

The Fine-tuning of the Minimal Supersymmetric Standard Model: Constraints by the LHC, Future Colliders and Dark Matter searches

DESY workshop on fine-tuning and naturalness
Hamburg 2018

<https://arxiv.org/abs/1612.06333>
and follow up

<https://arxiv.org/abs/1605.02797>
and follow ups

Melissa van Beekveld, Wim
Beenakker, Sascha Caron, Ruud
Peeters, Roberto Ruiz de Austri

Approach

- This talk: Use worldwide data and we search for the minimal “fine-tuning (FT) of the Higgs mass” given the model (today Supersymmetry)
 - *Search for minimum in high dimensional new physics model space*
 - *Taking into account all data constraints (e.g. LHC and DM experiments)*

→ Determine parameters yielding lowest FT

Our interests:

1. Does this yield a good Dark Matter candidate?
2. When/how is this model found at the LHC ?
3. Which future collider / machine is best to constrain FT?
4. FT measures and does this all make sense ?

Model(s)

1. **MSSM19@SUSY** scale and **MSSM19@GUT** scale:

Model with phenomenological most relevant parameters
(for Dark Matter, LHC and fine-tuning)

2. Constrained models @GUT scale

e.g. mSugra

MSSM19:

*10 sfermion masses (1st and 2nd gen. degenerated), 3 gaugino masses (M_1, M_2, M_3),
tan beta, mixed higgs mass parameter μ , heavy CP odd higgs M_A , 3 trilinear couplings)*

Fine tuning problem

Every beyond the SM theory “coupling” to (any?) of the SM particles and defined at the scale Λ will contribute to the Higgs mass:

$$\text{Higgs Mass}^2 \approx X + \text{Quantum Corrections } (\Lambda)^2$$

$$\text{Quantum Corrections } (\Lambda)^2 \approx \text{coupling} * \Lambda^2$$

t'Hooft: “Natural” if symmetry increases if parameter $\rightarrow 0$

→ No symmetry in the SM to protect spin 0 particles (Higgs mass)

Fine tuning in SUSY

Higgs mass related to Z mass

→ Fine tuning of Higgs mass can be rewritten in fine-tuning of Z mass:

Maybe formulate as:

Can we actually predict the Z mass in a SUSY model?

Approach: By brute force we **calculate** the Z mass as a function of the SUSY-scale MSSM parameters defined at SUSY scale beyond tree level:

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2.$$

$$-\mathcal{L}_{\text{higgsino mass}} = \mu(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0) + \text{c.c.},$$

as well as Higgs squared-mass terms in the scalar potential

$$-\mathcal{L}_{\text{supersymmetric Higgs mass}} = |\mu|^2(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2).$$

Other parts are stemming soft SUSY breaking parameters

Fine tuning in SUSY

Approach: By brute force we **calculate** the Z mass as a function of the SUSY-scale MSSM parameters defined at SUSY scale beyond tree level:

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Stop contributions are in here

Pro: Agnostic of GUT scale
SUSY model
Contra: Agnostic of GUT
scale SUSY model

Fine tuning in SUSY

If one of these single contributions is large

→ The other contributions have to be “fine-tuned” to compensate

→ FT measure : Maximum contribution relative to the squared Z-mass/2

$$\text{FT} = \Delta_{\text{EW}} = \max_i \left| \frac{C_i}{m_Z^2/2} \right|, \quad (2)$$

where the C_i are defined as:

$$C_{m_{H_d}} = \frac{m_{H_d}^2}{\tan^2 \beta - 1}, \quad C_{m_{H_u}} = \frac{-m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1}, \quad C_\mu = -\mu^2$$
$$C_{\Sigma_d^d} = \frac{\max(\Sigma_d^d)}{\tan^2 \beta - 1}, \quad C_{\Sigma_u^u} = \frac{-\max(\Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1}.$$

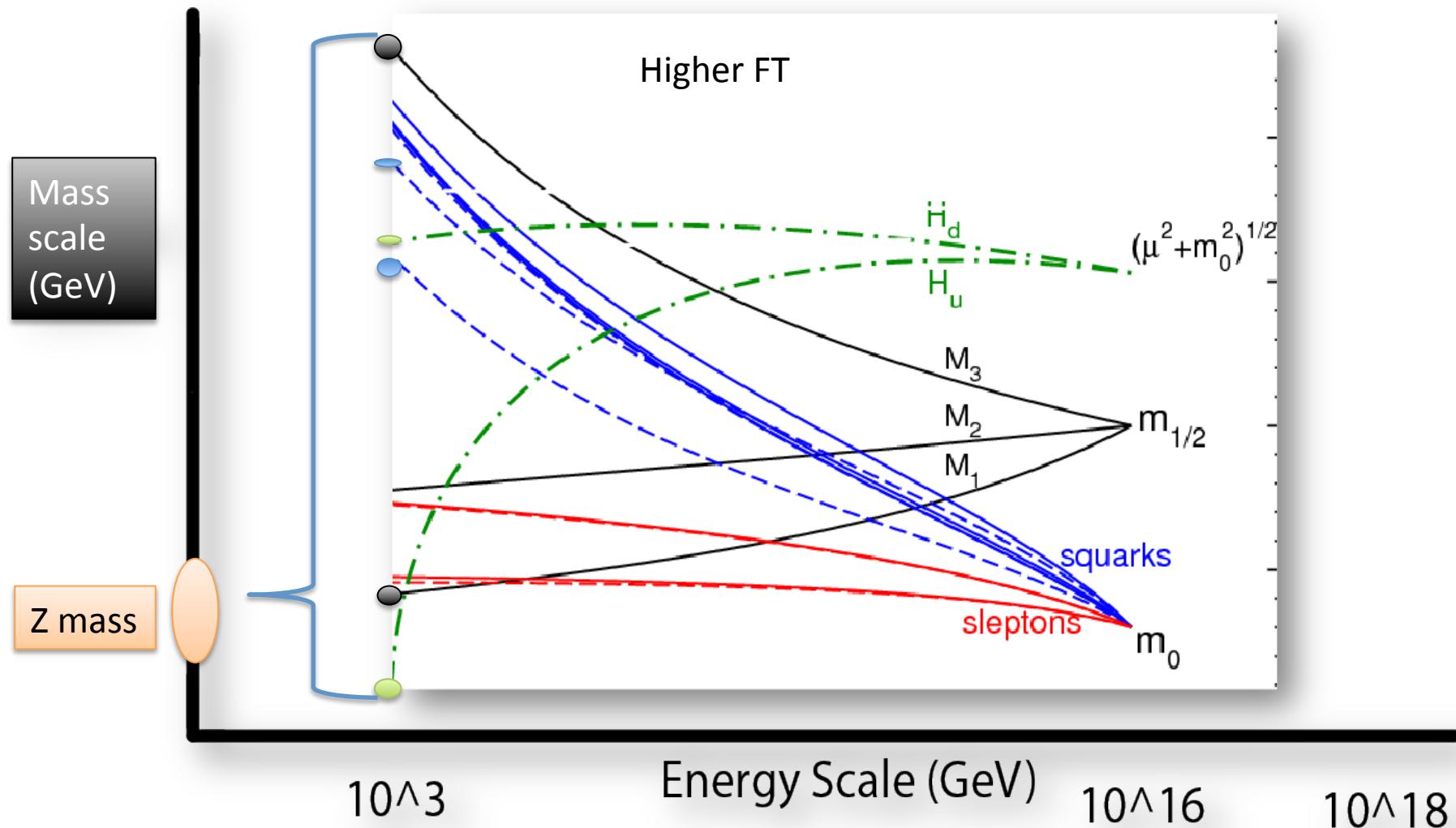
OK, but we could have other FT measures... ?
Does this make sense ?

Extra Info: Other FT measures

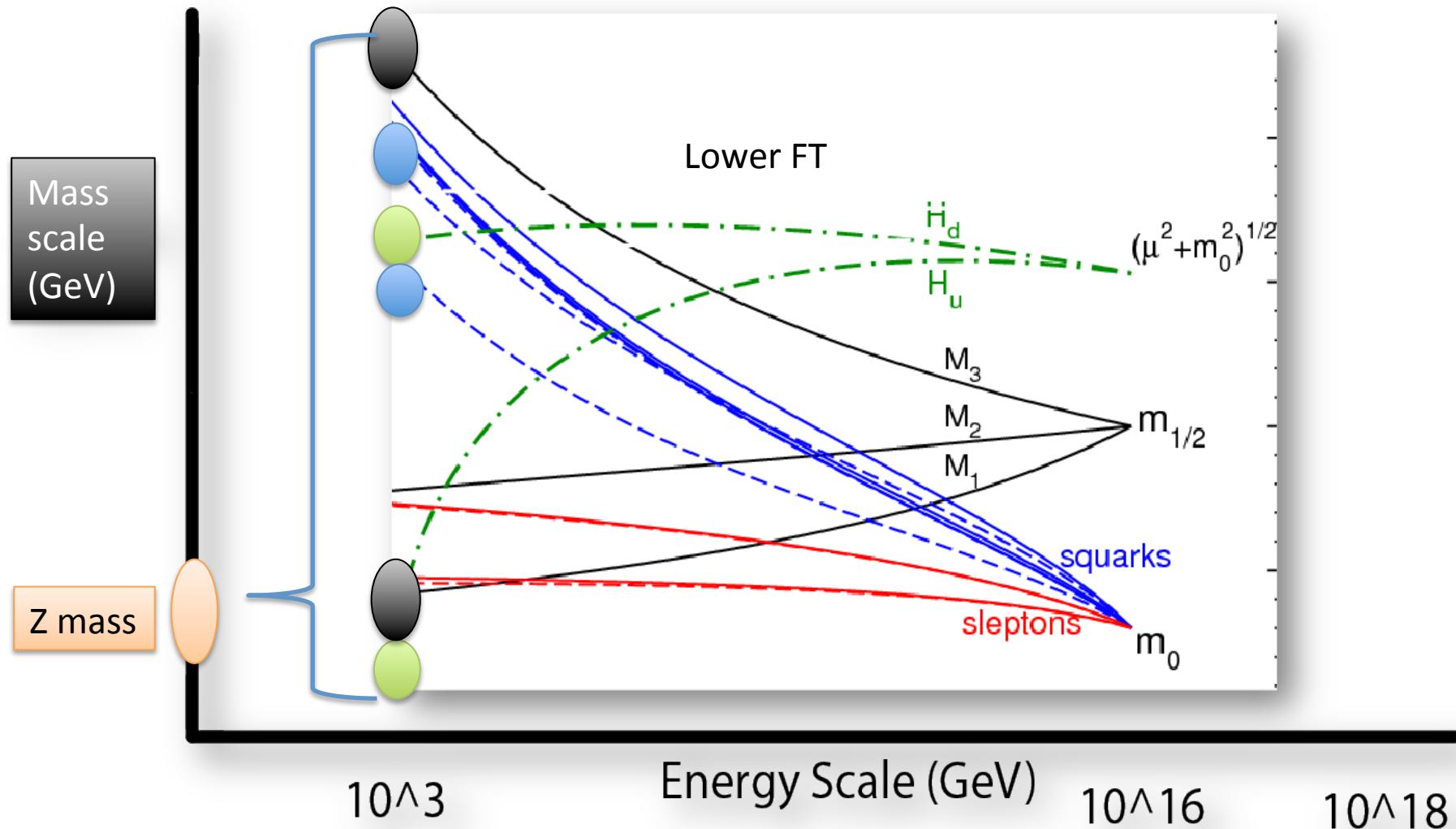
- Barberie Giudice : Similar measure if defined at **SUSY scale**: $|\frac{\partial \ln y}{\partial \ln x_0}|$ with y observable, x_0 parameter
(to consider this we apply a similar measure as well at SUSY scale by demanding that M_Z should not change by more than FT with parameter variations → i.e. small perturbations should have small impact, i.e. no chaotic behavior)
- Bayesian measures: Given GUT scale parameter possibilities → Are SM values probable ?
→ *Marginalization over parameters yields connection to Barberie Giudice measure and Baer measure (“size of parameter space”)*

→ We believe that the scale where FT is defined (GUT or SUSY) is more relevant than the “measure”

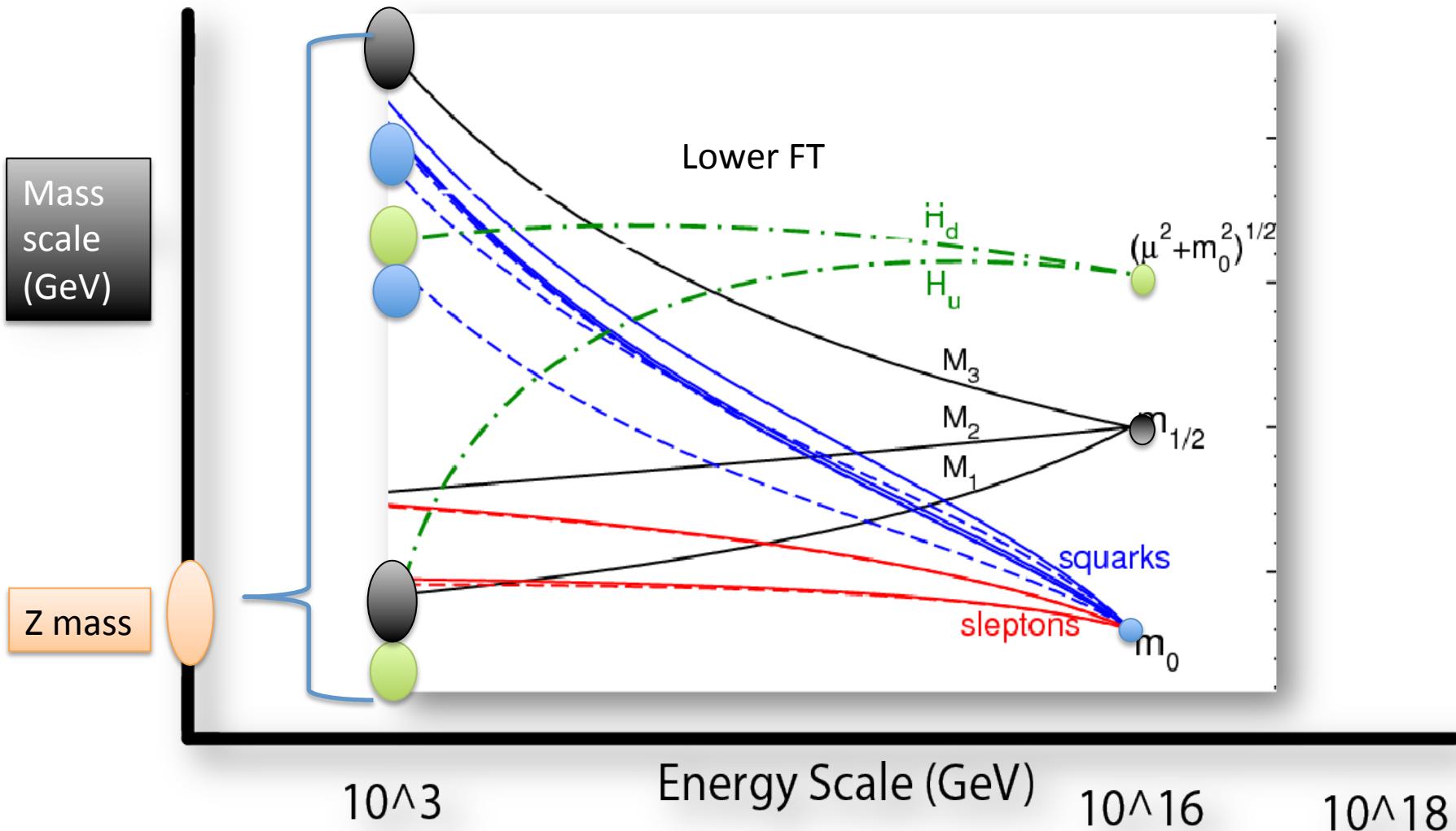
Measure at SUSY scale: We want to find “stable” MSSM parameter sets at the SUSY scale?
“Stable”: We can vary the parameters a “tiny bit” (e.g. 1%) and Z mass changes by similar amount



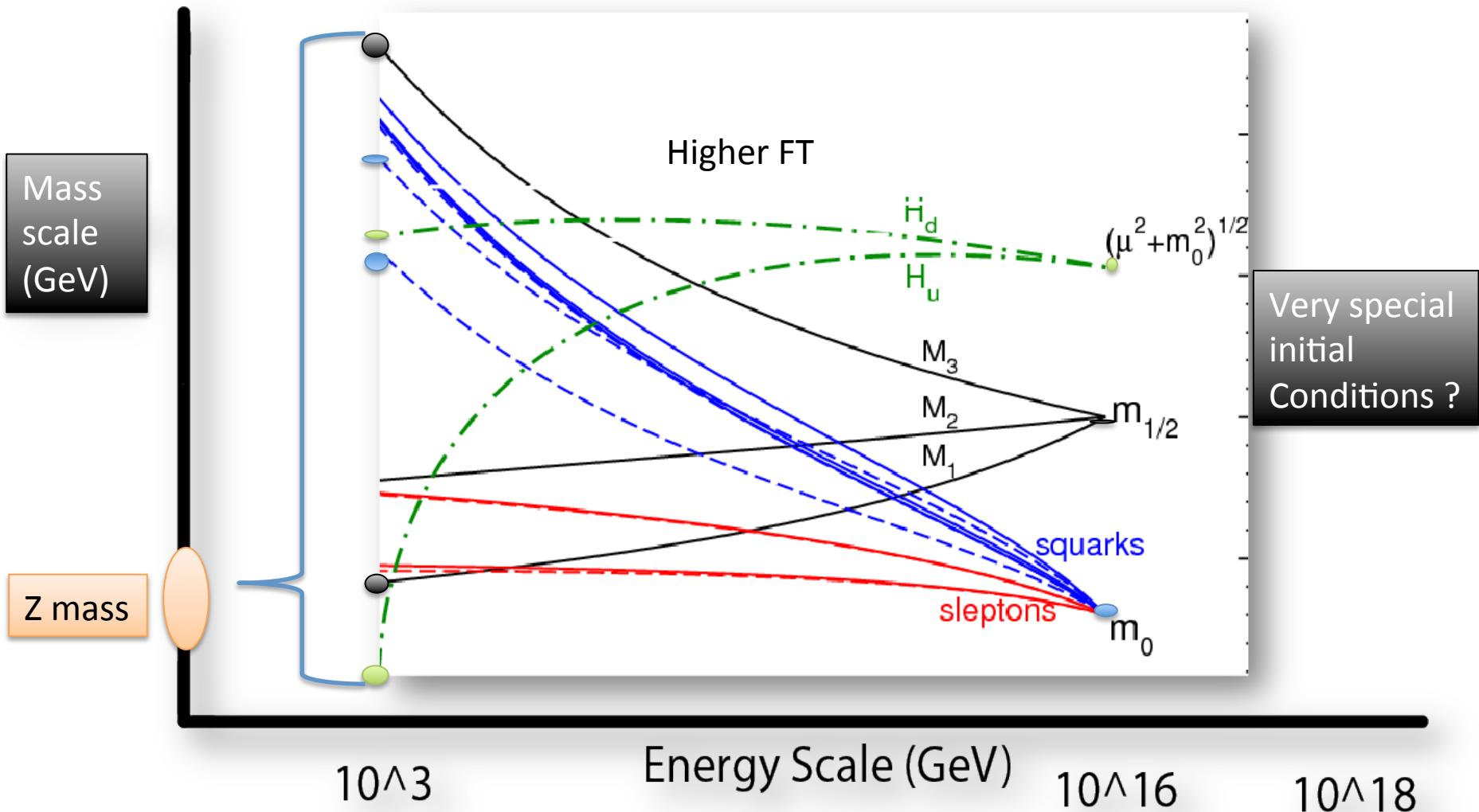
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Models defined at the GUT scale will likely be more “fine-tuned” (at GUT scale)
High scale \rightarrow low scale connected by RGE running



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High scale \rightarrow low scale connected by RGE running



Differences in FT measures

- **Vary parameters at SUSY scale (1 TeV) (Baer et al)**
- **Vary parameters at the GUT scale and define a GUT scale model (Barberie, Giudice et al.)**

New: Minimizing FT

Idea:

Use a smart minimum finding algorithm for large parameter spaces (gaussian particle filtering)

