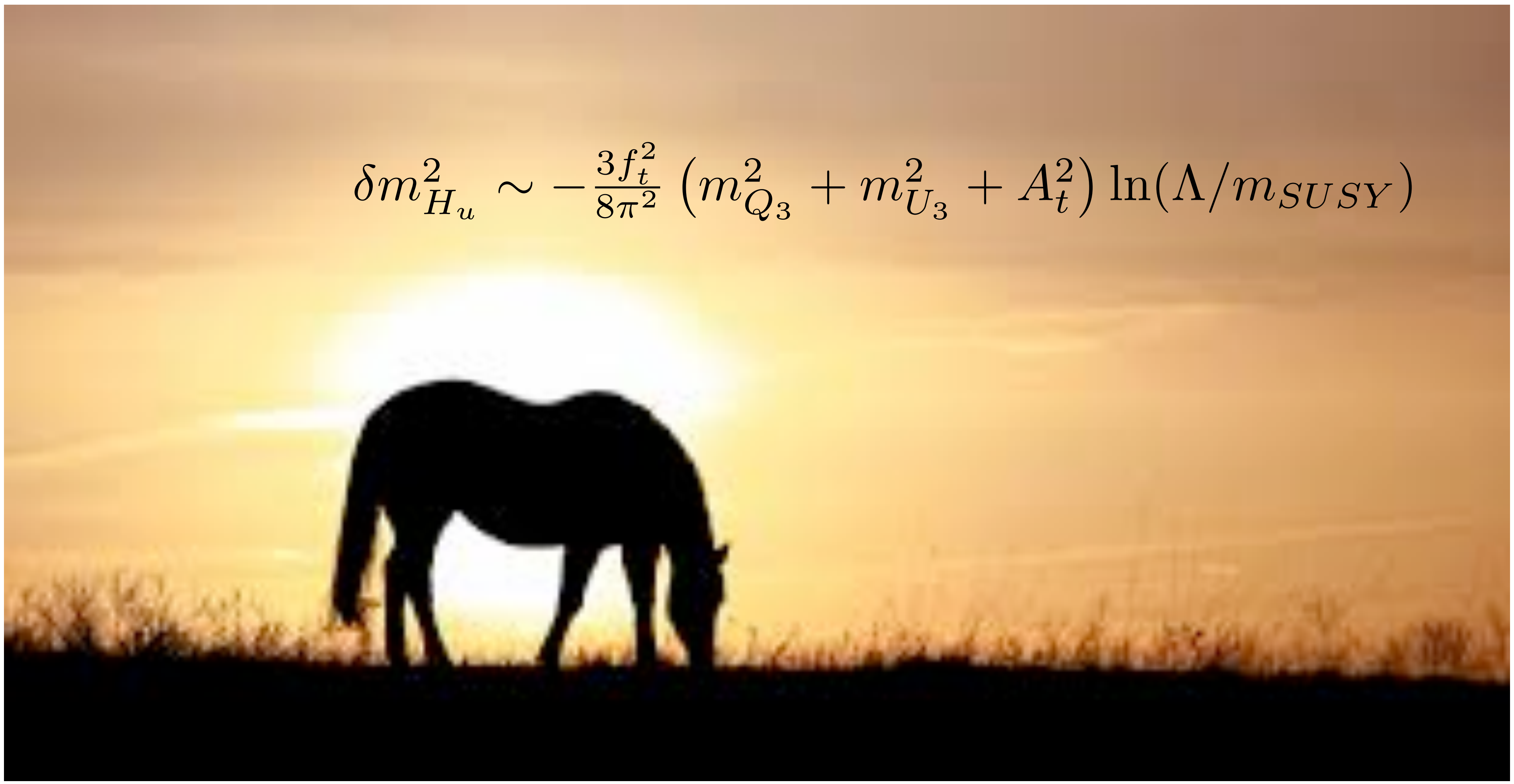


green: excluded by Xe-100

annihilation rate is high but rescaling is squared

Gamma-ray sky signal is factor 10-20 below current limits

Recommendation: put this horse out to pasture



R.I.P.

sub-TeV 3rd generation squarks not required for naturalness