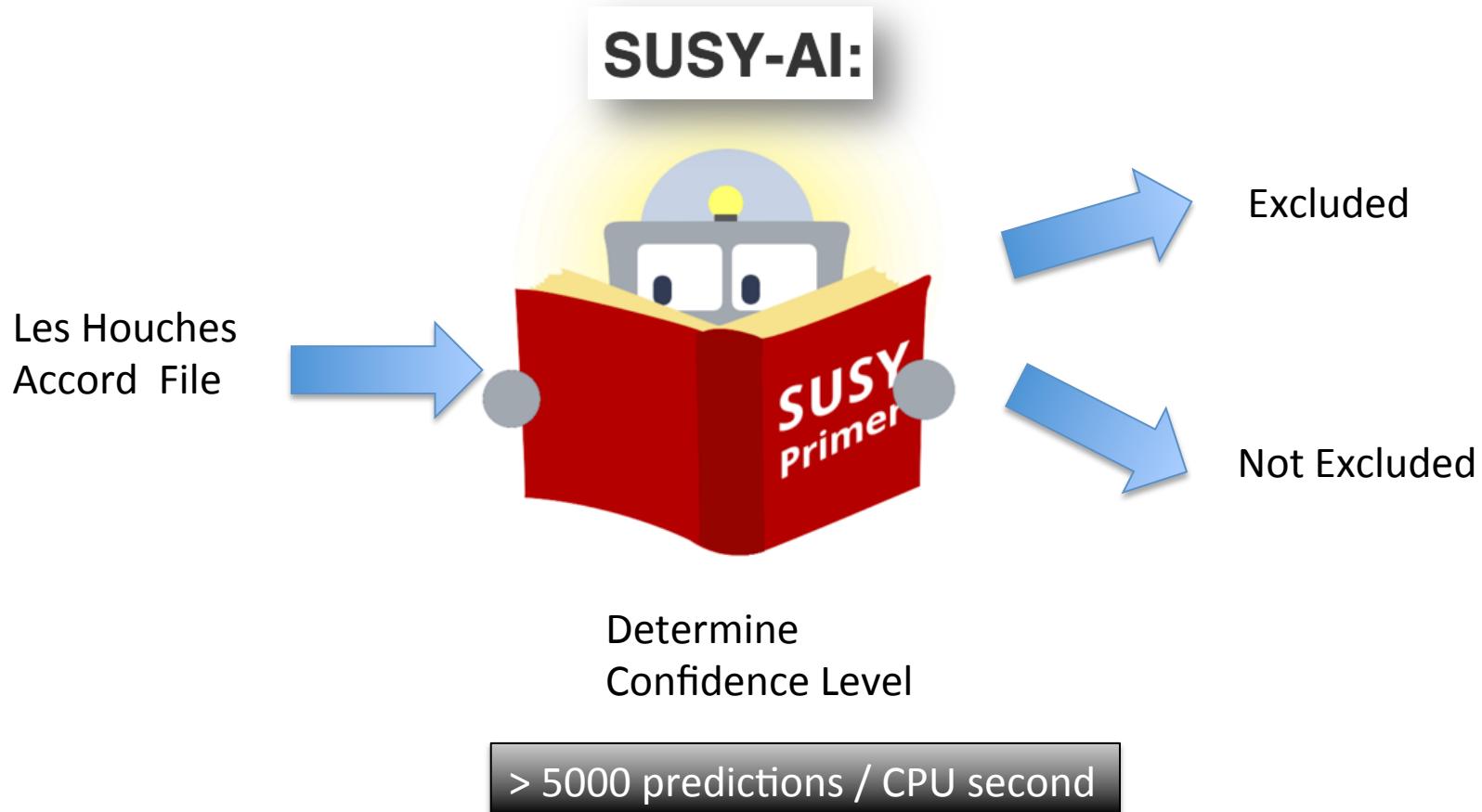
→ Finding minimum, i.e. result does not depend on sampling prior.

External constraints

- We apply all constraints from all experimental data (with 2 sigma)
- We apply constraints from direct detection experiments (Xenon, Lux) with 3 sigma
- Apply electroweak precision measurements, rare decays, Higgs mass, Higgs couplings
- We apply LHC bounds ... **How do we do this?**

Machine Learning LHC results

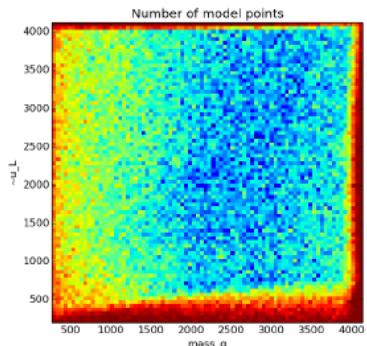
- ATLAS (JHEP 1510 (2015) 134) released limits of 200 signal regions for about 300000 MSSM points
- We used them to construct a “Random Forest” of Decision Trees



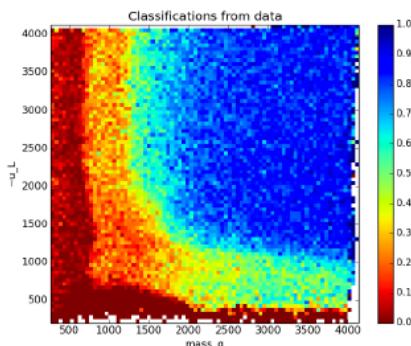


Difference between
classification and prediction

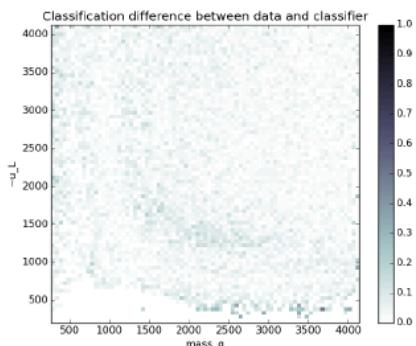
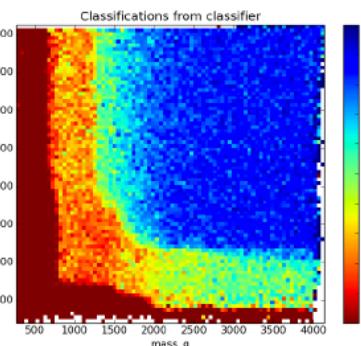
Number of model points



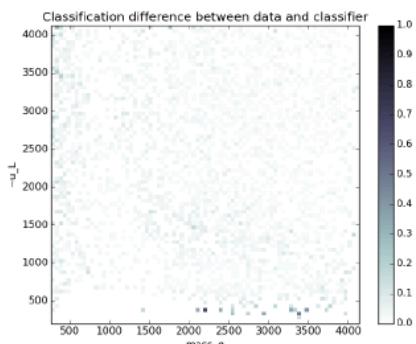
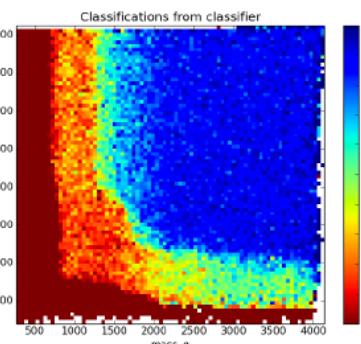
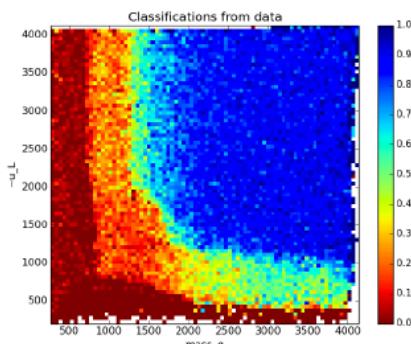
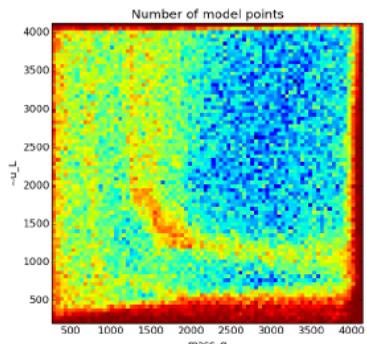
True classification



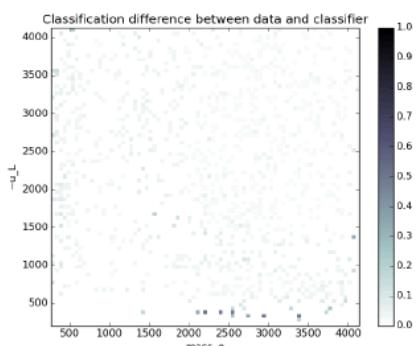
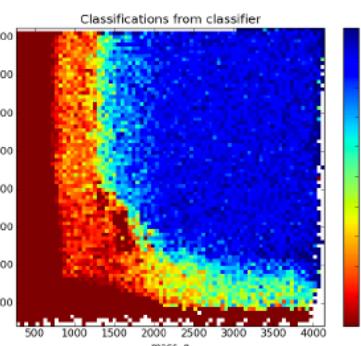
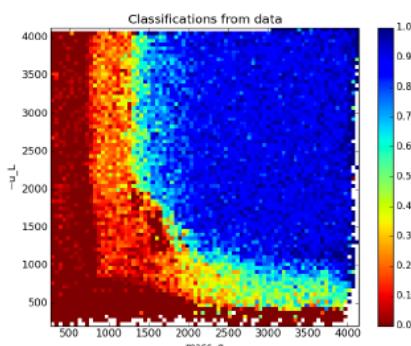
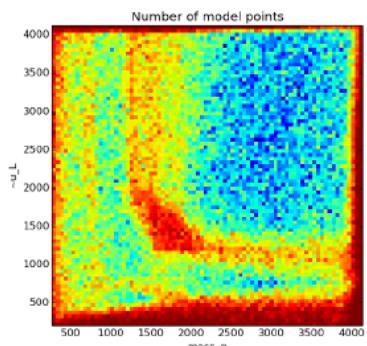
Prediction by classifier



All data



95CL



99CL

Test data: Accuracy > 93%

OK, what is the result of this ?

What is the minimum ?

SUSY scale measure sees no problem with FT so far

→ Found solutions with FT around 3,5 ...

**Let us first look at the Dark Matter experiments
(personal interest ;) ...**

Wait: What to expect for relic DM density ?

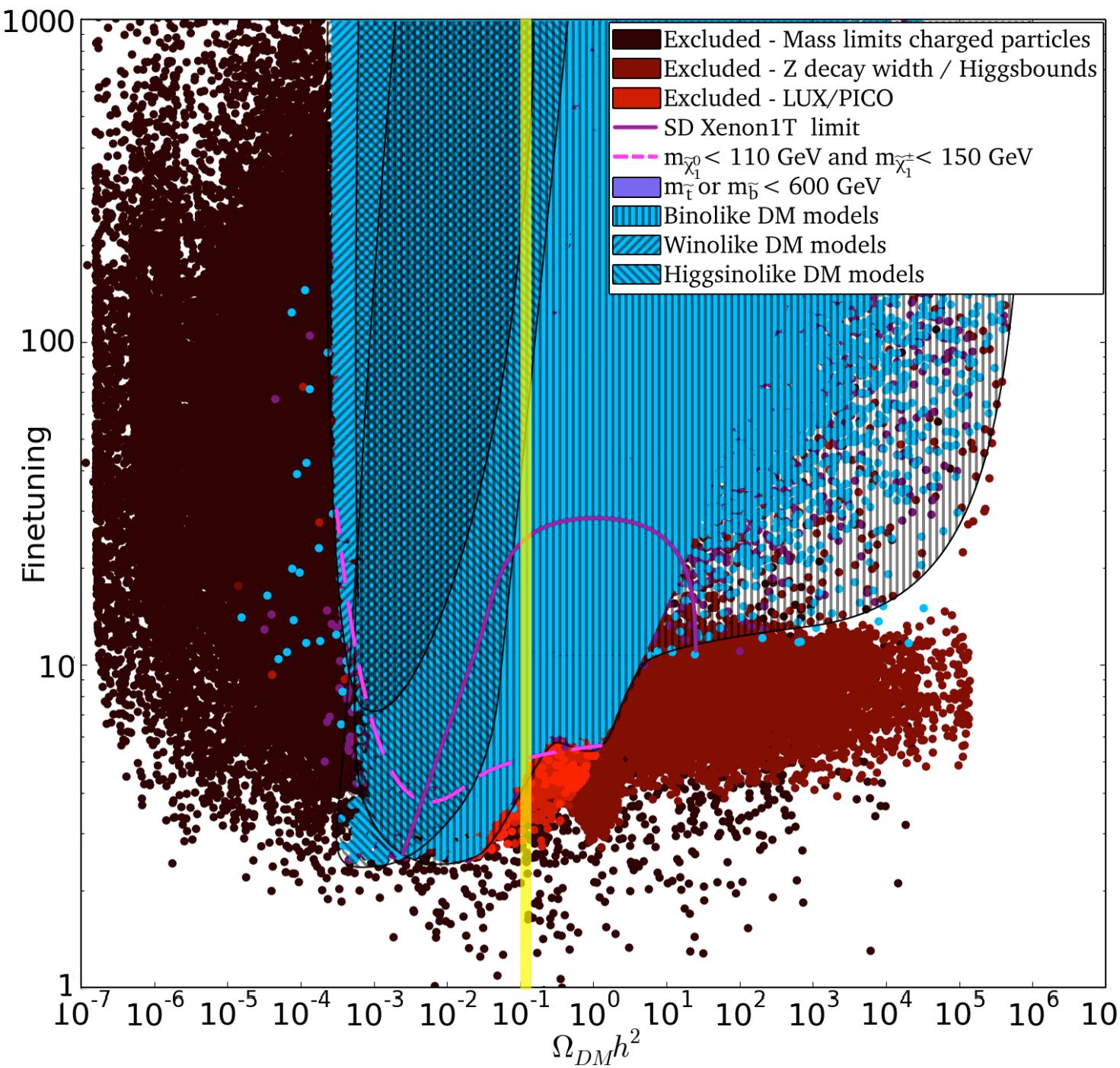
- → 14 orders of magnitude variations in DM relic density with Neutralino DM at 10-1000 GeV

$\Omega_{DM} * h^2 = 0.12$ as required by observations

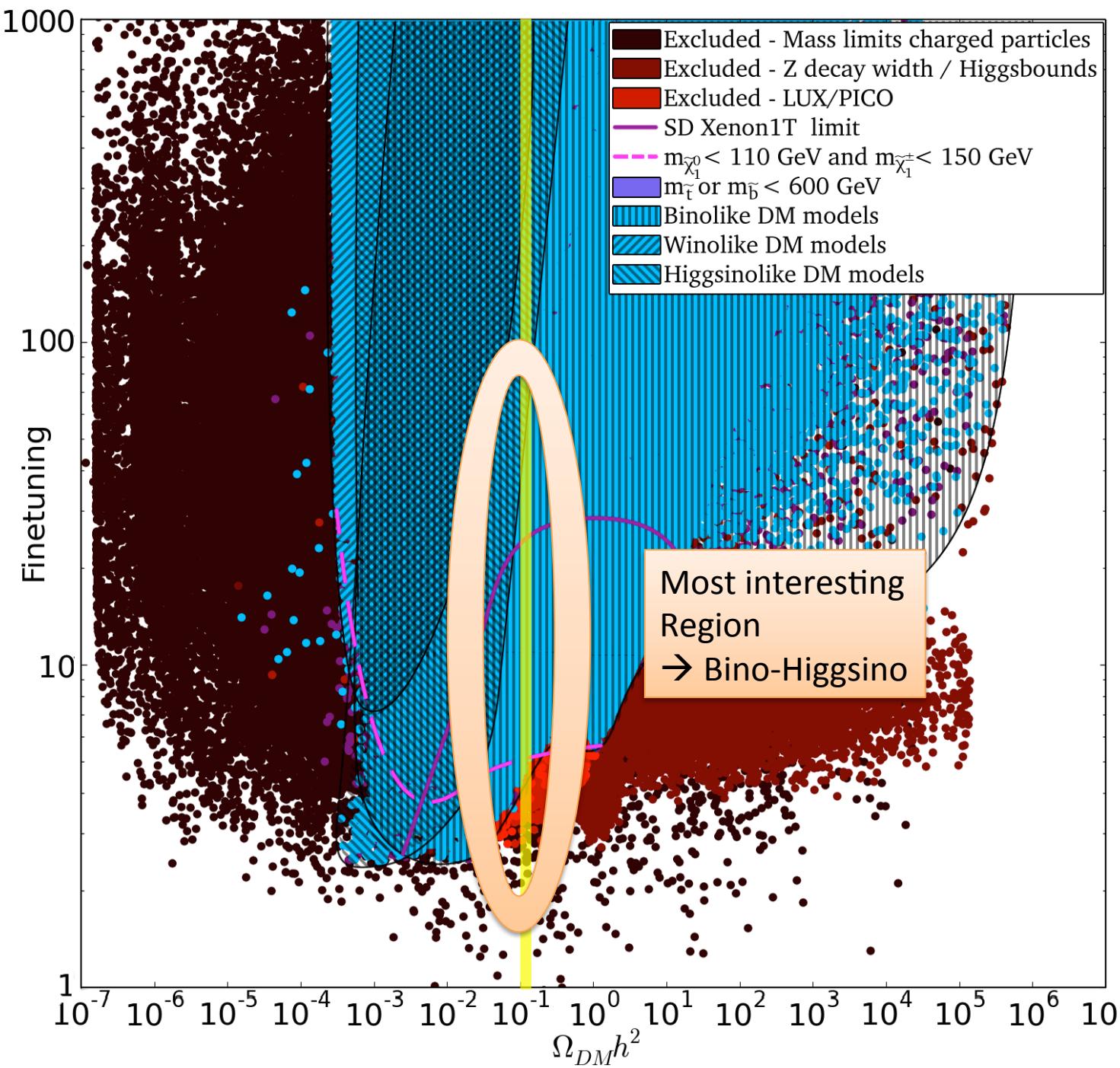
Here Ω_{DM} is the dark matter density in units of the critical density and $h = H_0/(100 \text{ km/s per Mpc}) = 0.68$ with H_0 the Hubble constant

Literature often states that low FT SUSY has a problem since 100 GeV Higgsino leads to a too small DM density → Not true in MSSM19 (low M1, low mu)

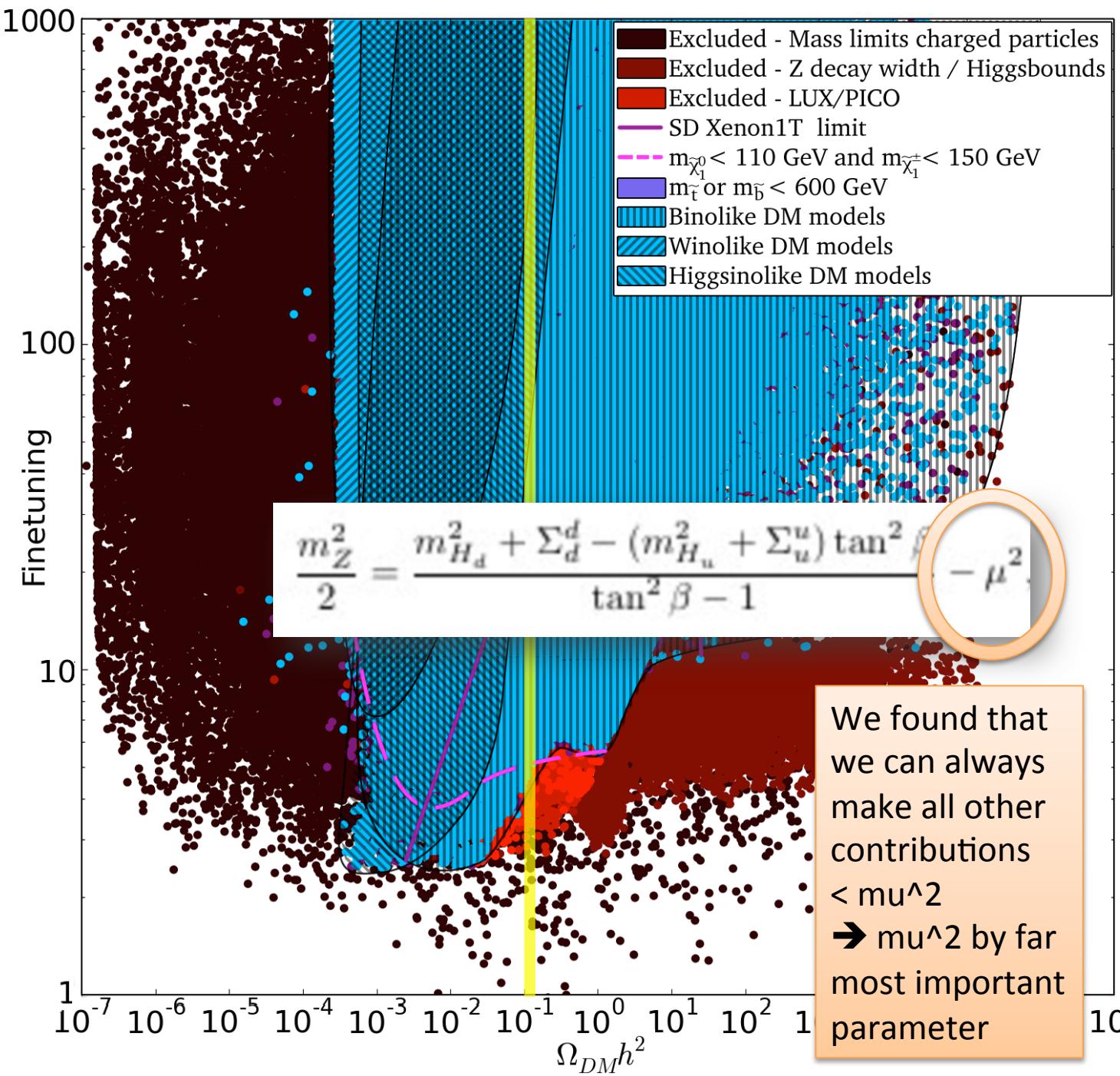
Dark Matter relic density



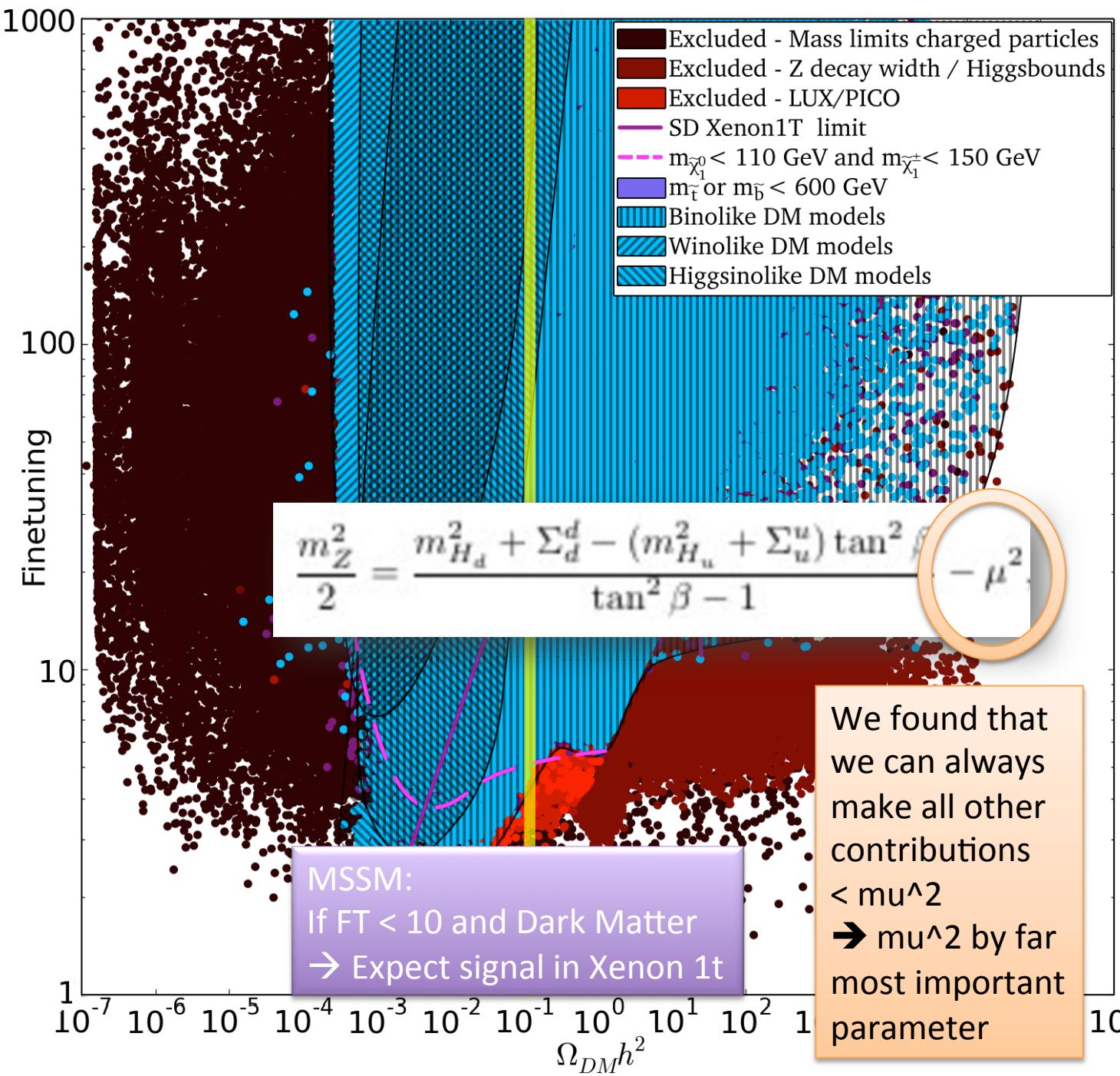
Dark Matter relic density



Dark Matter relic density



Dark Matter relic density



What about the LHC impact ?

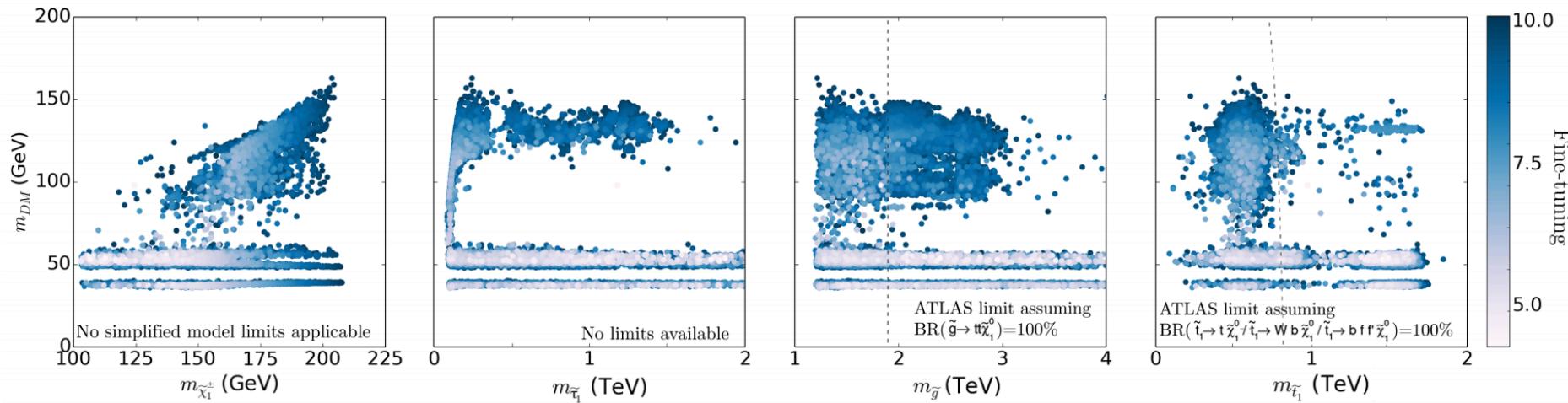


Figure 3. Lightest chargino, stau, gluino and stop mass versus the DM mass for lowest-FT natural models satisfying all constraints (including the dark matter relic density). The FT is shown in color scale. The ATLAS 13 TeV search limits, produced using simplified SUSY models, are also shown for comparison. However, as explained in the text, these limits actually are not applicable to the majority of our models [63–66].

Actually the LHC had NO impact on the FT@SUSY measure !!!

FT in the minimum is dominated by mu !

We can always minimize all other contributions to FT for a given mu.

What about future direct detection Experiments ?

They constrain mu via the higgsino
component in spin-dependent
DM-nucleon scattering.

Fine-tuning impact: Dark Matter direct detection

Dark Matter spin-dependent (SD) cross sections

→ Z coupling to Higgsino component constrains μ

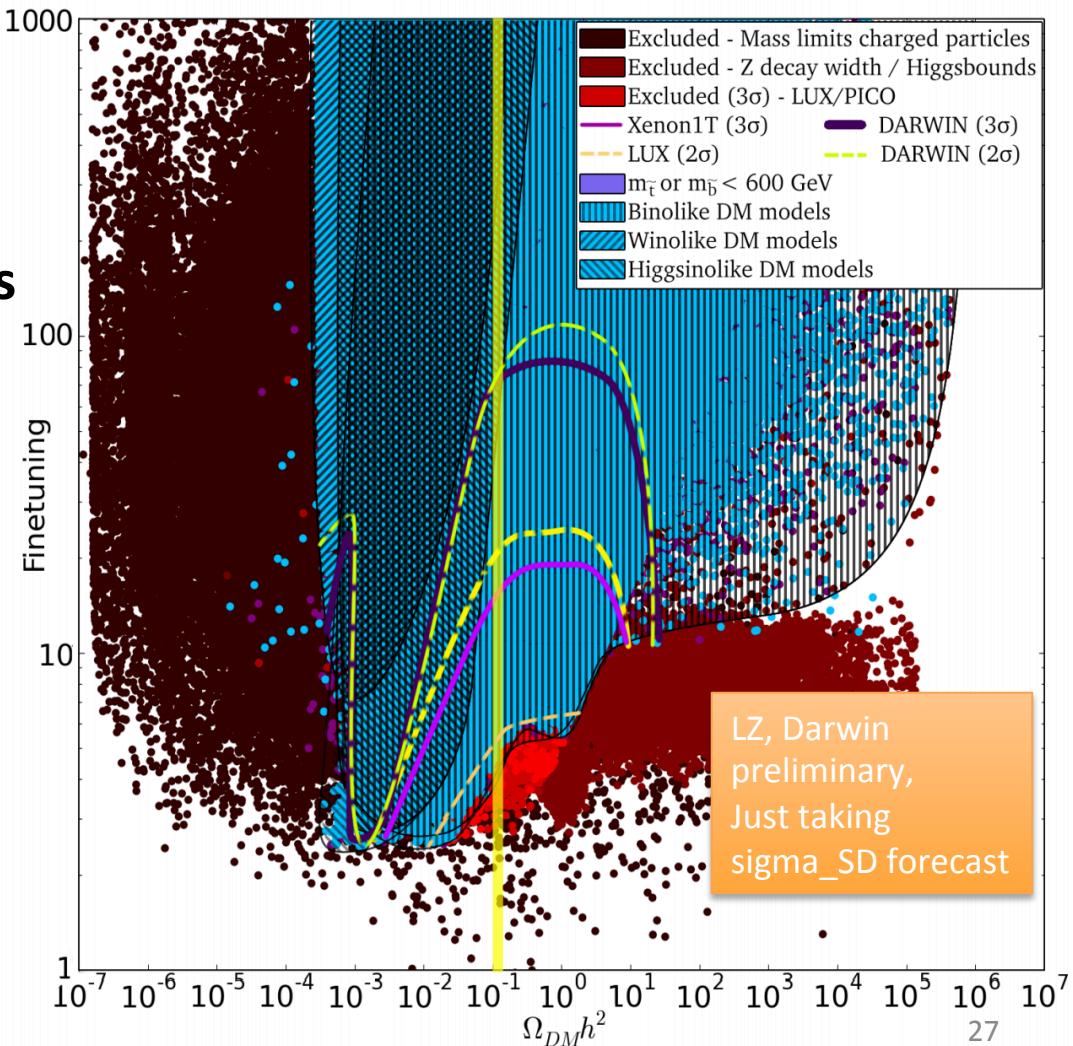
→ FT minimum fully **depends on lowest possible μ !**

We find that SD DM-nucleon cross section has **enormous impact on FT for all MSSM SUSY theories with DM**

Rough numbers:

Xenon1T: FT > 30

LZ (preliminary) : FT>60



Fine-tuning impact: Dark Matter direct detection

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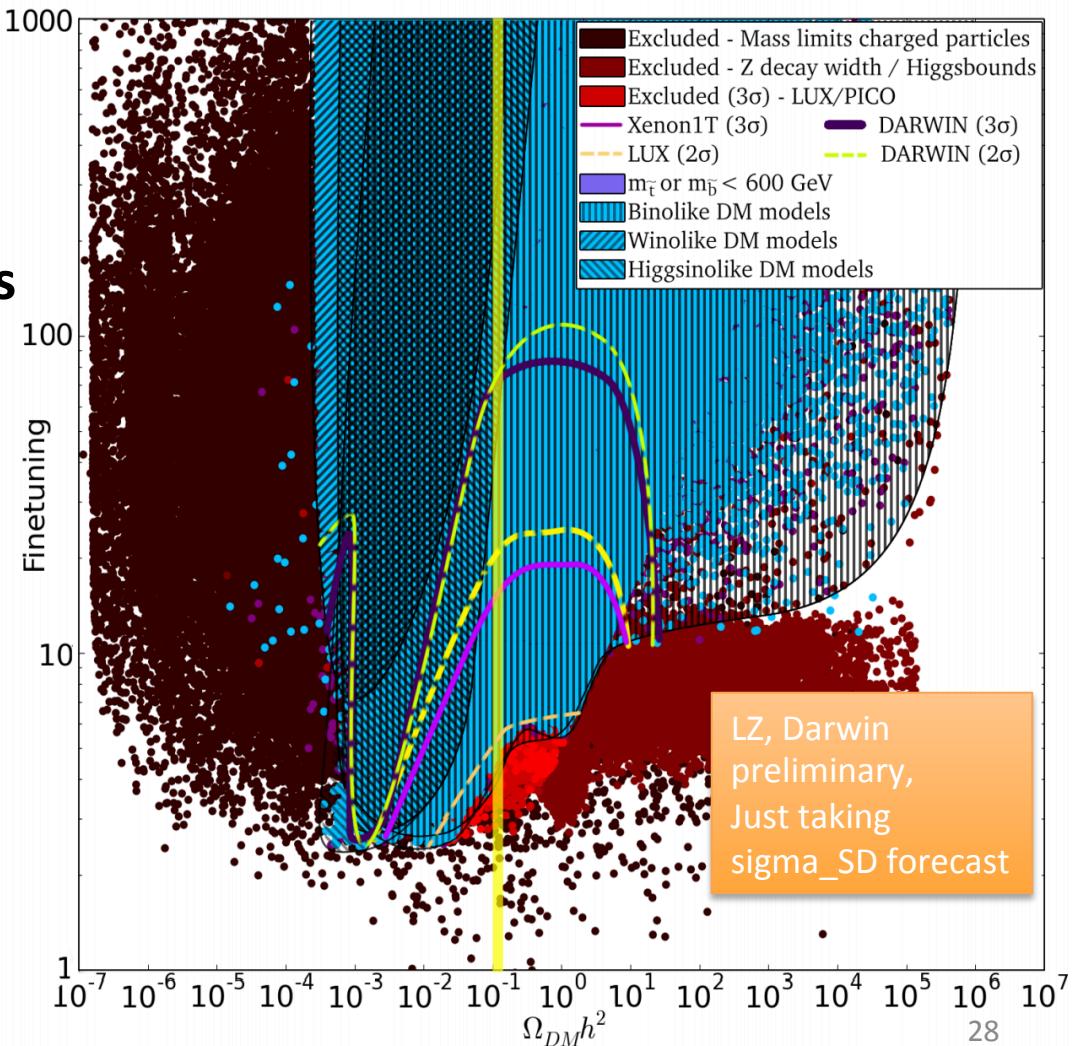
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What about future e+e- colliders ?

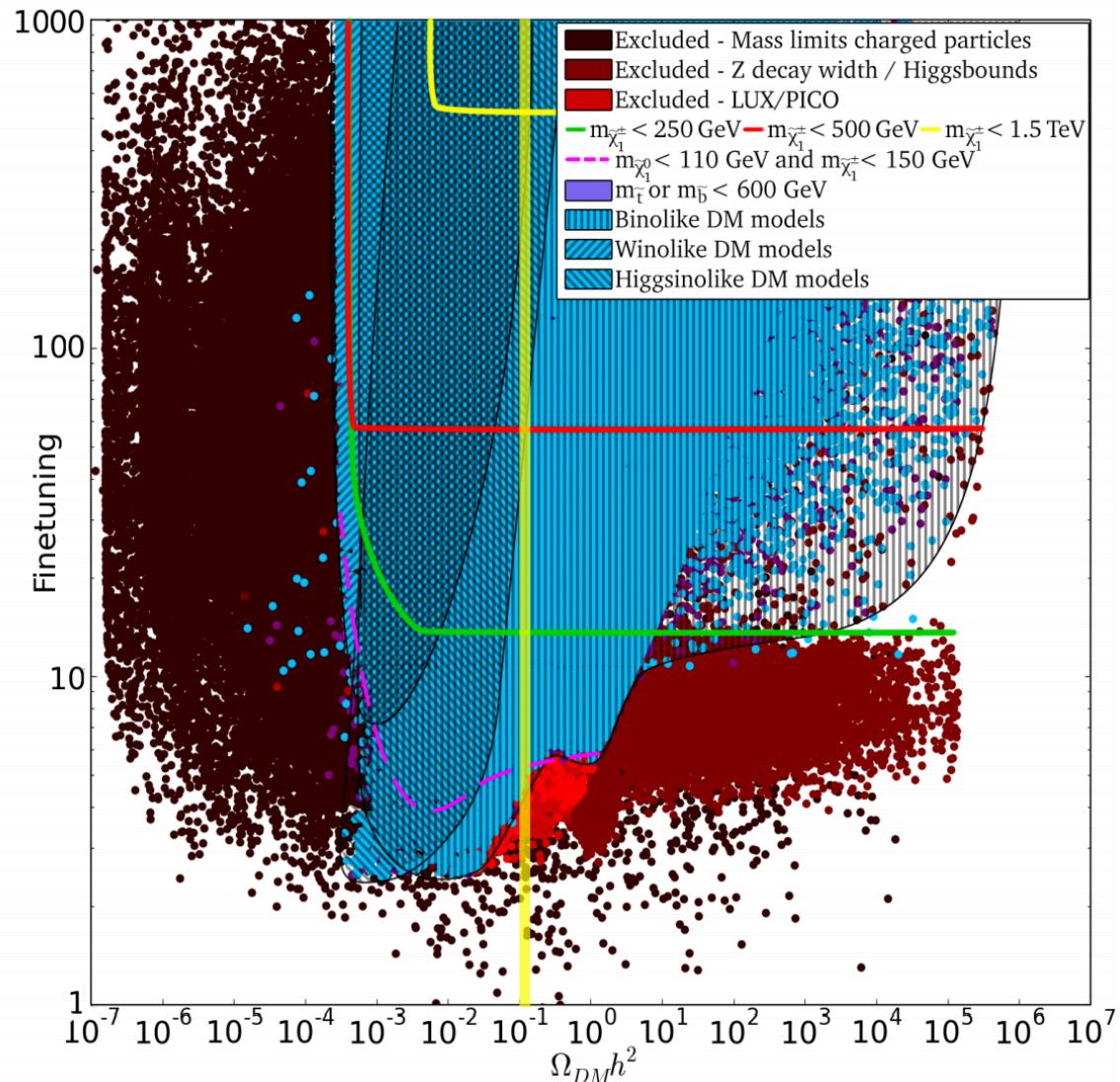
As LEP they can constrain mu via
Higgsino pair production !

Future experiments: e+ e- Colliders

Consider e+e- collider
chargino searches
(a la LEP)

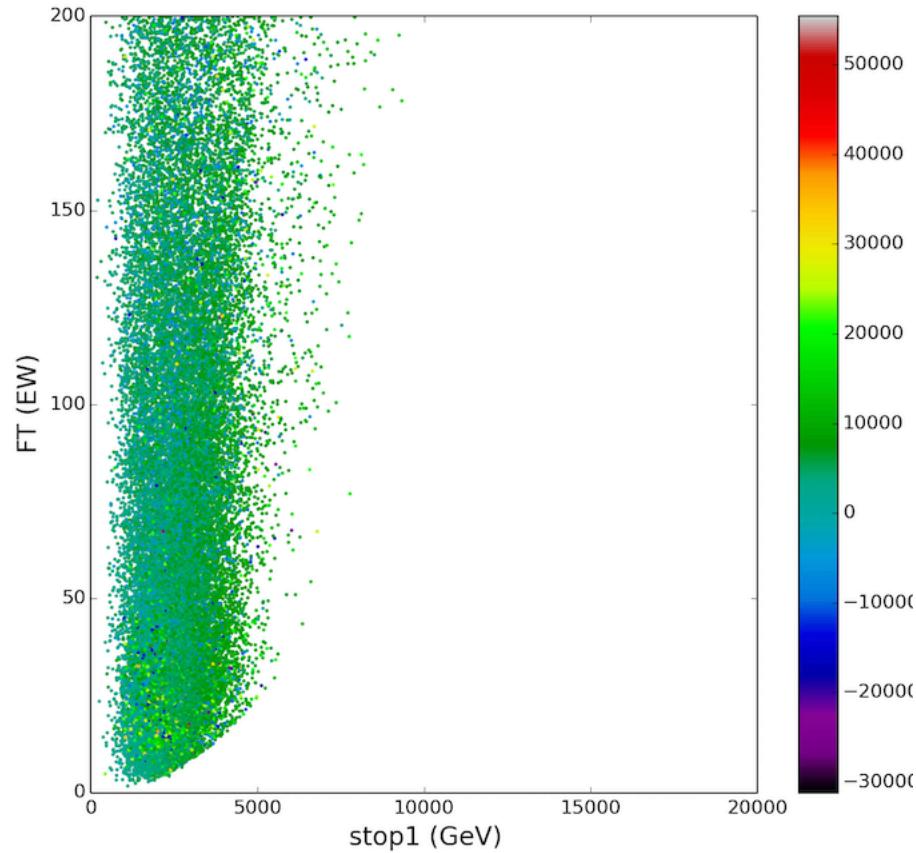
→ Almost model
independent limit
on mu (minimum
of FT depends on
mu !)

3 different CMS
energies...



Future hadron colliders via stop or gluino?

stop reach for FT(EW)

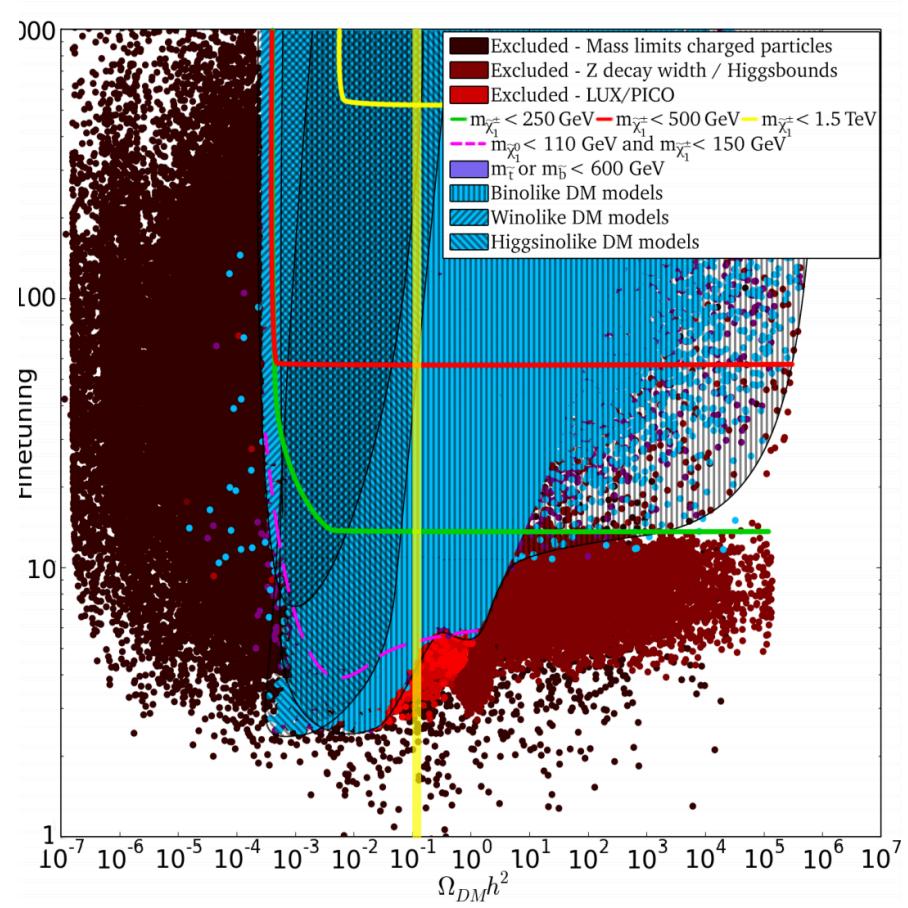
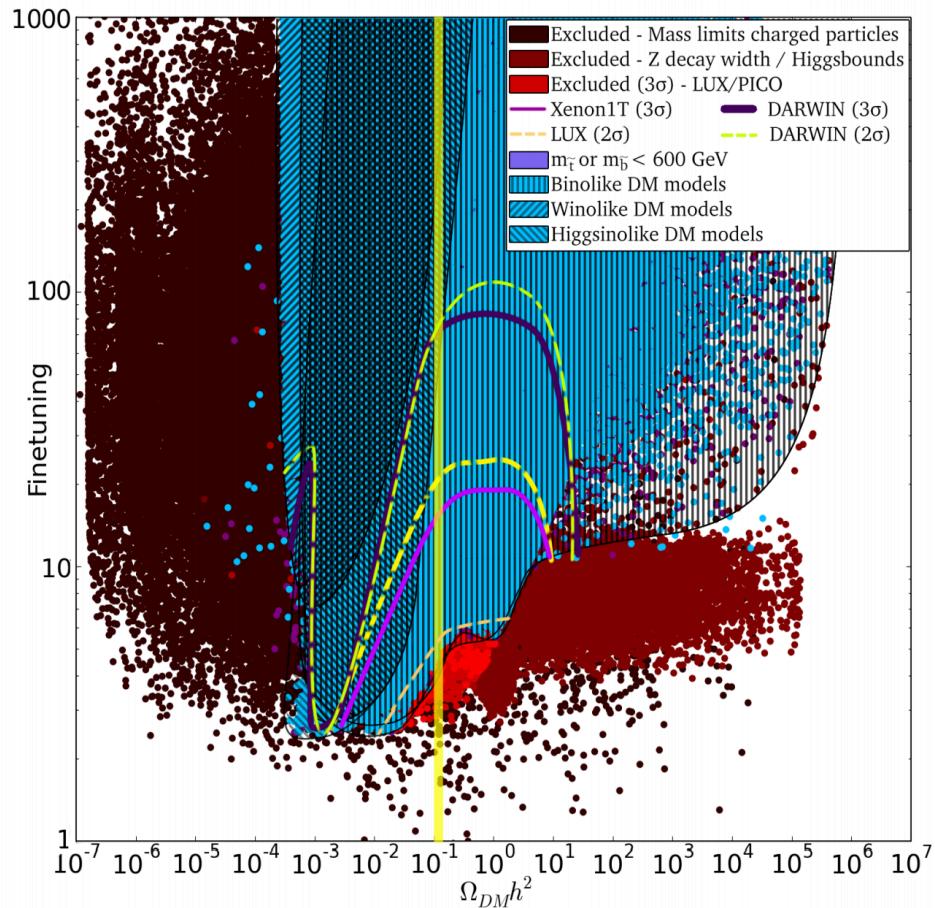


LHC-HL 1.5 TeV → FT > 3.5

LHC-HE 5 TeV → FT > 20

FCC-pp 10 TeV → FT> 100

MSSM: e+e- vs DM direct detection



Points in the minimum

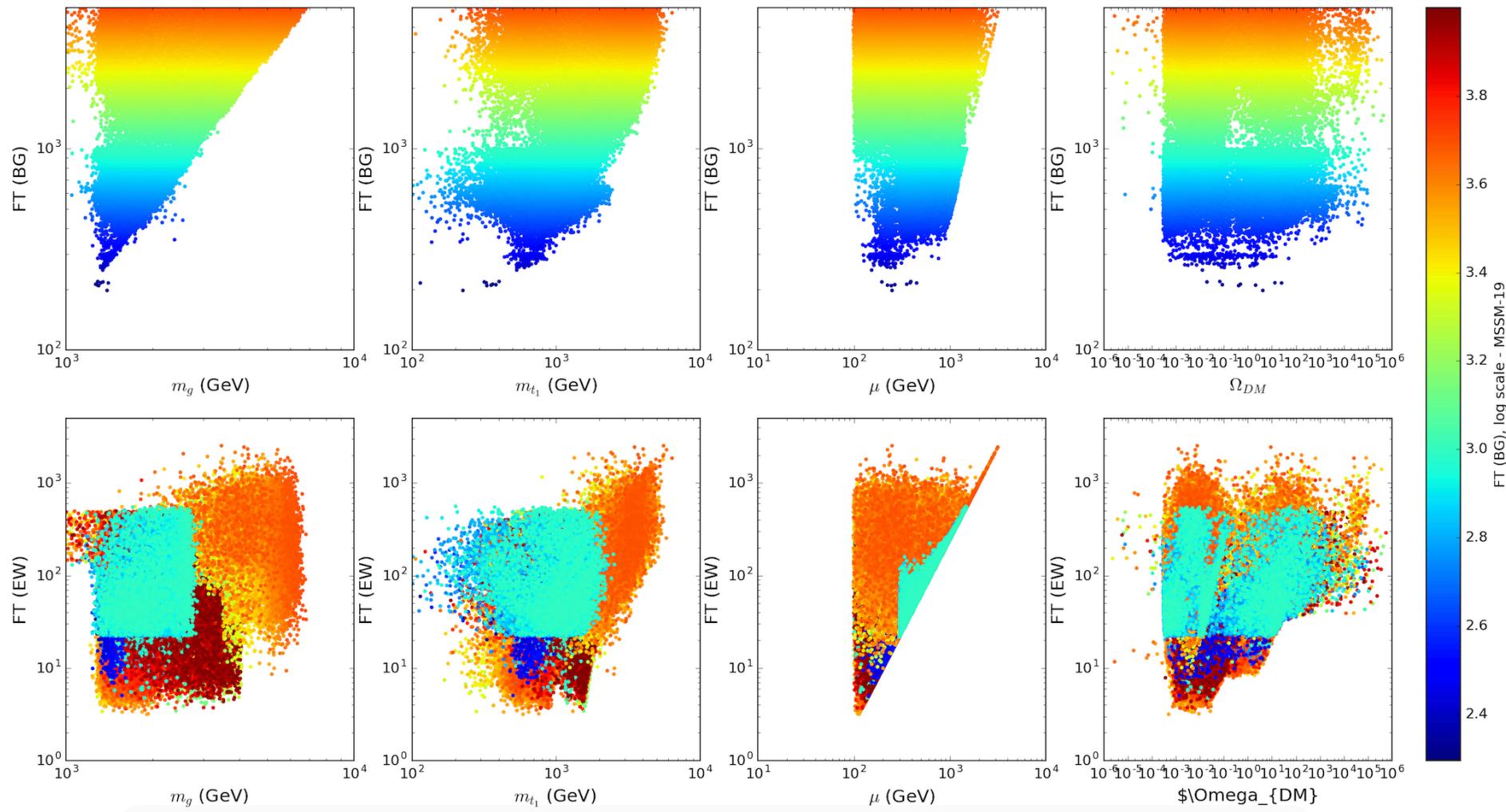
- All points with low FT and good DM relic density have Bino-Higgsino DM with mass between 45- 200 GeV
- SUSY scale measure: Stop mass can be large (>1 TeV), Gluino mass can be large (>2 TeV)
- → Next: Search for related GUT scale theory requiring also low GUT scale FT !
- That theory is not the cMSSM / mSUGRA

All this was the Baer et al SUSY scale FT

What if we define a similar measure at GUT scale ?

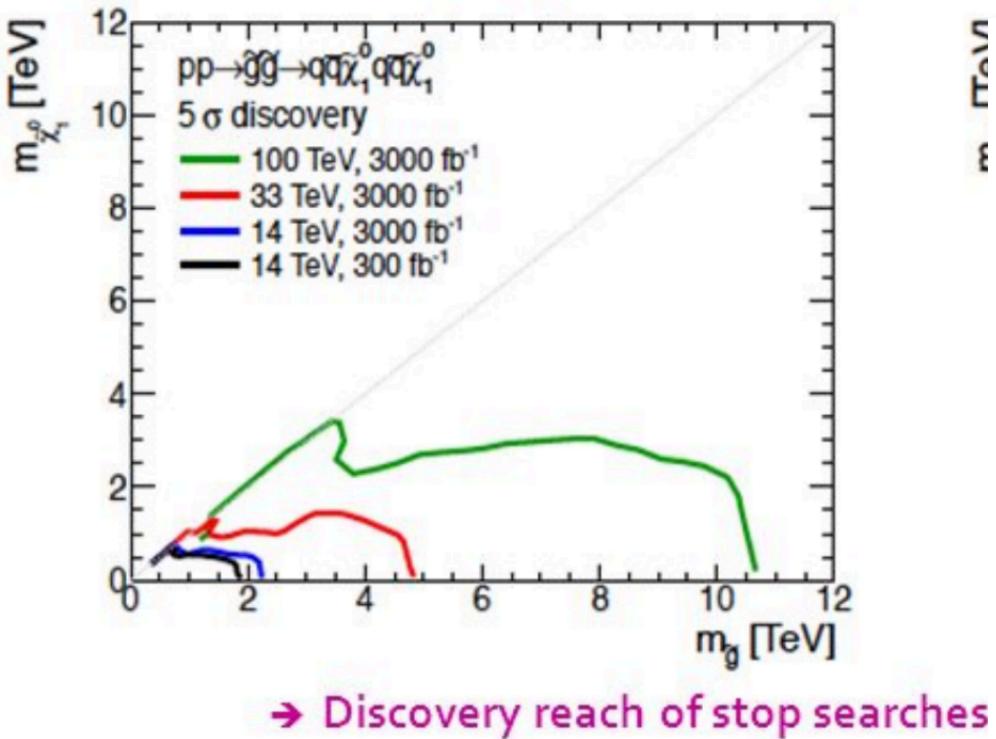
Here MSSM19@GUT → all MSSM19 parameters assumed to be INDEPENDENT (likely not...)

GUT scale FT measure for MSSM19



Limits

→ Example: gluino discovery reach ~ 11 TeV at FCC-hh (5 TeV at HE-LHC)

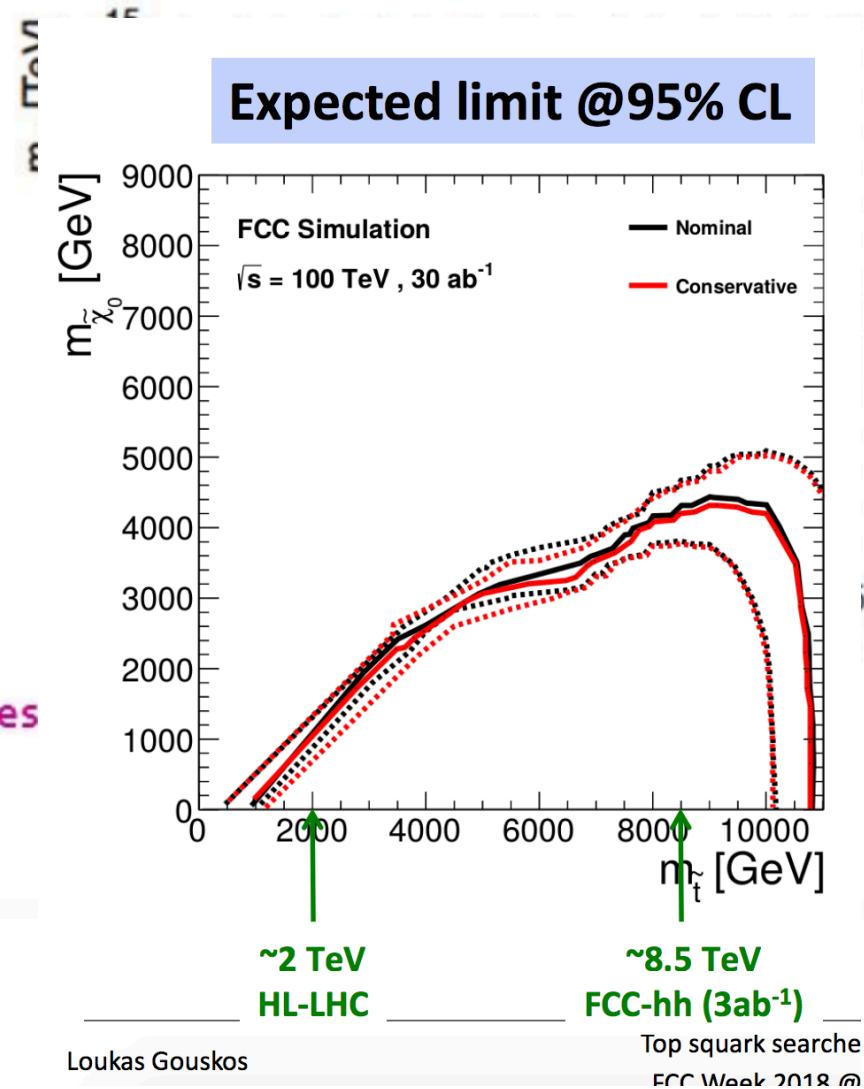


→ Discovery reach of stop searches

Up to 3 TeV with HE-LHC

Up to 6 TeV with FCC-hh

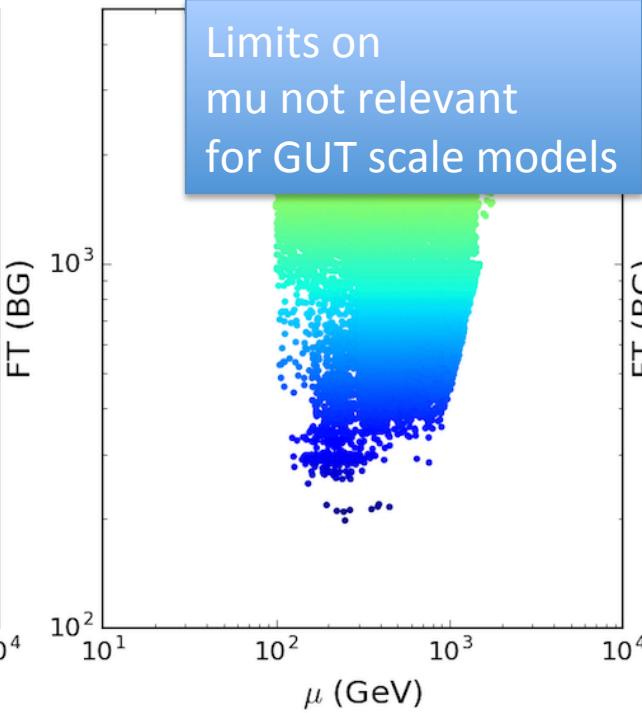
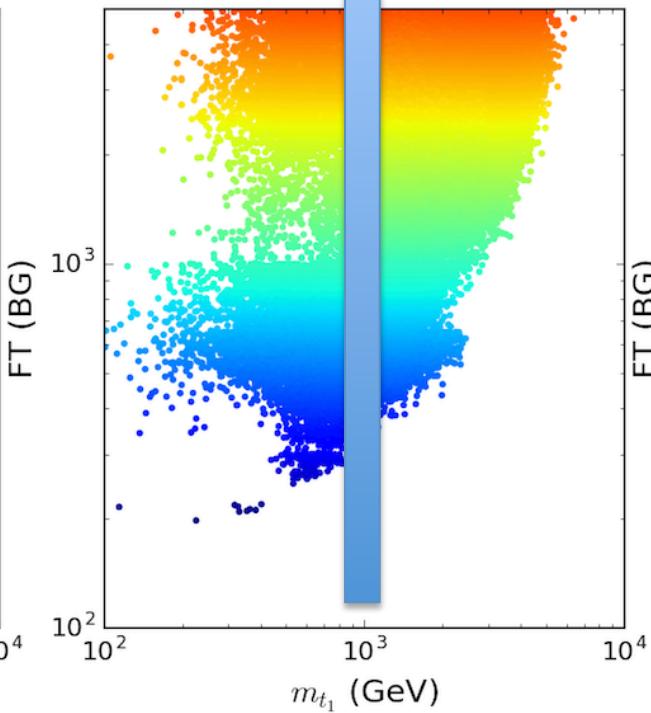
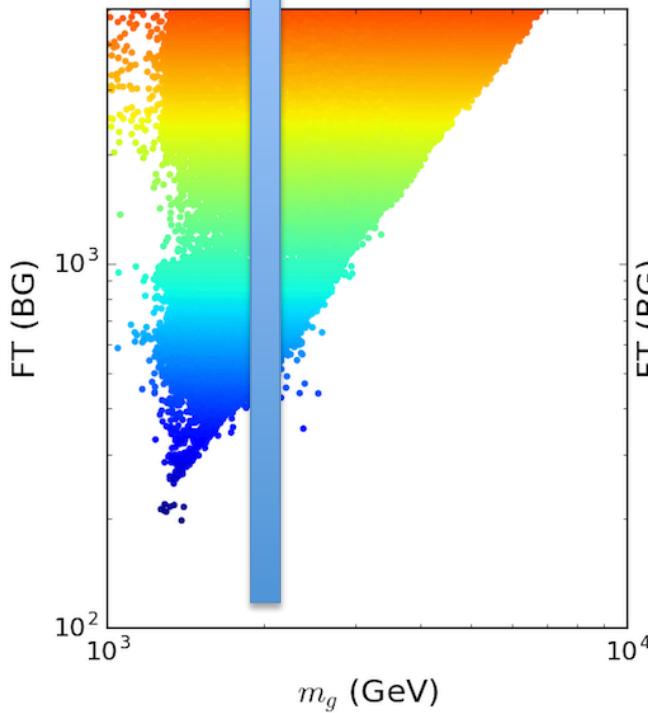
Patrick Janot slide



Loukas Gouskos

Impact Gluino vs stop

Expected LHC-HL limits

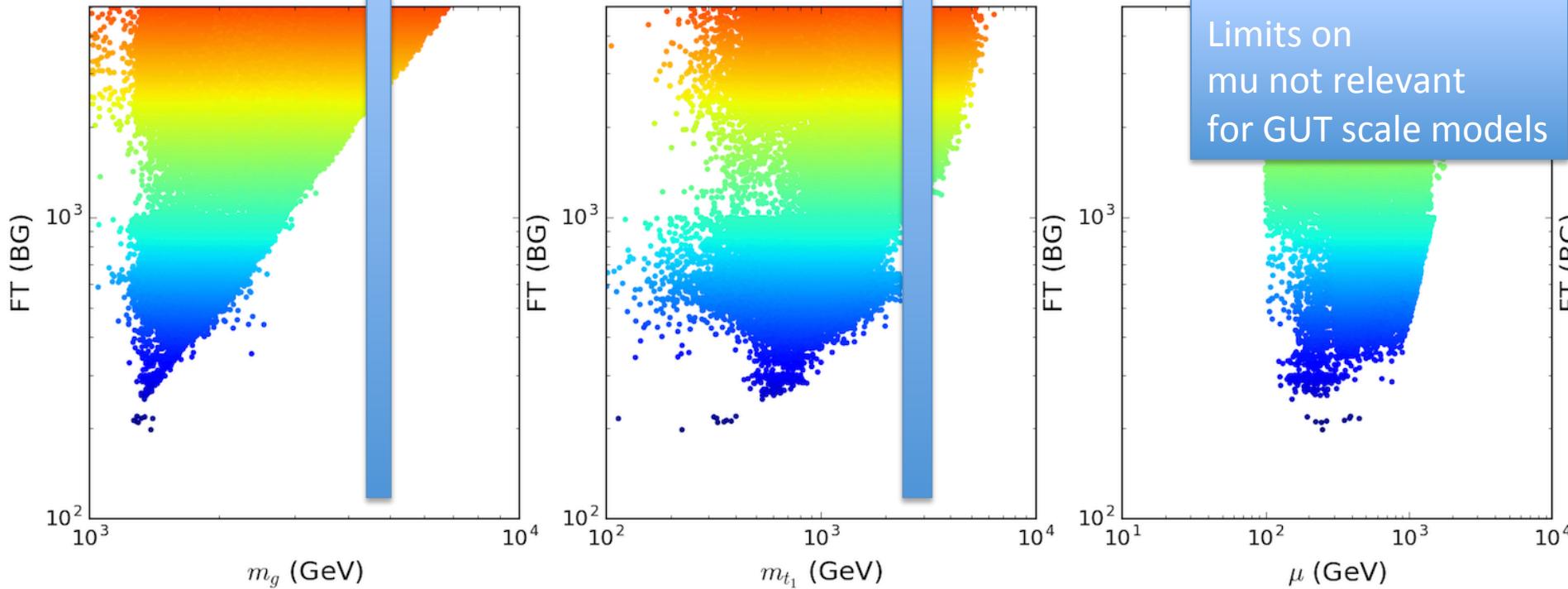


LHC HL → increasing FT from 200-300 to about 500 mainly by gluino limits

(all plots by Melissa van Beekveld !!)

Impact Gluino vs stop

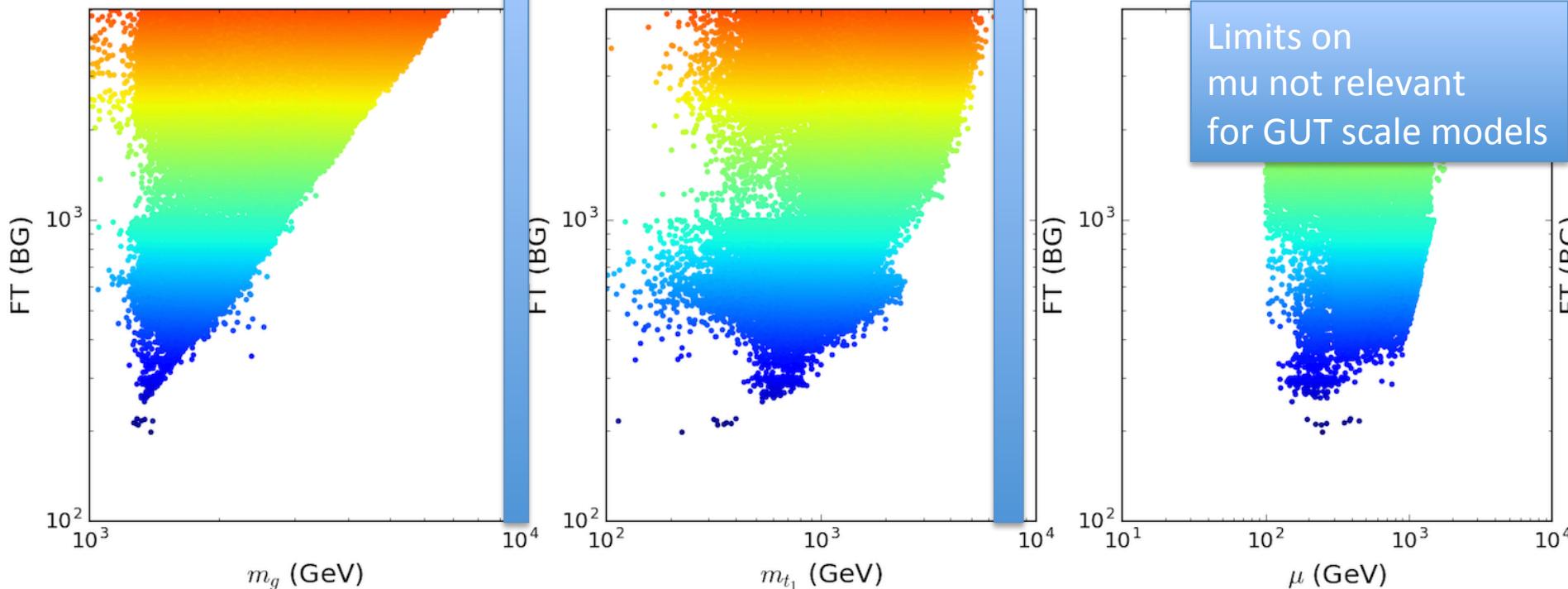
Expected LHC-HE limits



LHC HE → increasing FT from 500 **to about 2000 mainly by gluino limits**

Impact Gluino vs stop

Expected FCC-pp limits



FCC-pp → increasing FT to about **10000 (mainly gluino limits)**

Differences in FT measures

- **Vary parameters at SUSY scale (1 TeV) (Baer et al)**

→ We find that only mu is relevant

→ Stop and gluino limits have no big impact

- **Vary parameters at the GUT scale and define a GUT scale model (Barberie, Giudice et al.)**

→ Usually stop and gluino have largest impact

→ However we do not know the GUT model, could define a model where variation of parameters all yield the M_Z scale....

Conclusion: All constraints on FT

- FT@SUSY for MSSM in minimum around 3.5 !!!! → **Bino-Higgsino** Dark Matter
- FT@GUT depends on GUT model → for MSSM > 200, however FT@GUT can be smaller than this...
- Interesting: FT@SUSY will get strongly constrained by direct DM searches (spin dependent)
- FT@SUSY → Limited by mu (**not** by gluino and stop)
- FT@GUT → GUT model dependent and limited by gluino limit (and stop_1)

Conclusion

- FT(EW) → best constrain → direct detection and spin-dependent cross section (NOT independent)
- FT(EW) → high energy e+e- machine
- FT(GUT) → high energy pp machine

**Low FT SUSY+ DM still possible
(predicts to see signal soon...)**

Extra Slides

NUHM-8

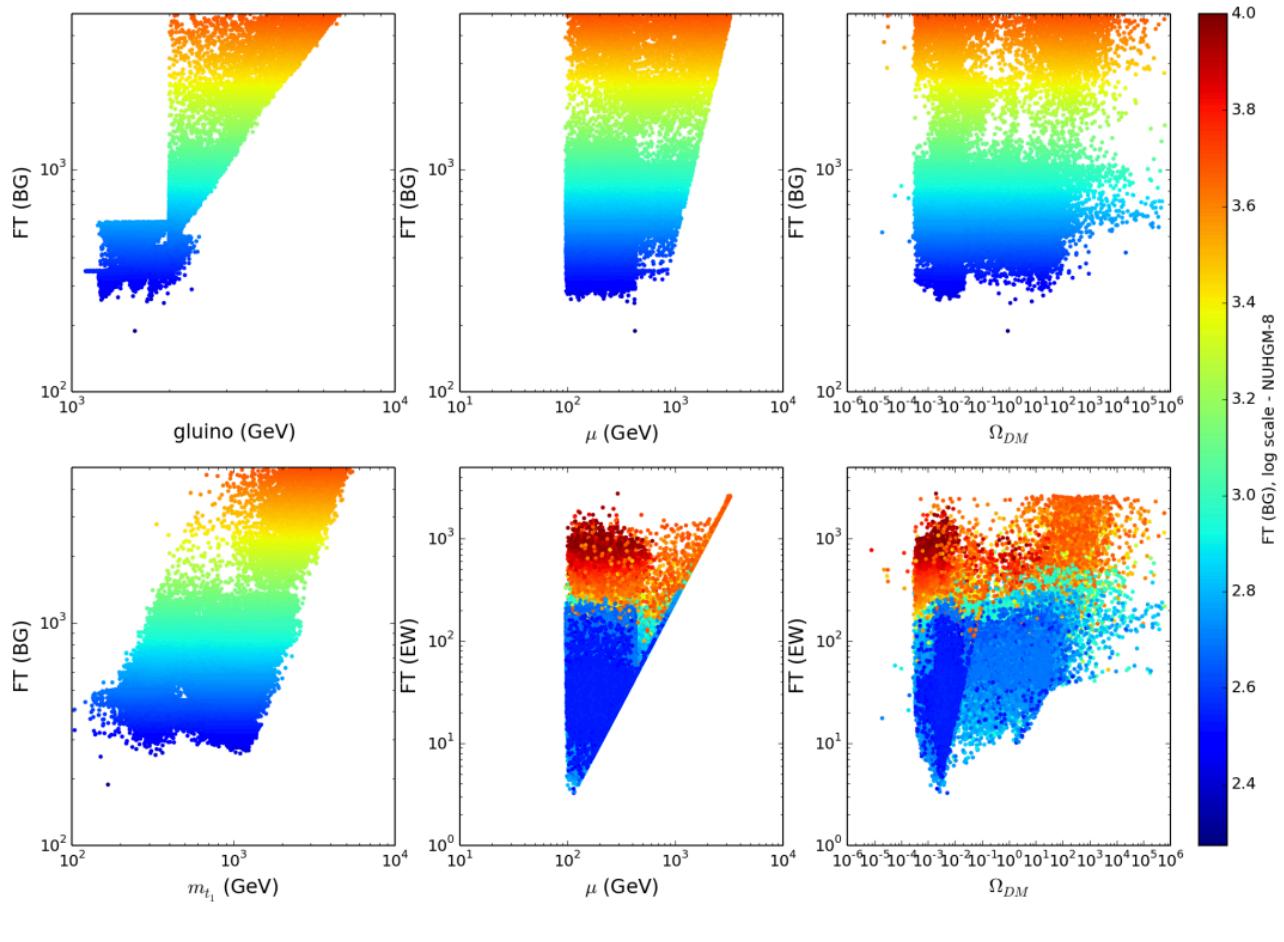


Figure 5: Generated points for the NUHGM GUT model. From left to right, top to bottom: BG FT vs gluino mass, BG FT vs μ , BG FT vs Ωh^2 , BG FT vs m_{t_1} , EW FT vs μ , EW FT vs Ωh^2 .

NUHM-8

- **NUHGM-8:** is defined by a three mass parameters for the scalar sector: $m_{0,L}$, which gives mass to all SUSY sparticles from the left-handed quarks, $m_{0,R}$, which gives mass to all SUSY sparticles from the right-handed quarks and M_A . The gaugino masses are not required to unify and are given by three different parameters M_1 , M_2 and M_3 . Furthermore there is one trilinear soft term A_0 , the supersymmetric higgsino mass term μ and $\tan \beta$ and we demand gauge coupling unification at the high scale M_{GUT} . Apart from μ and $\tan \beta$, these values are all defined at the high scale M_{GUT} . **MB: CHECK IS THIS TRUE? SHOULD MA BE INCLUDED HERE?**

mSUGRA

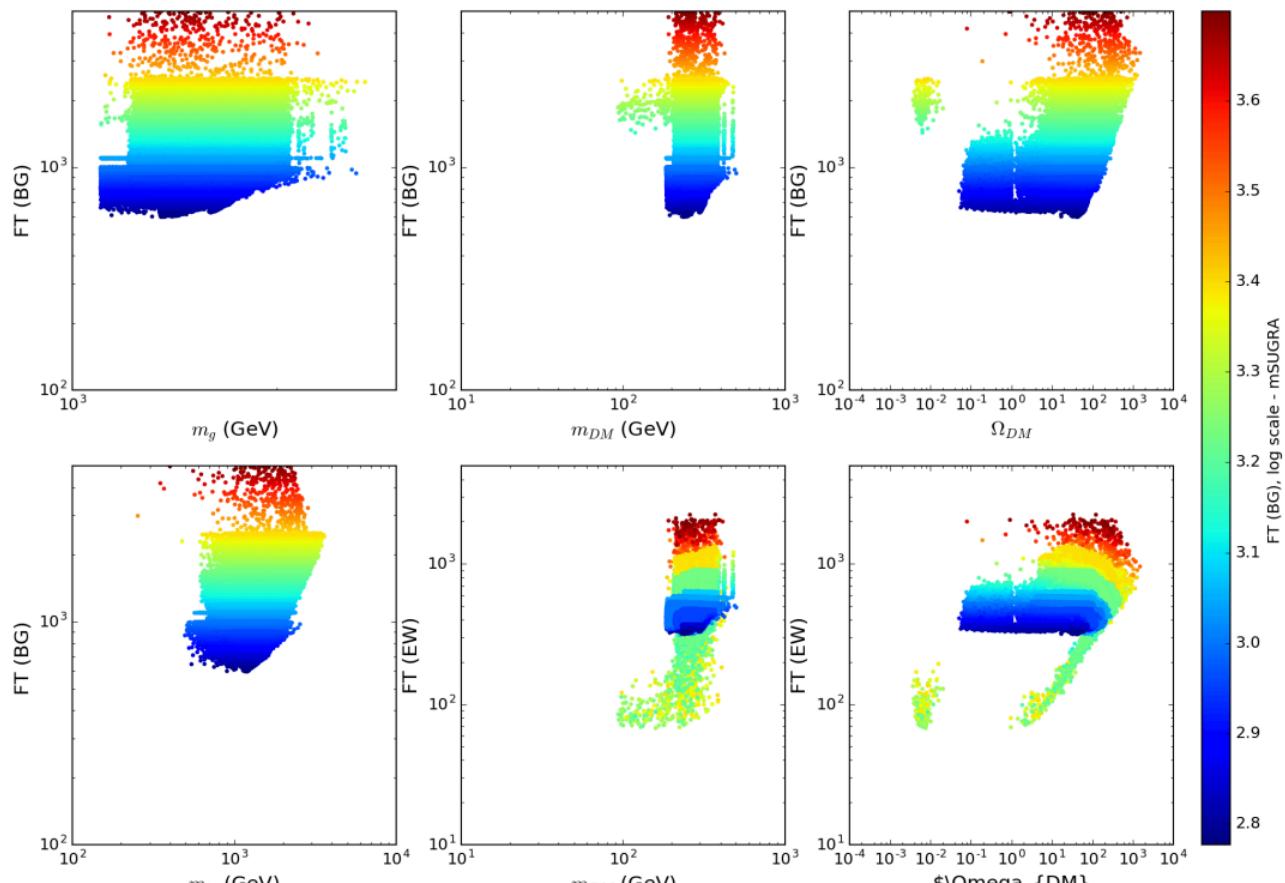


Figure 6: Generated points for the mSUGRA GUT model. From left to right, top to bottom: BG FT vs gluino mass, BG FT vs μ , BG FT vs Ωh^2 , BG FT vs m_{t_1} , EW FT vs μ , EW FT vs Ωh^2 .

3.1. Calculation of the FT

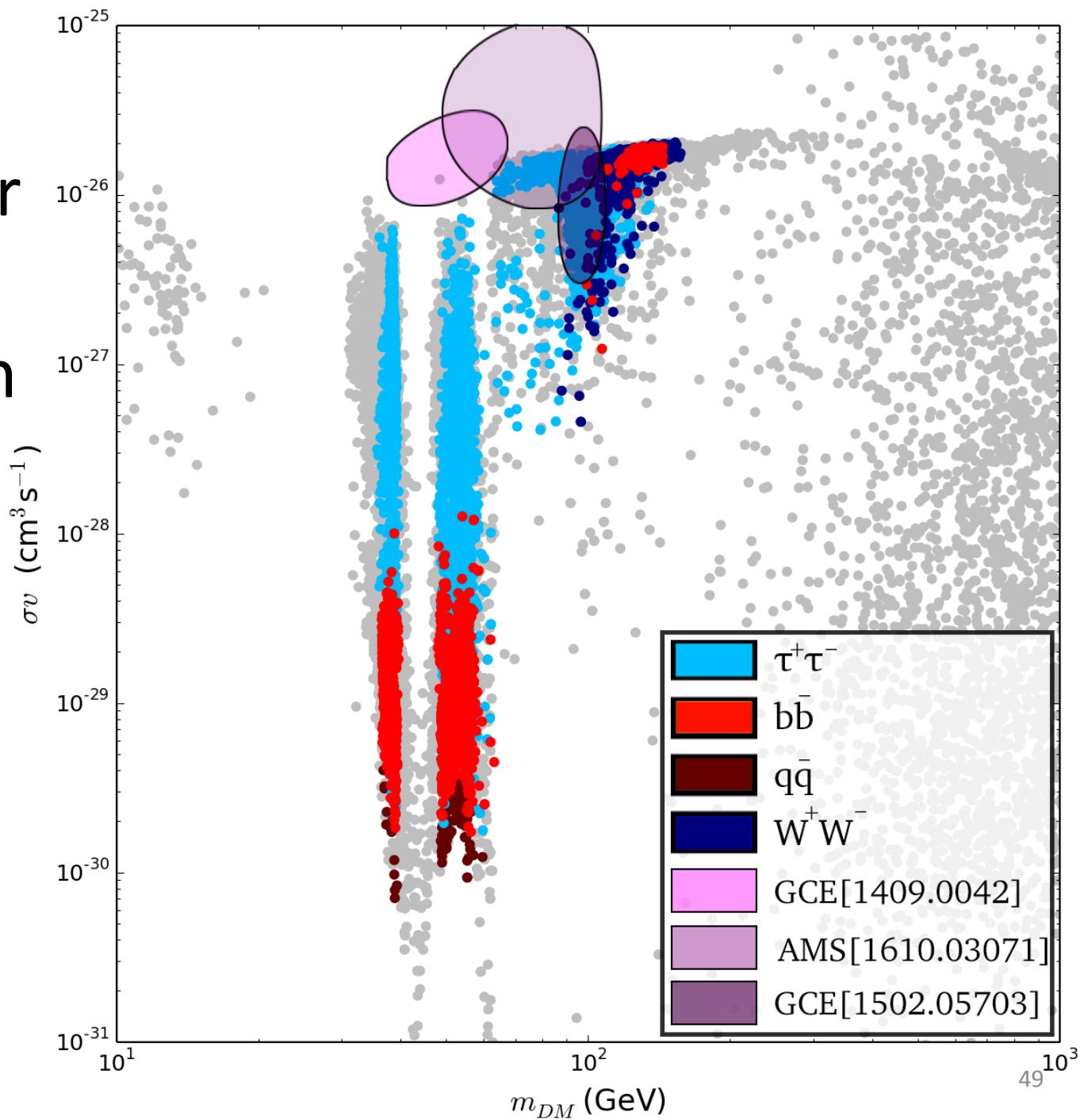
We have two separate calculations for the FT measure. The EW measure is calculated simply by taking calculating the effective potential terms and determining the maximal contribution via equation 2. The high scale measure is calculated via the following procedure, implemented in softsusy:

1. The value of the considered parameter is varied at the high scale.
2. The two-loop RGEs are evaluated from the high scale to the SUSY scale.
3. The one-loop tadpole equations are calculated using the initial VEVs.
4. The one-loop corrected tadpole equations are solved.
5. The updated VEVs are used to re-calculate the one-loop tadpole equations. This is done until the solution for the VEVs has converged.
6. The high scale FT for the particular parameter is calculated from the change in the Z -mass because of the change in VEVs.

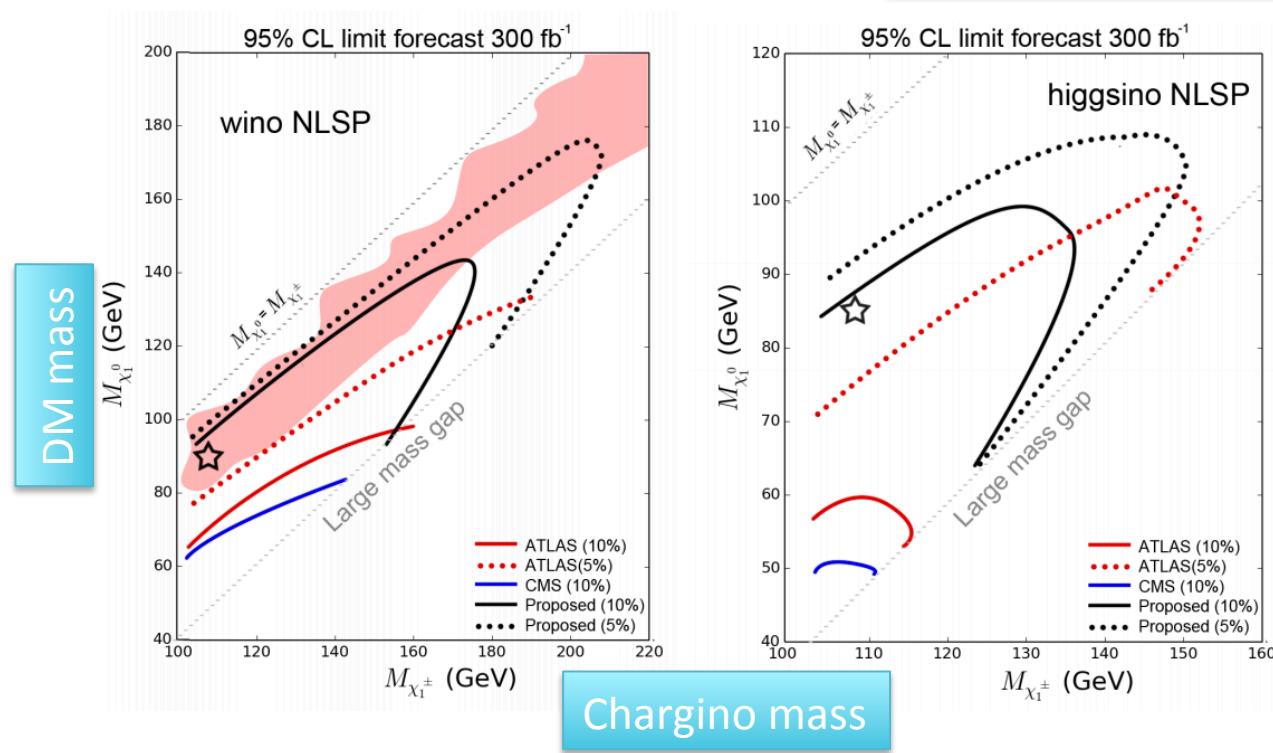
This is done for each independent parameter separately and the value for the FT is taken to be the maximal value of the parameter-specific FT values.

Dark Matter self annihilation

FT<10
and
correct
 $\omega^* h^2$

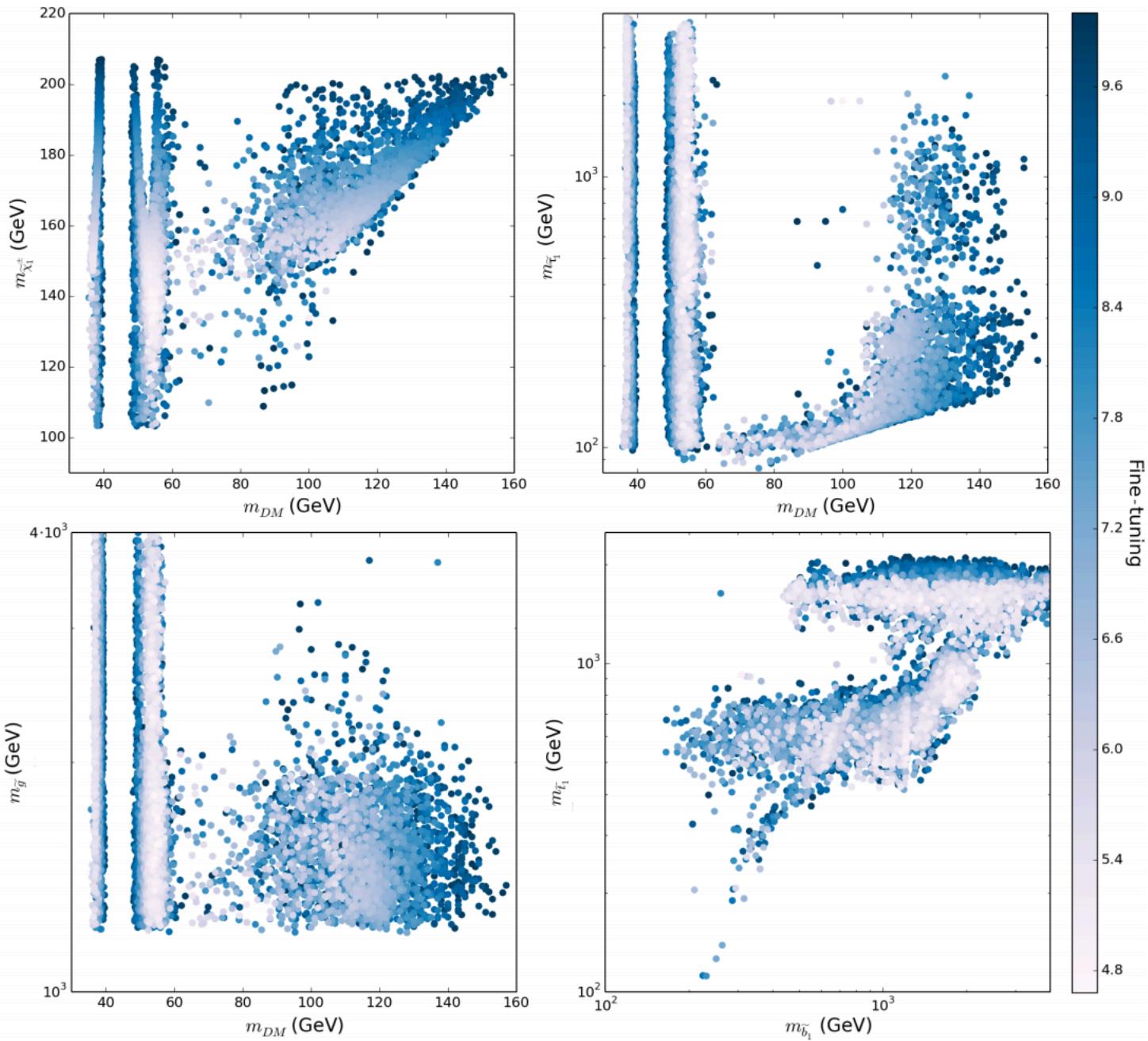


LHC ?

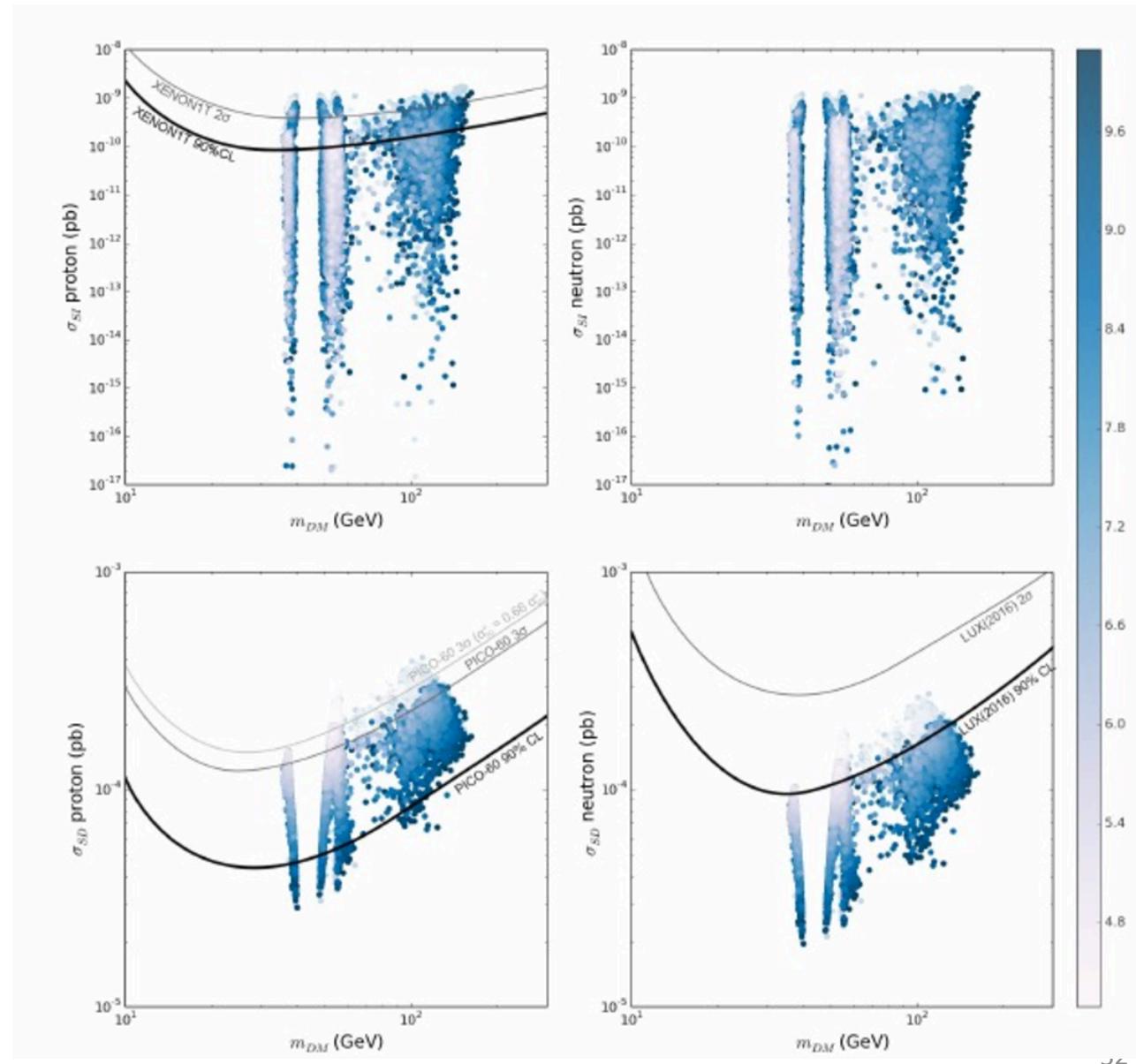


- Best: Chargino – Neutralino2 search
- Bino-Higgsino → **Lower cross section than simplified SUSY models**
- Needs dedicated search strategy → Our first proposal: **3 leptons + reversed missing momentum cut**
- **Conclusion 1: LHC so far blind to lowest FT models**
- **Conclusion 2: Need 30-300 fb-1 or more data and dedicated new search strategy**
- Events are there: Need to get smarter to get them out of the background ? Machine Learning ?

LOW
FT



Low FT models



Global MSSM fits

Given all world data:

What is the most “best fit” set of SUSY parameters ?

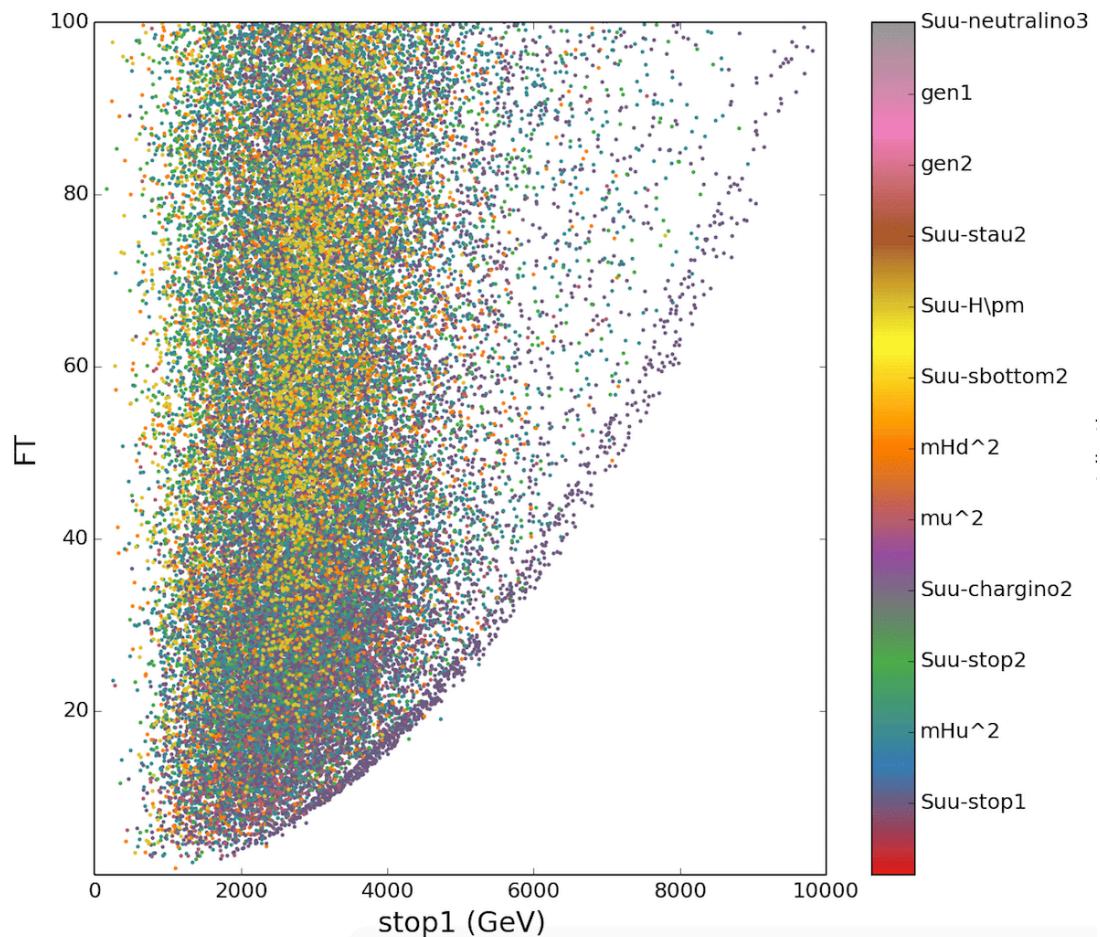
- All world data
- Attempts to include SUSY LHC limits
- 8-18 pMSSM parameters

Future experiments: Stops

- Stop contribution plays a role for FT

With FT measure at SUSY scale \rightarrow no large impact for 2 TeV stops

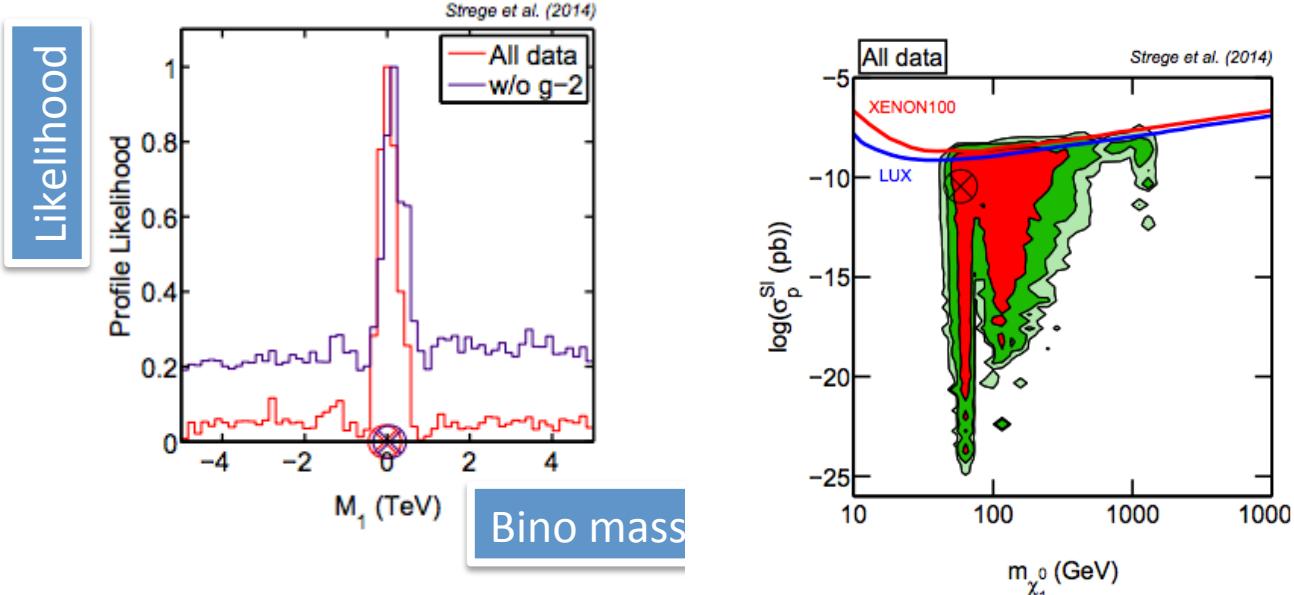
Larger impact with GUT scale measure e.g. via RGE contributions to m_{Hu} : How large ?



Global MSSM Fits for Dark Matter

JHEP 1409 (2014) 081

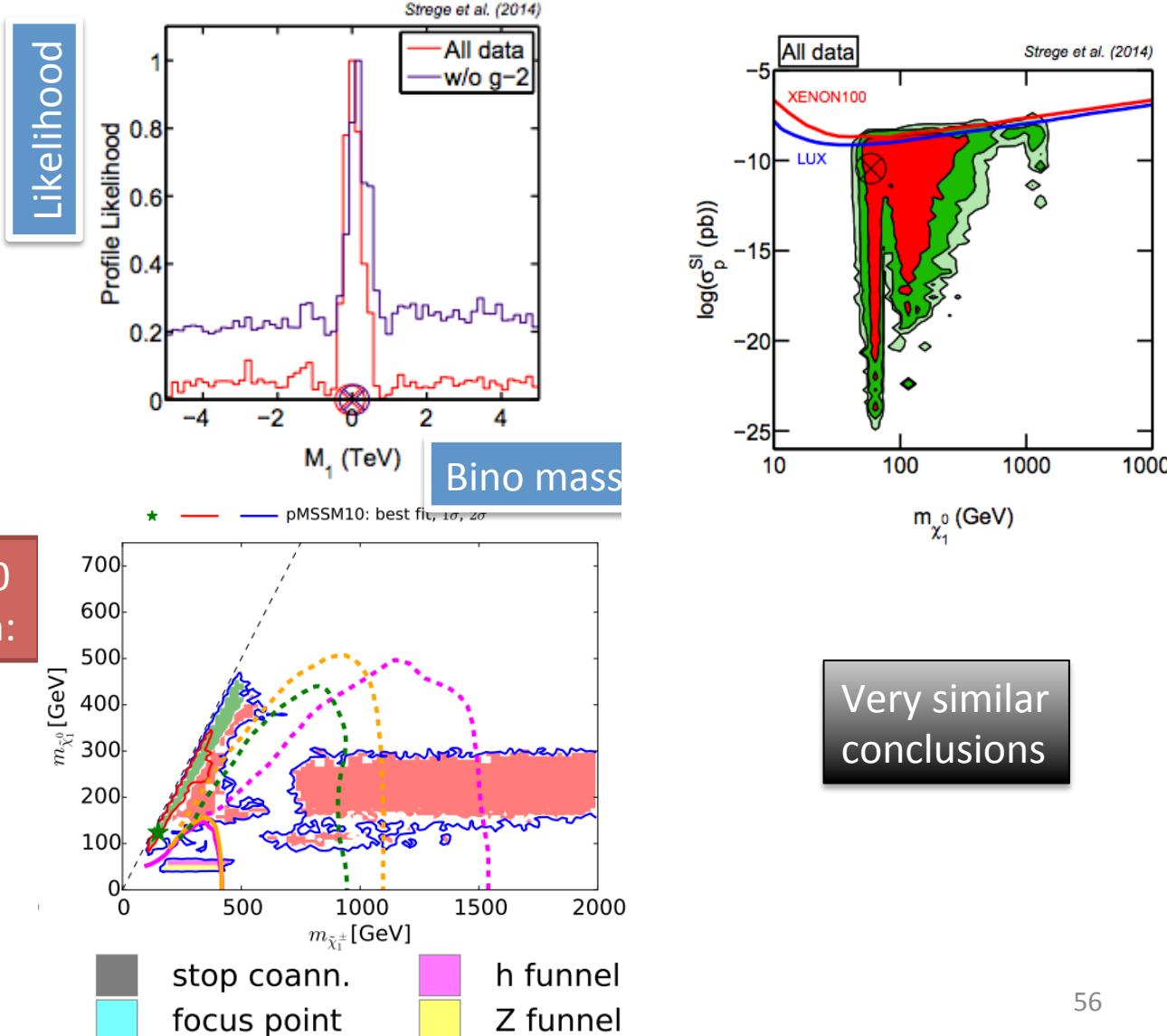
Fit in MSSM model with 18 parameters using all worldwide data, but no LHC and Fermi-LAT



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JHEP 1409 (2014) 081

Fit in MSSM model with 18 parameters using all worldwide data, but no LHC and Fermi-LAT

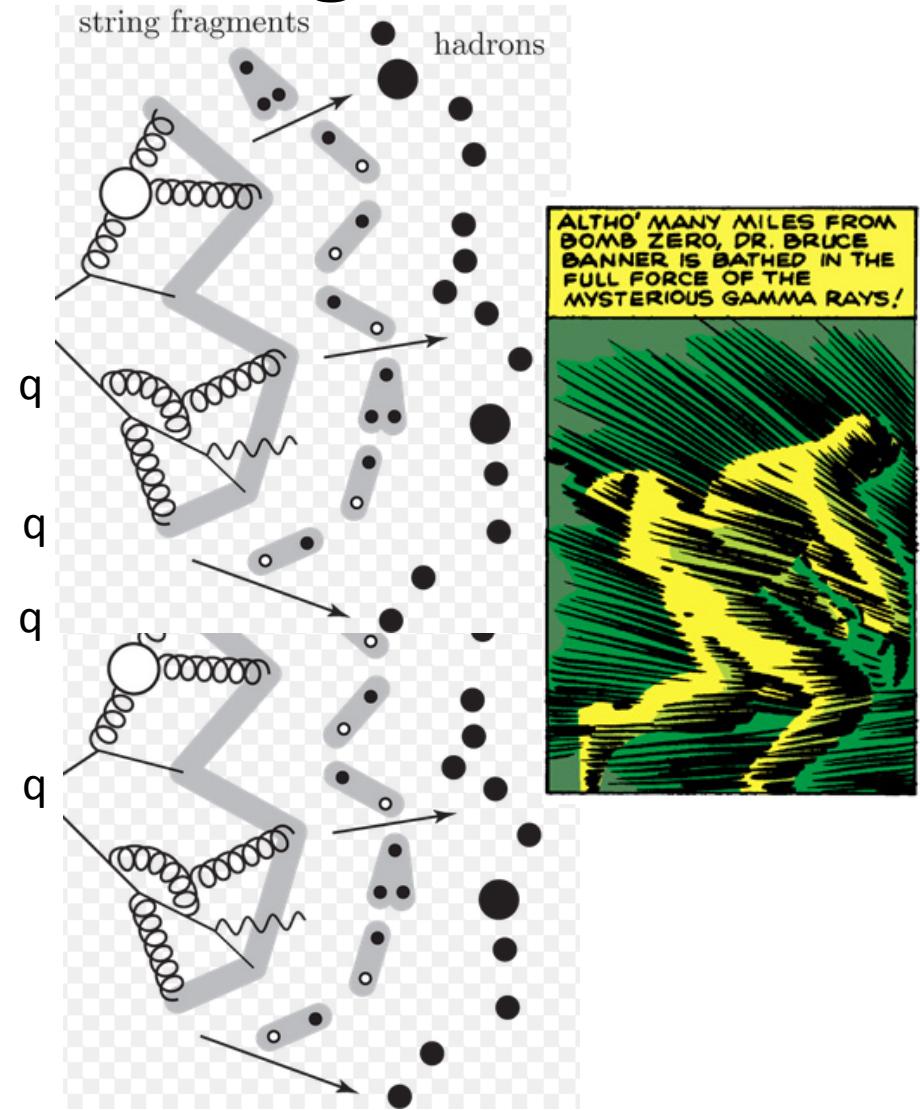
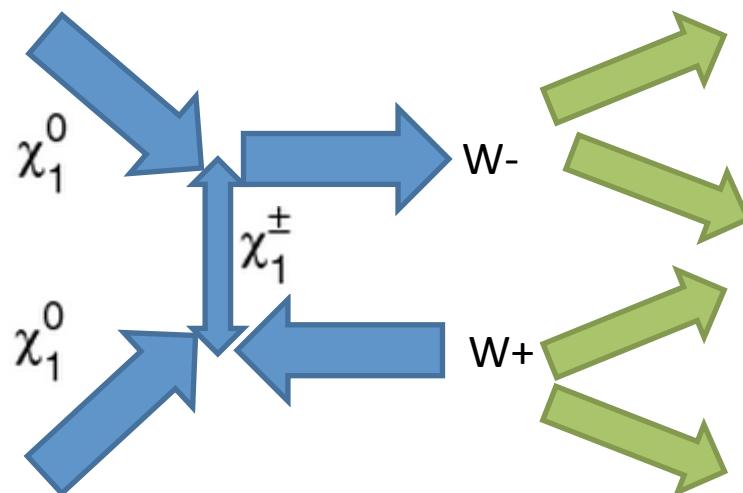


Eur.Phys.J. C75 (2015) 500
Mastercode collaboration:

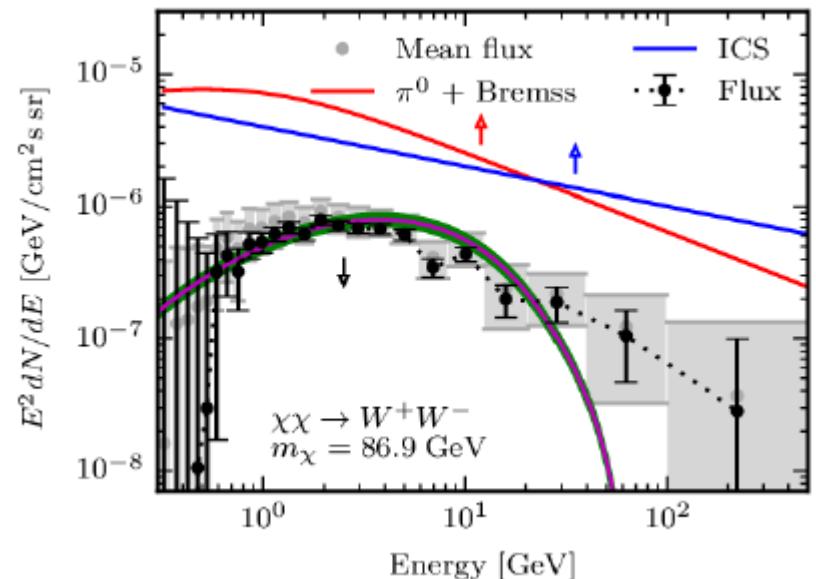
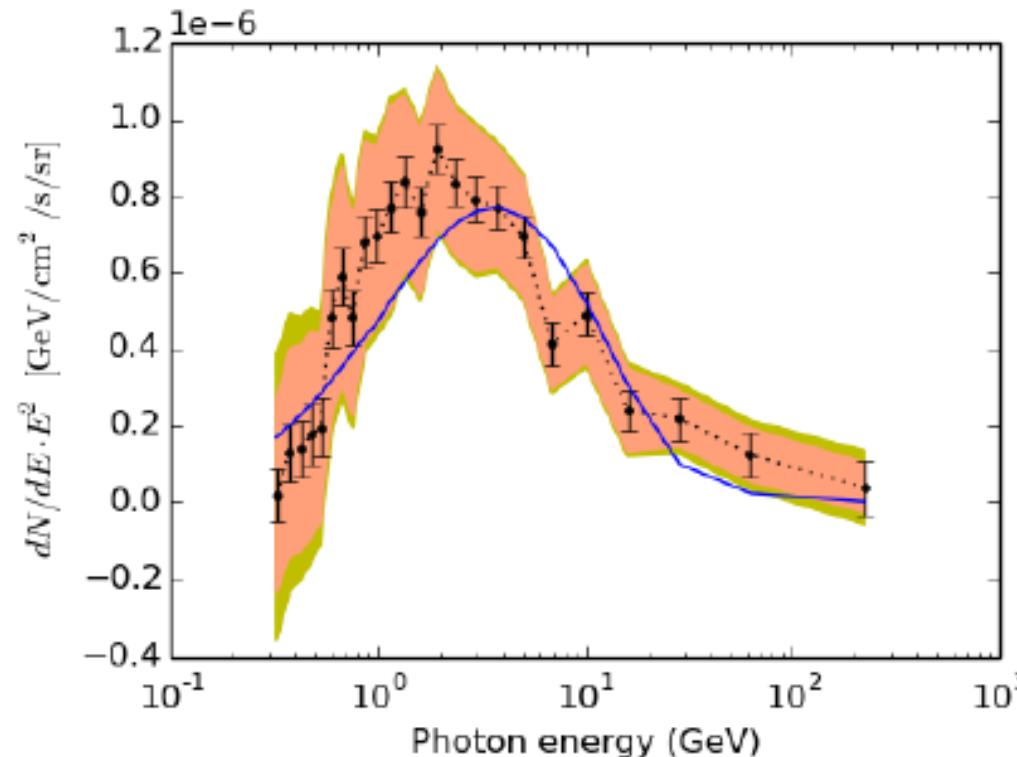
Fit in MSSM model with 10 parameters using all worldwide data, but no Fermi-LAT

Very similar conclusions

Signal Modelling



Signal Modelling



Shown are only Astronomy uncertainties which are highly correlated.

→ P-value of this fit : **0.3-0.4**

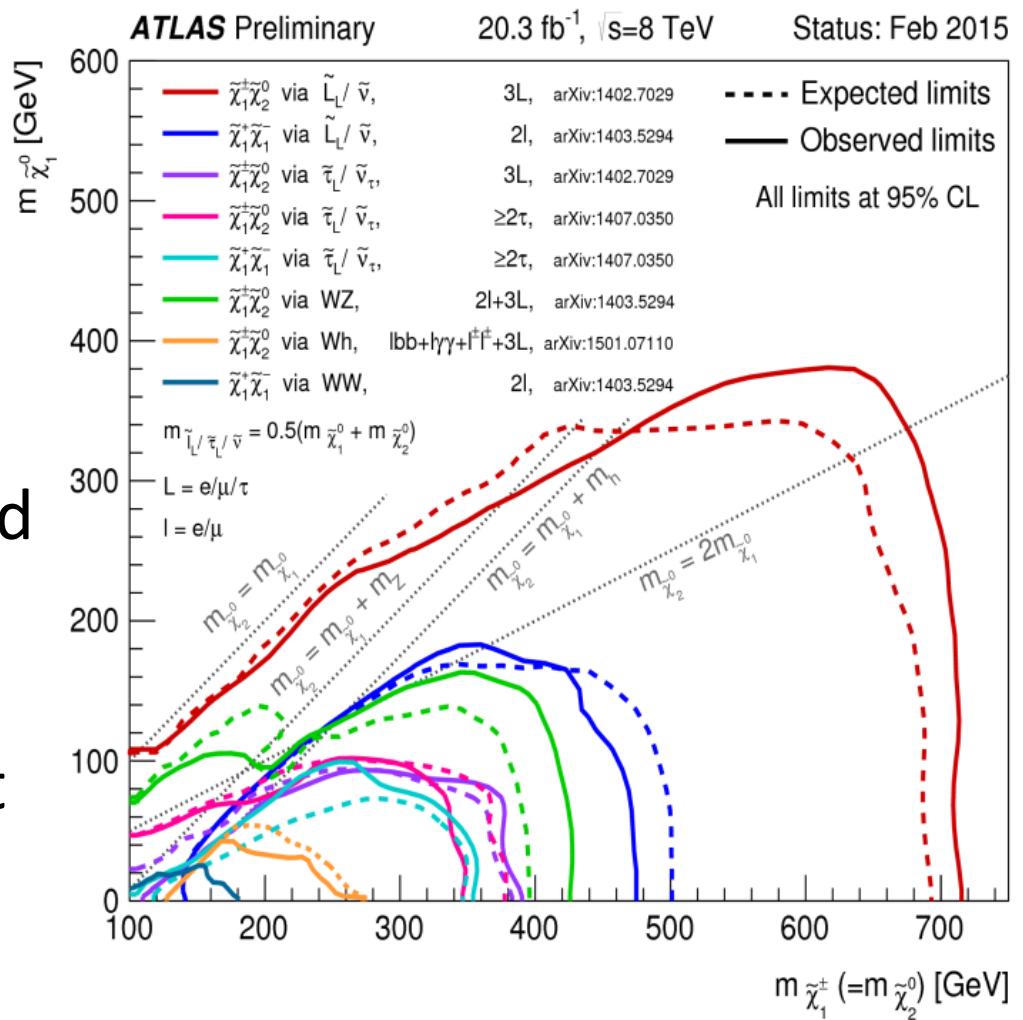
OK, they must have been excluded
already by LHC searches?

No !!!

Carefully checked
All 3 solutions !

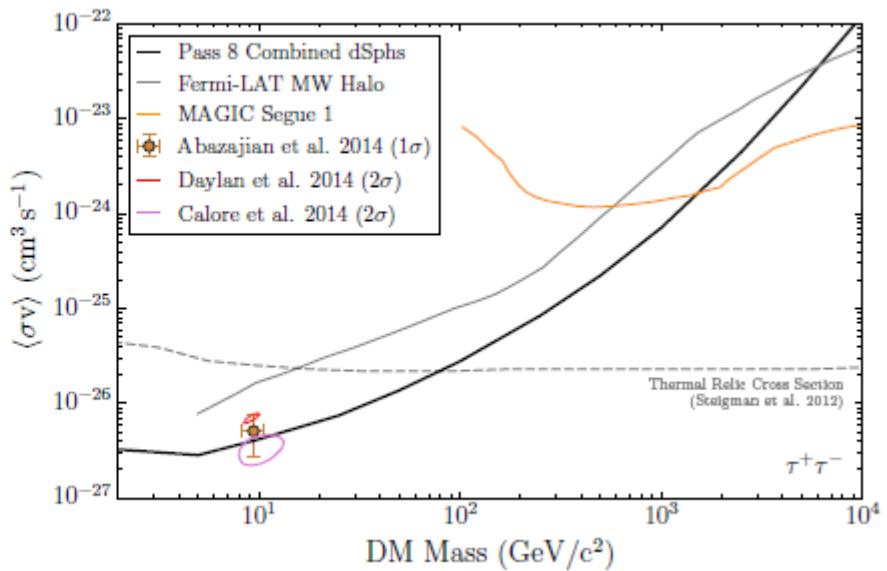
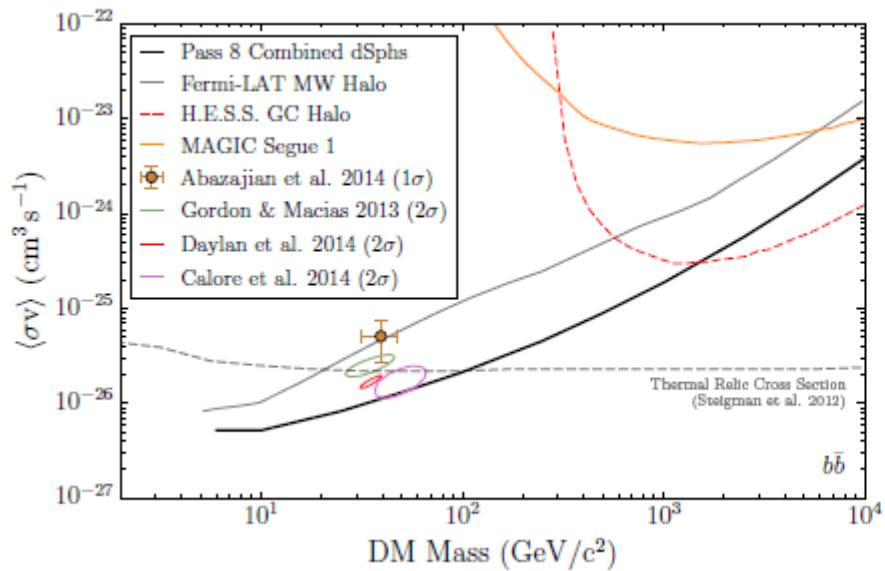
None of them is excluded
by LHC → see later

Solutions also consistent
with all precision
measurements



Dwarf galaxies...

New 6 years limits from 15 dSphs



<http://arxiv.org/pdf/1503.02641v1.pdf>

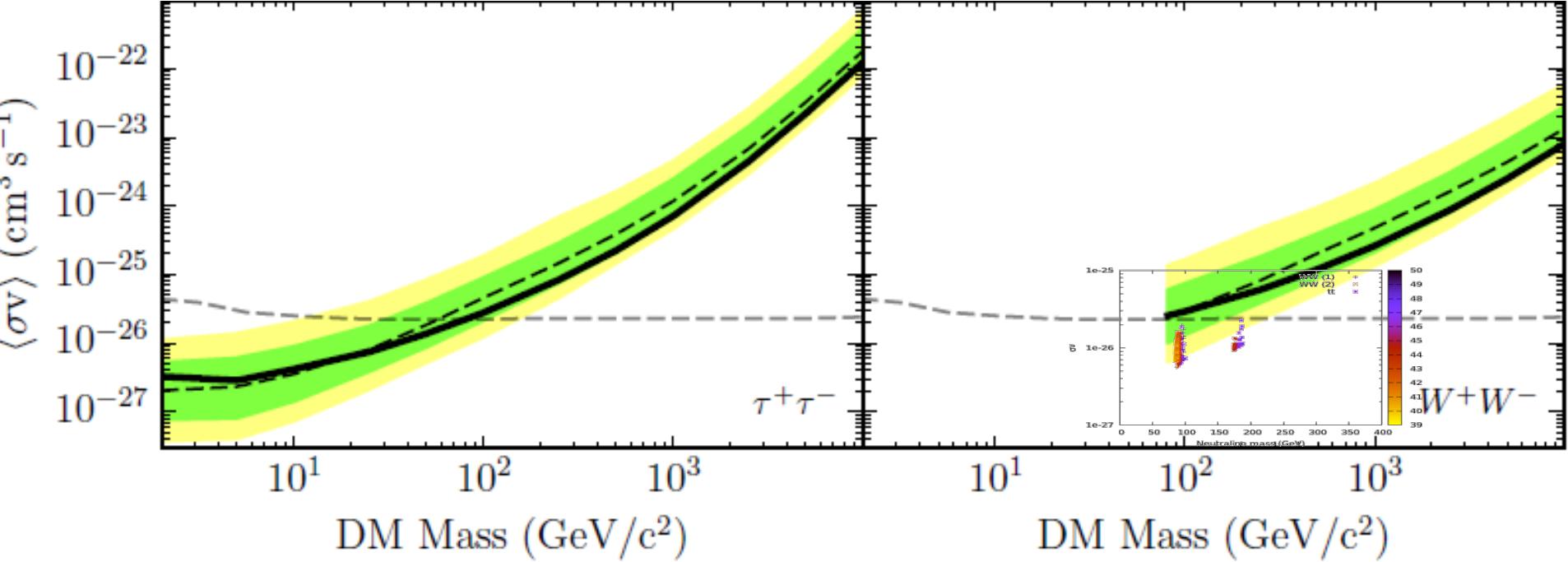


Fig. 8. DM annihilation cross-section constraints derived from the combined 15-dSph analysis for various channels

Our solutions are **not** excluded...

Relic Density MSSM

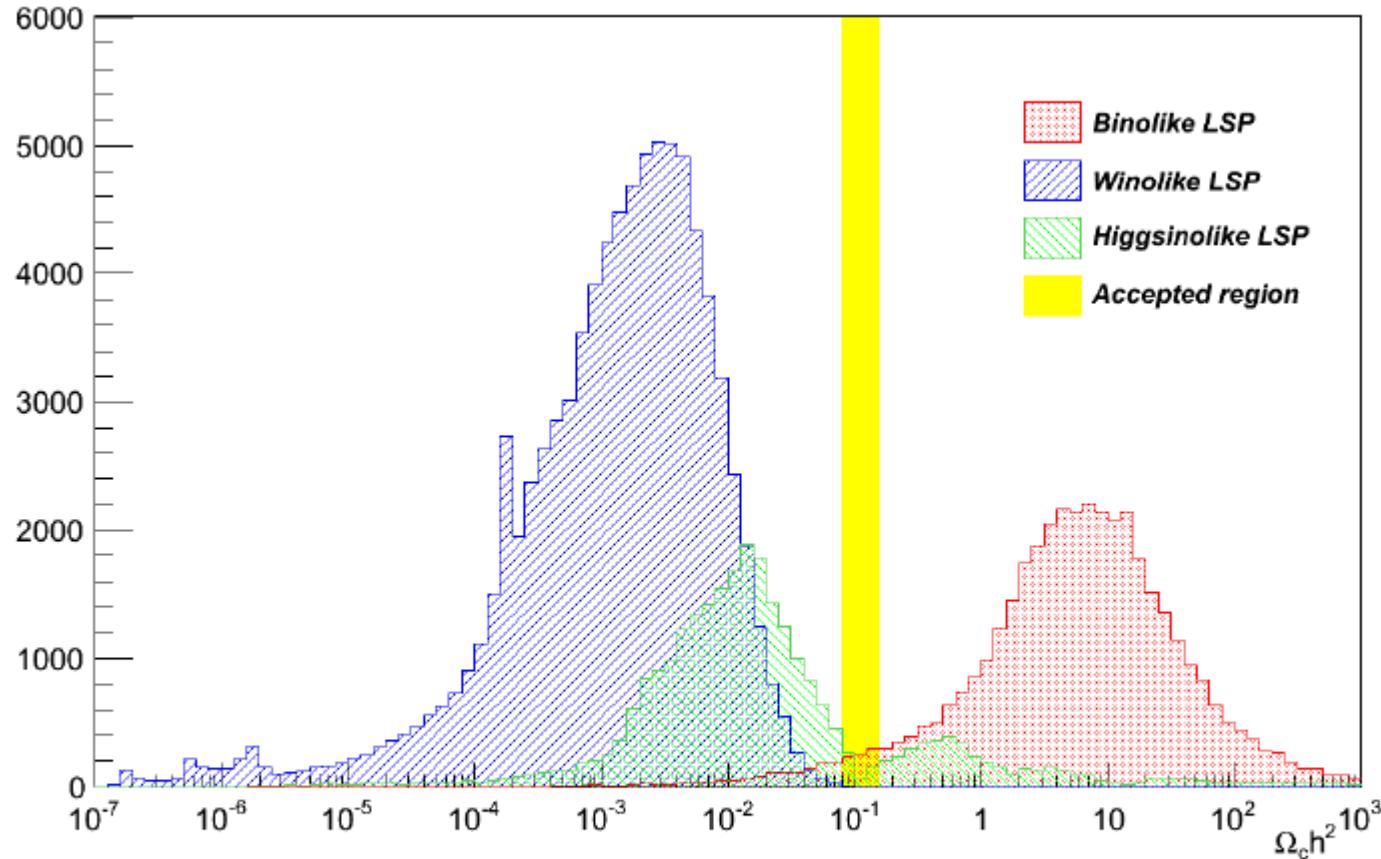
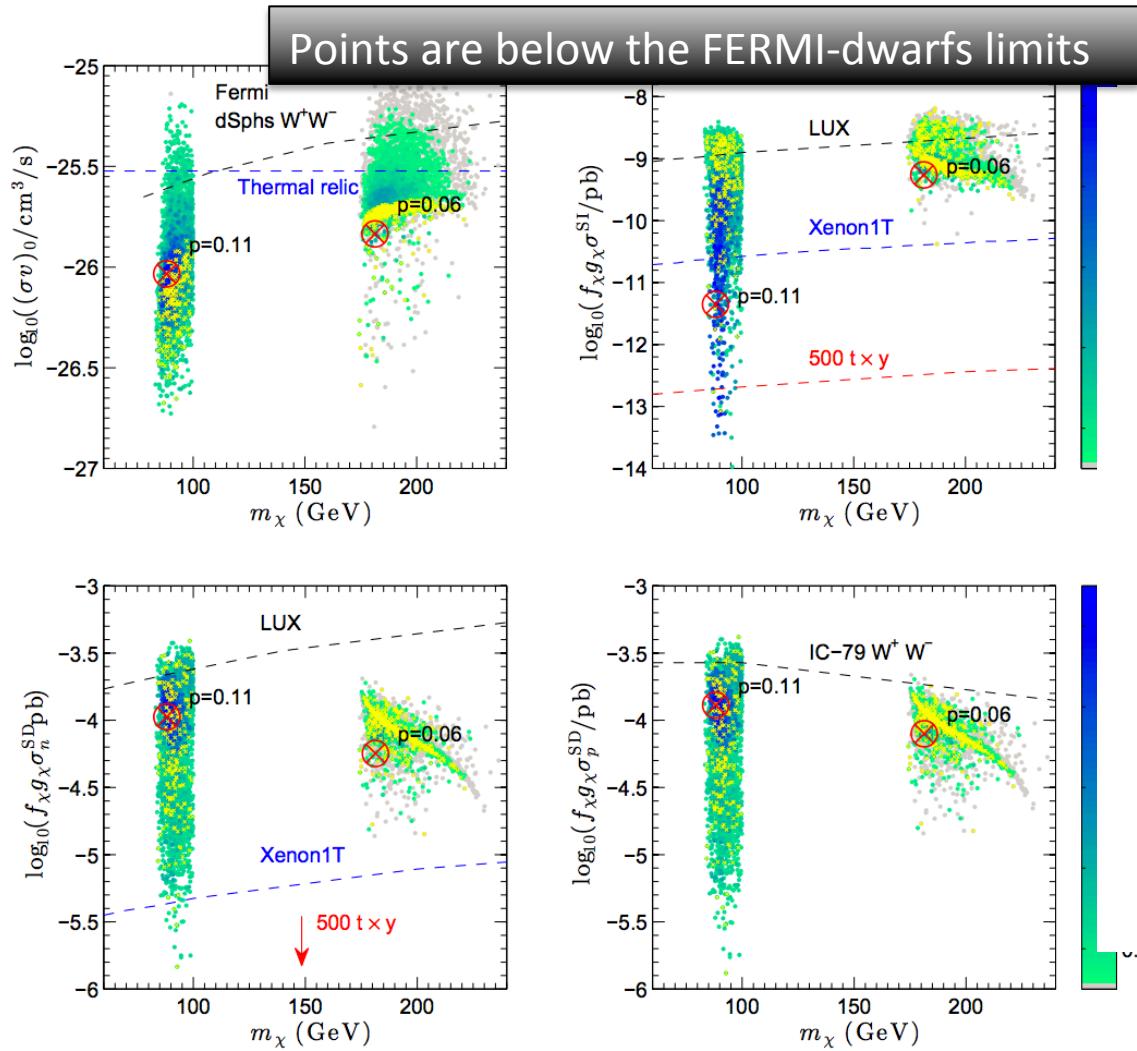


Figure 2: Dark Matter relic density $\Omega_c h^2$ obtained from the 19 parameter pMSSM models compared with the accepted region. The number of models is shown as a function of $\Omega_c h^2$.

Full MSSM19 fit (including GC excess)



colour indicates
p-value of the fit
yellow means
that points have
right Omega*h2
within 2 sigma

$$\rho_\chi / \rho_{\text{DM}} = \Omega_\chi / \Omega_{\text{DM}} \equiv f_\chi$$

Best fit points

- Best fit region of MSSM 19 fit with GC excess overlaps with MSSM 10 Mastercode solutions

Interesting:

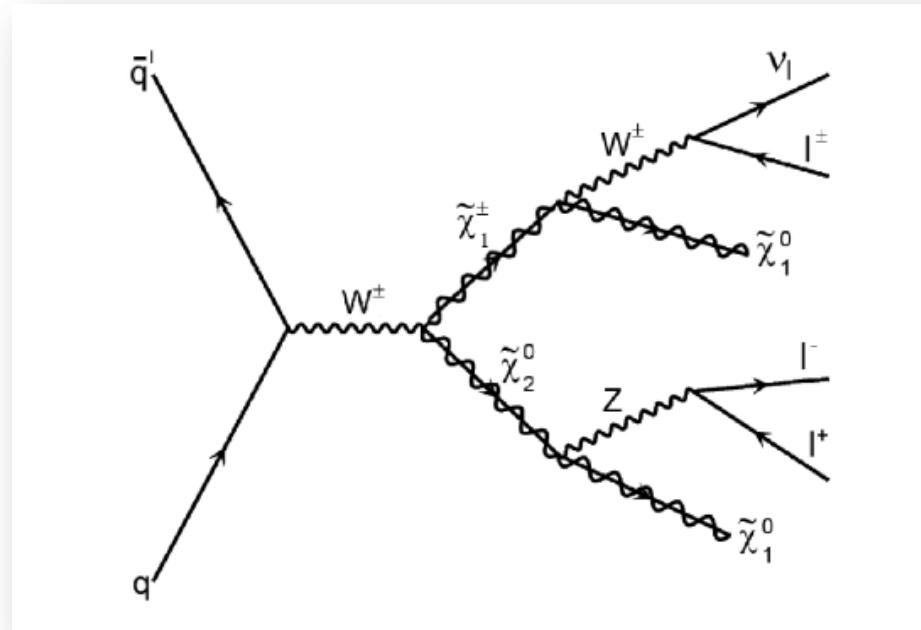
- Solutions not excluded yet at LHC
- **Even worse: No sensitivity** at LHC with 3000 fb-1
- Unless dedicated search done...

The case for a 100 GeV Bino

MSSM global fits and Galactic Center excess prefer region of approx. 100 GeV Bino Dark Matter, compressed with a chargino yielding the correct DM density

No sensitivity seen in:

- Monojets
 - Or other “typical” DM searches
- Solution 3leptons with NO MET and special angular cuts

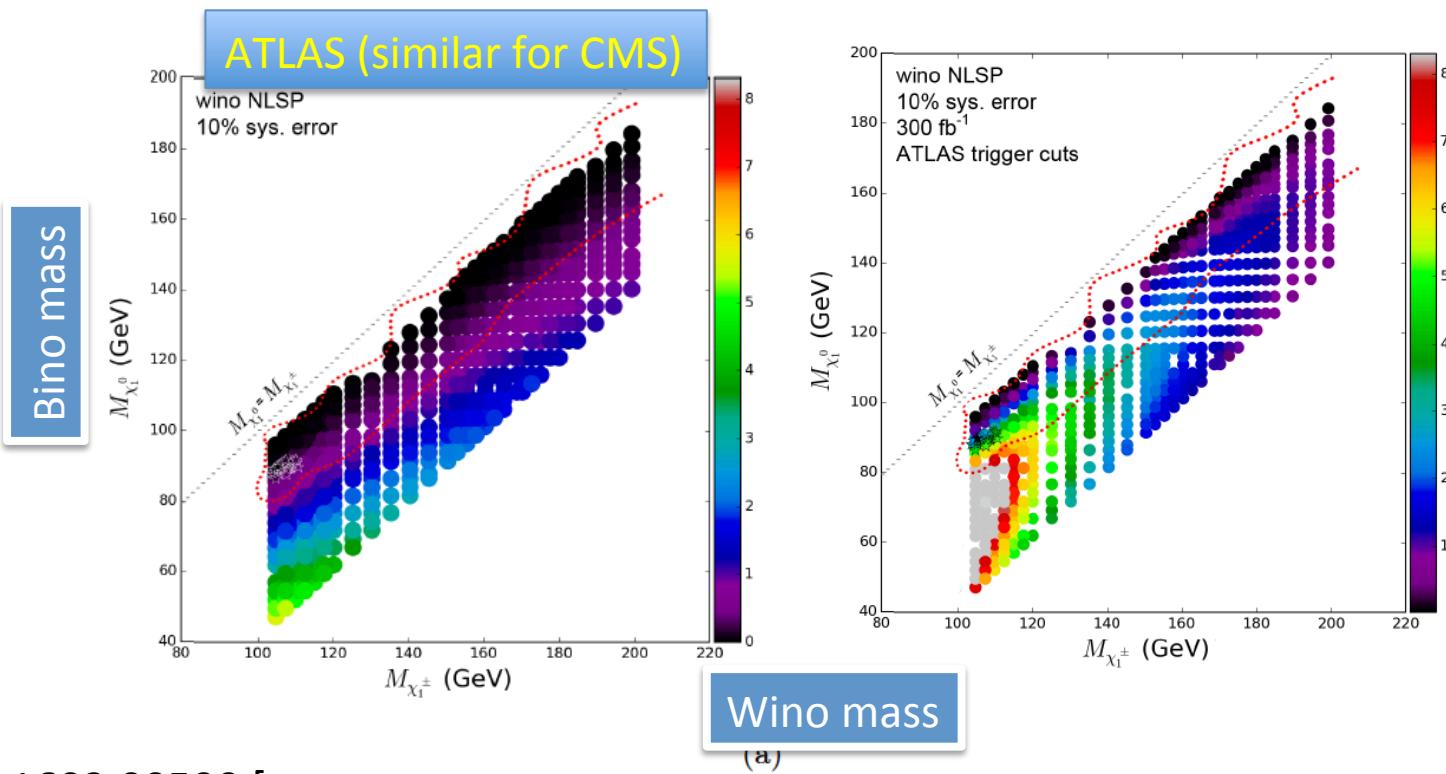


NEEDS
FULL
MODEL!!!

The case for a 100 GeV Bino

MSSM global fits and Galactic Center excess prefer region of approx. 100 GeV Bino Dark Matter, compressed with a chargino yielding the correct DM density

Dedicated new 3lepton search (“low MET”) would yield sensitivity in this Region!



Addition

- “Fitting” Dark Matter
- How can we include the LHC data?

Iterative Particle Filtering

A filter algorithm (you know the Kalman filter)

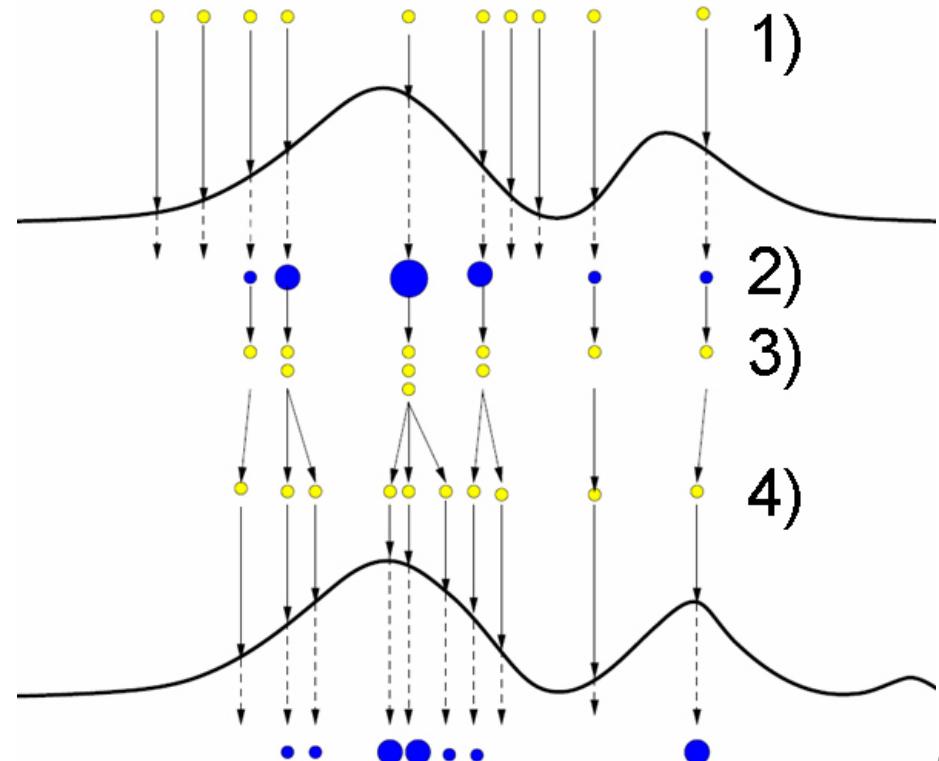
Usually used for e.g. “tracking objects” (your new car or drone)

Idea: importance sampling

→ *Generate recursively more points in interesting regions*

Set of particles
(parameter points)
to represent the
posterior density.

Still we need to evaluate
 $> 10^7$ points

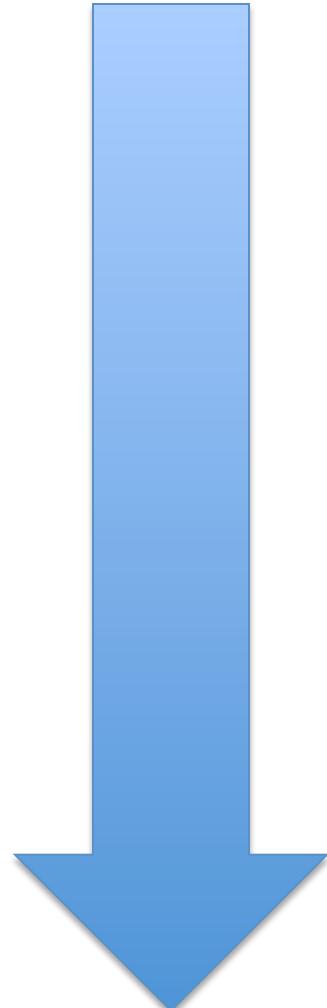


Curse of dimensionality and random sampling

- Volume of some solutions is 10^{-20} of parameter space (see later)
- Random sampling will not work to find solutions
- Random sampling good to get first (iteration) overview of parameter space

Which model ?

- Simplified models (1-3 parameters)
- Simple models (e.g. mSUGRA, 4-6 parameters)
- Models (MSSM, 7-20 parameters)
- Full models (SUSY ?, >20 parameters)

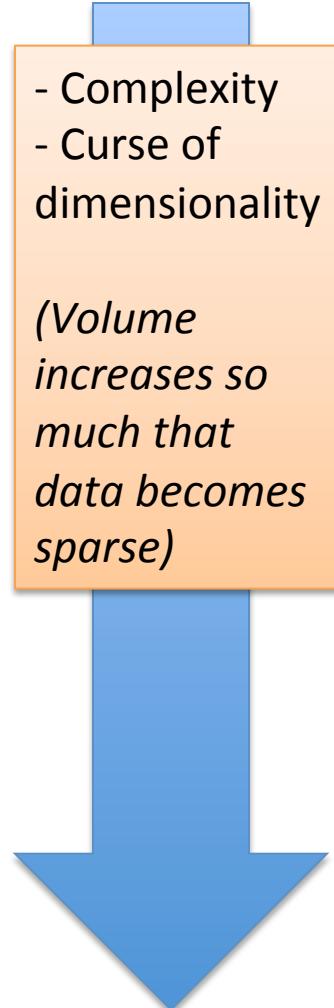


Which model ?

- Simplified models (1-3 parameters) * number of models
- Simple models (e.g. mSUGRA, 4-6 parameters) * number of models
- Models (MSSM, 7-20 parameters)
- Full models (SUSY ?, >20 parameters)

- Complexity
- Curse of dimensionality

(Volume increases so much that data becomes sparse)



Model candidate 1: Supersymmetry

- *Combination of space time and internal symmetries*
- Unification of couplings at large energies
- Possible GUT scale theory ? String Theory ...

→ But we see no SUSY yet !
→ Is there a problem for SUSY naturalness/
fine-tuning given non results at LHC ?

Today:

- Does it still solve the hierarchy/fine tuning ?
- Does it still give a **good** Dark Matter candidate?
→ Predict “Dark Matter relic density”

How many parameters ?

Simplified models * number of models

not equal

Full model